JEE Advanced 2024
Sample Paper - 5

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

Mathematics (MRQ)

1. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x - 2)^2$ is/are **[4]**

   a) y = 0 b) y = - 4 (x -1)

   c) y = 4(x - 1) d) y = - 30x - 50

2. The function f(x) = 1 + | sin x | is **[4]**

   a) continuous nowhere b) continuous everywhere

   c) not differentiable at x = 0 d) not differentiable at infinite number of points.

3. Let f(x) = $7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is(are) **[4]**

   a) $\int\limits_0^{\pi/4} f(x)dx = 1$ b) $\int\limits_0^{\pi/4} f(x)dx = 0$

   c) $\int\limits_0^{\pi/4} xf(x)dx = \frac{1}{12}$ d) $\int\limits_0^{\pi/4} xf(x)dx = \frac{1}{6}$

Mathematics (MCQ)

4. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, where $\binom{p}{q}$ = 0 if p > q, is maximum when m is equal to **[3]**

a) 15
b) 10
c) 5
d) 20

5. If f(x) = x^2 + 2bx + $2c^2$ and g(x) = - x^2 - 2cx + b^2, such that min f(x) > max g(x), then the relation between b and and c, is **[3]**

a) $|c| < |b|\sqrt{2}$
b) $|c| > |b|\sqrt{2}$
c) $0 < c < b\sqrt{2}$
d) No real value of b and c

6. Let P = (-1, 0), Q = (0, 0) and R = (3, $3\sqrt{3}$) be three points. Then the equation of the bisector of the angle PQR is: **[3]**

a) x + $\sqrt{3}$y = 0
b) $\frac{\sqrt{3}}{2}$x + y = 0
c) $\sqrt{3}$x + y = 0
d) x + $\frac{\sqrt{3}}{2}y$ = 0

7. Suppose f (x) = $(x + 1)^2$ for x $\geq$ -1. If g (x) is the function whose graph is reflection of the graph of f (x) with respect to the line y = x, then g (x) equals **[3]**

a) $\sqrt{x} - 1$, x $\geq$ 0
b) $\frac{1}{(x+1)^2}$, x > -1
c) $-\sqrt{x} - 1, x \geq 0$
d) $\sqrt{x+1}$, x $\geq$ -1

Mathematics (NUM)

8. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set X ∪ Y is _______. **[4]**

9. Consider a circle C_1 : x^2 + y^2 - 4x - 2y = α - 5. Let its mirror image in the line y = 2x + 1 be another circle C_2 : $5x^2$ + $5y^2$ - 10fx - 10gy + 36 = 0. Let r be the radius of C_2. Then α + r is equal to _______. **[4]**

10. In a triangle ABC, let A B = $\sqrt{23}$, BC = 3 and CA = 4. Then the value of $\frac{\cot A+\cot C}{\cot B}$ is **[4]**

11. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is: **[4]**

12. Let $f_1:(0,\infty)\rightarrow\mathbb{R}$ and $f_2:(0,\infty)\rightarrow\mathbb{R}$ be defined by $f_1(x)=\int_0^x\prod_{j=1}^{21}(t-j)^j dt, x>0$ and $f_2(x)$ = 98 $(x - 1)^{50}$ - 600$(x - 1)^{49}$ + 2450, x > 0, where for any positive integer n and real **[4]**

numbers $a_1, a_2, \ldots a_n$, $\Pi_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots a_n$, Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, i = 1, 2, in the interval $(0, \infty)$

The value of $2m_1 + 3n_1 + m_1n_1$ is ________.

13. Let ABC and ABC' be two non-congruent triangles with sides AB = 4, AC = AC' = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is **[4]**

Mathematics (MATCH)

14. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; k = 1, 2, ..., 9. **[3]**

List-I	List-II
(P) For each z_k there exists as z_j such that $z_k.z_j = 1$	(1) True
(Q) There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1.z = z_k$ has no solution z in the set of complex numbers	(2) False
(R) $\frac{\|1-z_1\|\|1-z_2\|\ldots\|1-z_9\|}{10}$ equal	(3) 1
(S) $1 - \sum\limits_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equal	(4) 2

a) (P) - (1), (Q) - (2), (R) - (3), (S) - (4)

b) (P) - (2), (Q) - (1), (R) - (3), (S) - (4)

c) (P) - (2), (Q) - (1), (R) - (4), (S) - (3)

d) (P) - (1), (Q) - (2), (R) - (4), (S) - (3)

15. Consider the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$. **[3]**

Let $H(\alpha, 0)$, $0 < \alpha < 2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G Suppose the straight line joining F and the origin makes an angle with the positive x-axis.

List - I	List - II
(I) If $\phi = \frac{\pi}{4}$, then the area of the triangle FGH is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q) 1
(III) If $\phi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV) If $\phi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

a) (I) $\rightarrow$ (R); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)

b) (I) $\rightarrow$ (R); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

d) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)

16. Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations **[3]**

x + y + z = 1

10x + 100y + 1000z = 0

qr x + pr y + pq z = 0.

List-I	List-II
(I) If $\frac{q}{r}$ = 10, then the system of linear equations has	(P) x = 0, y = $\frac{10}{9}$, z = $-\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq$ 100, then the system of linear equations has	(Q) x = $\frac{10}{9}$, y = $-\frac{1}{9}$, z = 0 solution
(III) If $\frac{p}{q} \neq$ 10, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q}$ = 10, then the system of linear equations has	(S) no solution
	(T) at least one solution

a) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

b) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

c) (I) $\rightarrow$ (T); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)

d) (I) $\rightarrow$ (T); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (T)

17. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes P_1 : 7x + y + 2z = 3, P_2 = 3x + 5y - 6 z = 4. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2. **[3]**

Match List I with List II and select the correct answer using the code given below the lists :

List I	List II
P. $a =$	1. 13
Q. $b =$	2. -3
R. $c =$	3. 1
S. $d =$	4. -2

a) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (4)

b) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (1), (S) $\rightarrow$ (3)

c) (P) $\rightarrow$ (1), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (4), (S) $\rightarrow$ (2)

d) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)

Physics (MRQ)

18. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod **[4]**

makes an angle 60° with vertical?
[g is the acceleration due to gravity]

a) The angular acceleration of the rod will be $\frac{2g}{L}$

b) The radical acceleration of the rod's center of mass will be $\frac{3g}{4}$

c) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

d) The normal reaction force from the floor on the rod $\frac{Mg}{16}$

19. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $\frac{v}{10}$, where v is the speed of propagation of the wave. If a = 10^{-3}m and v = 10 ms^{-1}, then λ and f are given by **[4]**

a) f = 10^4Hz

b) f = $\frac{10^3 Hz}{2\pi}$

c) $\lambda = 2\pi \times 10^{-2}$ m

d) λ = 10^{-3}m

20. H^+, He^+ and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 au, 4 amu and 16 amu respectively. Then **[4]**

a) H^+ will be deflected most

b) O^{2+} will be deflected most

c) He^+ and O^{2+} will be deflected equally

d) all will be deflected equally

Physics (MCQ)

21. Dimensions of electrical resistance is: **[3]**

a) $[ML^{-1}t^3A^2]$

b) $[ML^3T^{-3}A^{-2}]$

c) $[ML^2T^{-3} A^{-1}]$

d) $[ML^2 T^{-3} A^{-2}]$

22. Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m respectively. The ratio of the time taken by them to reach the ground is: **[3]**

a) $\frac{5}{4}$

b) $\frac{12}{5}$

c) $\frac{5}{12}$

d) $\frac{4}{5}$

23. If the orbital velocity of the moon is increased by 41.4% of its present value, then the: **[3]**

a) moon will become a stationary satellite

b) moon will leave its orbit and escape into space

c) moon will orbit around the earth with double velocity

d) radius of moon's orbit will become double

24. An electron initially at rest falls a distance of 1.5 cm in a uniform electric field of magnitude 2×10^4 N/C. The time taken by the electron to fall this distance is: **[3]**

a) 1.3×10^2 s

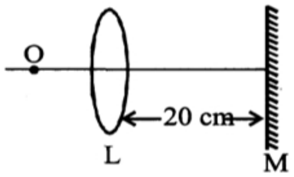
b) 2.1×10^{-12} s

c) 1.6×10^{-10} s

d) 2.9×10^{-9} s

Physics (NUM)

25. An object is placed on the principal axis of convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is _______ cm. **[4]**

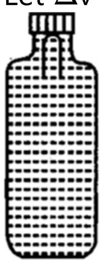


26. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is: **[4]**

27. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})$ kg ms^{-2} with k = 1 kg s^{-2}. At time t = 0, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right) m$ and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right) ms^{-1}$. Let v_x and v_y denote the x and y components of the particle's velocity, respectively. **Ignore gravity**. When z = 0.5 m, the value of $(xv_y - yv_x)$ is _______ m^2x^{-1}. **[4]**

28. A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is v_0 = 3.3 cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $P_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$. **[4]**

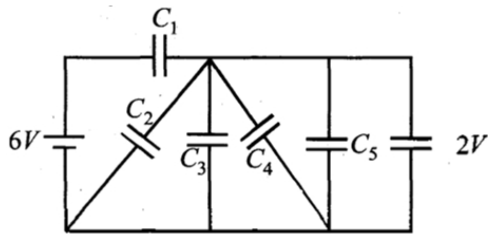
Let Δv = X cc and $\Delta p = Y \times 10^3$ Pa.



