WORK, POWER & ENERGY

1. WORK :

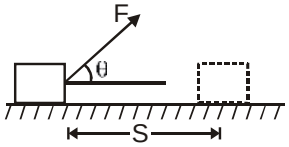
Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

1.1 Work done by constant force

Consider an object undergoes a displacement S along a straight line while acted on a force F that makes an angle θ with S as shown.

The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement.

i.e., $W = FS\cos\theta$ ...(1)



Work done is a scalar quantity and its S.I. unit is N-m or joule (J).

2. UNITS OF WORK :

In cgs system, the unit of work is erg.

One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.

$\therefore$ Relation between joule and erg

1 joule = 1 newton × 1 metre

1 joule = 10^7 erg

Dimensions of Work :

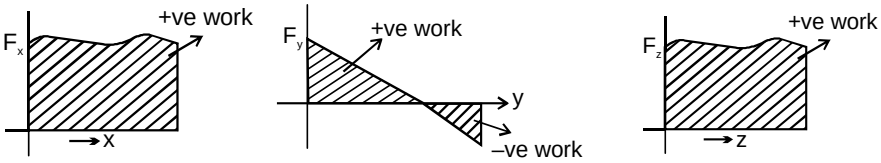
$[\text{Work}] = [\text{Force}]\,[\text{Distance}] = [MLT^{-2}]\,[L] = [ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time, On the basis of dimensional formula, the unit of work is kg $m^2\,s^{-2}$.

Note that 1 kg $m^2 s^{-2}$ = (1 kg m s^{-2}) m = 1 N m = 1 J.

3. AREA UNDER FORCE DISPLACEMENT CURVE :

Graphically area under the force-displacement is the work done



The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

4. ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertiable.

There are various forms of energy. Heat, electricity, light, sound and chemical energy are all familiar forms. In studying mechanics, we are however concerned chiefly with mechanical energy. This type of energy is a property of movement or position.

4.1 Kinetic Energy

Kinetic energy (K.E.), is the capacity of a body to do work by virtue of its motion.

If a body of mass m has velocity v its kinetic energy is equivalent to the work, which an external force would have to do to bring the body from rest up to its velocity v.

The numerical value of the kinetic energy can be calculated from the formula

$$K.E. = \frac{1}{2}mv^2$$

- Since both m and v^2 are always positive, K.E. is always positive and does not depend upon the direction of motion of the body.

4.2 Potential Energy

Potential energy is energy of the body by virtue of its position. A body is capable to do work by virtue of its position, configuration or state of strain.

Now relation between Potential energy and work done is

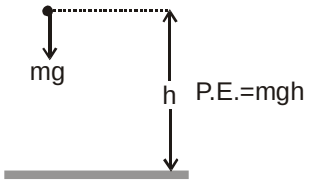
$W.D = -\Delta U$

where ΔU is change in potential energy

There are two common forms of potential energy, gravitational and elastic.

Important points related to Potential energy :

1. Potential energy is a straight function (defined only for position)
2. Potential energy of a point depends on a reference point
3. Potential energy difference between two position doesn't depend on the frame of reference.
4. Potential energy is defined only for conservative force because work done by conservative force is path independent.
5. If we define Potential energy for non conservative force then we have to define P.E. of a single point through different path which gives different value of P.E. at single point that doesn't make any sense.



fig(a)

5. POWER

Power is defined as the time rate of doing work.

When the time taken to complete a given amount of work is important, we measure the power of the agent doing work.

The average power $(\bar{P} \text{ or } P_{av})$ delivered by an agent is given by

$$\bar{P} \text{ or } P_{av} = \frac{\Delta W}{\Delta t} = \frac{\text{Total work done}}{\text{Total time}}$$

where ΔW is the amount of work done in time Δt.

Power is the ratio of two scalars-work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less.

5.1 Unit of Power :

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

1 watt = 1 joule/second = 10^7 erg/second

Also, 1 watt = $\frac{1\,\text{newton} \times 1\,\text{metre}}{1\,\text{second}}$ = 1 N ms^{-1}.

Dimensional formula of power

$$[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$