Communications Test 5

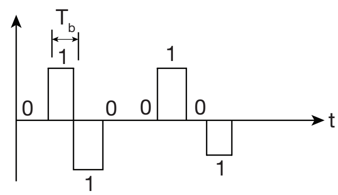
Number of Questions: 25 **Time: 60 min.**

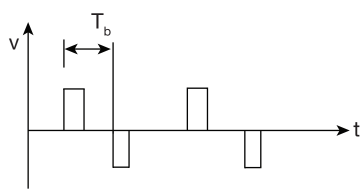
Directions for questions 1 to 25: Select the correct alternative from the given choices.

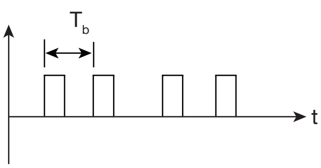
1. An analog voltage in the 0 to 16V is divided in 32 equal intervals for conversion to 4-bit digital output. The maximum quantization error (in V) is

(A) 0.75 (B) 0.5
(C) 0.25 (D) 0.2

2. For the binary sequence 01100101. The AMI R_z signaling format is

(A) 

(B) 

(C) 

(D) None of these

3. For an independent input and output, The mutual information is equal to

(A) sum of entropies of transmitter and receiver
(B) self information of transmitter and receiver
(C) sum of receiver entropy and noise entropy
(D) zero

4. Binary data is transmitted using PSK signaling scheme with $S_1(t) = A \text{ Cos } \omega_c t$, $S_2(t) = -A \text{ Cos } \omega_c t$ $0 \le t \le T_b$ where T_b (bit duration) is equal to 0.2 ms. The carrier frequency $f_c = 5 f_b$. The carrier amplitude at the receiver input is 2 mv and the PSD of AWGN at the input is 10^{-11} W/Hz.

The probability of error of the optimum filter is ______

(A) $\frac{1}{2}\varepsilon rfc\sqrt{2}$ (B) $\frac{1}{2}\varepsilon rfc\sqrt{20}$

(C) $\varepsilon rfc\sqrt{20}$ (D) $\varepsilon rfc\sqrt{10}$

5. Probability of error in DPSK is

(A) $\frac{1}{2}\ \varepsilon rfc\sqrt{\frac{Eb}{\eta}}$ (B) $\varepsilon rfc\sqrt{\frac{Eb}{\eta}}$

(C) $\frac{1}{2}e^{-\frac{Eb}{2\eta}}$ (D) $\frac{1}{2}e^{-\frac{Eb}{\eta}}$

6. Optimum filter is matched filter when

(A) Input noise is white
(B) Impulse response of the filter is real
(C) Impulse response of the filter is imaginary
(D) None of these

7. The technique that is applicable to digital signals only is

(A) FDMA (B) TDMA
(C) CDMA (D) SDMA

8. When the entire frequency band is filled, then the technique use is ______

(A) TDMA (B) CDMA
(C) SDMA (D) None

9. Frequency re-use is associated with

(A) FDMA (B) TDMA
(C) CDMA (D) SDMA

10. Precise time synchronization is important in

(A) FDMA (B) TDMA
(C) CDMA (D) B and C

11. A binary symmetric channel (BSC) has a transition probability of $\frac{1}{10}$ of the binary transmit symbol X is such that $P(X = 0) = \frac{4}{5}$. Then the probability of error for an optimum receiver will be

(A) $\frac{1}{10}$ (B) $\frac{1}{8}$

(C) $\frac{1}{5}$ (D) $\frac{4}{5}$

12. A source generates four symbols with probabilities, 0.125, 0.125, 0.25, 0.5 assuming independent generation of symbols, the most efficient source encoder would have average bit rate is

(A) 5000 bits/sec (B) 6000 bits/sec
(C) 7000 bits/sec (D) 8000 bits/sec

13. An AWGN communication channel having signal to noise ratio SNR >> 2 and bandwidth B has capacity C_1 of the SNR is becomes the 4 times keeping B constant, the resulting capacity C_2 is given by-

(A) $C_2 \approx 3C_1$ (B) $C_2 \approx C_1 + 0.3B$
(C) $C_2 = C_1 + B$ (D) $C_2 = C_1 + 2B$

