Exercise 2.4

Chapter 2 Derivatives Exercise 2.4 1E

f (x) = 3 x2 -2cos x

Differentiating with respect to x

f '(x) = 6x-2(-sinx) = 6 x + 2sin x

Chapter 2 Derivatives Exercise 2.4 2E

Consider the function

$f(x)=\sqrt{x}\sin x$.

The product rule:

If *u* and *v* are both differentiable, then

$$\frac{d}{dx}[u(x)v(x)]=u(x)\frac{d}{dx}[v(x)]+v(x)\frac{d}{dx}[u(x)]$$

To find differentiate of the given function:

$f'(x)=\sqrt{x}\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(\sqrt{x})$ By using the product rule

$=x^{1/2}\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x^{1/2})$ By using the radical rule: $\sqrt[n]{a}=a^{1/n}$

$=x^{1/2}\cdot\cos x+\sin x\frac{d}{dx}(x^{1/2})$ Since $\frac{d}{dx}(\sin x)=\cos x$

$=x^{1/2}\cos x+\sin x\cdot\frac{1}{2}x^{-1/2}$ By using the power rule: $\frac{d}{dx}(x^n)=nx^{n-1}, n\in\mathbb{R}$

$=x^{1/2}\cos x+\frac{1}{2}x^{-1/2}\sin x$ Simplify.

Therefore, the differentiate of the given function is

$$\boxed{f'(x)=x^{1/2}\cos x+\frac{1}{2}x^{-1/2}\sin x}.$$

Chapter 2 Derivatives Exercise 2.4 3E

Differentiation is the process of finding the derivative of a function.

Consider the function

$$f(x)=\sin x+\frac{1}{2}\cot x.$$

By the formula,

The sum rule:

If u and v are both differentiable, then

$$\frac{d}{dx}[u(x)+v(x)]=\frac{d}{dx}[u(x)]+\frac{d}{dx}[v(x)]$$

The constant multiple rule:

If u is differentiable and c is a constant, then

$$\frac{d}{dx}[cu(x)]=c\frac{d}{dx}[u(x)]$$

Calculate the differentiation of a function as,

By the sum rule,

$$f'(x)=\frac{d}{dx}(\sin x)+\frac{d}{dx}\left(\frac{1}{2}\cot x\right)$$

By the constant multiple rule,

$$f'(x)=\frac{d}{dx}(\sin x)+\frac{1}{2}\frac{d}{dx}(\cot x)$$

By the formula,

$$\frac{d}{dx}(\sin x)=\cos x,\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$f'(x)=\cos x+\frac{1}{2}\cdot\left(-\csc^2 x\right)$$
$$=\cos x-\frac{1}{2}\csc^2 x$$

Therefore the required solution is $\boxed{f'(x)=\cos x-\frac{1}{2}\csc^2 x}$

Chapter 2 Derivatives Exercise 2.4 4E

To find the derivative of the function

$$y=2\sec x-\csc x$$

Differentiate both sides of it with respect to x, then

$$\frac{d}{dx}(y)=\frac{d}{dx}(2\sec x-\csc x)$$

Use difference rule of differentiation

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

for $f(x)=2\sec x$ and $g(x)=\csc x$

So,

$$y'=\frac{d}{dx}(2\sec x)-\frac{d}{dx}(\csc x)$$
$$=2\frac{d}{dx}(\sec x)-\frac{d}{dx}(\csc x) \quad \text{since } \frac{d}{dx}(kf(x))=k\frac{d}{dx}(f(x))$$

Use the derivative of Trigonometric functions,

$$\frac{d}{dx}(\sec x)=\sec x\tan x \text{ and } \frac{d}{dx}(\csc x)=-\csc x\cot x$$

Therefore,

$$y' = 2\frac{d}{dx}(\sec x) - \frac{d}{dx}(\csc x)$$
$$= 2\cdot\sec x\tan x - (-\csc x\cot x)$$
$$= 2\sec x\tan x + \csc x\cot x$$

Hence,

$$y' = \boxed{2\sec x\tan x + \csc x\cot x}$$

Chapter 2 Derivatives Exercise 2.4 5E

Given that $y = \sec\theta\tan\theta$

Then $\frac{dy}{d\theta} = \frac{d}{d\theta}(\sec\theta\tan\theta)$

Rule: $\frac{d}{dx}(fg) = fg' + f'g$.

Using the above Rule, we have

$$\frac{dy}{d\theta} = \sec\theta\frac{d}{d\theta}\tan\theta + \tan\theta\frac{d}{d\theta}\sec\theta$$
$$= \sec\theta\sec^2\theta + \tan\theta[\sec\theta\tan\theta]$$
$$= \sec^3\theta + \sec\theta\tan^2\theta$$
$$= \sec\theta(\sec^2\theta + \tan^2\theta)$$

$$\boxed{\therefore \frac{dy}{d\theta} = \sec\theta(\sec^2\theta + \tan^2\theta)}$$

Chapter 2 Derivatives Exercise 2.4 6E

$$g(t) = 4\sec t + \tan t$$
$$g'(t) = 4\sec t\tan t + \sec^2 t$$

Chapter 2 Derivatives Exercise 2.4 7E

The measure of rate of change of a quantity with respect to some other quantity is called as the derivative and the method to determine the derivative is called differentiation.

Consider the formulae of differentiation shown below:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$
$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

Consider the function:

$$y = c\cos t + t^2\sin t$$

Use the above formulae to find $\frac{dy}{dt}$:

$$\frac{dy}{dt} = \frac{d}{dt}[c\cos t + t^2\sin t]$$
$$= \frac{d}{dt}[c\cos t] + \frac{d}{dt}[t^2\sin t]$$
$$= c\frac{d}{dt}(\cos t) + t^2\frac{d}{dt}(\sin t) + \sin t\frac{d}{dt}(t^2)$$
$$= -c\sin t + t^2(\cos t) + \sin t(2t)$$

Simplify the above equation further:

$$\frac{dy}{dt} = -c\sin t + t^2\cos t + 2t\sin t$$
$$= t^2\cos t + (2t - c)\sin t$$
$$= -c\sin t + t(t\cos t + 2\sin t)$$

Hence, the final expression is $\boxed{\frac{dy}{dt} = -c\sin t + t(t\cos t + 2\sin t)}$.

Chapter 2 Derivatives Exercise 2.4 8E

We have $y = u(a\cos u + b\cot u)$

Now we differentiate with respect to u as follows:

$$\frac{dy}{dx} = (u)'(a\cos u + b\cot u) + (u)(a\cos u + b\cot u)' \quad \text{[Product rule]}$$

$$= a\cos u + b\cot u + ua(\cos u)' + bu(\cot u)' \quad \text{[Constant multiple and sum rule]}$$

$$= \boxed{a\cos u + b\cot u - au\sin u - bu\csc^2 u} \quad \left[\text{Since } (\cos u)' = -\sin u \text{ and } (\cot u)' = -\csc^2 u\right]$$

Chapter 2 Derivatives Exercise 2.4 9E

Consider the function $y = \frac{x}{2 - \tan x}$

Need to find differentiate the function.

Differentiate with respect to x

$$y = \frac{x}{2 - \tan x}$$

Let $u = x, v = 2 - \tan x$

$$f'(x) = \frac{(2 - \tan x)\frac{d}{dx}(x) - x\frac{d}{dx}(2 - \tan x)}{(2 - \tan x)^2} \quad \text{since } \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

Continuation to the above steps,

$$= \frac{(2 - \tan x)1 - x(0 - \sec^2 x)}{(2 - \tan x)^2} \quad \text{since } \frac{d}{dx}(\tan x) = \sec^2 x$$

$$= \frac{(2 - \tan x)1 - x(-\sec^2 x)}{(2 - \tan x)^2}$$

$$= \frac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}$$

Therefore,

$$y' = \boxed{\frac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}}.$$

Chapter 2 Derivatives Exercise 2.4 10E

Let $y = \sin\theta\cos\theta$

Formula: $\frac{d}{dx}(fg) = f'g + fg'$

Using the above formula

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(\sin\theta\cos\theta)$$
$$= \sin\theta\frac{d}{d\theta}\cos\theta + \cos\theta\frac{d}{d\theta}\sin\theta$$
$$= \sin\theta(-\sin\theta) + \cos\theta(\cos\theta)$$
$$= \cos^2\theta - \sin^2\theta$$

$$\boxed{\therefore \frac{dy}{d\theta} = \cos^2\theta - \sin^2\theta}$$

Chapter 2 Derivatives Exercise 2.4 11E

$$f(\theta)=\frac{\sec\theta}{1+\sec\theta}$$

Using quotient rule,

$$f'(\theta)=\frac{(\sec\theta)'(1+\sec\theta)-(\sec\theta)(1+\sec\theta)'}{(1+\sec\theta)^2}$$

We know that $\boxed{\frac{d}{dx}(\sec x)=\sec x\tan x}$

$$f'(\theta)=\frac{\sec\theta\tan\theta(1+\sec\theta)-\sec\theta\sec\theta\tan\theta}{(1+\sec\theta)^2}$$

$$f'(\theta)=\frac{\sec\theta\tan\theta+\sec^2\theta\tan\theta-\sec^2\theta\tan\theta}{(1+\sec\theta)^2}$$

$$f'(\theta)=\frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$$

Therefore $f'(\theta)=\boxed{\frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}}$

Chapter 2 Derivatives Exercise 2.4 12E

Let $y=\frac{\cos x}{1-\sin x}$

Formula: $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{gf'-fg'}{g^2}$

Using the above formula

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\cos x}{1-\sin x}\right)$$

$$=\frac{(1-\sin x)\frac{d}{dx}\cos x-\cos x\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$=\frac{(1-\sin x)(-\sin x)-\cos x(-\cos x)}{(1-\sin x)^2}$$

$$=\frac{-\sin x+\sin^2 x+\cos^2 x}{(1-\sin x)^2}$$

$$=\frac{1-\sin x}{(1-\sin x)^2}$$

$$=\frac{1}{1-\sin x}$$

$$\boxed{\therefore \frac{dy}{dx}=\frac{1}{1-\sin x}}$$

Chapter 2 Derivatives Exercise 2.4 13E

The measure of rate of change of a quantity with respect to some other quantity is called as the derivative and the method to determine the derivative is called differentiation.

