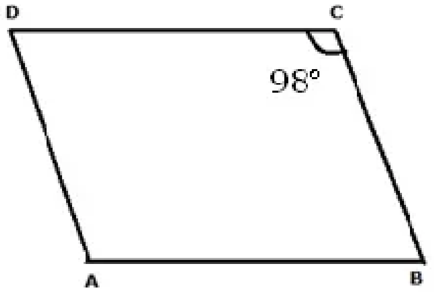
Chapter 19. Quadrilaterals

Ex 19.1

Answer 2.



ABCD is a parallelogram

$\angle C = 98°$

$\therefore \angle A = \angle C = 98°$ (opposite angles of a parallelogram are equal)

Now,

$\angle A + \angle B + \angle C + \angle D = 360°$ (Sum of the angles of a quadrilateral = 360°)

$98° + \angle B + 98° + \angle D = 360°$

$\angle B + 196° + \angle D = 360°$

$\angle B + \angle D = 360° - 196°$

$\angle B + \angle D = 164°$

But $\angle B = \angle D$ (opposite angles of a parallelogram are equal)

$\Rightarrow 2\angle B = 164°$

$\Rightarrow \angle B = 82° = \angle D$

Therefore, $\angle B = 82°$, $\angle A = 98°$

Answer 4.

In ΔBDC,

$\angle BDC + \angle DCB + \angle CBD = 180^\circ$

$2a + 5a + 3a = 180^\circ$

$10a = 180^\circ$

$\Rightarrow a = 18^\circ$

$\angle BDC = 2a = 2 \times 18^\circ = 36^\circ$

$\angle DCB = 5a = 5 \times 18^\circ = 90^\circ$

$\angle CBD = 3a = 3 \times 18^\circ = 54^\circ$

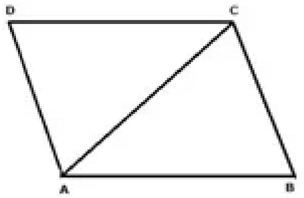
$\angle DAB = \angle DCB = 90^\circ$ (opposite angles of parallelogram are equal)

$\angle DBA = \angle BDC = 36^\circ$ (alternate angles since $AB||CD$)

$\angle BDA = \angle CBD = 54^\circ$ (alternate angles since $AB||CD$)

Therefore, $\angle DAB = \angle DCB = 90^\circ$, $\angle DBA + \angle CBD = 90^\circ$, $\angle BDA + \angle BDC = 90^\circ$

Answer 6.



ABCD is a parallelogram.

Let $\angle CAB = x^\circ$

Then, $\angle ABC = 5x^\circ$ and $\angle BCA = 3x^\circ$

In ΔABC,

$\angle CAB + \angle ABC + \angle BCA = 180^\circ$ (sum of angles of triangle = 180°)

$x^\circ + 5x^\circ + 3x^\circ = 180^\circ$

$9x^\circ = 180^\circ$

$x^\circ = 20^\circ$

$\Rightarrow \angle CAB = x^\circ = 20^\circ$

$\Rightarrow \angle ABC = 5x^\circ = 5 \times 20^\circ = 100^\circ$

$\Rightarrow \angle BCA = 3x^\circ = 3 \times 20^\circ = 60^\circ$

Now,

$\angle ADC = \angle ABC = 100^\circ$ (opposite angles of a parallelogram are equal)

$\angle ACD = \angle CAB = 20^\circ$ (Alternate angles since BC||AD)

$\angle CAD = \angle BCA = 60^\circ$ (Alternate angles since BC||AD)

Therefore, $\angle ADC = \angle ABC = 100^\circ$, $\angle ACD + \angle BCA = 80^\circ$, $\angle CAD + \angle CAB = 80^\circ$

Answer 7.

PQRS is a parallelogram.

