Sample Question Paper - 11
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

OR

If $\frac{3}{5}$, a, 4 are three consecutive terms of an A.P., then find the value of a.

2. A vertical pole is 100 metres high. Find the angle subtended by the pole at a point on the ground $100\sqrt{3}$ meters from the base of the pole.

3. PQ is a tangent to a circle with centre O at point P. If $\Delta\, OPQ$ is an isosceles triangle, then find $\angle OQP$.

4. If the radius of the sphere is increased by 100%, then how much volume of the corresponding sphere is increased ?

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. What is the length of the wire?

6. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the find the mean of the last three observation

OR

Find the class-marks of the classes 10-25 and 35-66.

Section B

7. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

8. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many litres of water is being pumped out through this pipe in one hour?

9. Compute the mode for the following frequency distribution:

Size of items (in cm)	0- 4	4- 8	8- 12	12-16	16-20	20-24	24-28
Frequency	5	7	9	17	12	10	6

10. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

OR

Compute the mode for the following frequency distribution:

Size of items (in cm)	0- 4	4- 8	8- 12	12-16	16-20	20-24	24-28
Frequency	5	7	9	17	12	10	6

Section C

11. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20} - S_{10})$

12. Draw two tangents to a circle of radius 4 cm, which are inclined to each other at an angle of $60°$.

OR

Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

13. Model Rocketry : A model rocket is a small rocket designed to reach low altitudes and be recovered by a variety of means. Flying model rockets is a relatively safe and inexpensive way for person to learn the basics of forces and the response of a vehicle to external forces. Like an airplane, a model rocket is subjected to the forces of weight, thrust, and aerodynamics during its flight.

Shalvi is a member of first rocket club of India named STAR Club. She launches her latest rocket from a large field. At the moment its fuel is exhausted, the rocket has a velocity of 240 ft/sec and an altitude of 544 ft. After t sec, its height $h(t)$ above the ground is given by the function $h(t) = -16t^2 + 240t + 544$.

(i) What is the maximum height attained by the rocket?

(ii) How many seconds was the rocket airborne after its fuel was exhausted?

14. Well Embankment : Well embankment is a raised wall that is built around the well. These are often constructed using soils obtained from a digging well. It provide protection to person from felling into the well.

A well of diameter 6 m is dug 14 m deep. $\frac{1}{15}$ of the earth taken out is spread evenly all around the well to form a embankment.

(i) Find the volume of the earth taken out.

(ii) If the height of embankment is 1.2 m, what is the width of the embankment ?

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

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SECTION A

1. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

Sol :

Given, $(a-b)^2$,(a^2+b^2) and $(a+b)^2$.

Common difference,

$$\begin{aligned} d_1 &= (a^2+b^2)-(a-b)^2 \\ &= (a^2+b^2)-(a^2+b^2-2ab \\ &= a^2+b^2-a^2-b^2+2ab \\ &= 2ab \end{aligned}$$

and $$\begin{aligned} d_2 &= (a+b)^2-(a^2+b^2) \\ &= a^2+b^2+2ab-a^2-b^2 = 2ab \end{aligned}$$

Since, $d_1 = d_2$, thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

or

If $\frac{3}{5}$, a, 4 are three consecutive terms of an A.P., then find the value of a.

Sol :

If $\frac{3}{5}$, a, 4 are in A.P., then

$$2a = \frac{3}{5}+4$$

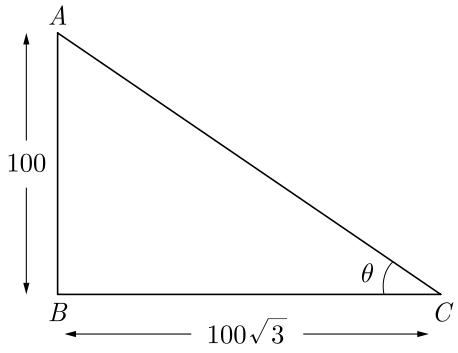
$$2a = \frac{23}{5}$$

$$a = \frac{23}{10}$$

2. A vertical pole is 100 metres high. Find the angle subtended by the pole at a point on the ground $100\sqrt{3}$ meters from the base of the pole.

Sol :

Let the angle be θ. As per given in question, we have drawn figure below



In ΔABC,

$$\begin{aligned} \tan\theta &= \frac{AB}{BC} = \frac{100}{100\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} = \tan 30^\circ \\ &= \tan 30^\circ \\ \tan\theta &= \tan 30^\circ \end{aligned}$$

Thus $$\theta = 30^\circ$$

3. PQ is a tangent to a circle with centre O at point P. If $\Delta\, OPQ$ is an isosceles triangle, then find $\angle OQP$.