Consider the formulae of differentiation shown below:

$$\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+f'(x)g(x)$$

$$\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right]=\frac{g(t)\frac{d}{dt}[f(t)]-f(t)\frac{d}{dt}[g(t)]}{[g(t)]^2}$$

Consider the function shown below:

$$y=\frac{t\sin t}{1+t}$$

Use the above formulae to find $\frac{dy}{dt}$:

$$\frac{d}{dt}(y)=\frac{d}{dt}\left(\frac{t\sin t}{1+t}\right)$$

$$\frac{d}{dt}\left(\frac{\sin t}{1+t}\right)=\frac{(1+t)\frac{d}{dt}(t\sin t)-t\sin t\frac{d}{dt}(1+t)}{(1+t)^2}$$

$$=\frac{(1+t)\left[t\frac{d}{dt}(\sin t)+\sin t\frac{d}{dt}(t)\right]-t\sin t(1)}{(1+t)^2}$$

$$=\frac{(1+t)\left[t(\cos t)+\sin t(1)\right]-t\sin t}{(1+t)^2}$$

Simplify the above equation:

$$\frac{d}{dt}\left(\frac{\sin t}{1+t}\right)=\frac{t(\cos t)(1+t)+\sin t(1+t)-t\sin t}{(1+t)^2}$$

$$=\frac{t(\cos t)(1+t)+\sin t+t\sin t-t\sin t}{(1+t)^2}$$

$$=\frac{t(\cos t)(1+t)+\sin t}{(1+t)^2}$$

$$=\frac{(t+t^2)(\cos t)+\sin t}{(1+t)^2}$$

Hence, the final expression is $\boxed{\frac{d}{dt}(y)=\frac{(t+t^2)\cos t+\sin t}{(1+t)^2}}$.

Chapter 2 Derivatives Exercise 2.4 14E

Consider the following function:

$$y=\frac{1-\sec x}{\tan x}$$

The objective is to find the derivative of the function.

The quotient rule:

If *u* and *v* are both differentiable, then

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]=\frac{v(x)\frac{d}{dx}[u(x)]-u(x)\frac{d}{dx}[v(x)]}{[v(x)]^2},v(x)\neq 0$$

The difference rule:

If *u* and *v* are both differentiable, then

$$\frac{d}{dx}[u(x)-v(x)]=\frac{d}{dx}[u(x)]-\frac{d}{dx}[v(x)]$$

Now differentiate y with respect to x.

$$\frac{dy}{dx}=\frac{\tan x\frac{d}{dx}(1-\sec x)-(1-\sec x)\frac{d}{dx}(\tan x)}{\tan^2 x}$$ Use the quotient rule

$$=\frac{\tan x\left[\frac{d}{dx}(1)-\frac{d}{dx}(\sec x)\right]-(1-\sec x)\frac{d}{dx}(\tan x)}{\tan^2 x}$$ Use the difference rule

$$=\frac{\tan x\cdot(0-\sec x\tan x)-(1-\sec x)\cdot\sec^2 x}{\tan^2 x}$$ Apply differentiation

$$=\frac{-\sec x\tan^2 x-\sec^2 x+\sec^3 x}{\tan^2 x}$$ Multiply

$$=\frac{\sec x\left(\sec^2 x-\tan^2 x\right)-\sec^2 x}{\tan^2 x}$$ Factor

$$=\frac{\sec x-\sec^2 x}{\tan^2 x}$$ Use the identity: $\sec^2 x-\tan^2 x=1$

Therefore, the derivative of the given function is $\boxed{\frac{dy}{dx}=\frac{\sec x(1-\sec x)}{\tan^2 x}}$.

Chapter 2 Derivatives Exercise 2.4 15E

Consider the following function:

$$h(\theta)=\theta\csc\theta-\cot\theta$$

The objective is to find the derivative of the function.

The product rule states that,

If u and v are both differentiable, then

$$\frac{d}{dx}[u(x)v(x)]=v(x)\frac{d}{dx}[u(x)]+u(x)\frac{d}{dx}[v(x)]$$

The difference rule:

If u and v are both differentiable, then

$$\frac{d}{dx}[u(x)-v(x)]=\frac{d}{dx}[u(x)]-\frac{d}{dx}[v(x)]$$

Now differentiate $h(\theta)$ with respect to θ.

$$\frac{d}{d\theta}(h(\theta))=\frac{d}{d\theta}(\theta\csc\theta-\cot\theta)$$

$$=\frac{d}{d\theta}(\theta\csc\theta)-\frac{d}{d\theta}(\cot\theta)$$ Apply the difference rule

$$=\frac{d}{d\theta}(\theta)\cdot\csc\theta+\theta\frac{d}{d\theta}(\csc\theta)-\frac{d}{d\theta}(\cot\theta)$$ Apply the product rule

$$=\csc\theta-\theta\csc\theta\cot\theta+\csc^2\theta$$ $\left[\begin{array}{l}\text{Use } \frac{d}{dx}\csc x=-\csc x\cot x\\ \text{and } \frac{d}{dx}\cot x=-\csc^2 x\end{array}\right]$

$$=\csc\theta(1-\theta\cot\theta+\csc\theta)$$ Factor

Therefore, the derivative of the given function is $\boxed{h'(\theta)=\csc\theta(1-\theta\cot\theta+\csc\theta)}$.

Chapter 2 Derivatives Exercise 2.4 16E

Given function $y = x^2 \sin x \tan x$

Differentiate y with respect to x using product rule, we have

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(x^2 \sin x \tan x\right) \\
&= x^2 \frac{d}{dx}(\sin x \tan x) + \sin x \tan x \frac{d}{dx}\left(x^2\right) \\
&= x^2\left[\sin x \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(\sin x)\right] + (\sin x \tan x)2x \\
&= x^2\left[\sin x\left(\sec^2 x\right) + \tan x(\cos x)\right] + (\sin x \tan x)2x \\
&= x^2\left[\sin x \sec^2 x + \cos x \tan x\right] + (\sin x \tan x)2x \\
&= x^2\left[\sin x \sec^2 x + \cos x \frac{\sin x}{\cos x}\right] + (\sin x \tan x)2x \quad \left(\because \ \tan x = \frac{\sin x}{\cos x}\right)
\end{aligned}$$

Therefore $\frac{dy}{dx} = x^2\left(\sin x \sec^2 x + \sin x\right) + 2x \sin x \tan x$

Chapter 2 Derivatives Exercise 2.4 17E

To prove $\quad \frac{d}{dx}(\csc x) = -\csc x \cot x$

Let $\quad f(x) = \csc x = \frac{1}{\sin x}$

Differentiating with respect to x by quotient rule

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\left(\frac{1}{\sin x}\right) \\
&= \frac{(\sin x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x)}{(\sin x)^2} \\
&= \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} \\
&= \frac{0 - (\cos x)}{(\sin x)^2} \\
&= \frac{-\cos x}{(\sin x)^2} \\
&= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \\
&= -\cot x \csc x
\end{aligned}$$

Or, $\boxed{\frac{d}{dx}(\csc x) = -\cot x \csc x}$

Chapter 2 Derivatives Exercise 2.4 18E

To prove $\quad \frac{d}{dx}(\sec x) = \sec x \tan x$

Let $\quad f(x) = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

Differentiating with respect to x, by chain rule, we get

$$\begin{aligned}
f'(x) &= -1(\cos x)^{-2} \frac{d}{dx}(\cos x) \\
&= -(\cos x)^{-2}(-\sin x) \\
&= \frac{\sin x}{\cos^2 x} \\
&= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \\
&= \sec x \tan x
\end{aligned}$$

Thus, $\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x}$

Chapter 2 Derivatives Exercise 2.4 19E

We have to prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$

Let $f(x) = \cot x = \frac{\cos x}{\sin x}$

Differentiating with respect to x by quotient rule

$$f'(x) = \frac{(\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

Or $\boxed{\frac{d}{dx}(\cot x) = -\csc^2 x}$

Chapter 2 Derivatives Exercise 2.4 20E

We have $f(x) = \cos x$

Then by the definition of derivative we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

We use $\cos(A+B) = \cos A.\cos B - \sin A \sin B$

Thus

$$f'(x) = \lim_{h\to 0} \frac{\cos x.\cos h - \sin x.\sin h - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x.\cos h - \cos x}{h} - \lim_{h\to 0} \frac{\sin x.\sin h}{h}$$

$$= \lim_{h\to 0} \cos x\left(\frac{\cos h - 1}{h}\right) - \lim_{h\to 0} \sin x \frac{\sin h}{h}$$

$$= \lim_{h\to 0} \cos x.\lim_{h\to 0} \frac{\cos h - 1}{h} - \lim_{h\to 0} \sin x.\lim_{h\to 0} \frac{\sin h}{h}$$

We have $\lim_{h\to 0} \frac{\sin h}{h} = 1$ and $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

Thus $f'(x) = \cos x.0 - \sin x.1$

$$\Rightarrow f'(x) = 0 - \sin x$$

$$\Rightarrow \boxed{f'(x) = -\sin x}$$

Chapter 2 Derivatives Exercise 2.4 21E

Problem: Find Equation of Tangent Line to y = sec(x) at the point (π /3,2)

In order to find the equation for the Tangent Line, use the point-slope form of a line:

(y - y1) = m(x - x1)

You are given (x1,y1) as (π /3,2), but the slope is unknown.