Let $\angle RPQ = 3x^\circ$

Then, $\angle PQR = 8x^\circ$ and $\angle QRP = 4x^\circ$

In ΔPQR,

$\angle RPQ + \angle PQR + \angle QRP = 180^\circ$ (sum of angles of triangle = 180°)

$3x^\circ + 8x^\circ + 4x^\circ = 180^\circ$

$15x^\circ = 180^\circ$

$x^\circ = 12^\circ$

$\Rightarrow \angle RPQ = 3x^\circ = 3 \times 12^\circ = 36^\circ$

$\Rightarrow \angle PQR = 8x^\circ = 8 \times 12^\circ = 96^\circ$

$\Rightarrow \angle QRP = 4x^\circ = 4 \times 12^\circ = 48^\circ$

Now,

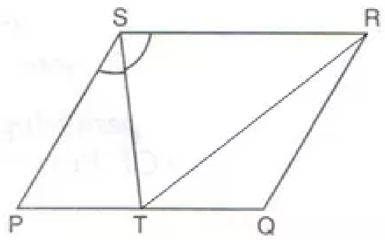
$\angle PSR = \angle PQR = 96^\circ$ (opposite angles of a parallelogram are equal)

$\angle RPS = \angle QRP = 48^\circ$ (Alternate angles since QR||PS)

$\angle PRS = \angle RPQ = 36^\circ$ (Alternate angles since QR||PS)

Therefore, $\angle PSR = \angle PQR = 96^\circ$, $\angle RPS + \angle RPQ = 84^\circ$, $\angle PRS + \angle QRP = 84^\circ$

Answer 8.



(i) $\angle PST = \angle TSR$ ....(i)

$\angle PTS = \angle TSR$ ....(ii)(alternate angles $\because SR||PQ$)

From (i) and (ii)

$\angle PST = \angle PTS$

Therefore, PT = PS

But PT = QT (T is midpoint of PQ)

And PS = QR (PS and QR are opposite and equal sides of a parallelogram)

Hence, QT = QR

(ii) Since QT = QR

$\angle QTR = \angle QRT$

But $\angle QTR = \angle TRS$ (alternate angles $\because SR||PQ$)

$\Rightarrow \angle QRT = \angle TRS$

Therefore, RT bisects $\angle R$

(iii) $\angle PST = \angle TSR$

$\angle QRT = \angle TRS$

$\angle QRS + \angle PSR = 180°$ (adjacent angles of ||gm are supplementary)

Multiplying by $\frac{1}{2}$

$\frac{1}{2}\angle QRS + \frac{1}{2}\angle PSR = \frac{1}{2} \times 180°$

$\angle TSR + \angle TRS = 90°$

In ΔSTR,

$\angle TSR + \angle RTS + \angle TRS = 180°$

$90° + \angle RTS = 180°$

$\angle RTS = 90°$

Answer 9.

(i) Since PC = BC (AD is half of AB and BC=AD and DC=AB)

$\angle CPB = \angle CBP$

But $\angle CPB = \angle PBA$ (alternate angles $\because$ DC||AB)

$\Rightarrow \angle CBP = \angle PBA$

Therefore, BP bisects $\angle ABC$

(ii) $\angle DAP = \angle PAB$

$\angle CBP = \angle PBA$

$\angle DAB + \angle CBA = 180^\circ$ (adjacent angles of ||gm are supplementary)

Multiplying by $\frac{1}{2}$

$\frac{1}{2}\angle DAB + \frac{1}{2}\angle CBA = \frac{1}{2} \times 180^\circ$

$\angle PAB + \angle PBA = 90^\circ$

In ΔAPB,

$\angle PAB + \angle PBA + \angle APB = 180^\circ$

$90^\circ + \angle APB = 180^\circ$

$\angle APB = 90^\circ$

Therefore, $\angle APB$ is a right angle.

Answer 10.

In quadrilateral APCQ,

AP||QC (since AB||CD)

$AP = \frac{1}{2}AB$ (given)

$CQ = \frac{1}{2}CD$ (given)

But AB = CD

$\Rightarrow$ AP = CQ

Therefore, APCQ is a parallelogram.

Answer 11.