The value of Y is _______.

29. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _______. **[4]**

30. In the following circuit C_1 = 12 μF, C_2 = C_3 = 4 μF and C_4 = C_5 = 2 μF. The charge stored in C_3 is ________ μC. **[4]**



Physics (MATCH)

31. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process. **[3]**

List-I	List-II
(I) 10^{-3} kg of water at 100°C is converted to steam at the same temperature, at a pressure of 10^5 Pa. The volume of the system changes from 10^{-6} m^3 to 10^{-3} m^3 in the process. Latent heat of water = 2250 kJ/kg.	(P) 2 kJ
(II) 0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume R = 8.0 $Jmol^{-1}$ K^{-1}.	(Q) 7 kJ
(III) One mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}m^3$ and pressure 2 kPa to volume $\frac{V}{8}$.	(R) 4 kJ
(IV) Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion.	(S) 5 kJ
	(T) 3 kJ

Which one of the following options is correct?

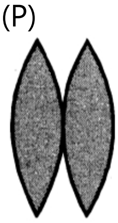
a) (I) $\rightarrow$ (P); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (Q)

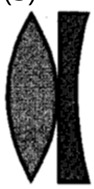
b) (I) $\rightarrow$ (S); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (P)

c) (I) $\rightarrow$ (T); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (Q)

d) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)

32. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. **[3]**

List-I	List-II
(P) 	(1) 2r

(Q) 	(2) $\frac{r}{2}$
(R) 	(3) -r
(S) 	(4) r

a) P - 4, Q - 1, R - 2, S - 3

b) P - 2, Q - 4, R - 3, S - 1

c) P - 2, Q - 1, R - 3, S - 4

d) P - 1, Q - 2, R - 3, S - 4

33. List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option. **[3]**

List - I	List - II
(P) ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{91}\mathrm{~Pa}$	(1) one a particle and one β^{+} particle
(Q) ${}^{214}_{82}\mathrm{~Pb} \rightarrow {}^{210}_{82}\mathrm{~Pb}$	(2) three β^{-} particles and one a particle
(R) ${}^{210}_{81}\mathrm{Tl} \rightarrow {}^{206}_{.82}\mathrm{~Pb}$	(3) two β^{-} particles and one a particle
(S) ${}^{228}_{91}\mathrm{~Pa} \rightarrow {}^{224}_{88}\mathrm{Ra}$	(4) one a particle and one β^{-} particle
	(5) one a particle and two β^{+} particles

a) $\mathrm{P} \rightarrow 5, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 3,\ \mathrm{S} \rightarrow 2$

b) $\mathrm{P} \rightarrow 5, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 1,\ \mathrm{S} \rightarrow 4$

c) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 2,\ \mathrm{S} \rightarrow 1$

d) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 2,\ \mathrm{S} \rightarrow 5$

34. A musical instrument is made using four different metal strings 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List - I gives the above four strings while list - II lists the magnitude of some quantity. **[3]**

List-I	List-II
(I) String - 1 (μ)	(P) 1
(II) String - 2 (2μ)	(Q) $\frac{1}{2}$

List-I	List-II
(III) String - 3 (3μ)	(R) $\frac{1}{\sqrt{2}}$
(IV) String - 4 (4μ)	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) $\frac{1}{16}$

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

a) (I) → (P), (II)→ (Q), (III)→ (T), (IV)→ (S)

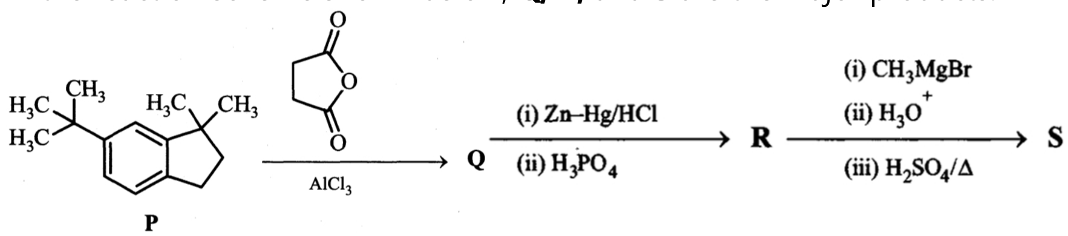
b) (I) → (Q), (II)→ (P), (III)→ (R), (IV)→ (T)

c) (I) → (Q), (II)→ (S), (III)→ (R), (IV)→ (P)

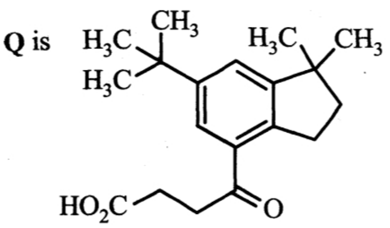
d) (I) → (P), (II)→ (R), (III)→ (S), (IV)→ (Q)

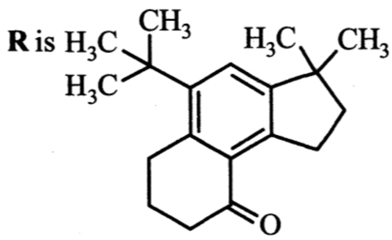
Chemistry (MRQ)

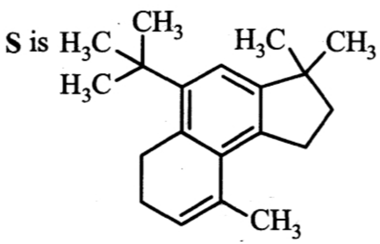
35. In the reaction scheme shown below, **Q, R,** and **S** are the major products. **[4]**

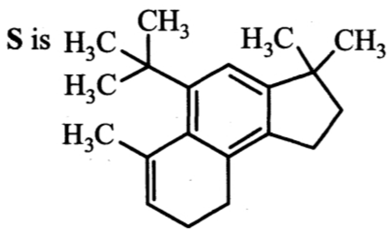


The correct structure of

a) 

b) 

c) 

d) 

36. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are) **[4]**

a) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

b) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used

c) The activation energy of the reaction is unaffected by the value of the steric factor

d) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

37. In thermodynamics, the P - V work done is given by $w = -\int dV P_{\text{ext}}$. **[4]**

For a system undergoing a particular process, the work done is, $w = -\int dV \left(\frac{RT}{V-b} - \frac{a}{V^2}\right)$

This equation is applicable to a

a) Process that is reversible and isothermal.

b) Process that is reversible and adiabatic.

c) Process that is irreversible and at constant pressure.

d) System that satisfies the van der Waals equation of state.