To find the slope, m, at x= π /3 requires evaluating the derivative of y = sec(x) at x= π /3.

Diferentiating y = sec(x) yields:

$\frac{d}{dx}$ sec(x) = sec(x)tan(x)

Evaluating the derivative sec(x)tan(x) at x= π /3 yields:

sec(π /3) * tan(π /3) = 2 $\sqrt{3}$

Therefore, the slope, m, is 2 $\sqrt{3}$

Substituting into the point-slope form of a line:

(y - 2) = 2 $\sqrt{3}$ (x - π /3)

Solving for y:

y = 2 $\sqrt{3}$ x -(2/3) $\sqrt{3}$ π + 2

Chapter 2 Derivatives Exercise 2.4 22E

$$y=(1+x)\cos x$$
$$\frac{dy}{dx}=(1+x)'\cos x+(1+x)(\cos x)'$$
$$=\cos x-(1+x)\sin x$$

The slope of the tangent at (0, 1) is

$$m=\left.\frac{dy}{dx}\right|_{x=0}=\cos 0-(1+0)\sin 0=1$$

The equation of the tangent at (0, 1) is

$$y-y_1=m(x-x_1)$$
$$\Rightarrow y-1=1(x-0)$$
$$\Rightarrow y=x+1$$

Chapter 2 Derivatives Exercise 2.4 23E

Given curve is $y = \cos x - \sin x$

Then $\frac{dy}{dx} = -\sin x - \cos x$

$\therefore$ Slope of the tangent line at $(\pi, -1)$ is

$$m = \left(\frac{dy}{dx}\right)_{x=\pi} = -\sin \pi - \cos \pi = 1$$

The equation of the tangent line is

$$(y+1) = 1(x-\pi)$$

$$\Rightarrow \quad y = x - \pi - 1$$

$$\boxed{\therefore\ y = x - \pi - 1}$$

Chapter 2 Derivatives Exercise 2.4 24E

Let $y = x + \tan x$

Then $\frac{dy}{dx} = 1 + \frac{d}{dx}\tan x$

$$= 1 + \sec^2 x$$

$\therefore$ Slope of the tangent line at (π, π) is

$$m = \left(\frac{dy}{dx}\right)_{x=\pi} = 1 + \sec^2 \pi = 2$$

$\therefore$ The equation of the tangent line is

$$(y-\pi) = 2(x-\pi)$$

$$\Rightarrow \quad y - \pi = 2x - 2\pi$$

$$\Rightarrow \quad y = 2x - 2\pi + \pi = 2x - \pi$$

$$\boxed{\therefore y = 2x - \pi}$$

Chapter 2 Derivatives Exercise 2.4 25E

(a) Given $y = (\sin x)2x$

the point is $\left(\frac{\pi}{2}, \pi\right)$

$$\frac{dy}{dx} = \frac{d}{dx}(2x \sin x)$$

$$= 2\frac{d}{dx}(x \sin x)$$

$$= 2\left[x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x)\right]$$

$$= 2[x \cos x + \sin x]$$

$$\left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2}, \pi\right)} = 2\left(\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$$

$$= 2(1)$$

$$= 2$$

$$= m$$

Equation of the tangent line is

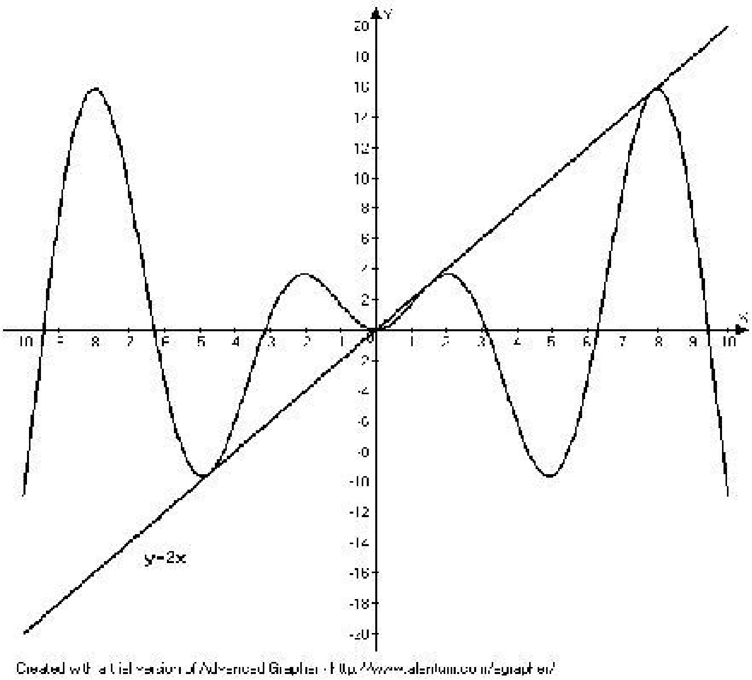
$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - \pi = 2\left(x - \frac{\pi}{2}\right)$$

$$\Rightarrow \quad y - \pi = 2x - \pi$$

$$\Rightarrow \quad y = 2x$$

(b) Graph of the curve and the tangent line is shown below.



Chapter 2 Derivatives Exercise 2.4 26E

(a)

To find the equation of the tangent line to the curve

$y = 3x + 6\cos x$

At the point $(\pi/3, \pi + 3)$ it is need to find the slope of the curve at this point.

Recall that, slope of the tangent line to the curve $y = f(x)$ at the point (x_1, y_1) is

$$m = \left(\frac{dy}{dx}\right)_{x=x_1}$$

As $y = 3x + 6\cos x$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(3x + 6\cos x) \\ &= \frac{d}{dx}(3x) + \frac{d}{dx}(6\cos x) \quad \text{using } \frac{d}{dx}(f+g) = \frac{d}{dx}(f) + \frac{d}{dx}(g) \\ &= 3\frac{dx}{dx} + 6\frac{d}{dx}(\cos x) \quad \text{using } \frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x)) \\ &= 3 - 6\sin x\end{aligned}$$

So,

$$\frac{dy}{dx} = 3 - 6\sin x$$

Slope of the tangent line to the curve $y = 3x + 6\cos x$ at the point $(\pi/3, \pi + 3)$ is

$$\begin{aligned}m &= \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} \\ &= 3 - 6\sin\frac{\pi}{3} \\ &= 3 - 6\cdot\frac{\sqrt{3}}{2} \\ &= 3 - 3\sqrt{3} \\ &= 3(1 - \sqrt{3})\end{aligned}$$

Equation of the tangent line, to the curve $y = f(x)$ at the point (x_1, y_1) have slope m is

$$y - y_1 = m(x - x_1)$$

Therefore, equation of the tangent line to the curve $y = 3x + 6\cos x$ have slope $m = 3(1-\sqrt{3})$ at the point $(\pi/3, \pi+3)$ is

$$y - (\pi+3) = 3(1-\sqrt{3})(x - \pi/3)$$

$$y - (\pi+3) = 3(1-\sqrt{3})\left(\frac{3x-\pi}{3}\right)$$

$$y - (\pi+3) = (1-\sqrt{3})(3x-\pi)$$

$$(1-\sqrt{3})3x - y + (\pi+3) - (1-\sqrt{3})\pi = 0$$

Continuation to the above

$$(1-\sqrt{3})3x - y + (\pi+3) - (1-\sqrt{3})\pi = 0$$

$$3(1-\sqrt{3})x - y + 3 + \sqrt{3}\pi = 0$$

$$3(1-1.732)x - y + 3 + (1.732)(3.141) = 0$$

$$3(-0.732)x - y + 3 + 5.44 = 0$$

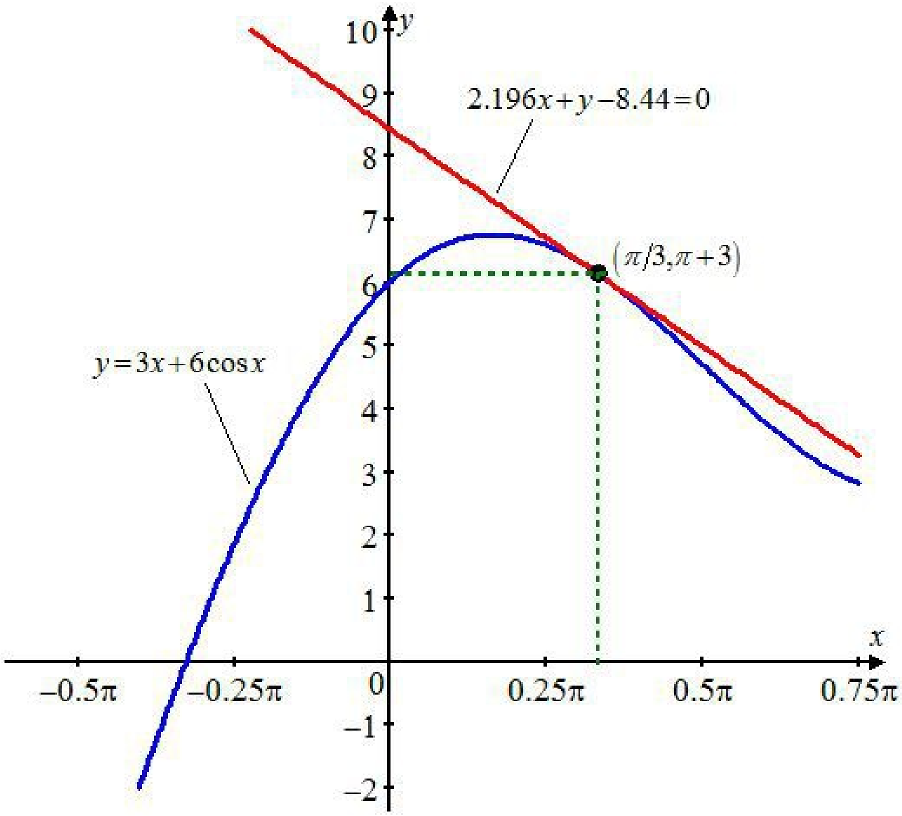
$$-2.196x - y + 8.44 = 0$$

$$2.196x + y - 8.44 = 0$$

Hence, equation of the tangent line is $\boxed{2.196x + y - 8.44 = 0}$

(b)

Graph of the tangent line to the given curve at the given point is shown as follows.