(i)In ΔSNR and ΔQMP

$\angle SNR = \angle QMP$ (right angles)

$\angle SRN = \angle MPQ$ (alternate angles since PQ||SR)

$\therefore \Delta SNR \sim \Delta QMP$

$\angle RSN = \angle PQM$ ....(i)

In ΔSNR and ΔQMP

$\angle SRN = \angle MPQ$

$\angle RSN = \angle PQM$ (from (i))

PQ = SR (PQRS is a parallelogram)

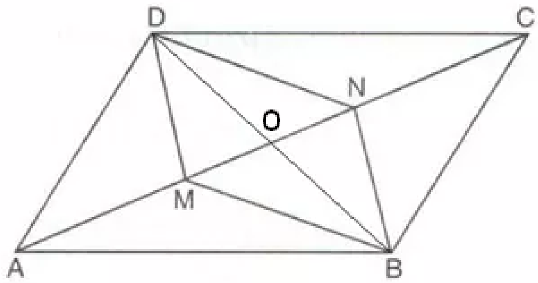
Therefore, $\Delta SNR \cong \Delta QMP$ (ASA axiom)

(ii) Since $\Delta SNR \cong \Delta QMP$

Hence, SN = QM

Answer 12.

Join BD.



The diagonals of a parallelogram bisect each other.

Therefore, AC and BD bisect each other.

$\Rightarrow$ OA = OC

But AM = CN

Therefore, OA – AM = OC – CN

$\Rightarrow$ OM = ON

Therefore, in quadrilateral BMDN,

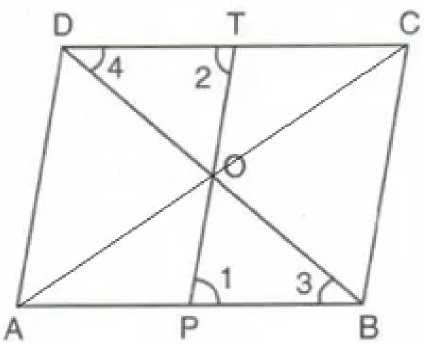
OM = ON and OD = OB

$\Rightarrow$ Diagonals MN and BD bisect each other

$\Rightarrow$ BMDN is a parallelogram.

Answer 14.

Join AC



Since AC and BD are diagonals of a parallelogram, AC and BD bisect each other.

$\Rightarrow$ OA = OC and OD = OB ....(i)

AP = CT

But AB = CD

$\Rightarrow$ PB = DT

In ΔDOT and ΔPOB,

PB = DT

$\angle 1 = \angle 2$ (alternate angles since AB||CD)

$\angle 3 = \angle 4$ (alternate angles since AB||CD)

Therefore, $\Delta DOT \cong \Delta POB$

Hence, OT = OP ......(ii)

From (i) and (ii)

OD = OB and OT = OP

Therefore, PT and BD bisect each other.

Answer 15.

PQ = QT

But PQ = SR (PQRS is a parallelogram)

Therefore, QT = SR

In ΔSOR and ΔQOT

QT = SR

$\angle 3 = \angle 4$ (vertically opposite angles)

$\angle 1 = \angle 2$ (alternate angles since PQ||SR)

Therefore, $\Delta SOR \cong \Delta QOT$

Hence, OS = OT and OR = OQ

Therefore, ST bisects QR.

Ex 19.2

Answer 1.

In ΔQOM,

$\angle OQM = 45^\circ$ (In square diagonals make 45° with the sides)

$OQ = MQ$

$\Rightarrow \angle QOM = \angle QMO$ (i) (equal sides have equal angles opposite to them)

$\angle QOM + \angle QMO + \angle OQM = 180^\circ$

$\angle QOM + \angle QOM + 45^\circ = 180^\circ$

$2\angle QOM = 180^\circ - 45^\circ$

$\angle QOM = 67.5^\circ$

In ΔQOR,

$\angle QOR = 90^\circ$ (diagonals bisect at right angles)

$\angle QOM + \angle MOR = 90^\circ$

$67.5^\circ + \angle MOR = 90^\circ$

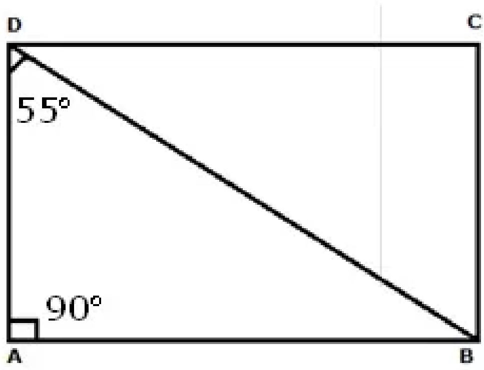
$\angle MOR = 22.5^\circ$

In ΔROS,

$\angle OSR = 45^\circ$ (In square diagonals make 45° with the sides)

$\Rightarrow \angle QSR = 45^\circ$

Answer 2.



