Sample Question Paper - 39
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, how high is the kite?

2. Solve for x : $25x^2 - 10a^2x + (a^4 - b^4) = 0$

OR

Is the following situation possible? The sum of ages of a mother and her daughter is 25 years. Five years ago the product of their ages was 58.

3. Find the mean age (in years) from the frequency distribution given below :

Class (age in years)	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	4	14	22	16	6	5	3

4. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then find the length of the wire.

OR

The ratio of the length of a rod and its shadow is $1:\sqrt{3}$. Find the altitude of the sun.

5. Divide a line segment of length 9 cm internally in the ratio 4 : 3.

6. From the following data find the median age of 100 residents of a colony who took part in swachch bharat abhiyan :

Age (in yrs.)	0-10	10-20	20-30	30-40	40-50	50-60
Number of residents	4	6	20	10	7	3

SECTION - B

7. A toy is in the form of a cone mounted on a hemisphere of radius 7 cm. The total height of the toy is 14.5 cm. Find the volume of the toy. $\left(\text{Take } \pi = \frac{22}{7}\right)$.

8. In a violent storm, a tree got bent by the wind. The top of the tree meets the ground at an angle of 30°, at a distance of 30 m from the root. At what height from the bottom did the tree get bent? What was the original height of the tree? [Use $\sqrt{3} = 1.73$]

9. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. Find the capacity of the capsule.

OR

A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 12 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy, if the total height of the toy is 30 cm.

10. *ABC* is right angled triangle, right angled at *A*. A circle is inscribed in it. The length of two sides containing the right angles are 5 cm and 12 cm. Find the radius of the circle.

SECTION - C

11. Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first installment of ₹ 1000. If he increases the installment by ₹ 100 every month, what amount will be paid by him in the 30^{th} installment?

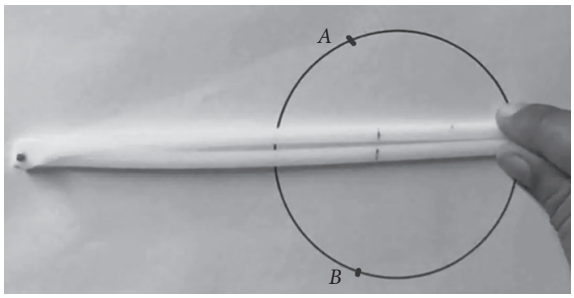
OR

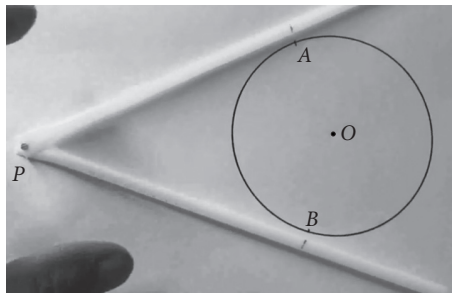
In an A.P., show that the sum of the terms equidistant from the beginning and the end is the same and is equal to the sum of first and last terms.

12. The sum of the areas of two squares is 452 m^2. If the difference of their perimeters is 8 m, then find the sides of the two squares.

Case Study - 1

13. Prem did an activity on tangents drawn to a circle from an external point using 2 straws and a nail for maths project as shown in figure.





Based on the above information, answer the following questions.

(i) On the basis of which of the following congruency criterion, $\Delta OAP \cong \Delta OBP$?

(ii) If $\angle AOB = 150°$, then find measure of $\angle APB$.

Case Study - 2

14. As the demand for the products grew, a manufacturing company decided to hire more employees. For which they want to know the mean time required to complete the work for a worker.

The following table shows the frequency distribution of the time required for each worker to complete a work.

Time (in hours)	15-19	20-24	25-29	30-34	35-39
Number of workers	10	15	12	8	5

Based on the above information, answer the following questions.

(i) If x_i's denotes the class marks and f_i's denotes the corresponding frequencies for the given data, then find the value of $\sum x_i f_i$.