Chapter 2 Derivatives Exercise 2.4 27E

Consider the function $f(x)=\sec x-x$

(a)

Need to find differentiate the function.

Differentiate with respect to x

$$\frac{d}{dx}f(x)=\frac{d}{dx}(\sec x-x)$$

$$\frac{d}{dx}f(x)=\frac{d}{dx}(\sec x)-\frac{d}{dx}(x)$$

Since $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

Continuation to the above steps,

$$f'(x)=\sec x\tan x-1$$

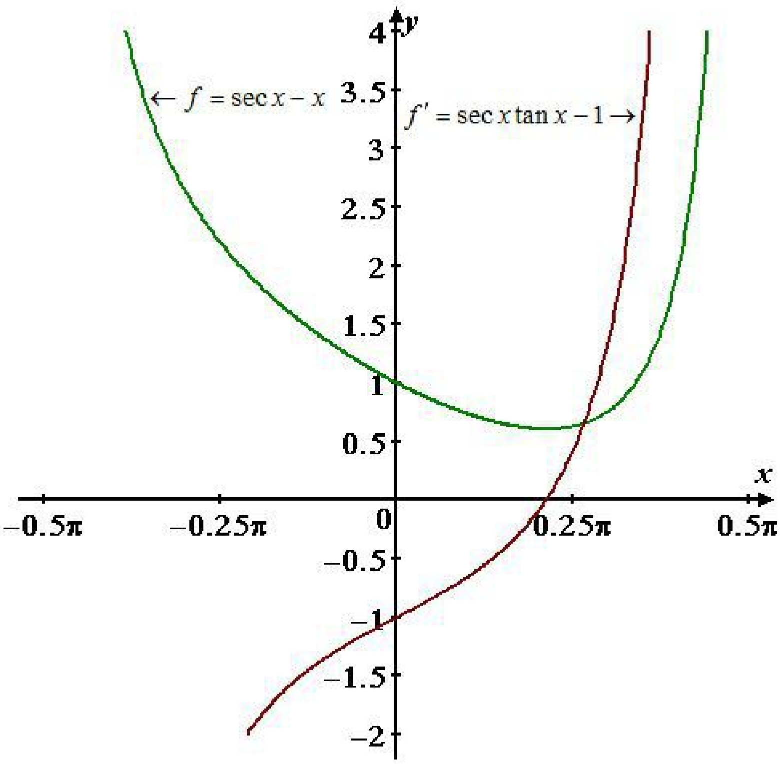
Since $\frac{d}{dx}(\sec x)=\sec x\tan x, \frac{d}{dx}(x)=1$

Therefore,

$$f'(x)=\boxed{\sec x\tan x-1}.$$

(b)

Graph of the functions $f(x)$ and $f'(x)$ for $|x|<\frac{\pi}{2}$



Chapter 2 Derivatives Exercise 2.4 28E

(A)

We have $f(x)=\sqrt{x}\sin x$

Then $f'(x)=\frac{d}{dx}\left(\sqrt{x}.\sin x\right)$

By the product law we have $\frac{d}{dx}(f(x).g(x))=f'(x).g(x)+g'(x)f(x)$

So $\quad f'(x) = \sin x.\frac{d}{dx}\sqrt{x} + \sqrt{x}.\frac{d}{dx}\sin x$

$$= \frac{1}{2}\sin x.\frac{1}{\sqrt{x}} + \sqrt{x}.\cos x$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x}$$

(B)

We draw the graph of $f(x) = \sqrt{x}\sin x$ and $f'(x)$ on the same screen we see that there is no tangent can be at $x = 0$ so here $f'(x)$ is not defined and where the slope of tangent is positive $f'(x)$ is also positive, for negative slope $f'(x)$ is also negative. This verifies that our answer in part (A) is reasonable

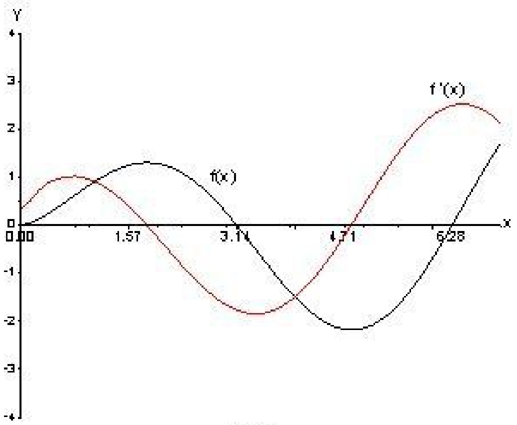


Fig.1

Chapter 2 Derivatives Exercise 2.4 29E

Product Rule: If $u(x)$ and $v(x)$ are differentiable, then

$$\frac{d}{dx}[u(x)v(x)] = u(x)\frac{d}{dx}[v(x)] + v(x)\frac{d}{dx}[u(x)]$$

Consider the function:

$$H(\theta) = \theta\sin\theta$$

Apply the product rule to differentiate the above function.

$$\begin{aligned} H'(\theta) &= \theta\frac{d(\sin\theta)}{d\theta} + \sin\theta\frac{d(\theta)}{d\theta} \\ &= \theta\cos\theta + \sin\theta \end{aligned}$$

Now, again differentiate the function.

$$\begin{aligned} H''(\theta) &= \theta\frac{d(\cos\theta)}{d\theta} + \cos\theta\frac{d(\theta)}{d\theta} + \frac{d(\sin\theta)}{d\theta} \\ &= \theta(-\sin\theta) + \cos\theta + \cos\theta \\ &= -\theta\sin\theta + 2\cos\theta \end{aligned}$$

Hence,

$$\boxed{\begin{aligned} H'(\theta) &= \theta\cos\theta + \sin\theta \\ H''(\theta) &= -\theta\sin\theta + 2\cos\theta \end{aligned}}$$

Chapter 2 Derivatives Exercise 2.4 30E

Let $f(t) = \csc t$

Then $f'(t) = \frac{d}{dt}\csc t$

$= -\csc t \cot t$

$f''(t) = \frac{d}{dt} f'(t)$

$= \frac{d}{dt}(-\csc t \cot t)$

$= -\left[\left(\frac{d}{dt}\cot t\right)\csc t + \cot t \frac{d}{dt}\csc t\right]$

$= -\left[\left(-\csc^2 t\right)\csc t + \cot t\left(-\csc t \cot t\right)\right]$

$= \csc^3 t + \csc t \cot^2 t$

$\therefore\ f''\left(\frac{\pi}{6}\right) = \csc^3\left(\frac{\pi}{6}\right) + \csc\left(\frac{\pi}{6}\right)\cot^2\left(\frac{\pi}{6}\right)$

$= \frac{1}{(0.5)^3} + \frac{1}{0.5}\left(\sqrt{3}\right)^2$

$= 8 + (2)\left(\sqrt{3}\right)^2$

$= 8 + 2(3)$

$= 8 + 6$

$= 14$

$\boxed{\therefore\ f''\left(\frac{\pi}{6}\right) = 14}$

Chapter 2 Derivatives Exercise 2.4 31E

Consider the function $f(x) = \frac{\tan x - 1}{\sec x}$

(a)

Need to find differentiate the function.

Differentiate with respect to x

$f(x) = \frac{\tan x - 1}{\sec x}$

Let $u = \tan x - 1, v = \sec x$

$$f'(x) = \frac{(\sec x)\frac{d}{dx}(\tan x - 1) - (\tan x - 1)\frac{d}{dx}(\sec x)}{(\sec x)^2}$$

Since $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$

Continuation to the above steps,

$$=\frac{(\sec x)(\sec^2 x-0)-(\tan x-1)(\sec x\tan x)}{(\sec x)^2}$$

Since $\frac{d}{dx}(\tan x)=\sec^2 x, \frac{d}{dx}(\sec x)=\sec x\tan x$

$$=\frac{(\sec x)(\sec^2 x)-(\tan x-1)(\sec x\tan x)}{\sec^2 x}$$

$$=\frac{(\sec x)(\sec^2 x-\tan x(\tan x-1))}{\sec^2 x}$$

$$=\frac{(\sec^2 x-\tan^2 x+\tan x)}{\sec x}$$

$$=\frac{(1+\tan x)}{\sec x} \qquad \text{since } \sec^2 x-\tan^2 x=1$$

Therefore,

$$f'(x)=\boxed{\frac{(1+\tan x)}{\sec x}}.$$

(b)

Consider the function $f(x)=\frac{\tan x-1}{\sec x}$

Need to simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, then find $f'(x)$.

$$f(x)=\frac{\left(\frac{\sin x}{\cos x}\right)-1}{\left(\frac{1}{\cos x}\right)} \quad \text{Since } \tan x=\frac{\sin x}{\cos x}, \sec x=\frac{1}{\cos x}$$

$$f(x)=\frac{\left(\frac{\sin x}{\cos x}\right)-\left(\frac{\cos x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)} \quad \text{Since } \frac{\cos x}{\cos x}=1$$

$$f(x)=\frac{\left(\frac{\sin x-\cos x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)}$$

$$f(x)=\sin x-\cos x$$

Need to find differentiate the function.