In ΔABD,

∠ADB = 55°

∠DAB = 90° (in rectangle angle between two sides is 90°)

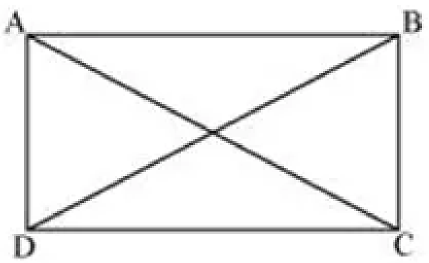
∠ADB + ∠DAB + ∠ABD = 180°

55° + 90° + ∠ABD = 180°

∠ABD = 180° - 145°

∠ABD = 35°

Answer 3.



Let ABCD be a parallelogram.

In ΔABC and ΔDCB,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given)

∴ ΔABC ≅ΔDCB (By SSS Congruence rule)

⇒ ∠ABC = ∠DCB

It is known that the sum of the measures of angles on the same side of transversal is 180°.

$\angle ABC + \angle DCB = 180^\circ$ (AB || CD)

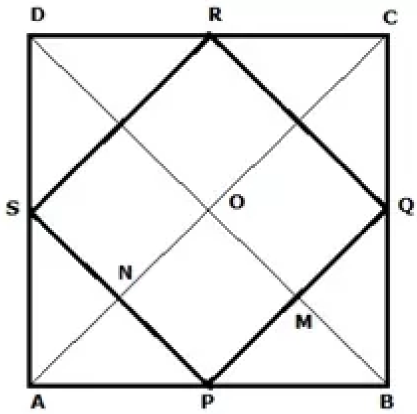
$\Rightarrow \angle ABC + \angle ABC = 180^\circ$

$\Rightarrow 2\angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 90^\circ$

Since ABCD is a parallelogram and one of its interior angles is 90°, ABCD is a rectangle.

Answer 4.



Join AC and BD

In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

Therefore, $PQ || AC$ and $PQ = \frac{1}{2}AC$.......(i)

In ΔADC, R and S are the mid-points of sides CD and AD respectively.

Therefore, $RS || AC$ and $RS = \frac{1}{2}AC$.........(ii)

From (i) and (ii)

$PQ || RS$ and $PQ = RS$ ....(iii)

Thus, in a quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

Since ABCD is a square

AB = BC = CD = DA

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD; \frac{1}{2}AB = \frac{1}{2}BC$

$\Rightarrow PB = RC; BQ = CQ$

Thus in ΔPBQ and ΔRCQ,

PB = RC

BQ = CQ

$\angle PBQ = \angle RCQ = 90^\circ$

Therefore, $\Delta PBQ \cong \Delta RCQ$

Hence, PQ = QR ....(iv)

From (iii) and (iv)

PQ = QR = RS

But PQRS is a parallelogram

$\Rightarrow QR = PS$

$\Rightarrow PQ = QR = RS = PS$ ....(v)

Now, PQ||AC

$\Rightarrow PM||NO$ ....(vi)

Since P and S are the mid-points of AB and AD respectively

PS||BD

$\Rightarrow PN||MO$ ....(vii)

Thus in quadrilateral PMON,

PM||NO (from (vi))

PN||MO (from (vii))