Chemistry (MCQ)

38. The octet rule is not valid for the molecule **[3]**

a) H_2O

b) O_2

c) CO

d) CO_2

39. When two reactants, A and B are mixed to give products, C and D, the reaction quotient, (Q) at the initial stages of the reaction: **[3]**

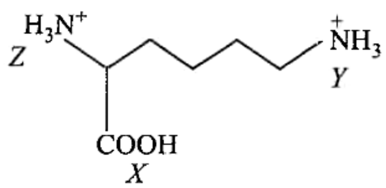
a) is zero

b) is independent of time

c) decreases with time

d) increases with time

40. **[3]**



Arrange in order of increasing acidic strength

a) X > Y > Z

b) Z < X > Y

c) X > Z > Y

d) Z > X > Y

41. The compound which reacts fastest with Lucas reagent at room temperature is **[3]**

a) 2-methyl propan-2-ol

b) 2-methyl propan-1-ol

c) butan-1-ol

d) butan-2-ol

Chemistry (NUM)

42. While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of 2 M H_2SO_4. The percentage of nitrogen present in organic compound is _______. **[4]**

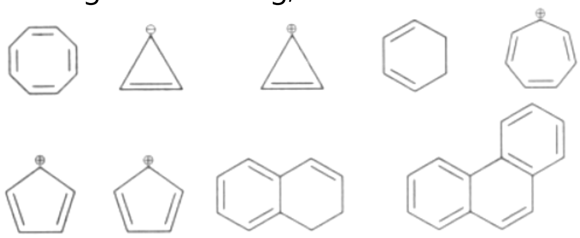
43. 0.5 g of fuming H_2SO_4 (Oleum) is diluted with water. This solution is completely neutralized by 26.7 mL of 0.4 N NaOH. Find the percentage of free SO_3 in the sample of oleum. **[4]**

44. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) of energy equivalent to 1560 kJ. He utilizes 50 percent of this gained energy in the event. In order to avoid the storage of energy in the body, calculate the weight of water he would need to perspire. The enthalpy of evaporation of water is 44 kJ/mole. **[4]**

45. Calculate the wave number for the shortest wavelength transition in the Balmer series of atomic hydrogen. **[4]**

46. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}\mathrm{U} \rightarrow {}^{214}_{82}\mathrm{Pb}$ is **[4]**

47. Among the following, the number of aromatic compound(s) is **[4]**



Chemistry (MATCH)

48. Match List I with List II: **[3]**

List I Test	List II Functional group/Class of Compound
(A) Molisch's Test	(I) Peptide
(B) Biuret Test	(II) Carbohydrate
(C) CarbylamineTest	(III) Primary amine
(D) Schiff's Test	(IV) Aldehyde

a) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)
b) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
c) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
d) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

49. LIST-I contains metal species and LIST-II contains their properties. **[3]**

LIST - I	LIST - II
(I) $[\mathrm{Cr(CN)_6}]^{4-}$	(P) t_{2g} orbitals contain 4 electrons
(II) $[\mathrm{RuCl_6}]^{2-}$	(Q) μ (spin- only) = 4.9 BM
(III) $[\mathrm{Cr(H_2O)_6}]^{2+}$	(R) low spin complex ion
(IV) $[\mathrm{Fe(H_2O)_6}]^{2+}$	(S) metal ion in 4+ oxidation state
	(T) d^4 species

[Given: Atomic number of Cr = 24,Ru = 44, Fe = 26]

a) I → R, S; II → P, T; III → P, Q; IV → Q, T

b) I → R, T; II → P, S; III → Q, T; IV → P, Q

c) I → P, R; II → R, S; III → R, T; IV → P, T

d) I → Q, T; II → S, T; III → P, T; IV → Q, R

50. The standard reduction potential data at 25^{o}C is given below: **[3]**

$E^o(Fe^{3+}, Fe^{2+})$ = + 0.77 V; $E^o(Fe^{2+}, Fe)$ = - 0.44 V; $E^o(Cu^{2+}, Cu)$ = + 0.34 V; $E^o(Cu^{+}, Cu)$ = + 0.52 V

$E^o[O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O]$ = +1.23 V; $E^o[O_2(g) + 2H_2O + 4e^- \rightarrow 4OH^-]$ = + 0.40 V

$E^o(Cr^{3+}, Cr)$ = -0.74 V; $E^o(Cr^{2+}, Cr)$ = -0.91 V

Match E^o of the redox pair in List I with the values given in List II and select the correct answer using the code given below the lists:

List I	List II
(P) $E^o(Fe^{3+}, Fe)$	(1) - 0.18 V
(Q) $E^o(4H_2O \rightleftharpoons 4H^+ + 4OH^-)$	(2) -0.8 V
(R) $E^o(Cu^{2+} + Cu \rightarrow 2Cu^+)$	(3) -0.04 V
(S) $E^o(Cr^{3+}, Cr^{2+})$	(4) -0.83 V

a) (P) - (3), (Q) - (4), (R) - (1), (S) - (2)

b) (P) - (2), (Q) - (3), (R) - (4), (S) - (1)

c) (P) - (1), (Q) - (2), (R) - (3), (S) - (4)

d) (P) - (4), (Q) - (1), (R) - (2), (S) - (3)

51. Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option. **[3]**

List- I	List- II
(P) $P_2O_3 + 3H_2O \rightarrow$	(1) $P(O)(OCH_3)Cl_2$
(Q) $P_4 + 3NaOH + 3H_2O \rightarrow$	(2) H_3PO_3
(R) $PCl_5 + CH_3COOH \rightarrow$	(3) PH_3
(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$	(4) $POCl_3$
	(5) H_3PO_4

a) P → 2; Q → 3; R → 1; S → 5

b) P → 2; Q → 3; R → 4; S → 5

c) P → 3; Q → 5; R → 4; S → 2

d) P → 5; Q → 2; R → 1; S → 3

JEE Advanced 2024

Sample Paper - 5

Solution

Mathematics (MRQ)

1. **(a)** y = 0

   (c) y = 4(x - 1)

   Explanation: If y - mx + c is tangent to y = x^2 then x^2 - mx - c = 0 has equal roots

   $\Rightarrow$ m^2 + 4c = 0 $\Rightarrow$ c = $-\frac{m^2}{4}$

   $\therefore$ y = mx $-\frac{m^2}{4}$ is tangent to y = x^2

   $\therefore$ This is also tangent to y = - $(x - 2)^2$

   $\Rightarrow$ mx $-\frac{m^2}{4}$ = -x^2 + 4x - 4

   $\Rightarrow$ x^2 + (m - 4)x + $\left(4-\frac{m^2}{4}\right)$= 0 has equal roots

   $\therefore$ m^2 - 8m + 16 = -m^2 + 16 $\Rightarrow$ m = 0, 4

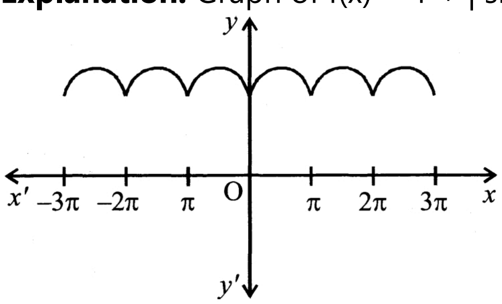
   $\therefore$ y = 0 or y = 4x - 4 are the tangents.

2. **(b)** continuous everywhere

   (c) not differentiable at x = 0

   (d) not differentiable at infinite number of points.

   Explanation: Graph of f(x) = 1 + | sin x | is as follows :



   From graph it is clear that function is continuous everywhere but not differentiable at integral multiples of π because at these points curve has sharp turnings.

3. **(b)** $\int\limits_0^{\pi/4} f(x)dx = 0$

   (c) $\int\limits_0^{\pi/4} xf(x)dx = \frac{1}{12}$

   Explanation: f(x) = $7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$

   $= \left(7\tan^4 x - 3\right)\left(\tan^4 x + \tan^2 x\right)$

   $= \left(7\tan^6 x - 3\tan^2 x\right)\sec^2 x$

   $\int_0^{\pi/4} f(x)dx = \left[\tan^7 x - \tan^3 x\right]_0^{\pi/4}$ = 1 - 1 = 0

   $\therefore \int_0^{\pi/4} xf(x)dx = \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4}$

   $- \int_0^{\pi/4}\left(\tan^7 x - \tan^3 x\right)dx$

   $= \int_0^{\pi/4}\tan^3 x\left(1 - \tan^2 x\right)\sec^2 x dx$

   $= \left[\frac{\tan^4 x}{4} - \frac{\tan^6 x}{6}\right]_0^{\pi/4} = \frac{1}{12}$

Mathematics (MCQ)

4. **(a)** 15

Explanation: $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ is the coefficient of x^m in the expansion of $(1 + x)^{10}$ (x + 1)^{20},

$\Rightarrow \sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ is the coefficient of x^m in the expansion of $(1 + x)^{30}$

i.e. $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i} = {}^{30}C_m = \binom{30}{m}$ ...(i)

and we know that, $\binom{n}{r}$is maximum, when $\binom{n}{r}_{\max} = \begin{cases} r = \frac{n}{2}, & \text{if } n \in \text{ even} \\ r = \frac{n\pm1}{2}, & \text{if } n \in \text{ odd} \end{cases}$

Hence, $\binom{30}{m}$ is maximum when m = 15

5.