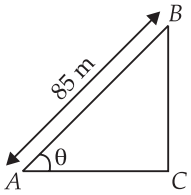
(ii) Find the mean time (in hrs) required to complete the work for a worker.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Length of the string of the kite from ground
$= AB = 85$ m



We have given, $\tan\theta = \frac{15}{8}$

$\Rightarrow \cot\theta = \frac{8}{15} \Rightarrow \text{cosec}^2\theta - 1 = \frac{64}{225}$

$\Rightarrow \text{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$

$\Rightarrow \text{cosec}\,\theta = \sqrt{\frac{289}{225}} = \frac{17}{15} \Rightarrow \sin\theta = \frac{15}{17}$

In ΔABC, $\sin\theta = \frac{BC}{AB} \Rightarrow \frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75$

$\therefore$ Height of kite = 75 m

2. We have, $25x^2 - 10a^2x + (a^4 - b^4) = 0$

$\Rightarrow 25x^2 - 5(a^2 - b^2)\,x - 5(a^2 + b^2)\,x + (a^4 - b^4) = 0$

$\Rightarrow 5x[5x - (a^2 - b^2)] - (a^2 + b^2)\,[5x - (a^2 - b^2)] = 0$

$\Rightarrow [5x - (a^2 - b^2)]\,[5x - (a^2 + b^2)] = 0$

$\Rightarrow 5x - (a^2 - b^2) = 0$ or $5x - (a^2 + b^2) = 0$

$\Rightarrow x = \frac{a^2 - b^2}{5}$ or $x = \frac{a^2 + b^2}{5}$

Hence, $\frac{a^2 - b^2}{5}$ and $\frac{a^2 + b^2}{5}$ are the roots of the given quadratic equation.

OR

Let age of mother be x years.

$\therefore$ Age of the daughter will be $(25 - x)$ years.

Five years ago,

Mother's age = $(x - 5)$ years

and daughter's age = $(25 - x - 5) = (20 - x)$ years

According to question, $(x - 5)\,(20 - x) = 58$

$\Rightarrow 20x - x^2 - 100 + 5x = 58 \Rightarrow x^2 - 25x + 158 = 0$

Here, $a = 1$, $b = -25$ and $c = 158$.

$\therefore D = b^2 - 4ac = (-25)^2 - 4\,(1)(158)$

$= 625 - 632 = -7 < 0$

$\therefore$ No real solution of the equation is possible.

Hence, the given situation is not possible.

3. The frequency distribution table for the given data can be drawn as :

Class	Modified class	(x_i)	(f_i)	$f_i x_i$
25-29	24.5-29.5	27	4	108
30-34	29.5-34.5	32	14	448
35-39	34.5-39.5	37	22	814
40-44	39.5-44.5	42	16	672
45-49	44.5-49.5	47	6	282
50-54	49.5-54.5	52	5	260
55-59	54.5-59.5	57	3	171
			$\Sigma f_i = 70$	$\Sigma f_i x_i = 2755$

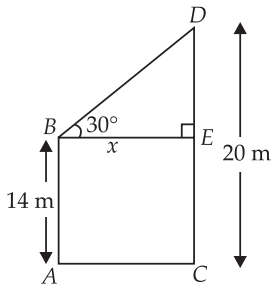
$\therefore$ Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{2755}{70} = 39.35$ years

4. Here, $CD = 20$ m [Height of big pole]

$AB = 14$ m [Height of small pole]

$DE = CD - CE \Rightarrow DE = CD - AB$ [$\because AB = CE$]

$\Rightarrow DE = 20 - 14 = 6$ m



In ΔBDE, $\sin 30° = \frac{DE}{BD}$

$\Rightarrow \frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12$ m

$\therefore$ Length of wire = 12 m

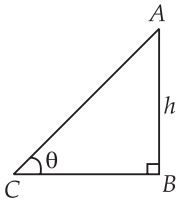
OR

Let AB be the rod and BC be its shadow.

In ΔABC,

$\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}}\right]$

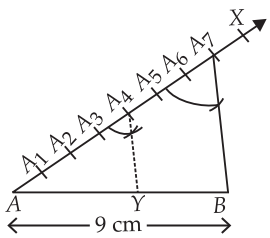


$\Rightarrow \theta = 30°$

5. Steps of construction :

Step-I : Draw a line segment $AB = 9$ cm.

Step-II : Draw a ray AX making an acute angle with AB.



Step-III : Locate $(4 + 3 = 7)$ points $(A_1, A_2, \ldots\ldots, A_7)$ on AX such that $AA_1 = A_1A_2 = \ldots\ldots = A_6A_7$.

Step-IV : Join A_7B.

Step-V : Through A_4, draw A_4Y parallel to A_7B, meeting AB at Y, such that $\angle AA_7B = \angle AA_4Y$

Hence, $AY : YB = 4 : 3$.

6. The frequency distribution table for the given data can be drawn as :

Class	Frequency (f_i)	Cumulative frequency (*c.f.*)
0-10	4	4
10-20	6	10
20-30	20	30
30-40	10	40
40-50	7	47
50-60	3	50
Total	**50**	

Here $\frac{N}{2} = \frac{50}{2} = 25$

$\therefore$ Median class is 20-30.