14. A fair coin is tossed repeatedly until a 'tail' appears for the first time. Let L be the number of tosses to get this first 'tail'. The entropy $H(L)$ in bits is __________

(A) 1 (B) 2
(C) 4 (D) 6

15. A source produces 3 symbols with probability 0.25, 0.25 and 0.5. For this source a practical coding scheme has an average code word length of 4 bits/symbols. The efficiency of code is ______

(A) 3/2 (B) 3/4
(C) 5/8 (D) 3/8

16. The signal power received for a desired signal of 100 kb/s bit rate signal is 10 mW. The chip frequency used is 100 MHz. the processing gain is, if received power is 1W.

(A) 10 (B) 100
(C) 1000 (D) 10000

17. An analog signal is bandlimited to B Hz, sampled at the Nyquist rate and the samples are quantized into 4 levels. The quantization levels Q_1, Q_2, Q_3 and Q_4 are assumed independent and occur with probabilities $P_1 = P_4 = \frac{1}{4}$ and $P_2 = P_3 = \frac{3}{4}$. The information rate of the source is ______

(A) 2 bits/message (B) 1.8 bits/message
(C) 1.71 bits/message (D) 1.31 bits/message

18. Given an AWGN channel with 8 kHz bandwidth and the noise power spectral density n/2 = 10^{-12} W/Hz. The signal power is required at the receiver is 0.1 mW. the capacity of the channel is

(A) 54.44 b/s (B) 108.88 b/s
(C) 50 b/s (D) 40 b/s

19. A pulse wave form is given as

$$S(t) = \begin{cases} e^{-2t}, & 0 \leq + \leq T \\ 0, & otherwise \end{cases}$$

The matched filter output over (O, T) is ________

(A) $\frac{e^{-2t}}{2} \sin h(2t)$ (B) $e^{-2t} \sin h(4t)$

(C) $\frac{e^{-2T}}{2} \cos(2t)$ (D) None

20. If a signal $S(t) = e^{-2t}u(t)$ is applied to the input of a low pass filter having $|H(\omega)| = \frac{2b}{\sqrt{\omega^2 + 4b^2}}$

Then the value of b is 50%, if the input energy is transformed to the output.

(A) $1 - 2\sqrt{2}$ (B) $1 - \sqrt{\frac{1}{2}}$

(C) $1 + \frac{1}{\sqrt{2}}$ (D) B and C

21. Coherent QPSK modulation is used to transmit 4 equiprobable symbol waveforms

$S_1(t) = \alpha \text{ Cos } 2\pi ft$

$S_2(t) = \alpha \text{Cos}\left(2\pi\, ft + \frac{\pi}{2}\right)$

$S_3(t) = \alpha \text{ Cos}(2\pi ft + \pi)$

$S_4(t) = \alpha \text{Cos}\left(2\pi\, ft + \frac{3\pi}{2}\right)$

Where $\alpha = 2\, mv$

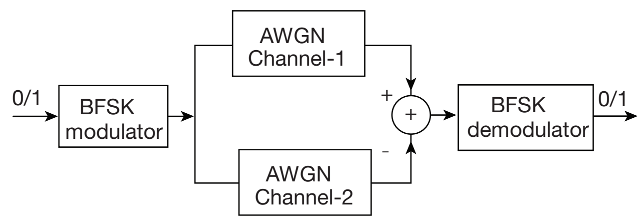
In AWGN channel with PSD $\frac{N_o}{2} = 0.4 \times 10^{-12}$ W/Hz.

Using an optimal receiver and relation and relation $Q(v) = \frac{1}{\sqrt{2\pi}} \int_u e^{-\frac{4^2}{2}}$ du. The bit error probability for a data rate of 4000 kpbs is

(A) $Q(2)$ (B) $Q(1)$

(C) $Q\left(\frac{1}{\sqrt{2}}\right)$ (D) $Q\left(\frac{2}{\sqrt{2}}\right)$

22. In $Q\left(\sqrt{a}\right)$ is the BER of a BFSK system over an AWGN channel with two sided noise power spectral density $N_o/2$. The parameter α is function of bit energy and noise power spectral density.