(b) $|c| > |b|\sqrt{2}$

Explanation: Given f(x) = x^2 + 2bx + $2c^2$ and g(x) = $-x^2$ - 2cx + b^2

Then, f(x) is minimum and g(x) is maximum at $\left(x = \frac{-b}{4a} \text{ and } f(x) = \frac{-D}{4a}\right)$, respectively.

$\therefore \quad \min f(x) = \frac{-(4b^2-8c^2)}{4} = (2c^2 - b^2)$

$\therefore \quad \max g(x) = \frac{-(4c^2+4b^2)}{4(-1)} = (b^2 + c^2)$

and min f(x) > max g(x)

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow \quad |c| > \sqrt{2}|b|$

6.

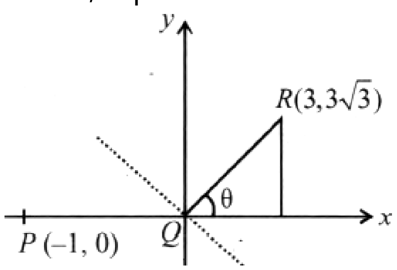
(c) $\sqrt{3}$x + y = 0

Explanation:

tan θ = $\sqrt{3} \Rightarrow \theta$ = $60^{\circ} \Rightarrow \angle PQR$ = 120°

$\therefore$ Slope of bisector of $\angle PQR$ = tan 120°

Hence, equation of bisector is $\sqrt{3}x$ + y = 0



7. **(a)** $\sqrt{x} - 1$, x $\geq$ 0

Explanation: It is only to find the inverse.

Let y = f (x) = $(x + 1)^2$, for x $\geq$ - 1

$\pm\sqrt{y} = x + 1$, x $\geq$ -1

$\Rightarrow \sqrt{y}$ = x + 1 $\Rightarrow$ y $\geq$ 0 ,x + 1 $\geq$ 0

$\Rightarrow$ x = $\sqrt{y}$ - 1

$\Rightarrow f^{-1}(y)$ = $\sqrt{y}$ - 1

$\Rightarrow f^{-1}(x) = \sqrt{x} - 1 \Rightarrow x \geq 0$

Mathematics (NUM)

8. 3748

Explanation:

The given sequences upto 2018 terms are 1, 6, 11, 16, ..., 10086 and 9, 16, 23, ..., 14128

The common terms are 16, 15, 86, ... upto n terms, where $T_n \leq 10086$

$\Rightarrow 16 + (n - 1)\, 35 \leq 10086$

$\Rightarrow 35n - 19 \leq 10086$

$\Rightarrow n \leq \frac{10105}{35} = 288.7$

$\therefore n = 288$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2018 + 2018 - 288 = 3748$

9. 2.0

Explanation:

$x^2 + y^2 - 4x - 2y + 5 - \alpha = 0$

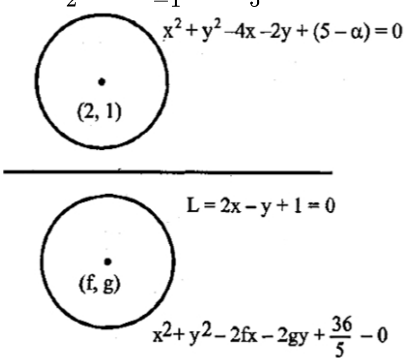
$C_1(2, 1)$ & $r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

Image of (2, 1) in given line will be

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$

$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$



$\Rightarrow x = 2 - \frac{16}{5} = \frac{-6}{5}, y = 1 + \frac{8}{5} = \frac{13}{5}$

So, $x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$, $C_2(f, g)$ &

$r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$ [radius of both circle will be same]

$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}\ \left[\because f = -\frac{6}{5},\ g = \frac{13}{5}\right]$

$= \frac{36+169-180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$

$\therefore \alpha + r + 2$

10. 2.0

Explanation:

Given that

$c = \sqrt{23}$; a = 3; b = 4

We have $\cot A = \frac{\cos A}{\sin A} = \frac{b^2+c^2-a^2}{2bc \sin A}$

$= \frac{b^2+c^2-a^2}{2.2\triangle}\ \left\{\triangle = \frac{1}{2} bc \sin A\right\}$

$\therefore \text{Cot } A = \frac{b^2+c^2-a^2}{4\triangle}$

Similarly, $\cot B = \frac{a^2+c^2-b^2}{4\triangle}$ & $\cot C = \frac{a^2+b^2-c^2}{4\triangle}$

Now $\frac{\cot A+\cot C}{\cot B} = \frac{b^2+c^2-a^2+a^2+b^2-c^2}{a^2+c^2-b^2}$

$= \frac{2b^2}{a^2+c^2-b^2} = \frac{2(16)}{9+23-16} = \frac{32}{16}$ = 2

11. 4

Explanation:

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$ = cos $\frac{\pi}{3} = \frac{1}{2}$

Given $p\vec{a} + q\vec{b} + r\vec{c} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c}$

Taking its dot product with $\vec{a}, \vec{b}, \vec{c}$, we get

$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = p|\vec{a}|^2 + q(\vec{b}\cdot\vec{a}) + r \mid \vec{c} - \vec{a}\Big)$

= p + $\frac{1}{2}$q + $\frac{1}{2}$r ...(1)

Given that $\frac{1}{2}$p + q + $\frac{1}{2}$r = 0 ...(2)

and $\frac{1}{2}$p + $\frac{1}{2}$q + r = $[\vec{a}\vec{b}\vec{c}]$ ...(3)

From (1) and (3),p = r Using (2) q = - p

$\therefore \frac{p^2+2q^2+r^2}{q^2} = \frac{p^2+2p^2+p^2}{p^2}$ = 4

12. 57.0

Explanation:

$f_1(x) = \int_0^x \prod_{j=1}^{21}(t-j)^j dt$

$f_1'(x) = \prod_{i=1}^{21}(x-j)^j = (x-1)(x-2)^2(x-3)^3\ldots(x-21)^{21}$

Checking the sign scheme of f'_1 (x) at x = 1, 2, 3,... 21

We get

f_1(x) has local minima at x = 1, 5, 9, 13, 17, 21 and local maxima at 3, 7, 11, 15, 19.

⇒ m_1 = 6, n = 5

So, $2m_1 + 3n_1 + m_1n_1$

= 2 × 6 + 3 × 5 + 6 ×5

= 57

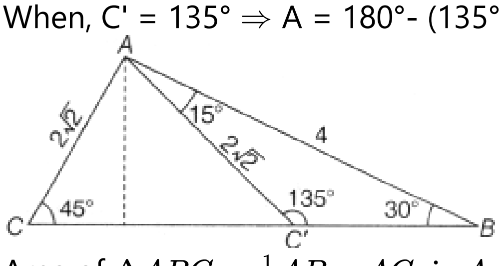
13. 4

Explanation:

In ΔABC, by sine rule, $\frac{a}{\sin A} = \frac{2\sqrt{2}}{\sin 30^\circ} = \frac{4}{\sin C}$

⇒ C = 45°, C' = 135°

When, C' = 135° ⇒ A = 180°- (135° + 30°) = 15°



Area of $\Delta ABC = \frac{1}{2}AB\times AC\sin A$

$= \frac{1}{2}\times 4\times 2\sqrt{2}\sin(105^\circ)$

$= 4\sqrt{2}\times\frac{\sqrt{3}+1}{2\sqrt{2}}$

$= 2(\sqrt{3}+1)$sq. units

Area of $\Delta ABC' = \frac{1}{2}AB\times AC\sin A$

$= \frac{1}{2}\times 4\times 2\sqrt{2}\sin(15^\circ)$

$= 2(\sqrt{3}-1)$sq. units

Difference of areas of triangle
$= |2(\sqrt{3}+1) - 2(\sqrt{3}-1)|$ = 4 sq. units

Mathematics (MATCH)

14. **(a)** (P) - (1), (Q) - (2), (R) - (3), (S) - (4)

Explanation: (P) $\rightarrow$ (1) : z_k = cos$\frac{2k\pi}{10} + i\sin\frac{2k\pi}{10}$, k = 1 to 9

$\therefore z_k = e^{i\frac{2k\pi}{10}}$

Now $z_k z_j = 1 \Rightarrow z_j = \frac{1}{z_k} = e^{-i\frac{2k\pi}{10}} = \overline{z_k}$

We know if z_k is 10^{th} root of unity so will be $\bar{z}_k$.