Differentiate with respect to x

$$f(x)=\sin x-\cos x$$

$$f'(x)=\cos x-(-\sin x) \quad \text{Since } \frac{d}{dx}(\sin x)=\cos x, \frac{d}{dx}(\cos x)=-\sin x$$

$$f'(x)=\cos x+\sin x$$

Therefore,

$$f'(x)=\boxed{\cos x+\sin x}.$$

(c)

Need to prove parts (a) and (b) are equivalent.

From part (a)

$$f'(x)=\frac{(1+\tan x)}{\sec x}$$

Need to simplify the expression for $f'(x)$ by writing it in terms of $\sin x$ and $\cos x$

$$f'(x)=\frac{\left(1+\left(\frac{\sin x}{\cos x}\right)\right)}{\left(\frac{1}{\cos x}\right)}$$ Since $\tan x=\frac{\sin x}{\cos x}, \sec x=\frac{1}{\cos x}$

$$f'(x)=\frac{\left(\left(\frac{\cos x}{\cos x}\right)+\left(\frac{\sin x}{\cos x}\right)\right)}{\left(\frac{1}{\cos x}\right)}$$ Since $\frac{\cos x}{\cos x}=1$

$$f'(x)=\frac{\left(\frac{\cos x+\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)}$$

$$f'(x)=\cos x+\sin x$$

Hence, $\boxed{\text{parts (a) and (b) are equivalent}}$.

Chapter 2 Derivatives Exercise 2.4 32E

Given $f\left(\frac{\pi}{3}\right)=4, f'\left(\frac{\pi}{3}\right)=-2$

$g(x)=f(x)\sin x$ and $h(x)=\frac{\cos x}{f(x)}$

(a) $g'(\pi/3)=\frac{d}{dx}(g(x))$ at $x=\pi/3$

$$\begin{aligned} g'(x) &= \frac{d}{dx}(f(x)\sin x) \\ &= f(x)\frac{d}{dx}(\sin x)+\sin x f'(x) \\ &= f(x)\cos x+\sin x f'(x) \end{aligned}$$

$$\begin{aligned} g'(\pi/3) &= f\left(\frac{\pi}{3}\right)+\cos\frac{\pi}{3}+\sin\frac{\pi}{3}f'(\pi/3) \\ &= 4\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{2}(-2) \\ &= \boxed{2-\sqrt{3}} \end{aligned}$$

(b) $h'(\pi/3)$

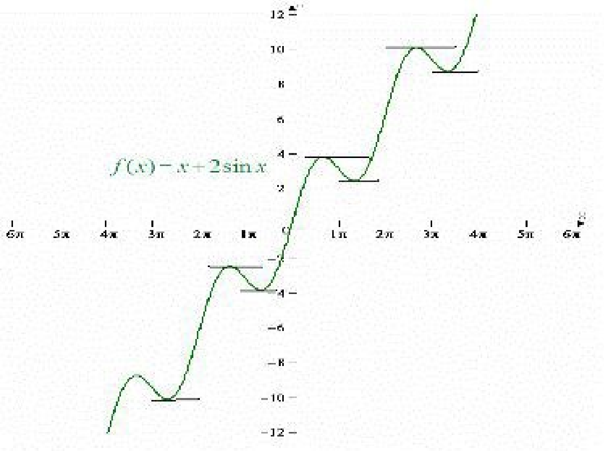
$$\begin{aligned} h'(x) &= \frac{d}{dx}\left(\frac{\cos x}{f(x)}\right) \\ &= \frac{f(x)\frac{d}{dx}(\cos x)-\cos x\frac{d}{dx}(f(x))}{(f(x))^2} \\ &= \frac{f(x)(-\sin x)-\cos x f'(x)}{(f(x))^2} \end{aligned}$$

$$h'(\pi/3) = \frac{f(\pi/3)(-\sin\pi/3) - \left(\cos\frac{\pi}{3}\right)f'(\pi/3)}{\left(f(\pi/3)\right)^2}$$

$$= \frac{4\left(-\sqrt{3}/2\right) - \frac{1}{2}(-2)}{16}$$

$$= \boxed{\frac{-2\sqrt{3}+1}{16}}$$

Chapter 2 Derivatives Exercise 2.4 33E



We have $f(x) = x + 2\sin x$

Differentiating with respect to x

$$f'(x) = 1 + 2\cos x$$

For horizontal tangents, we must have

$$f'(x) = 0$$

So $1 + 2\cos x = 0$

$$\Rightarrow \quad \cos x = -\frac{1}{2}$$

$$\Rightarrow \quad x = \frac{2\pi}{3}$$

Or $\boxed{x = 2n\pi + \frac{2\pi}{3}}$

Or we can write $\boxed{x = (2n+1)\pi \pm \frac{\pi}{3}}$ Where n is an integer

Chapter 2 Derivatives Exercise 2.4 34E

The equation of the curve $y = \frac{(\cos x)}{(2+\sin x)}$

The slope of the tangent at x will be the derivative of y with respect to x then slope of the curve

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(\cos x)}{(2+\sin x)}\right]$$

$\Rightarrow$By using Quotient rule

$$\frac{dy}{dx} = \frac{(2+\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(2+\sin x)}{(2+\sin x)^2}$$

$$= \frac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2}$$

$$\left[\frac{d}{dx}\cos x = -\sin\right]$$

$$\left[\frac{d}{dx}\sin x = \cos x\right]$$

$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - (\sin^2 x + \cos^2 x)}{(2+\sin x)^2}$$

We have $\sin^2 A + \cos^2 A = 1$

Then $\boxed{\frac{dy}{dx} = \frac{-2\sin x - 1}{(2+\sin x)^2}}$

Now the condition is that the tangent is horizontal so slope of the tangent $= 0$

Thus $\frac{dy}{dx} = 0$

$$\Rightarrow \frac{-2\sin x - 1}{(2+\sin x)^2} = 0$$

$\Rightarrow -2\sin x - 1 = 0$
$\Rightarrow -2\sin x = 1$
$\Rightarrow \sin x = -\frac{1}{2}$

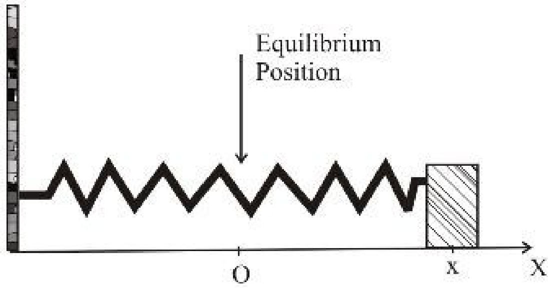
This gives $\Rightarrow x = \ldots\ldots, \frac{-17\pi}{6}, \frac{-13\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \ldots\ldots$

So the points are

$$\ldots\ldots, \left(\frac{-13\pi}{6}, \frac{1}{\sqrt{3}}\right), \left(\frac{-5\pi}{6}, -\frac{1}{\sqrt{3}}\right), \left(\frac{-\pi}{6}, \frac{1}{\sqrt{3}}\right), \left(\frac{7\pi}{6}, -\frac{1}{\sqrt{3}}\right), \left(\frac{11\pi}{6}, \frac{1}{\sqrt{3}}\right), \ldots\ldots$$

at which the tangents of the curve $y = \frac{(\cos x)}{(2+\sin x)}$ are horizontal.

Chapter 2 Derivatives Exercise 2.4 35E



(A) The equation of motion is $x(t) = 8\sin t$

The velocity of the mass at time t is

$$x'(t) = \frac{d(x(t))}{t} = \frac{d}{dt}(8\sin t)$$

$$= 8\frac{d}{dt}\sin t$$

$$= 8\cos t \qquad * \boxed{\frac{d}{dx}\sin x = \cos x}$$

Then velocity $\boxed{= 8\cos t}$ cm/Sec

(B)

Position of the mass at $t = \frac{2\pi}{3}$ is

$$x\left(\frac{2\pi}{3}\right) = 8\sin\left(\frac{2\pi}{3}\right) \qquad \left(\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}\right)$$

$$= 8.\frac{\sqrt{3}}{2}$$

$$\Rightarrow \boxed{x\left(\frac{2\pi}{3}\right) = 4\sqrt{3}} \text{ cm}$$

And the velocity of the mass at $t = \frac{2\pi}{3}$

$$x'\left(\frac{2\pi}{3}\right) = 8\cos\left(\frac{2\pi}{3}\right) \qquad \left(\cos\frac{2\pi}{3} = -\frac{1}{2}\right)$$

$$= 8(-\frac{1}{2})$$

$$\Rightarrow \boxed{\text{Velocity} = -4 \text{ cm/s}} \text{ to the left.}$$

Chapter 2 Derivatives Exercise 2.4 36E

Equation of the motion $s = 2\cos t + 3\sin t$

(a)

Find the velocity at time t

Recollect that the velocity at time t is $v(t) = \frac{ds}{dt}$

Equation of the motion $s = 2\cos t + 3\sin t$.