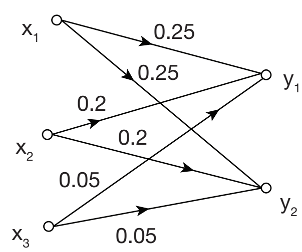


Then the BER rate of this system is

(A) zero (B) $Q\left(2\sqrt{a}\right)$

(C) $Q\sqrt{2a}$ (D) None of these

23. The mutual information for the channel shown in figure is



(A) 1.36 bits/message (B) 1 bits/message
(C) zero (D) None

24. If probability p is 0.4 then the channel capacity for the BSC (Binary Symmetric Channel) is

(A) 0.029 bits/message (B) 3.97 bits/message
(C) 0.28 bits/message (D) 3.72 bits/message

25. An M-level FSK modulation scheme is used to transmit independent binary digits over a band-pass channel with bandwidth 200 kHz. The bit rate is 300 kbps and the system characteristic is a raised cosine spectrum with 100% excess bandwidth. The minimum value of M is _____.

(A) 3 (B) 8
(C) 16 (D) 4

Answer Keys

1. C	**2.** B	**3.** D	**4.** B	**5.** D	**6.** A	**7.** C	**8.** C	**9.** D	**10.** B
11. A	**12.** C	**13.** D	**14.** B	**15.** D	**16.** C	**17.** C	**18.** B	**19.** A	**20.** D
21. B	**22.** C	**23.** C	**24.** A	**25.** B					

Hints and Explanations

Solutions for questions 1 to 25:

1. The maximum quantization error is $Q_e = \dfrac{\Delta}{2}$

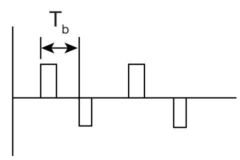
Where $\Delta = \dfrac{Dynamic\ range}{2^n\ or\ L}$

$$\Delta = \frac{16}{2^5} = \frac{16}{32} = 0.5$$

$$\therefore \quad Q_e = \frac{0.5}{2} = 0.25$$

Choice (C)

2. Since R_2 format so pulse will return to zero axis in between the bit time period so



Choice (B)

3. For an independent input and output the mutual information is zero. Choice (D)

4. $\dfrac{N_o}{2} = 10^{-11}$ W/Hz

$N_o = 2 \times 10^{-11}$ W/Hz

$$E_b = \frac{A^2}{2} \times T_b$$

$$= \frac{2\times10^{-3}\times2\times10^{-3}}{2}\times0.2\times10^{-3} = 0.4 \times 10^{-9}$$

$$P_e = \frac{1}{2}\ \varepsilon rfc\sqrt{\frac{E_b}{N_o}}$$

$$= \frac{1}{2}\ \varepsilon rfc\sqrt{\frac{0.4\times10^{-9}}{2\times10^{-11}}}$$

$$= \frac{1}{2}\ \varepsilon rfc\sqrt{\frac{4}{2}\times10}$$

$$= \frac{1}{2}\ \varepsilon rfc\sqrt{20}$$

Choice (B)

5. Choice (D)

6. Choice (A)

7. FDMA is applicable for analog signals.
TDMA is applicable for analog and digital signals.
CDMA is applicable for digital signals only.
Choice (C)

8. When entire frequency band is filled with TDMA, CDMA and FDMA and additional capacity is to be achieved then SDMA (Space Division Multiple Access) is used with FDMA, TDMA or CDMA. Choice (C)

9. Choice (D)

10. **FDMA:** Each earth station is assigned a specific uplink and downlink carrier frequency. So no need of timing synchronization.
TDMA: In this method uplink station is assigned a non-overlapping time slot. So timing synchronization is needed.
CDMA: In CDMA, entire bandwidth is used by all the time by every user. So no need of timing synchronization. Choice (B)