So, PMON is a parallelogram

$\Rightarrow \angle MPN = \angle MON$

$\Rightarrow \angle MPN = \angle BOA$

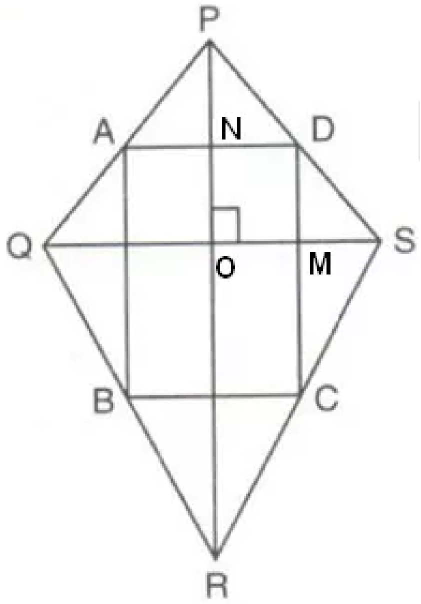
$\Rightarrow \angle MPN = 90^\circ (\because \perp \text{ diagonals})$

$\Rightarrow \angle QPS = 90^\circ$

Thus, PQRS is a quadrilateral such that PQ=QR=RS=PS and $\angle QPS = 90^\circ$

Hence, PQRS is a square.

Answer 5.



In ΔPQS, A and D are mid points of sides QP and PS respectively.

Therefore, AD || QS and $AD = \frac{1}{2}QS$ ....(i)

In ΔQRS

B and C are the mid points of QR and RS respectively

Therefore, BC || QS and $BC = \frac{1}{2}QS$ ....(ii)

From equations (i) and (ii),

AD || BC and AD = BC

As in quadrilateral ABCD one pair of opposite sides are equal and parallel to each other, so, it is a parallelogram.

The diagonals of quadrilateral PQRS intersect each other at point O.

Now in quadrilateral OMDN

ND || OM (AD || QS)

DM || ON (DC || PR)

So, OMDN is parallelogram

$\angle MDN = \angle NOM$

$\angle ADC = \angle NOM$

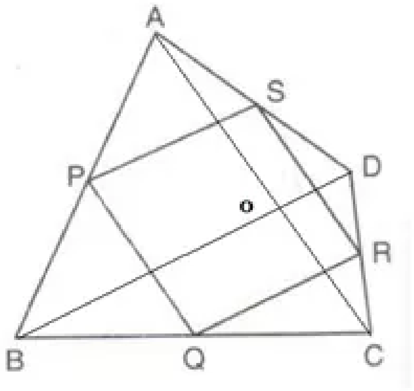
But, $\angle NOM = 90^\circ$ (diagonals are perpendicular to each other)

$\Rightarrow \angle ADC = 90^\circ$

Clearly ABCD is a parallelogram having one of its interior angle as 90°.

Hence, ABCD is rectangle.

Answer 6.



Join AC and BD

In ΔABC,

P and Q are mid-points of AB and BC respectively.

Therefore, $PQ || AC$ and $PQ = \frac{1}{2} AC$ ....(i)

In ΔADC,

S and R are mid-points of AD and DC respectively.

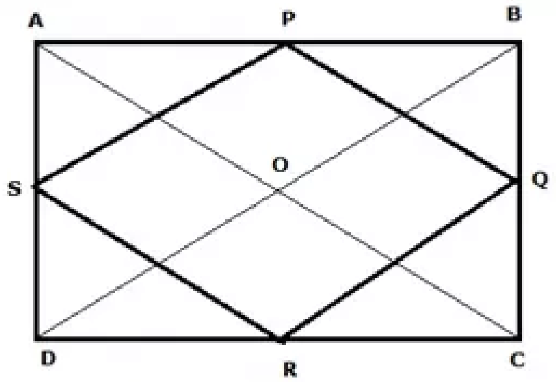
Therefore, $SR || AC$ and $SR = \frac{1}{2} AC$ ....(ii)

From (i) and (ii)

$PQ || SR$ and $PQ = SR$

Therefore, PQRS is a parallelogram.

Answer 7.



Let us join AC and BD

In ΔABC

P and Q are the mid-points of AB and BC respectively

Therefore, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ (mid-point theorem) ... (1)

Similarly in ΔADC

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ (mid-point theorem) ..... (2)

Clearly, $PQ \parallel SR$ and $PQ = SR$

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

Therefore, $PS \parallel QR$ and $PS = QR$ (opposite sides of parallelogram)... (3)

Now, in ΔBCD, Q and R are mid points of side BC and CD respectively.