Differentiate with respect to t on both sides to get the following:

$\frac{ds}{dt} = -2\sin t + 3\cos t$ Since $\frac{d}{dt}(\cos t) = -\sin t, \frac{d}{dt}(\sin t) = \cos t$

Therefore, the velocity at time t is $v(t) = \boxed{-2\sin t + 3\cos t}$.

Find the acceleration at time t

Recollect that the acceleration at time t is the following:

$$\begin{aligned} s(t) &= \frac{d}{dt}(v) \\ &= \frac{d}{dt}\left(\frac{ds}{dt}\right) \quad \text{since } v = \frac{ds}{dt} \\ &= \frac{d^2s}{dt^2} \end{aligned}$$

The velocity at time t is $\frac{ds}{dt} = -2\sin t + 3\cos t$

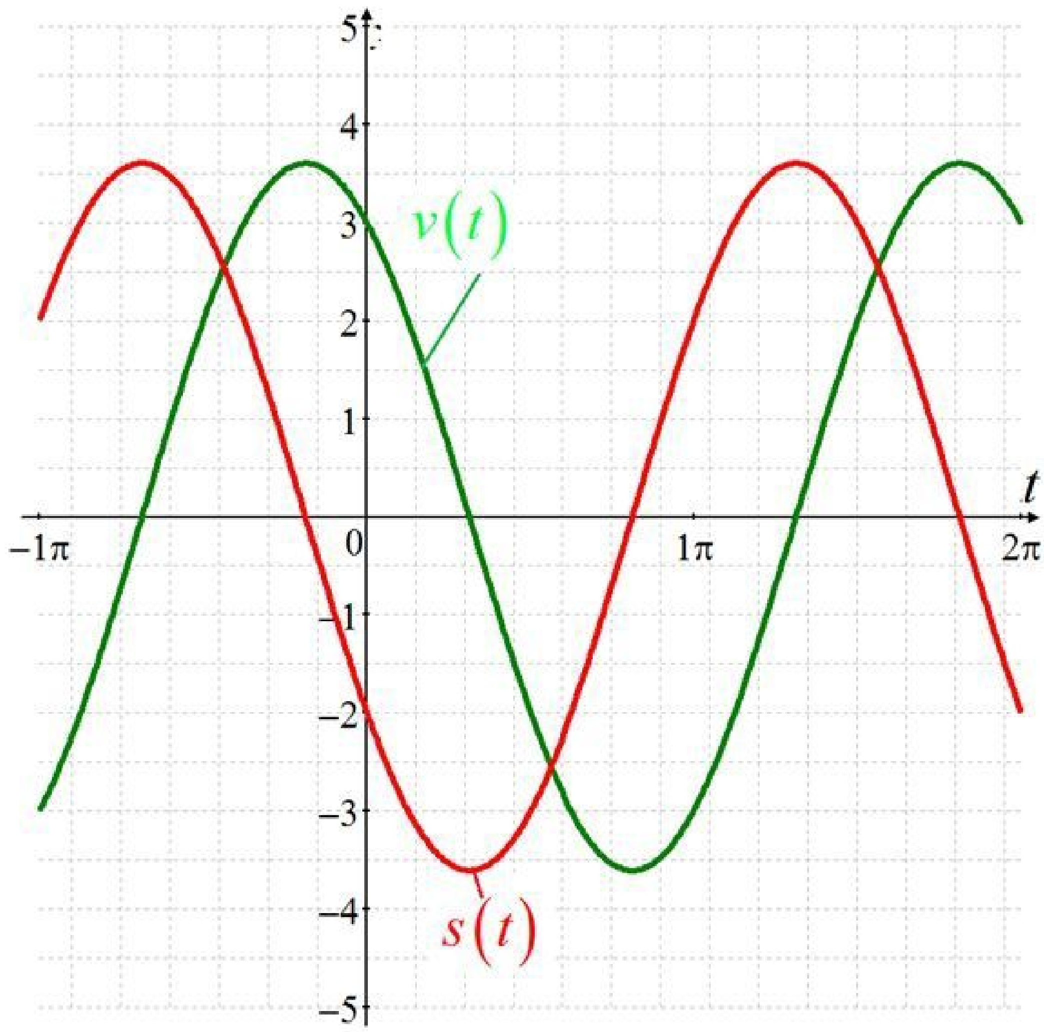
Differentiate with respect to t on both sides to get the following:

$\frac{d^2s}{dt^2} = -2\cos t - 3\sin t$ Since $\frac{d}{dt}(\cos t) = -\sin t, \frac{d}{dt}(\sin t) = \cos t$

Therefore, the acceleration at time t is $s(t) = \boxed{-2\cos t - 3\sin t}$.

(b)

Graph the velocity and acceleration functions.



(c)

Find the mass pass through the equilibrium position for the first time.

At equilibrium, $s(t)=0$

$-2\cos t-3\sin t=0$

$-3\sin t=2\cos t$ Add on both sides with $2\cos t$

$\sin t=-\frac{2}{3}\cos t$ Divide on both sides by -3

$\tan t=-\frac{2}{3}$ Divide on both sides by $\cos t$

$\tan^{-1}(\tan t)=\tan^{-1}\left(-\frac{2}{3}\right)$ Take $\tan^{-1}$ on both sides

$t=\tan^{-1}\left(-\frac{2}{3}\right)$ Since $\tan^{-1}(\tan t)=t$

$t=\pi-\tan^{-1}\left(\frac{2}{3}\right)$ Since $\tan^{-1}(-x)=\pi-\tan^{-1}(x)$

$t=3.142-0.58$ Since $\tan^{-1}\left(\frac{2}{3}\right)=0.58$

$t\approx 2.56$

Therefore, the mass pass through the equilibrium position for the first time is $\boxed{2.56\text{seconds}}$

(d)

At extreme position, $v(t)=0$

$-2\sin t+3\cos t=0$

$-2\sin t=-3\cos t$ Subtract on both sides with $3\cos t$

$\sin t=\frac{3}{2}\cos t$ Divide on both sides by -2

$\tan t=\frac{3}{2}$ Divide on both sides by $\cos t$

By the right triangle with base 2 and altitude 3

Recollect the Pythagoras theorem.

$(\text{hyp})^2=(\text{side})^2+(\text{side})^2$

$(\text{hyp})^2=(2)^2+(3)^2$

$(\text{hyp})^2=4+9$

$(\text{hyp})^2=13$

$\text{hyp}=\sqrt{13}$ Take root on both sides

$$\cos t=\frac{\text{adj}}{\text{hyp}}$$
$$=\frac{2}{\sqrt{13}}$$

And

$$\sin t=\frac{\text{opp}}{\text{hyp}}$$
$$=\frac{3}{\sqrt{13}}$$

Substitute these values in $s=2\cos t+3\sin t$

$$s=2\left(\frac{2}{\sqrt{13}}\right)+3\left(\frac{3}{\sqrt{13}}\right)$$

$$s=\left(\frac{4}{\sqrt{13}}\right)+\left(\frac{9}{\sqrt{13}}\right)$$

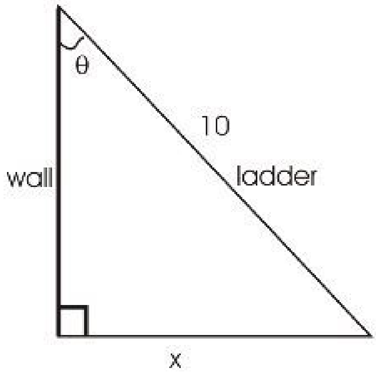
$$s=\frac{13}{\sqrt{13}}$$

$s=\frac{13}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}$ Rationalize with $\sqrt{13}$

$$s=\sqrt{13}$$

$$s\approx\boxed{3.6\text{ cm}}.$$

Chapter 2 Derivatives Exercise 2.4 37E



From the figure,

$$x = 10\sin\theta$$

Differentiating with respect to θ

$$\Rightarrow \quad \frac{dx}{d\theta} = 10\cos\theta$$

When $\theta = \pi/3$

Then $\frac{dx}{d\theta} = 10\cos(\pi/3)$

$$= \boxed{5 \text{ ft/rad}}$$

Chapter 2 Derivatives Exercise 2.4 38E

Magnitude of the force is,

$$F = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

a)

To find the rate of change of F with respect θ, differentiate the function F with respect to θ

$$F = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

$$\frac{dF}{d\theta} = \frac{d}{d\theta}\left(\frac{\mu W}{\mu\sin\theta + \cos\theta}\right)$$

$$= \mu W \frac{d}{d\theta}\left(\frac{1}{\mu\sin\theta + \cos\theta}\right)$$

$$= \mu W\left[-\frac{1}{(\mu\sin\theta + \cos\theta)^2}\right]\cdot\frac{d}{d\theta}(\mu\sin\theta + \cos\theta)$$

$$\left[\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2} \text{ and use chain rule}\right]$$

Simplify further.

$$= \mu W\left[-\frac{1}{(\mu\sin\theta + \cos\theta)^2}\right](\mu\cos\theta - \sin\theta)$$

$$= \frac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

$$= \frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

Therefore rate of change of F with respect θ is,

$$\frac{dF}{d\theta} = \boxed{\frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2}}$$

b)

To find when the rate of change equal to 0, set $\frac{dF}{d\theta}=0$ and solve for θ

$$\begin{aligned}\frac{dF}{d\theta}&=0\\ \frac{\mu W(\sin\theta-\mu\cos\theta)}{(\mu\sin\theta+\cos\theta)^2}&=0\\ \mu W(\sin\theta-\mu\cos\theta)&=0\\ (\sin\theta-\mu\cos\theta)&=0\\ \sin\theta&=\mu\cos\theta\\ \tan\theta&=\mu\end{aligned}$$