$\therefore$ Median $= 20 + \left(\frac{25-10}{20}\right) \times 10$

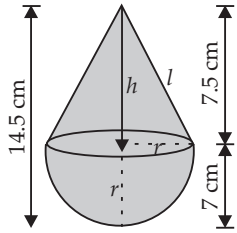
$= 20 + 7.5 = 27.5$

7. Radius of the hemisphere, $r = 7$ cm

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (7)^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{44 \times 49}{3} = \frac{2156}{3}\text{ cm}^3$



Height of cone, $h = 14.5 - 7$

$= 7.5$ cm and

radius of cone = 7 cm

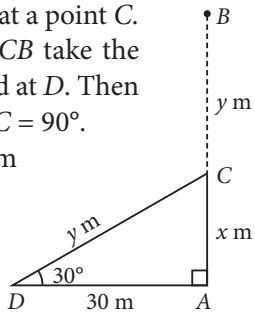
Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7.5$

$= \frac{1155}{3}\text{cm}^3$

Volume of the toy = Volume of hemisphere + Volume of cone $= \frac{2156}{3} + \frac{1155}{3} = \frac{3311}{3} = 1103.67\text{cm}^3$

8. Let AB be the tree, got bent at a point C. After getting bent, let the part CB take the position CD, meeting the ground at D. Then $AD = 30$ m, $\angle ADC = 30°$, $\angle DAC = 90°$.

Let $AC = x$ m and $CD = CB = y$ m



In ΔDAC, $\frac{AC}{AD} = \tan 30°$

$\Rightarrow \frac{x}{30} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow x = 10\sqrt{3} = 17.3$

In ΔDAC, $\frac{DC}{AD} = \sec 30° \Rightarrow \frac{y}{30} = \frac{2}{\sqrt{3}}$

$\Rightarrow y = 30 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} = 34.6$

$\therefore$ $AB = AC + CB = x + y = 17.3 + 34.6 = 51.9$ m

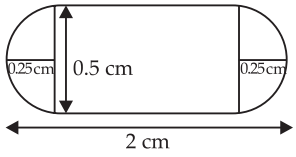
Hence, the tree got bent at the height of 17.3 m from the bottom and the original height of the tree was 51.9 m.

9. Given, diameter of cylinder

= Diameter of hemisphere = 0.5 cm

$\therefore$ Radius of cylinder (r) = radius of hemisphere (r)

$= \frac{0.5}{2} = 0.25$ cm



and total length of capsule = 2 cm

$\therefore$ Length of cylindrical part of capsule, h

$= 2 - (0.25 + 0.25) = 1.5$ cm

Now, capacity of capsule

= Volume of cylindrical part + 2 × volume of hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \frac{22}{7}[(0.25)^2 \times 1.5 + \frac{4}{3} \times (0.25)^3]$

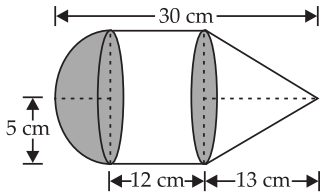
$= \frac{22}{7}[0.09375 + 0.02083] = \frac{22}{7} \times 0.11458 = 0.36\text{ cm}^3$

Hence, the capacity of capsule is 0.36 cm^3.

OR

Radius of the hemisphere = 5 cm

Radius of the base of the cylinder = 5 cm



Radius of the base of the cone = 5 cm

Height of the hemisphere = 5 cm

Let the height of the cone be h cm.

Then, $(5 + 12 + h)$ cm = 30 cm $\Rightarrow h = 13$

$\therefore$ Height of the cone = 13 cm

Slant height of the cone,

$l = \sqrt{5^2 + (13)^2}\text{ cm} = \sqrt{194}\text{ cm} = 14\text{ cm}$ (Approx.)

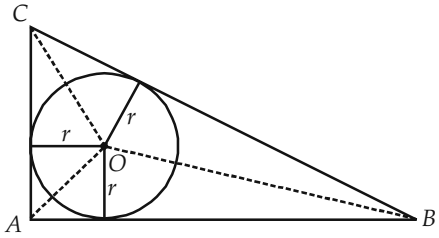
Surface area of the toy = curved surface area of hemisphere + curved surface area of the cylinder + curved surface area of the cone

$= (2\pi r^2 + 2\pi rh + \pi rl)$

$= [2\pi \times (5)^2 + 2\pi \times 5 \times 12 + \pi \times 5 \times 14]\text{cm}^2$

$= (240\pi)\ \text{cm}^2 = \left(240 \times \frac{22}{7}\right)\text{cm}^2 = 754.28\ \text{cm}^2$

10. Given that ΔABC is right angled at A



Let $AC = 5$ cm and $AB = 12$ cm

and $BC^2 = AC^2 + AB^2 = 25 + 144 = 169 \Rightarrow BC = 13$ cm

Join OA, OB, OC.