$\therefore$ For every z_k, there exist $z_j = \bar{z}_k$

Such that $z_k \cdot z_j = z_k \cdot \bar{z}_k$ = 1

Hence the statement is true.

(Q) $\rightarrow$ (2) $z_1 = z_k$ $\Rightarrow z = \frac{z_k}{z_1}$ for $z_1 \neq 0$

$\therefore$ We can always find a solution of $z_1.z = z_k$

Hence the statement is false.

(R) $\rightarrow$ (3): We know z^{10} -1 = (z - z_1)...(z - z_9)

$\Rightarrow$ (z - z_1) (z - z_2)...(z - z_9) $= \frac{z^{10}-1}{z-1}$

= 1 + z + z^2 + ...z^9

For z = 1, we get (1 - z_1) (1 - z_2)...(1 - z_9) = 10

$\therefore \frac{|1-z_1||1-z_2|\ldots|1-z_9|}{10}$ = 1

(S) $\rightarrow$ (4): 1, Z_1, Z_2, ..., Z_9 are 10th roots of unity.

$\therefore Z^{10}$ - 1 = 0

From equation 1 + Z_1 + Z_2 +...+ Z_9 = 0,

Re(1) + Re(Z_1) + Re(Z_2) +...+ Re(Z_9) = 0

$\Rightarrow$ Re(Z_1) + Re(Z_2) + ... Re(Z_9) = -1

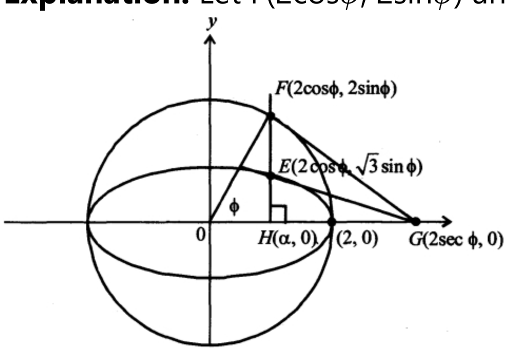
$\Rightarrow \sum_{K=1}^{9} \cos\frac{2k\pi}{10} = -1 \Rightarrow 1 - \sum_{K=1}^{9} \cos\frac{2k\pi}{10}$ = 2

Hence ((P) - (1), (Q) - (2), (R) - (3), (S) - (4)) is the correct option.

15.

(c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

Explanation: Let F(2cosϕ, 2sinϕ) and E(2cosϕ, $\sqrt{3}$ sin ϕ)



$\alpha \equiv$ cosϕ

Tangent at E(2cosϕ, $\sqrt{3}$ sinϕ) to ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

i.e. $\frac{x\cos\phi}{2} + \frac{y\sin\phi}{\sqrt{3}} = 1$ intersect x-axis at G(2secϕ, 0)

Area of triangle FGH = $\frac{1}{2}$HG $\times$ FT

= $\frac{1}{2}$(2 secϕ - 2cosϕ) 2sinϕ; $\triangle$ = 2sin$^2\phi$ · tan ϕ

$\triangle$ = (1 -cos2ϕ) · tanϕ

I. If $\phi = \frac{\pi}{4}, \triangle = 1 \rightarrow (Q)$

II. If $\phi = \frac{\pi}{3}, \triangle = 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \sqrt{3} = \frac{3\sqrt{3}}{2} \rightarrow (T)$

III. If $\phi = \frac{\pi}{6}, \triangle = 2 \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} \rightarrow (S)$

IV. If $\phi = \frac{\pi}{12}, \triangle = \left(1 - \frac{\sqrt{3}}{2}\right) \cdot (2 - \sqrt{3}) = \frac{(2-\sqrt{3})^2}{2} \rightarrow (P)$

16. **(a)** (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

Explanation: We have system of linear equations

x + y + z = 1 ...(i)

10x + 100y + 1000z = 0

x + 10y + 100z = 0 ...(ii)

qrx + pry + pqz = 0 ...(iii)

$\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r}$ = 0 ($\because$ p, q, r $\neq$ 0)

Let p = $\frac{1}{a+9d}$, q = $\frac{1}{a+99d}$, r = $\frac{1}{a+999d}$

Now, equation (iii) is

(a + 9d)x + (a + 99d)y + (a + 999d)z = 0

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 100 \\ a+9d & a+99d & a+999d \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 10 & 100 \\ 0 & a+99d & a+999d \end{vmatrix} = 900(d - a)$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 100 \\ a+9d & 0 & a+999d \end{vmatrix} = 990(a - d)$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 0 \\ a+9d & a+99d & 0 \end{vmatrix} = 90(d - a)$$

Let option I: If $\frac{q}{r}$ = 10 $\Rightarrow$ a = d

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

Since eq. (i) and eq. (ii) represents non-parallel planes and eq. (ii) and eq. (iii) represents same plane

$\Rightarrow$ Infinitely many solutions

So, option I $\rightarrow$ P, Q, R, T

Option II: $\frac{p}{r} \neq 100 \Rightarrow a \neq d$

$\Delta = 0$, $\Delta_x, \Delta_y, \Delta_z \neq 0$

No solution

So, option II $\rightarrow$ S

Option III: $\frac{p}{q} \neq 10 \Rightarrow a \neq d$

No solution

So, option III $\rightarrow$ S

Option IV: If $\frac{p}{q}$ = 10 $\Rightarrow$ a = d

Infinitely many solution
Hence, IV $\rightarrow$ P, Q , R , T

17.

(d) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)

Explanation: Let any point on L_1 is (2λ + 1, -λ, λ - 3) and that on L_2 is (μ + 4, μ - 3, 2μ - 3)

For point of intersection of L_1 and L_2

2λ + 1 = μ + 4, -λ = μ - 3, λ - 3 = 2μ - 3

$\Rightarrow \lambda = 2, \mu = 1$

$\therefore$ Intersection point of L1 and L_2 is (5, -2, -1)

Equation of plane passing through, (5, -2, -1) and perpendicular to P_1 & P_2 is given by

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow$ x - 3y - 2z = 13

$\therefore$ a = 1, b = -3, c = -2, d = 13

or (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)

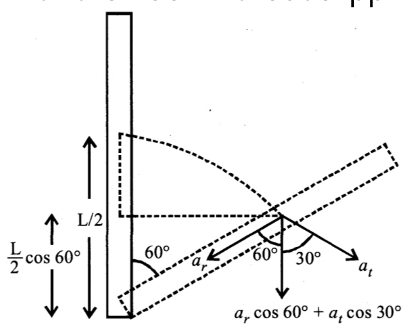
Physics (MRQ)

18. **(b)** The radical acceleration of the rod's center of mass will be $\frac{3g}{4}$

(c) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(d) The normal reaction force from the floor on the rod $\frac{Mg}{16}$

Explanation: The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping.



Gain in kinetic energy = loss in potential energy

$\frac{1}{2}I\omega^2$ = mg $\frac{l}{2}$ (1 - cos 60°)

$\therefore \frac{ml^2}{3}\omega^2$ = mg $\frac{l}{2} \Rightarrow \omega = \sqrt{\frac{3\text{ g}}{2l}}$

Now, τ = Iα

$\therefore$ mg $\times \frac{l}{2}$sin 60° = $\frac{1}{3}$m$l^2\alpha \Rightarrow \alpha = \frac{3\sqrt{3}}{4}\frac{\text{g}}{1}$

Further a_t = $\frac{l}{2}\alpha = \frac{3\sqrt{3}g}{8}$

Also a_r = $\omega^2\frac{l}{2} = \frac{3\text{ g}}{2l} \times \frac{l}{2} = \frac{3\text{ g}}{4}$

For vertical motion of centre of mass

mg - N = m($a_r$cos 60° + a_t cos 30°)

$\therefore$ mg - N = m$\left[\frac{3g}{4} \times \frac{1}{2} + \frac{3\sqrt{3}g}{8} \times \frac{\sqrt{3}}{2}\right]$

$\therefore$ M = $\frac{\text{Mg}}{16}$

19. **(b)** f = $\frac{10^3 Hz}{2\pi}$

(c) $\lambda = 2\pi \times 10^{-2}$ m

Explanation: For a transverse sinusoidal wave travelling on a string, the maximum velocity $v_{max} = a\omega$.