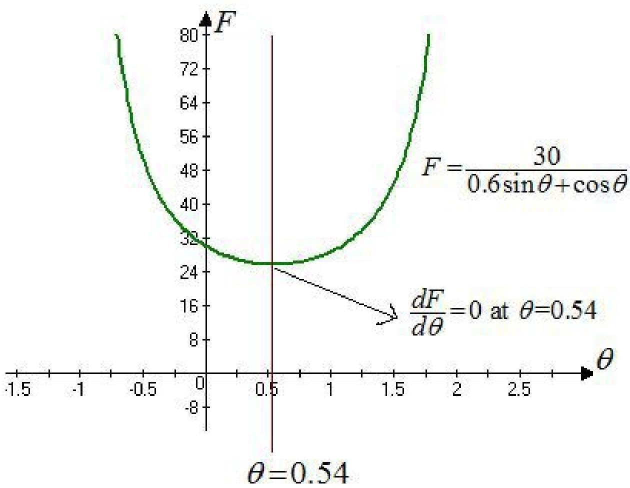
Or $\theta=\tan^{-1}\mu$

Therefore the rate of change is equal to zero when $\theta=\boxed{\tan^{-1}\mu}$

c)

Recall that extreme values for a function occur at the points where the derivative is zero.

Take $W=50,\mu=0.6$ and sketch the graph of F



Observe from the graph, minimum value is occurring at $\theta=0.54$, so $\frac{dF}{d\theta}=0$ for $\theta=0.54$

From part (b), $\frac{dF}{d\theta}=0$ for $\theta=\tan^{-1}\mu$

Set $\mu=0.6$ and find θ

$$\begin{aligned}\theta&=\tan^{-1}\mu\\ &=\tan^{-1}(0.6)\\ &\approx 0.54\end{aligned}$$

Therefore, from the graph we observed that $\frac{dF}{d\theta}=0$ for $\theta=0.54$ and setting $\mu=0.6$ in part (b), we obtained $\theta=0.54$ where $\frac{dF}{d\theta}=0$

Therefore the conclusion of part (c) is in agreement with that of part (b)

Chapter 2 Derivatives Exercise 2.4 39E

Consider the limit:

$$\lim_{x\to 0}\frac{\sin 3x}{x}.$$

The objective is to evaluate the limit.

Multiply numerator and denominator by 3 in order to put the function in a form in which we can use the following limit.

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1 \;\ldots\ldots (1)$$

The function becomes,

$$\begin{aligned}\frac{\sin 3x}{x} &= \frac{3}{3}\cdot\frac{\sin 3x}{x}\\ &= 3\cdot\frac{\sin 3x}{3x}\end{aligned}$$

If we let $\theta = 3x$ then $\theta\to 0$ as $x\to 0$.

$$\begin{aligned}\lim_{x\to 0}\frac{\sin 3x}{x} &= \lim_{x\to 0}3\times\frac{\sin 3x}{3x} && \text{Divide and Multiply with 3}\\ &= 3\lim_{3x\to 0}\frac{\sin 3x}{3x} && [\text{as } x\to 0, 3x\to 0]\\ &= 3\lim_{\theta\to 0}\frac{\sin\theta}{\theta} && [\text{as } x\to 0, \theta\to 0]\\ &= 3\times 1 && \text{from } (1)\\ &= 3\end{aligned}$$

Hence the result is $\boxed{3}$.

Chapter 2 Derivatives Exercise 2.4 40E

$$\lim_{x\to 0}\frac{\sin 4x}{\sin 6x}$$

$$= \lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{4x}{6x}\times\frac{6x}{\sin 6x}$$

$$= 1\times\frac{4}{6}\times 1 \qquad \left(\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$$

$$= \frac{2}{3}$$

Chapter 2 Derivatives Exercise 2.4 41E

Consider the limit $\lim_{t\to 0}\frac{\tan 6t}{\sin 2t}$.

The objective is to evaluate the limit.

Recollect the following result:

$$\lim_{x\to 0}\frac{\sin x}{x}=1. \;\cdots\cdots(1)$$

Use the above result to find the given limit as follows:

$$\lim_{t\to 0}\frac{\tan 6t}{\sin 2t}=\lim_{t\to 0}\left[\tan 6t\cdot\frac{1}{\sin 2t}\right]$$
$$=\lim_{t\to 0}\left[\frac{\sin 6t}{\cos 6t}\cdot\frac{1}{\sin 2t}\right]$$
$$=\lim_{t\to 0}\left[\frac{\sin 6t}{6t}\times 6t\cdot\frac{1}{\cos 6t}\cdot\frac{1}{\frac{\sin 2t}{2t}\times 2t}\right]$$
$$=\lim_{t\to 0}\left[\frac{\sin 6t}{6t}\cdot\frac{1}{\cos 6t}\cdot\frac{1}{\frac{\sin 2t}{2t}}\times\frac{6\cancel{t}}{2\cancel{t}}\right]$$
$$=\lim_{6t\to 0}\frac{\sin 6t}{6t}\cdot\lim_{6t\to 0}\frac{1}{\cos 6t}\cdot\frac{1}{\lim_{2t\to 0}\frac{\sin 2t}{2t}}\times 3 \text{ Since } 6t\to 0 \text{ and } 2t\to 0 \text{ as } t\to 0.$$
$$=1\cdot\frac{1}{\cos 0}\cdot\frac{1}{1}\times 3 \qquad \text{Use equation } (1).$$
$$=1\cdot\frac{1}{1}\cdot\frac{1}{1}\times 3$$
$$=3$$

Thus, the value of the limit is $\lim_{t\to 0}\frac{\tan 6t}{\sin 2t}=\boxed{3}$.

Chapter 2 Derivatives Exercise 2.4 42E

Consider the following limit,

$$\lim_{\theta\to 0}\frac{\cos\theta-1}{\sin\theta}$$

The objective is to evaluate the limit.

Multiply numerator and denominator by θ in order to put the function in a form in which we can use the following limits.

$$\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}=0$$
$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

Therefore, the value of the limit is,

$$\lim_{\theta\to 0}\frac{\cos\theta-1}{\sin\theta}=\lim_{\theta\to 0}\left(\frac{\cos\theta-1}{\sin\theta}\cdot\frac{\theta}{\theta}\right)$$
$$=\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}\cdot\lim_{\theta\to 0}\frac{\theta}{\sin\theta}$$
$$=\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}\cdot\frac{1}{\lim_{\theta\to 0}\frac{\sin\theta}{\theta}}$$
$$=(0)\cdot(1)$$
$$=\boxed{0}$$

Chapter 2 Derivatives Exercise 2.4 43E

$$\lim_{x\to 0} \frac{\sin 3x}{5x^3-4x}$$
$$= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x}.3x}{5x^3-4x}$$
$$= \lim_{x\to 0} \frac{\sin 3x}{3x}. \lim_{x\to 0} \frac{3x}{5x^3-4x}$$
$$= \lim_{3x\to 0} \frac{\sin 3x}{3x}. \lim_{x\to 0} \frac{3}{5x^2-4}$$
$$= 1.\frac{-3}{4}$$
$$= \frac{-3}{4}$$
$$\boxed{\therefore \lim_{x\to 0} \frac{\sin 3x}{5x^3-4x} = \frac{-3}{4}}$$

Chapter 2 Derivatives Exercise 2.4 44E

$$\lim_{x\to 0} \frac{\sin 3x \sin 5x}{x^2}$$
$$= \lim_{x\to 0} \frac{\sin 3x}{3x}.3 \lim_{x\to 0} \frac{\sin 5x}{5x}.5$$
$$= 1.3.5$$
$$= 15$$
$$\boxed{\therefore \lim_{x\to 0} \frac{\sin 3x \sin 5x}{x^2} = 15}$$

Chapter 2 Derivatives Exercise 2.4 45E

We have to evaluate $\lim_{\theta\to 0} \frac{\sin\theta}{\theta+\tan\theta}$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta+\tan\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta+\frac{\sin\theta}{\cos\theta}}$$
$$= \lim_{\theta\to 0} \frac{\sin\theta}{\frac{\theta\cos\theta+\sin\theta}{\cos\theta}}$$
$$= \lim_{\theta\to 0} \frac{\sin\theta\cos\theta}{\theta.\cos\theta+\sin\theta}$$
$$= \lim_{\theta\to 0}\left[\frac{\sin\theta}{\theta}\cdot\frac{\cos\theta}{\left(\cos\theta+\frac{\sin\theta}{\theta}\right)}\right]$$
$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\frac{\lim_{\theta\to 0}\cos\theta}{\left(\lim_{\theta\to 0}\cos\theta+\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\right)}$$
$$= 1\cdot\frac{\cos 0}{(\cos 0+1)}$$
$$= 1\times\frac{1}{(1+1)}$$

Thus $\boxed{\lim_{\theta\to 0}\frac{\sin\theta}{\theta+\tan\theta} = 1/2}$

Chapter 2 Derivatives Exercise 2.4 46E

Find the limit, $\lim_{x\to 0}\frac{\sin(x^2)}{x}$.