Let the radius of the inscribed circle be r

Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA

$$\Rightarrow \frac{1}{2} \times AB \times AC = \frac{1}{2}(12 \times r) + \frac{1}{2}(13 \times r) + \frac{1}{2}(5 \times r)$$

$\Rightarrow 12 \times 5 = r \times \{12 + 13 + 5\} \Rightarrow 60 = r \times 30$

$\Rightarrow r = 2$ cm

11. Given that,

Jaspal Singh takes total loan = ₹ 118000

He repays his total loan by paying every month.

His first installment = ₹1000

Second installment = 1000 + 100 = ₹ 1100

Third installment = 1100 + 100 = ₹ 1200 and so on

Let its 30th installment be n,

Thus, we have 1000, 1100, 1200, ..., which forms an A.P., with first term, $a = 1000$ and

common difference, $d = 1100 - 1000 = 100$

n^{th} term of an A.P., $a_n = a + (n - 1)d$

For 30th installment, $a_{30} = 1000 + (30 - 1)100$

$= 1000 + 29 \times 100 = 1000 + 2900 = 3900$

So, ₹ 3900 will be paid by him in the 30th installment.

OR

Let the A.P. be a, $a + d$, $a + 2d$ ...., $(l - 2d)$, $(l - d)$, l, where a is the first term and l is the last term of A.P.

We know that p^{th} term from the beginning

$= a + (p - 1)d$ ...(i)

We find p^{th} term from the end.

Writing the A.P. in reverse order we have, $l, l - d, l - 2d$, ..., $a + 2d$, $a + d$, a.

It is an A.P. with $a = l$ and common difference (D)

$= l - d - l = -d$.

$\therefore$ p^{th} term from end $= a + (p - 1)\ D$

$= l - (p - 1)d$. ...(ii)

Adding (i) and (ii), we get

(p^{th} term from beginning) + (p^{th} term from end) $= a + l$

Hence proved.

12. Let x m be the length of the side of first square and y m be the length of the side of the second square such that $x > y$.

According to the question,

$x^2 + y^2 = 452$ ...(i)

and $4x - 4y = 8$

$\Rightarrow x - y = 2$

$\Rightarrow x = 2 + y$ ...(ii)

From (i) and (ii), we have $(2 + y)^2 + y^2 = 452$

$\Rightarrow 4 + 4y + y^2 + y^2 = 452 \Rightarrow 2y^2 + 4y + 4 - 452 = 0$

$\Rightarrow 2y^2 + 4y - 448 = 0 \Rightarrow y^2 + 2y - 224 = 0$

$\Rightarrow y^2 + 16y - 14y - 224 = 0$

$\Rightarrow y(y + 16) - 14(y + 16) = 0$

$\Rightarrow (y - 14)(y + 16) = 0 \Rightarrow y - 14 = 0$ or $y + 16 = 0$

$\therefore$ $y = 14$ [$\because y \neq -16$, as side cannot be negative]

Therefore, $x = 2 + y = 2 + 14 = 16$

Hence, sides of two squares are 16 m and 14 m respectively.

13. (i) In ΔOAP and ΔOBP,

$\angle OAP = \angle OBP = 90°$

[Since, radius at the point of contact is perpendicular to tangent]

$OP = OP$ (Common)

$OA = OB$ (Radii of circle)

So, $\Delta OAP \cong \Delta OBP$ (By RHS congruency criterion)

(ii) In quadrilateral $OAPB$, $\angle AOB = 150°$ [Given]

$\angle OAP = \angle OBP = 90°$

$\therefore$ $\angle APB = 360° - 90° - 90° - 150° = 30°$

14. (i) Let us consider the following table :

Modified class	**Class mark (x_i)**	**Frequency (f_i)**	$x_i f_i$
14.5-19.5	17	10	170
19.5-24.5	22	15	330
24.5-29.5	27	12	324
29.5-34.5	32	8	256
34.5-39.5	37	5	185
Total		$\sum f_i = 50$	$\sum x_i f_i = 1265$

$\therefore$ $\sum x_i f_i = 1265$

(ii) Mean $(\bar{x}) = \frac{\sum x_i f_i}{\sum f_i} = \frac{1265}{50} = 25.3$

Thus, the mean time to complete the work for a worker = 25.3 hrs