11. $P[X = 0] = \dfrac{4}{5}$

$P[X = 1] = 1 - P[X = 0] = \dfrac{1}{5}$

Transition probability $= P(1/0) = P(0/1) = \dfrac{1}{10}$

So probability of error for optimum receiver will be

$Pe = P(0)\ P(1/0) + P(1).P(0/1)$

$$= \frac{4}{5}\times\frac{1}{10}+\frac{1}{5}\times\frac{1}{10} = \frac{4}{50}+\frac{1}{50}$$

$$= \frac{5}{50} = \frac{1}{10}$$

Choice (A)

12. Bit rate $= H.R_b$

$$H = 0.125\log_2\frac{1}{0.125}+0.125\log_2\frac{1}{0.125}+$$

$$0.25\log_2\frac{1}{0.25}+0.5\log_2\frac{1}{0.5}$$

$= 0.125 \log_2 2^3 + 0.12 \log_2 2^3 + 0.25 \log_2 2^4 + 0.5 \log_2 2$
$= 0.375 + 0.375 + 0.5 + 0.5 = 1.750$
Bit rate $= 1.75 \times 4000 = 7000$ bits/sec Choice (C)

13. As we know that

$$C_1 = B\log_2\left(1+\frac{S}{N}\right)$$

$$\because \frac{S}{N} \gg 2$$

So $C_1 = B\log_2\left(\dfrac{S}{N}\right)$

and $C_2 = B\log_2\left(\frac{4S}{N}\right)$

$= B\log_2\left(\frac{S}{N}\right) + 2B\log_2 2 = C_1 + 2B$ Choice (D)

14. If $P = 1$, i.e., if 1 toss is required to get first tail, then probability to get first tail is $P_1 = \frac{1}{2}$

If $P = 2$, i.e., two tosses are required to get first tail.

The probability to get the first tail is $P_2 = \frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

If $P = 3$, i.e., if 3 tosses are required to get first tail.

$P_3 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

Entropy $H = \sum_{i=1}^{H} P_i \log = \frac{1}{P_i}$

$$=\frac{1}{2}\log_2\frac{1}{1/2}+\frac{1}{4}\log_2\frac{1}{1/4}+\frac{1}{8}\log_2\frac{1}{1/8}+\frac{1}{16}\log_2\frac{1}{1/16}....$$

$$=\frac{1}{2}\times1+\frac{1}{4}\times2+\frac{1}{8}\times3+\frac{1}{16}\times4...$$

$$=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}.....$$

It is the A.G.P and of the A.G.P is like this

$ab + (a + d)\,br + (a + 2d)br^2....$

Then sum $= \frac{a.b}{1-r}+\frac{dbr}{(1-r)^2}$

here $a = 1$ $d = 1$

$b = \frac{1}{2}$ $r = ½$

$$H=\frac{1.\frac{1}{2}}{\left(1-\frac{1}{2}\right)}+\frac{1.\frac{1}{2}.\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}=1+1=2$$ Choice (B)

15. Entropy $H = -\sum_{i=1}^{n} P_i\log_2(P_i)$

$$H=-\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{4}\log_2\left(\frac{1}{4}\right)+\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right]$$

$$=+\left[\frac{1}{2}+\frac{1}{4}\times2+\frac{1}{4}\times2\right]=+\frac{3}{2}$$

Code efficiency $\eta = \frac{H}{L}\times100\% = \frac{3/2}{4}=\frac{3}{8}$ Choice (D)

16. chip frequency $f_c = 10^8$ Hz

Signal frequency $f_b = 10^5$ Hz

Processing gain $G = \frac{f_e}{f_b} = \frac{10^8}{10^5}=1000$

Choice (C)

17. The average information H is

$$H = P_1\log_2\frac{1}{P_1} + P_2\log_2\frac{1}{P_2} + P_3\log_2\frac{1}{P_3} + P_4\log_2\frac{1}{P_4}$$

$$=\frac{1}{4}\log_2 4+\frac{3}{4}\log_2\frac{4}{3}+\frac{3}{4}\log_2\frac{4}{3}+\frac{1}{4}\log_2 4$$

$$=\frac{2}{4}+\frac{2}{4}+\frac{3}{4}\times2+\frac{3}{4}\times2-\frac{3}{4}\times1.54-\frac{3}{4}\times1.54$$

$$=\frac{1}{2}+\frac{1}{2}+\frac{3}{2}+\frac{3}{2}-1.15-1.15$$

= 1.7 bits/message Choice (C)