Given maximum velocity = $\frac{v}{10} = \frac{10}{10} = 1\ m/s$

$\therefore a\omega = 1 \Rightarrow 10^{-3} \times 2\pi v = 1\ [\because \omega = 2\pi v]$

$\Rightarrow v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} Hz$

And, $\lambda = \frac{v}{v} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\ m$

20. **(a)** H^+ will be deflected most

(c) He^+ and O^{2+} will be deflected equally

Explanation: When the charged particles enter a magnetic field then a force acts on the particle which will act as a centripetal force.

qvB = $\frac{mv^2}{r}$ $\Rightarrow$ r = $\frac{mv}{qB}$ or, r = $\frac{\sqrt{2mk}}{qB}$

$\therefore$ r $\propto \frac{\sqrt{m}}{q}$ [Kinetic energy, 'k' and 'B' are same]

$r_{H^+} \propto \frac{\sqrt{1}}{1}; r_{He^+} \propto \frac{\sqrt{4}}{1}; r_{O^{++}} \propto \frac{\sqrt{16}}{2}$

$\Rightarrow r_{H+} \propto 1; r_{He+} \propto 2; r_{O++} \propto 2$

Hence He^+ and O^{++} will be deflected equally.

H^+ will be deflected the most since its radius is smallest.

Physics (MCQ)

21.

(d) $[ML^2 T^{-3} A^{-2}]$

Explanation: According to ohm's law,

v = RA or R = $\frac{V}{A}$

Dimensions of V = $\frac{W}{q} = \frac{[ML^2T^{-2}]}{[AT]}$

$\therefore$ R = $\frac{[ML^2T^{-2}/AT]}{[A]}$ = $[ML^2T^{-3}A^{-2}]$

22.

(d) $\frac{4}{5}$

Explanation: Time taken by a body to fall a height h to reach the ground is,

$t = \sqrt{\frac{2h}{g}}$

$\therefore \frac{t_A}{t_B} = \frac{\sqrt{2h_A/g}}{\sqrt{2h_B/g}} = \sqrt{\frac{h_A}{h_B}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

23.

(b) moon will leave its orbit and escape into space

Explanation: New orbital velocity = v_O + $\frac{41.4}{100}v_o$

= $1.414v_o$ = $\sqrt{2}v_0$ = v_e

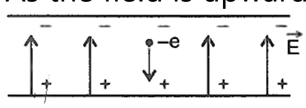
i.e., the moon will leave its orbit and escape into space.

24.

(d) 2.9×10^{-9} s

Explanation:

As the field is upward,



so, the negatively charged electron experiences a downward force of magnitude eE, where E is the magnitude of the electric field. The acceleration of the electron is,

$a_e = \frac{eE}{m_e}$

where m_e is the mass of the electron.

Starting from rest, the time taken by the electron to fall through a distance h is given by:

$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$

$= \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^4}}$

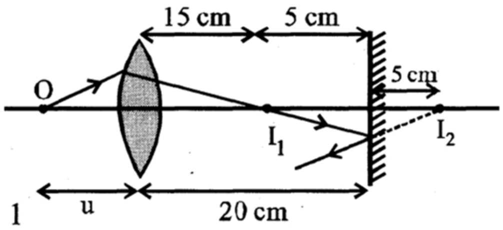
$= 2.9 \times 10^{-9}$ s

Physics (NUM)

25. 30.0

Explanation:

How I_1 is image formed by lens and I_2 is image formed by mirror.



f = 10 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{15} - \frac{1}{u} = \frac{1}{10}$

$\Rightarrow \frac{1}{u} = \frac{1}{10} - \frac{1}{15} \Rightarrow$ u = 30 cm

26. 2

Explanation:

From (i) Stefan-Boltzmann law, P = σAT^4 and (ii) Wein's displacement law = $\lambda_m \times$ T = constant

$\frac{P_A}{P_B} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$

$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$

$\therefore \frac{\lambda_A}{\lambda_B} = 2$

27. 3

Explanation:

Here, $\vec{F} = -K\vec{r}$. So force passes through origin.

$\tau_{origin} = 0 \Rightarrow$ angular momentum about origin will be conserved

So, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$

$\Rightarrow \hat{k}\left[\frac{1}{\sqrt{2}} \times \sqrt{2} - (-\sqrt{2}) \times \sqrt{2}\right] = \hat{k}\left(xv_y - yv_x\right)$

$\Rightarrow xv_y - yv_x = 3$

28. 10.0

Explanation:

Isothermal process for air, temperature is constant.

$\therefore$ From $P_1V_1 = P_2V_2$

$10^5 \times (3.3) = P_2(3) \Rightarrow P_2 = 1.1 \times 10^5$

$\Delta P = P_2 - P_1 = 1.1 \times 10^5 - 10^5 = 0.1 \times 10^5$

or, $\Delta P = 10 \times 10^3$ Pascal = $Y \times 10^3$ Pascal

$\therefore Y = 10$

29. 27.0

Explanation:

We have,

$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So, $\frac{1}{\lambda_1} = R_H\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = R_H\left(\frac{8}{9}\right) \Rightarrow \lambda_1 = \frac{9}{8R_H}$

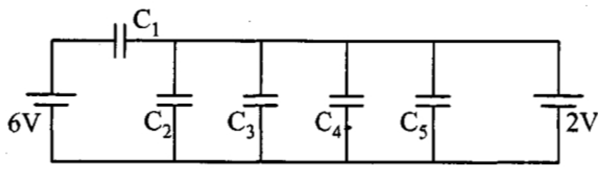
$\frac{1}{\lambda_2} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R_H\left(\frac{3}{4}\right) \Rightarrow \lambda_2 = \frac{4}{3R_H}$

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{\frac{9}{8R_H}}{\frac{4}{3R_H}} = \frac{27}{32}$

30. 8.0

Explanation:

The circuit can be redrawn as



So, charge stored in C_3 in given as

$Q_3 = C_3 \times 2V = 4\mu F \times 2V = 8\mu C$

Physics (MATCH)

31. **(a)** (I) → (P); (II) → (R); (III) → (T); (IV) → (Q)

Explanation:

I. By first law of thermodynamics.

$\Delta U = \Delta Q - \Delta W$

$= ML_V - P\Delta V$

$= 10^{-3} \times 2250 \times 10^3 - 10^5 \times \left(10^{-3} - 10^{-6}\right)$

= 2250 - 100 = 2150 J

= 2.15 kJ. So, (I) → (P)

II. $P = \frac{nRT}{V} = \frac{0.2 \times 8 \times 500}{V} = \frac{800}{V}$ Pa

$\Delta U = \frac{f}{2} P\Delta V = \frac{5}{2} \times \frac{800}{V} \times 2V$ = 4000 J = 4 kJ

So (II) → (R)

III. $PV^\gamma =$ const $\Rightarrow P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow 2 V^\gamma = P_2\left(\frac{V}{8}\right)^\gamma \Rightarrow P_2 = 2 \times 8^\gamma = 2 \times 8^{5/3}$ = 64 kPa

So, $\Delta U = \frac{f}{2}(P_2V_2 - P_1V_1)$

$= \frac{3}{2}\left(64 \times \frac{1}{24} - 2 \times \frac{1}{3}\right) \times 10^3$ = 3 kJ

So, (III) → (T)

IV. Here f = 7

So, $\Delta U = nC_V\Delta T = \frac{t}{2} nR\Delta T = \frac{7}{2} nR\Delta T$

and, $\Delta Q = nC_V\Delta T = \left(\frac{f}{2} + 1\right) nR\Delta T = \frac{9}{2} nR\Delta T = \frac{9}{2} \times \frac{2}{7}\ \Delta U = \frac{9}{7}\Delta U$

So, $\Delta U = \frac{7}{9}\Delta Q = \frac{7}{9} \times 9$ = 7 kJ. So (IV) → (Q)

32.