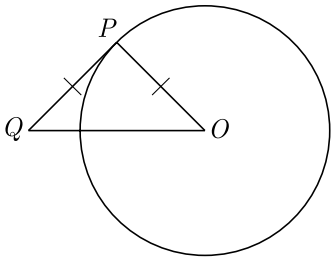
Sol :

Given PQ is a tangent to a circle with centre O at P and $\Delta\, OPQ$ is isosceles.

$$PQ = PQ$$

$$\angle Q = \angle O$$

We draw the figure as given below.



Since tangent is perpendicular to the radius at point of contact,

$$\angle P = 90^\circ$$

Now, in ΔOPQ,

$$\angle P + \angle Q + \angle O = 180^\circ \quad \text{[Angle sum properly]}$$

$$90^\circ + 2\angle Q = 180^\circ$$

$$2\angle Q = 90^\circ$$

$$\angle Q = 45^\circ$$

Thus $\angle OQP = 45^\circ$

4. If the radius of the sphere is increased by 100%, then how much volume of the corresponding sphere is increased ?

Sol :

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $V_{2r} = \frac{4}{3}\pi \times (2r)^3$

$$= \frac{4}{3}\pi \times 8r^3$$

Thus new volume is 8 times of original volume.

Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. What is the length of the wire?

Sol :

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l,

Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$

$$\pi \times \left(\frac{2}{2} \times \frac{1}{10}\right)^2 \times l = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$

$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 3^3$$

$$\frac{l}{100} = 4 \times 3^2 = 36$$

$$l = 3600\,\text{cm} = 36\,\text{m}$$

6. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the find the mean of the last three observation

Sol :

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation,

$$= \frac{x+5+x+7+x+10}{3} = \frac{5x+22}{3}$$

$$\frac{3x+22}{3} = \frac{3 \times 4 + 22}{3}$$

$$= \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

or

Find the class-marks of the classes 10-25 and 35-66.

Sol :

Class mark of $10-25$, $= \frac{10+25}{2}$

$$= \frac{35}{2} = 17.5$$

and class mark of $35-55$, $= \frac{35+55}{2}$

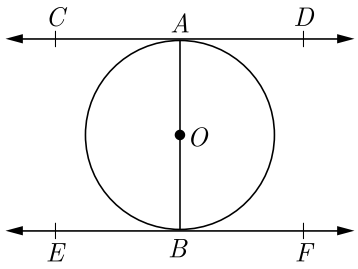
$$= \frac{90}{2} = 45$$

Section B

7. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol :

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^\circ$ and $\angle ABF = 90^\circ$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD \,||\, EF$ Hence Proved

8. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many litres of water is being pumped out through this pipe in one hour?

Sol :

Length of water that flows in 1 sec is 80 cm.

$$\text{Radius of pipe} = \frac{8}{2} = 4\,\text{cm}$$

Thus, volume of water flows in 1 sec

$$= \pi \times (4)^2 \times 80$$
$$= 128\pi\,\text{cm}^3$$

Volume of water flows in 1 hour

$$= 128\pi \times 60 \times 60$$
$$= 460800\pi\,\text{cm}^3$$
$$= 14469120\,\text{cm}^3$$
$$= 14469.12\,\text{L}$$

$[1\text{ Lit} = 1000\,\text{cm}^3]$

9. Compute the mode for the following frequency distribution:

Size of items (in cm)	0-4	4-8	8-12	12-16	16-20	20-24	24-28
Frequency	5	7	9	17	12	10	6

Sol :

Class 12-16 has the maximum frequency 17, therefore this is model class.

We have $l = 12$, $f_1 = 17$, $f_0 = 9$, $f_2 = 12$ and $h = 4$

$$\text{Mode} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 12 + \left(\frac{17 - 9}{2 \times 17 - 9 - 12}\right) \times 4$$
$$= 12 + \frac{8 \times 4}{13} = 12 + 2.46 = 14.46$$

10. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Sol :

Class	Class Mark	Frequency	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
	Total	$40 + f$	$704 + 20f$

We have $\sum f_i = 40 + f$

$\sum f_i x_i = 704 + 20f$

Mean, $M = \dfrac{\sum f_i x_i}{\sum f_i}$

$$18 = \frac{704 \times 20f}{40 + f}$$
$$720 + 18f = 704 + 20f$$
$$f = 8$$

or

Compute the mode for the following frequency distribution:

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$$= 12 + \frac{8 \times 4}{13} = 12 + 2.46 = 14.46$$

Section C

11. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20} - S_{10})$

Sol :

Let the first term be a, and common difference be d.