18. $B = 8$ kHz

$S = 0.1 \times 10^{-3}\ W$

$N = \eta B = 2\ (10^{-12})\ 8000$

$N = 8(10^{-9})$

$$\frac{S}{N}=\frac{0.1(10^{-3})}{8(10^{-9})} = 1.25(10^4)$$

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

$= 8000\log_2[1 + 1.25(10^4)] = 108.88$ b/s Choice (B)

19. For the given $S(t)$, the impulse response of the matched filter is

$h(t) = S(T - t);\ h(t) = e^{-2(T-t)}$

Now output $Z(t) = S(t) \times h(t)$

or $Z(t) = \int_{-\infty}^{+\infty} e^{-2\tau(\text{Tan})}.e^{-2(T-t+\tau(\text{Tan}))}dz$

$$= e^{-2T}\int_{-\infty}^{+\infty} e^{-4\tau(\text{Tan})}.e^{2t}\,dz$$

$$=e^{-2T}\int_{-\infty}^{+\infty} e^{2t-4\tau(\text{Tan})}\,dz = \frac{e^{-2T}}{-4}\left[e^{2t-4\tau(\text{Tan})}\right]_0^t$$

$$=\frac{-e^{-2T}}{4}\left[e^{-2t}-e^{+2t}\right]=\frac{e^{-2T}}{2}\left[\frac{e^{2t}-e^{-2t}}{2}\right]$$

$$=\frac{e^{-2T}}{2}\text{Sin}h(2(t)) = \frac{e^{-2T}}{2}\text{Sin}h(2t)$$ Choice (A)

20. Input signal $S(t) = e^{-2t}\,u(t)$

Input PSD; $S_{xx}(\omega) = \frac{1}{\omega^2+4}$

Given $\frac{1}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}|S_{xx}(\omega)|\,d\omega\right]=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|S_{yy}(\omega)|\,d\omega$

Output PSD $S_{yy}(\omega) = S_{xx}(\omega) \times |H(\omega)|^2$

$$S_{yy}(\omega)=\frac{4b^2}{\omega^2+4b^2}\times\frac{1}{\omega^2+4}$$

$$\Rightarrow\frac{1}{2}\int_{-\infty}^{+\infty}\left(\frac{1}{\omega^2+4}\right)d\omega=\int_{-\infty}^{+\infty}\frac{4b^2}{\omega^2+4b^2}\times\frac{1}{\omega^2+4}d\omega$$

$$\frac{1}{2}\times\frac{1}{2}\left[\tan^{-1}\frac{\omega}{2}\right]_{-\infty}^{+\infty}=\frac{4b^2}{4b^2-2}\int_{-\infty}^{+\infty}\frac{1}{\omega^2+4}-\frac{1}{\omega^2+4b^2}d\omega$$

$$=\frac{1}{4}(\pi)=\frac{4b^2}{4b^2-2}\left[\left[\frac{1}{2}\tan^{-1}\left(\frac{\omega}{2}\right)\right]_{-\infty}^{+\infty}-\left(\frac{1}{2b}\tan^{-1}\left(\frac{\omega}{2b}\right)\right)\right]_{-\infty}^{+\infty}$$

$$\Rightarrow\frac{\pi}{4}=\frac{4b^2}{4\mathbf{b}^2-2}\left[\frac{1}{2}\pi-\frac{1}{2b}\pi\right]$$

$$\Rightarrow\frac{\pi}{4}=\frac{4b^2}{4b^2-2}\pi\left[\frac{1}{2}-\frac{1}{2b}\right]$$

$$1=\frac{4b^2}{2b^2-1}\left[\frac{b-1}{b}\right];\ 1=\frac{4b}{2b^2-1}(b-1)$$

$2b^2 - 1 = 4b^2 - 4b$

$2b^2 - 4b + 1 = 0$

$b = 1\pm\sqrt{\frac{1}{2}}$ Choice (D)