(b) P - 2, Q - 4, R - 3, S - 1

Explanation: For double convex lens, (P) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\Rightarrow$ (1.5 - 1)$\left(\frac{1}{r} - \frac{1}{r}\right)$ = (1.5 - 1)$\left[\frac{2}{r}\right]$ = $\frac{1}{r}$ $\Rightarrow$ f = r

$\frac{1}{F_{\text{eq.}}}$ = $\frac{1}{f_1} + \frac{1}{f_2}$ = $\frac{1}{r} + \frac{1}{r}$ = $\frac{2}{r}$

$\therefore$ F_{eq} = $\frac{r}{2}$

For (Q) plano-convex lens $\frac{1}{f}$ = $(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

= (1.5 - 1)$\left[\frac{1}{\infty} - \frac{1}{-r}\right]$ = $\frac{0.5}{r}$ = $\frac{1}{2r}$ $\therefore$ f = 2r

$\frac{1}{F_{\text{eq.}}}$ = $\frac{1}{f_1} + \frac{1}{f_2}$ = $\frac{1}{2r} + \frac{1}{2r}$ = $\frac{2}{2r}$ = $\frac{1}{r}$ $\therefore$ $F_{eq.}$ = r

For (R) plano-concave lens

$\frac{1}{f}$ = (1.5 - 1)$\left(\frac{1}{-r} - \frac{1}{\infty}\right)$ $\Rightarrow$ f = -2r

$\frac{1}{F_{\text{eq.}}}$ = $\frac{1}{f} + \frac{1}{f}$ = $\frac{1}{-2r} + \frac{1}{-2r}$ $\Rightarrow$ $F_{eq.}$ = -r

For (S) combination of one double convex and one planoconcave lens

$\frac{1}{F_{\text{eq.}}}$ = $\frac{1}{r} + \frac{1}{-2r}$ = $\frac{1}{2r}$ $\Rightarrow$ $F_{eq.}$ = 2r

33.

(c) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 2,\ \mathrm{S} \rightarrow 1$

Explanation: In α-decay mass number (A) decreases by 4 units and atomic number (Z) decreases by 2 units. In β^-decay A does not change but Z increases by 1 unit.

In β^+decay A does not change but Z decreases by 1 unit.

(P) ${}_{92}\mathrm{U}^{238} \rightarrow {}_{91}\ \mathrm{Pa}^{234}$

$\mathrm{N}_1 = \frac{238-234}{4} = 1 \rightarrow 1\alpha$

$\mathrm{N}_2 - \mathrm{N}_3 = (92 - 91) - \left(\frac{4}{2}\right) = -1 \rightarrow 1\beta^-$

$\therefore 1\alpha$ and $1\beta^-$emission.

(Q) ${}_{82}\ \mathrm{Pb}^{214} \rightarrow_{82}\ \mathrm{Pb}^{210}$

$\mathrm{N}_1 = \frac{214-210}{4} = 1 \rightarrow 1\alpha$

$\mathrm{N}_2 - \mathrm{N}_3 = (82 - 82) - \left(\frac{4}{2}\right) = -2 \rightarrow 2\beta^-$

$\therefore 1\alpha$ and $2\beta^-$emission.

(R) ${}_{81}\ \mathrm{T}\ell^{210} \rightarrow_{82}\ \mathrm{Pb}^{206}$

$\mathrm{N}_1 = \frac{210-206}{4} = 1 \rightarrow 1\alpha$

$\mathrm{N}_2 - \mathrm{N}_3 = (81 - 83) - \frac{4}{2} = -3 \rightarrow 3\beta^-$

$\therefore 1\alpha$ and $3\beta^-$emission.

(S) ${}_{91}\ \mathrm{Pa}^{228} \rightarrow_{88} \mathrm{Ra}^{224}$

$\mathrm{N}_1 = \frac{228-224}{4} = 1\alpha$

$\mathrm{N}_2 - \mathrm{N}_3 = (91 - 88) - \frac{4}{2} = 1\beta^+$

$\therefore 1\alpha$ and $1\beta^+$emission.

34.

(d) (I) $\rightarrow$ (P), (II)$\rightarrow$ (R), (III)$\rightarrow$ (S), (IV)$\rightarrow$ (Q)

Explanation: Frequency, v = $\frac{1}{2\ell}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}}$ for first mode of vibration

For 'v' to be maximum, 'l' should be minimum.

String - 1 $\mathrm{f}_0 = \frac{1}{2\,\mathrm{L}_0}\sqrt{\frac{\mathrm{T}_0}{\mu}}$

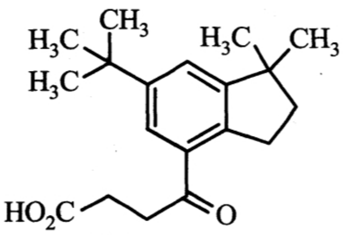
String - 2 $\mathrm{f}_2 = \frac{1}{2\,\mathrm{L}_0}\sqrt{\frac{\mathrm{T}_0}{2\mu}} = \frac{\mathrm{f}_0}{\sqrt{2}}$

String - 3 $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

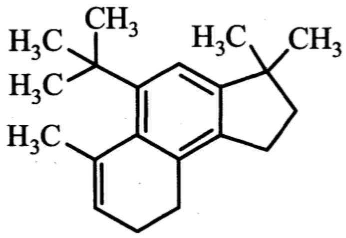
String - 4 $f_4 = \frac{1}{2\,L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

Chemistry (MRQ)

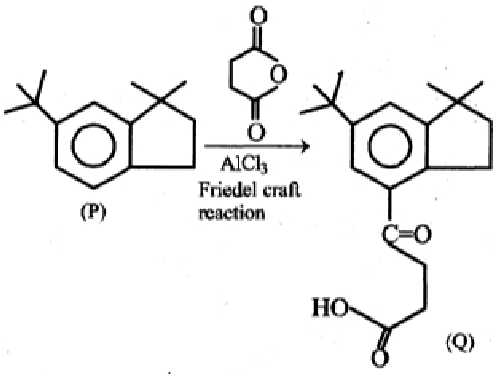
35. **(a)** **Q is**

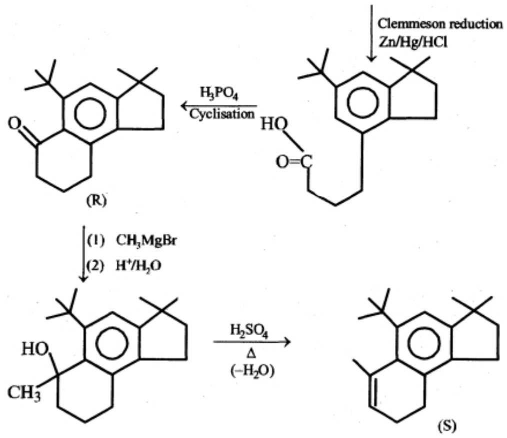


(d) **S is**



Explanation:





36. **(a)** Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

(c) The activation energy of the reaction is unaffected by the value of the steric factor

Explanation: According to Arrhenius equations

$k = Ae^{-Ea/RT}$

where, A = Frequency factor

Taking into account orientation factor,

$P = \frac{A}{Z}$ or A = PZ

$k = PZe^{-Ea/RT}$

where, P = steric factor, Z = collision frequency

The value of steric factor lies between 0 and 1 predicted by Arrhenius equation. Thus, the experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation.

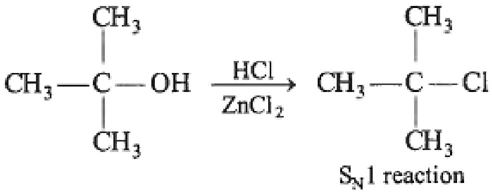
37. **(a)** Process that is reversible and isothermal.
(b) Process that is reversible and adiabatic.
(d) System that satisfies the van der Waals equation of state.
Explanation: P-V work done is applicable for reversible isobaric as well as isothermal and adiabatic process.
$w=-\int P_{\text{ext}}\cdot \mathrm{d}V$
For van der Waals equation,
$P_{\text{ext}}=P=\left(\frac{RT}{v-b}-\frac{a}{v^2}\right)$
$w=-\int dv\left(\frac{RT}{v-b}-\frac{a}{v^2}\right)$ ...(i)
Equation (i) is not applicable to irreversible process. Therefore work done is calculated assuming pressure is constant throughout the process.