Therefore, $QR \parallel BD$ and $QR = \frac{1}{2}BD$ (mid-point theorem) ... (4)

But diagonals of a rectangle are equal

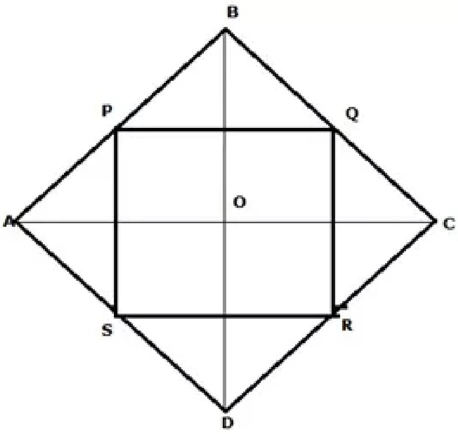
$\Rightarrow AC = BD$... (5)

Now, by using equation (1), (2), (3), (4), (5) we can say that

$PQ = QR = SR = PS$

So, PQRS is a rhombus.

Answer 8.



In ΔABC, P and Q are mid points of sides AB and BC respectively.

Therefore, PQ || AC and $PQ = \frac{1}{2}AC$ (using mid-point theorem) ... (1)

In ΔADC

R and S are the mid points of CD and AD respectively

Therefore, RS || AC and $RS = \frac{1}{2}AC$ (using mid-point theorem) ... (2)

From equations (1) and (2), we have

PQ || RS and PQ = RS

As in quadrilateral PQRS one pair of opposite sides are equal and parallel to each other, so, it is a parallelogram.

Let diagonals of rhombus ABCD intersects each other at point O.

Now in quadrilateral OMQN

MQ || ON (PQ || AC)

QN || OM (QR || BD)

So, OMQN is parallelogram

$\angle MQN = \angle NOM$

$\angle PQR = \angle NOM$

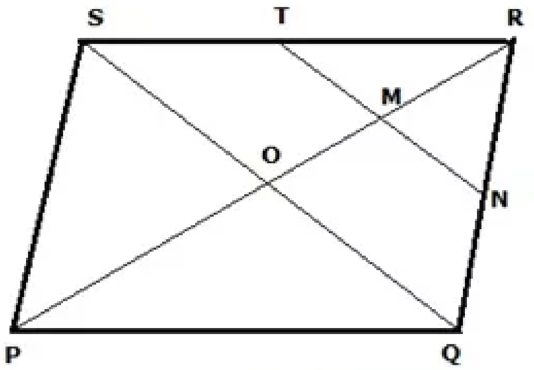
But, $\angle NOM = 90^\circ$ (diagonals of a rhombus are perpendicular to each other)

$\angle PQR = 90^\circ$

Clearly PQRS is a parallelogram having one of its interior angle as 90°.

Hence, PQRS is rectangle.

Answer 9.



Join PR to intersect QS at O

Diagonals of a parallelogram bisect each other.

Therefore, OP = OR

But $MR = \frac{1}{4}PR$

$\therefore MR = \frac{1}{4}(2 \times OR)$

$\Rightarrow MR = \frac{1}{2}OR$

Hence, M is the mid-point of OR.

In ΔROS, T and M are the mid-points of RS and OR respectively.

Therefore, $TM || OS$

$\Rightarrow TN || QS$

Also in ΔRQS, T is the mid-point of RS and $TN || QS$

Therefore, N is the mid-point of QR and $TN = \frac{1}{2}QS$

$\Rightarrow QN = RN$

Answer 10.

$KP = \frac{1}{3}KN$ (since KP: PN=1:2)

$MQ = \frac{1}{3}LM$ (since LQ: MQ=1:2)

Bit KN = LM (opposite sides of parallelogram KLMN)

$\Rightarrow \frac{1}{3}KN = \frac{1}{3}LM$

$\therefore KP = MQ$............(i)

Also, KN||LM

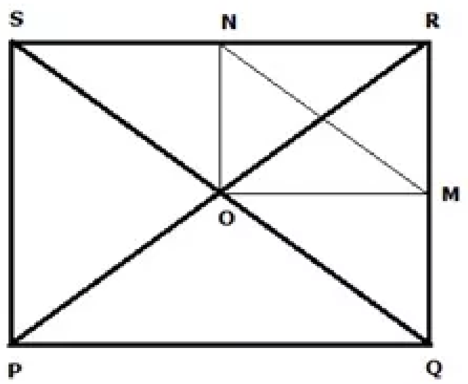
$\Rightarrow$KP||MQ ....(ii)

From (i) and (ii)

KP = MQ and KP||MQ

Hence, KQMP is a parallelogram.