21. $P_e=\frac{1}{2}\ \varepsilon rfc\left[\frac{E_b}{N_o}\right]$ for $QPSK$

$R_b = 4000 \times 10^3$ bps

$\frac{N_o}{2}\ 0.5 \times 10^{-12}$ W/Hz

$\alpha = 2 \times 10^{-3}$ V

$T_b=\frac{1}{R_b} = 0.25 \times 10^{-6}$

Bit energy $E_b=\frac{\alpha^2}{2}.T_b$

$=\frac{2\times2\times10^{-6}}{2}\times0.25\times10^{-6} = 0.5 \times 10^{-12}$

So $P_e=\frac{1}{2}\ \varepsilon rfc\left[\sqrt{\frac{0.5\times10^{-12}}{1.0\times10^{-12}}}\right]$

$=\frac{1}{2}\ \varepsilon rfc\left[\sqrt{\frac{1}{2}}\right]$

$=\frac{1}{2}\ \varepsilon rfc\left[\frac{1}{\sqrt{2}}\right]$

$P_e = Q(1)\left[\because\frac{1}{2}\varepsilon rfc\left[\frac{X}{\sqrt{2}}=Q(X)\right]\right]$

Choice (B)

22. Let the output of channel 1 is V Cos ωt and the output of channel 2 is $-V$ Cos ωt

Energy of channel 1 is $\frac{V^2}{2}.T_b = Eb$

Energy of channel 2 is $\frac{V^2}{2}T_b = Eb$

So energies are additive $2E$

BER at output of demodulator is $Q\left(\sqrt{2\alpha}\right)$

Choice (C)

23. The joint probability matrix for the channel is

$$P[X, Y] = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.25 & 0.25 \\ 0.2 & 0.2 \\ 0.05 & 0.05 \end{bmatrix} \end{array}$$

$P(x_1) = 0.25 + 0.25 = 0.5$

$P(x_2) = 0.2 + 0.2 = 0.4$

$P(x_3) = 0.05 + 0.05 = 0.1$

$P(y_1) = P(y_2) = 0.25 + 0.2 + 0.05 = 0.5$

So $H(x)=-\sum_{j=1}^{3}p(x_j)\log p(x_j)$

$= -[0.5 \log_2 0.5 + 0.4\log_2 0.4 + 0.1 \log_2 0.1]$

$= 0.5 + 0.528 + 0.332$

$= 1.36$ bits/message

$H(Y)=-\sum_{k=1}^{2}p(y_k)\log p(y_k)$

$= -[0.5 \log_2 0.5 + 0.5 \log_2 0.5]$

$= 1$ bit/messge

$H(X, Y)=\sum_{j=1}^{3}\sum_{k=1}^{2}-p(xj, yk)\log p(xj, yk)$

$= -[0.25 \log_2 0.25 + 0.25 \log_2 0.25 + 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.05 \log 0.05 + 0.05 \log_2 0.05]$

$= 0.5 + 0.5 + 0.464 + 0.464 + 0.216 + 0.216$

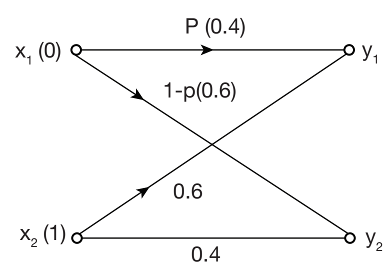
$= 2.36$ bits/message

So mutual information

$I(X; Y) = H(X) + H(Y) - H(X, Y)$

$= 1.36 + 1 - 2.36 = 0$ Choice (C)

24. A BSC can be shown as



Channel capacity $C = \log n - A$

$= 2 - H(Y/xj)\ (n = 2)$

$= 2 - [(p\log p + (1 - p)\log(1 - p))]$

$= 2 + p \log p + (1 - p)\log (1 - p)]$

$= 2 + 0.4 \log 0.4 + 0.6 \log 0.6$

$= 0.029$ bit/message Choice (A)

25. $B=\frac{R_b}{\log_2 M}(1+\alpha)$

$200=\frac{300\times2}{\log_2 M}$

$\log_2 M=\frac{300\times2}{200} = 3$

$M = 8$ Choice (B)