Chemistry (MCQ)

38.
(c) CO
Explanation: ∵ after forming the bonds, C has only $6e^-$ in its valence shell.

39.
(d) increases with time
Explanation: $A+B\rightleftharpoons C+D, Q=\frac{[C][D]}{[A][B]}]$
As time passes, amount of products 'C' and 'D' increases, hence Q increases.

40.
(c) X > Z > Y
Explanation: Carboxylic acid is stronger acid than ammonium ion, hence -COOH(X) is most acidic. Z (NH_3)is more acidic than Y (NH_3) due to - I effect of -COOH on Z. Hence, overall acid strength order is
X > Z > Y

41. **(a)** 2-methyl propan-2-ol
Explanation:
2-methyl propan-2-ol is a tertiary alcohol, will react fastest with Lucas reagent:


Chemistry (NUM)

42. 56.0
Explanation:
The formula used in Kjeldahl's method
% of 'N' = $\frac{1.4(\mathrm{N_1V_1})}{\mathrm{W}}$
N_1 = Normality of acid = 2×2 (N)
V_1 = Volume of acid used = 2.5 mL
W = Mass of organic compound = 0.25 g
% of 'N' = $\frac{1.4\times2.5\times2\times2}{0.25}$ = 56

43. 3.84
Explanation:

$N_1 = 1, V_1 = ?, N_2 = 26.7, V_2 = 0.4$

$N_1V_1 = N_2V_2$; $1 \times V_1 = 26.7 \times 0.4$

$V_1 = \frac{26.7 \times 0.4}{1} = 10.68$

49g ($\because$ eq wt of H_2SO_4 = 49) of H_2SO_4 will be neutralised by= 1N 1000 mL NaOH

$\therefore$ 0.5g of H_2SO_4 will be neutralised by

$= \frac{1000}{49} \times 0.5 = 10.20$ mL 1N NaOH

Volume of 1 N NaOH used by dissolved

SO_3 = 10.68 - 10.20 = 0.48 mL

$SO_3 + 2NaOH \rightarrow Na_2SO_4 + H_2O$

$\therefore$ Eq wt of $SO_3 = \frac{\text{Mol wt}}{2} = \frac{80}{2} = 40$

Wt of SO_3 in 0.48 mL of 1 M solution

$= \frac{40}{1000} \times 0.48 = 0.0192$ g

% of $SO_3 = \frac{0.0192}{0.5} \times 100 = 3.84\%$

44. 319.1

Explanation:

100 g of glucose = 1560 kJ

Energy utilised in body $= \frac{50}{100} \times 1560 = 780$ kJ

Energy left unutilised in body = 1560 - 780 = 780kJ

Energy to be given out = 1560 - 780 = 780 kJ

Enthalpy of evaporation of water = 44 kJ/mole = 44 kJ/18 g of water [1 mole H_2O = 18g water]

Hence amount water to be perspired to avoid storage of energy $= \frac{18}{44} \times 780 = 319.1$ g

45. 27419

Explanation:

The shortest wavelength transition in the Balmer series corresponds to the transition $n = 2 \rightarrow n = \infty$. Hence, $n_1 = 2$, $n_2 = \infty$

$\bar{v} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (109677 \text{ cm}^{-1}) \left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$= 27419.25 \text{ cm}^{-1}$

46. 8.0

Explanation:

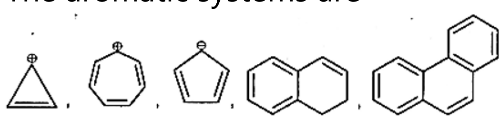
${}_{92}U^{238} \xrightarrow{-6\alpha} {}_{80}X^{214} \xrightarrow{-2\beta} {}_{82}Pb^{214}$

Hence, total number of particles emitted are 2 + 6 = 8.

47. 5

Explanation:

The aromatic systems are



Chemistry (MATCH)

48.

(d) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

Explanation: (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

49.

(b) I $\rightarrow$ R, T; II $\rightarrow$ P, S; III $\rightarrow$ Q, T; IV $\rightarrow$ P, Q

Explanation:

I. $[\mathrm{Cr(CN)_6}]^4 \Rightarrow \mathrm{Cr}^{2+}\left(d^4\right) \Rightarrow$ low spin Oh complex as CN^- is strong field ligand.

Electronic configuration $= t_{2g}^4 e_g^0; \Delta_0 > \mathrm{P}$;

$\mu_{\mathrm{S.o.}}$ = $\sqrt{2(2+2)}$ = 2.82BM

(P), (R), (T)

II. $[\mathrm{RuCl_6}]^{2-} \Rightarrow \mathrm{Ru}^{4+}\left(d^4\right) \Rightarrow$ Low spin Oh complex as **Ru** is of large size.

Electronic configuration $= t_{2g}^4 e_g^0; \Delta_0 > \mathrm{P}$;

$\mu_{\mathrm{S.o.}} = \sqrt{2(2+2)}$ = 2.82 BM

(P), (R), (S), (T)

III. $[\mathrm{Cr(H_2O)_6}]^{2+} \Rightarrow \mathrm{Cr}^{2+}\left(d^4\right) \Rightarrow$ high spin Oh complex as H_2O is weak field ligand.

Electronic configuration $= t_{2g}^3 e_g^1; \Delta_0 < \mathrm{P}$;

$\mu_{\mathrm{S.O.}} = \sqrt{4(4+2)}$ = 4.89 Bm

(Q), (T)

IV. $[\mathrm{Fe(H_2O)_6}]^{2+} \Rightarrow \mathrm{Fe}^{2+}\left(d^6\right) \Rightarrow$ high spin Oh complex

Electronic configuration $= t_{2g}^4 e_g^2; \Delta_0 < \mathrm{P}$;

$\mu_{\mathrm{S.o.}} = \sqrt{4(4+2)}$ = 4.89 Bm

(P), (Q)

50. **(a)** (P) - (3), (Q) - (4), (R) - (1), (S) - (2)

Explanation:

p. $Fe^{3+} \xrightarrow[n=1]{+0.77V} Fe^{2+} \xrightarrow[n=2]{-0.44V} Fe$; $Fe^{3+} \rightarrow Fe$: x V, $n=3$

$$\Delta G^o_{Fe^{3+}/Fe} = \Delta G^o_{Fe^{3+}/Fe^{2+}} + \Delta G^0_{Fe^{2+}/Fe}$$

$$\Rightarrow -3 \times FE^o_{(Fe^{+3}/Fe)} = -1 \times FE^o_{(Fe^{+3}/Fe^{+2})} + \left(-2 \times FE^o_{Fe^{+2}/Fe}\right)$$

$\Rightarrow 3 \times x = 1 \times 0.77 + 2 \times (-0.44)$

$\Rightarrow x = -\frac{0.11}{3} V \simeq -0.04$ V.

q.

$2H_2O \longrightarrow O_2 + 4H^+ + 4e^-$ $\quad E^o = -1.23V$

$4e + O_2 + 2H_2O \longrightarrow 4OH^-$ $\quad E^o = +0.40V$

$4H_2O \longrightarrow 4H^+ + 4OH^-$ $\quad E^o = -0.83V$

r.

$Cu^{2+} + 2e \longrightarrow Cu$ $\quad E^o = +0.34V$

$2Cu \longrightarrow 2Cu^+ + 2e$ $\quad E^o = -0.52V$

$Cu^{2+} + Cu \longrightarrow 2Cu^+$ $\quad E^o = -0.18V$

s. $Cr^{3+} \xrightarrow[n=1]{x} Cr^{2+} \xrightarrow[n=2]{-0.91V} Cr$; $Cr^{3+} \rightarrow Cr$: $-0.74V$, $n=3$

$x \times 1 + 2 \times (-0.91) = 3 \times (-0.74)$

$x - 1.82 = -2.22 \Rightarrow x = -0.4$ V

51.

(b) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 4; S $\rightarrow$ 5

Explanation: (P) $P_2O_3 + 3H_2O \rightarrow 2H_3PO_3$

(Q) $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

(R) $PCl_5 + CH_3COOH \rightarrow CH_3COCl + POCl_3 + HCl$

(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow 4Ag + 4HNO_3 + H_3PO_4$