Answer 11.



In ΔSRQ,

N and O are the mid-points of SR and PR respectively.

Therefore, ON||QR and $ON = \frac{1}{2}QR$ i.e. ON = MR ....(i)

PR = SQ (diagonals of rectangle are equal and bisect each other)

$\Rightarrow$ O is mid-point of SQ

In ΔRQS,

M and O are the mid-points of QR and SQ respectively.

Therefore, OM||SR and OM$=\frac{1}{2}$SR i.e. OM = NR ....(ii)

∠MRN = ∠QRS = 90° ....(iii) (PQRS is a rectangle)

From (i), (ii) and (iii)

Therefore, quadrilateral MONR has two opposite pairs of sides equal and parallel and an interior angle as right angle, so it is a rectangle.

In ΔSQR,

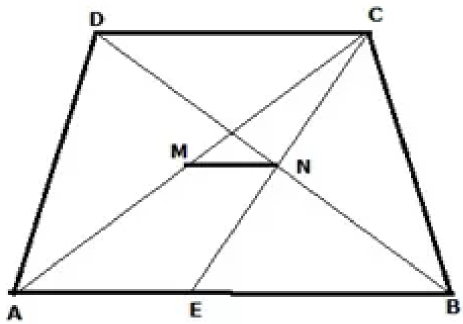
M and N are the mid-points of QR and SR respectively.

Therefore, MN||SQ and MN$=\frac{1}{2}$SQ

But SQ = PR

$\Rightarrow$MN$=\frac{1}{2}$PR

Answer 12.



Join AC and BD. M and N are mid-points of AC and BD respectively. Join MN. Draw a line CN cutting AB at E.

Now, in Δs DNC and BNE,

DN = NB	(N is the mid-point of BD, given)
∠ CDN = ∠EBN	(Alternate angles as DC || AB)
∠ DNC = BNE	(Vertically opposite angles)
⇒ Δ DNC ≅ Δ BNE	(By A-S-A Test)

⇒ DC = BE

By Mid-Point Theorem, in Δ ACE, M and N are mid-points

$MN = (\frac{1}{2})$ AE and MN||AE or MN||AB

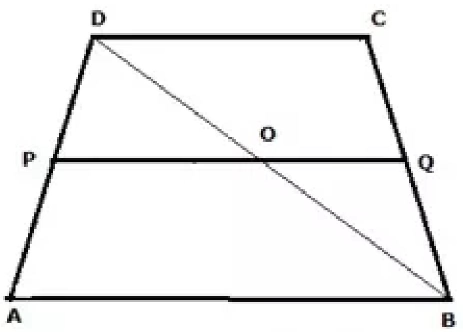
Also, AB||CD, therefore, MN||CD

$\Rightarrow$ $MN = (\frac{1}{2})$ [AB - BE]

$\Rightarrow$ $MN = (\frac{1}{2})$[AB - CD] (since BE=CD)

$\Rightarrow$ $MN = (\frac{1}{2})$ x Difference of parallel sides AB and CD

Answer 14.



PQ||DC $\Rightarrow$ OQ||DC||AB

Therefore, Q and O are mid-points of BC and BD respectively.

In ΔABD,

P and O are mid-points of AD and BD respectively

$\Rightarrow OP = \frac{1}{2}AB$ ....(i)

In ΔBCD,

Q and O are mid-points of BC and BD respectively

$\Rightarrow OQ = \frac{1}{2}CD$ ....(ii)

Adding (i) and (ii)

$OP + OQ = \frac{1}{2}AB + \frac{1}{2}CD$

$\Rightarrow PQ = \frac{1}{2}(AB+CD)$