Now $S_{30} = \frac{30}{2}(2a + 29d)$ ...(1)

$= 15(2a + 29d)$

$3(S_{20} - S_{10}) = 3\big[10(2a + 19d) - 5\ 2a + 9$

$= 3[20a + 190d - 10a - 45$

$= 3[10a + 145d]$

$= 15[2a + 29d]$ ...(2)

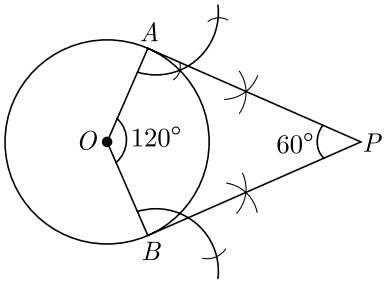
Hence $S_{30} = 3(S_{20} - S_{10})$

12. Draw two tangents to a circle of radius 4 cm, which are inclined to each other at an angle of 60°.

Sol :

Step of construction :

1. Draw a circle of radius 4 cm with O as centre.
2. Draw two radii OA and OB inclined to each other at an angle of 120°.
3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B. which meet at P.
4. PA and PB are the required tangents inclined to each other an angle of 60°.



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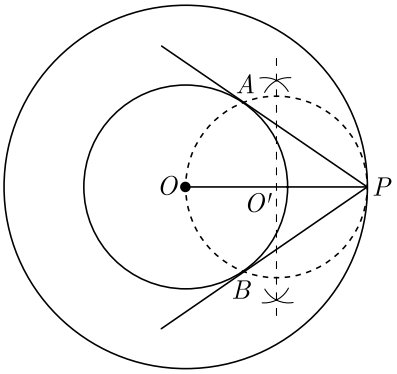
or

Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Sol :

Steps of Construction :

1. Draw a circle with radius 3 cm and centre O.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on the circumference of outer circle and join O to P.
4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B.
5. Join A to P and B to P. AP and BP are the required tangents.



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Shalvi is a member of first rocket club of India named STAR Club. She launches her latest rocket from a large field. At the moment its fuel is exhausted, the

rocket has a velocity of 240 ft/sec and an altitude of 544 ft. After t sec, its height $h(t)$ above the ground is given by the function $h(t) = -16t^2 + 240t + 544$.

(i) What is the maximum height attained by the rocket?

(ii) How many seconds was the rocket airborne after its fuel was exhausted?

Sol :

We have, $h(t) = -16t^2 + 240t + 544$

(i) $$\begin{aligned} h(t) &= -16t^2 + 240t + 544 \\ &= -16(t^2 - 15t) + 544 \\ &= -16(t^2 - 15t + 7.5^2 - 7.5^2) + 544 \\ &= -16(t - 7.5)^2 + 16 \times 7.5^2 + 544 \\ &= -16(t - 7.5)^2 + 900 + 544 \\ &= -16(t - 7.5)^2 + 1444 \end{aligned}$$

From above equation it is clear that $h(t)$ is maximum at $t = 7.5$ second and this maximum value is 1444 feet.

(ii) For airbonre time, h will be zero.

$$\begin{aligned} 0 &= -16t^2 + 240t + 544 \\ 0 &= t^2 - 15t - 34 \\ 0 &= (t - 17)(t + 2) \\ t &= 17 \text{ sec} \end{aligned}$$

14. Well Embankment : Well embankment is a raised wall that is built around the well. These are often constructed using soils obtained from a digging well. It provide protection to person from felling into the well.

A well of diameter 6 m is dug 14 m deep. $\frac{1}{15}$ of the earth taken out is spread evenly all around the well to form a embankment.

(i) Find the volume of the earth taken out.

(ii) If the height of embankment is 1.2 m, what is the width of the embankment ?

Sol :

(i) Depth of well, $d = 14$ m,

Radius, $r = \frac{6}{2} = 3$ m.

Volume of earth taken out,

$$\pi r^2 h = \frac{22}{7} \times (3)^2 \times 14 = 396 \text{ m}^3$$

(ii) Let w be the width of embankment. The radius of outer circle of embankment

$$= 3 + w$$

Area of upper surface of embankment

$$= \pi[(3 + w)^2 - (3)^2]$$

Volume of embankment $= \frac{1}{15}$ of volume of earth taken out

$$\begin{aligned} \pi[(3 + w)^2 - (3)^2] \times 1.2 &= \frac{1}{15} \times 396 \\ \pi(9 + w^2 + 6w - 9) \times 1.2 &= \frac{1}{15} \times 396 \\ \frac{22}{7}(w^2 + 6w) \times 1.2 &= \frac{1}{15} \times 396 \\ w^2 + 6w &= \frac{396 \times 7}{15 \times 1.2 \times 22} = 7 \\ w^2 + 6w - 7 &= 0 \\ (w + 7)(w - 1) &= 0 \\ \Rightarrow \quad w &= 1 \end{aligned}$$

Hence width of embankment is 1 m.