Use the following formulas:

(i) $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

(ii) $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$

Consider the expression,

$$\lim_{x\to 0}\frac{\sin\left(x^2\right)}{x}=\lim_{x\to 0}\frac{\sin\left(x^2\right)}{x^2}\cdot x$$

$=\left[\lim_{x^2\to 0}\frac{\sin\left(x^2\right)}{x^2}\right]\cdot\left[\lim_{x\to 0}x\right]$ Since $x^2\to 0$ as $x\to 0$

$$=(1)(0)$$
$$=0$$

Therefore, the limit is $\lim_{x\to 0}\frac{\sin\left(x^2\right)}{x}=\boxed{0}$.

Chapter 2 Derivatives Exercise 2.4 47E

L Hospital's Rule states that suppose that and g are differentiable functions with $g'(x)\neq 0$, and suppose $f(x)\to 0$ and $g(x)\to 0$.

As $x\to c^+$, $x\to c^-$, $x\to c$, $x\to\infty$ or $x\to-\infty$

If $\frac{f'(x)}{g'(x)}\to \mathrm{L}$, then $\frac{f(x)}{g(x)}\to \mathrm{L}$

If $\frac{f'(x)}{g'(x)}\to\infty$ or $-\infty$, then $\frac{f(x)}{g(x)}\to\infty$ or $-\infty$ respectively.

Consider,

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}$$

As $x\to\frac{\pi}{4}$, both numerator and denominator tend to 0.

The above limit is of $\frac{0}{0}$ indeterminate form.

Hence, use L'Hospital's rule

Let

$$f(x)=1-\tan x$$
$$f'(x)=0-\sec^2 x$$
$$=-\sec^2 x$$

And

$$g(x)=\sin x-\cos x$$
$$g'(x)=\cos x-(-\sin x)$$
$$=\cos x+\sin x$$

Use L'Hospital's rule;

$$\frac{f'(x)}{g'(x)}=\frac{-\sec^2 x}{\cos x+\sin x}$$

Continuation to the above steps,

$$\frac{f'(x)}{g'(x)} = \frac{-\sec^2 x}{\cos x + \sin x}$$

$$= \frac{-\sec^2\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)}$$

$$= \frac{-\left(\sqrt{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{-2}{\left(\frac{2}{\sqrt{2}}\right)}$$

$$\frac{f'(x)}{g'(x)} = -2 \cdot \frac{\sqrt{2}}{2}$$

$$= -\sqrt{2}$$

It follows from L'Hospital's rule that

$$\frac{1 - \tan x}{\sin x - \cos x} \to -\sqrt{2}$$

As

$$x \to \frac{\pi}{4}$$

Therefore,

$$\boxed{\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} = -\sqrt{2}}.$$

Chapter 2 Derivatives Exercise 2.4 48E

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2}$$

We factorize the denominator

$$\Rightarrow x^2 + x - 2 = x^2 + 2x - x - 2$$

$$= x(x+2) - 1(x+2)$$

$$= (x-1)(x+2)$$

$$\lim_{x \to 1} \frac{\sin(x-1)}{(x+2)(x-1)} = \lim_{x \to 1} \frac{1}{(x+2)} \cdot \lim_{x \to 1} \frac{\sin(x-1)}{(x-1)}$$

$$\left(\because \lim_{x \to a} f(x)g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)\right)$$

Let $x - 1 = \theta$

Here $x \to 1$ then $x - 1 = \theta \to 0$

$$= \lim_{x \to 1} \frac{1}{(x+2)} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= \frac{1}{(1+2)} \cdot 1 \qquad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$$

$$= \frac{1}{3}$$

$$\Rightarrow \boxed{\lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2} = \frac{1}{3}}$$

So we have

$$\lim_{x \to 1} \frac{\sin(x-1)}{(x-1)} = \lim_{x-1 \to 0} \frac{\sin(x-1)}{(x-1)}$$

If we compare this with

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \text{ Then } \theta = x - 1$$

So we have

$$\lim_{x\to1}\frac{\sin(x-1)}{(x-1)}=\lim_{x-1\to0}\frac{\sin(x-1)}{(x-1)}=1$$

So

$$\lim_{x\to1}\frac{\sin(x-1)}{x^2+x-2}=\lim_{x\to1}\frac{1}{(x+2)}.1$$

$$=\frac{1}{(1+2)}$$

$$=\frac{1}{3}$$

$$\Rightarrow\boxed{\lim_{x\to1}\frac{\sin(x-1)}{x^2+x-2}=\frac{1}{3}}$$

Chapter 2 Derivatives Exercise 2.4 49E

In order to find the 99th derivative, consider the function.

$$f(x)=\sin x$$

To determine first few derivatives of the function $f(x)=\sin x$, use following formulas.

$$\frac{d}{dx}(\cos x)=-\sin x \text{ and } \frac{d}{dx}(\sin x)=\cos x$$

$$f'(x)=\cos x$$
$$f''(x)=-\sin x$$
$$f'''(x)=-\cos x$$
$$f^{(4)}(x)=\sin x$$
$$f^{(5)}(x)=\cos x$$

Notice the pattern that a successive derivative occurs in a cycle of length 4. The successive derivative $f^{(n)}(x)=\sin x$ occurs when n is a multiple of 4.

The number 96 is the nearer to 99 which is a multiple of 4.
Therefore, it can be concluded that $f^{(96)}(x)$ will also be equal to $\sin x$.

$$f^{(96)}(x)=\sin x$$

To obtain the 99th derivative of $f(x)$, differentiate three more times.

$$f^{97}(x)=\cos x$$
$$f^{98}(x)=-\sin x$$
$$f^{99}(x)=-\cos x$$

Therefore, the 99th derivative of $\sin x$ is,

$$\boxed{\frac{d^{99}}{dx^{99}}(\sin x)=-\cos x}.$$

Chapter 2 Derivatives Exercise 2.4 50E

Given function is $x\sin x$

$$\frac{d}{dx}(x\sin x)=x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x)$$
$$=x\cos x+\sin x$$

$$\frac{d^2}{dx^2}(x\sin x)=\frac{d}{dx}(x\cos x+\sin x)$$
$$=\frac{d}{dx}(x\cos x)+\frac{d}{dx}(\sin x)$$
$$=x(-\sin x)+\cos x+\cos x$$
$$=2\cos x-x\sin x$$

$$\frac{d^3}{dx^3}(x\sin x)=\frac{d}{dx}(2\cos x-x\sin x)$$
$$=2(-\sin x)-\frac{d}{dx}(x\sin x)$$
$$=-2\sin x-(x\cos x+\sin x)$$
$$=-3\sin x-x\cos x$$

$$\frac{d^4}{dx^4}(x\sin x)=\frac{d}{dx}(-3\sin x-x\cos x)$$
$$=-3\cos x-\left[x(-\sin x)+\cos x\right]$$
$$=-4\cos x+x\sin x$$

$$\frac{d^5}{dx^5}(x\sin x)=\frac{d}{dx}(-4\cos x+x\sin x)$$
$$=4\sin x+x\cos x+\sin x$$
$$=5\sin x+x\cos x$$

We continue like this we get
$$\frac{d^{35}}{dx^{35}}(x\sin x)=\boxed{-35\sin x-x\cos x}$$

Chapter 2 Derivatives Exercise 2.4 51E

Given function is $y=A\sin x+B\cos x$

Then differentiating both sides w.r.t x, we get $\frac{dy}{dx}=y'=A\cos x-B\sin x$

Again differentiating both sides w.r.t x, we get $\frac{d^2y}{dx^2}=y''=-A\sin x-B\cos x$

$\therefore\ y''+y'-2y=-A\sin x-B\cos x+A\cos x-B\sin x-2\left[A\sin x+B\cos x\right]=\sin x$

$\Rightarrow y''+y'-2y=(-A-B-2A)\sin x+(-B+A-2B)\cos x=\sin x$

$\therefore\ -3A-B=1\ ,\ \ A-3B=0$

$\Rightarrow\ B=\frac{-1}{10}$ and $A=\frac{-3}{10}$

$$\boxed{\therefore\ A=\frac{-3}{10}\text{ and }B=\frac{-1}{10}}$$

Chapter 2 Derivatives Exercise 2.4 52E

Squeeze theorem: if $f(x)\le g(x)\le h(x)$ when x is near a (except possibly at a) and $\lim_{x\to a}f(x)=\lim_{x\to a}h(x)=L$, then $\lim_{x\to a}g(x)=L$

(a) we know that $\lim_{t\to 0}\frac{\sin t}{t}=1$ ...... (1) and $\lim_{x\to\infty}\frac{1}{x}=0$

So, we can write $x\to\infty=\frac{1}{x}\to 0$

Keeping this in view, consider $\lim_{x\to\infty}x\sin\frac{1}{x}$

$$=\lim_{\frac{1}{x}\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

This can otherwise be written as $\lim_{y\to 0}\frac{\sin y}{y}$ by assuming $\frac{1}{x}=y$

By (1), we get $\lim_{y\to 0}\frac{\sin y}{y}=1$

Thus, $\boxed{\lim_{x\to\infty}x\sin\frac{1}{x}=1}$

(b) We know that $-1 \le \sin t \le 1$ for any real number t

Replacing t with $\frac{1}{x}$, we get $-1 \le \sin\frac{1}{x} \le 1$

Multiplying throughout with x, we get $-x \le x\sin\frac{1}{x} \le x$

Applying limit throughout as x tends to 0, we get $\lim_{x\to 0}(-x) \le \lim_{x\to 0}\left(x\sin\frac{1}{x}\right) \le \lim_{x\to 0}(x)$

$\Rightarrow 0 \le \lim_{x\to 0}\left(x\sin\frac{1}{x}\right) \le 0$

By squeeze theorem, we get $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right) = 0$

Therefore, $\boxed{\lim_{x\to\infty} x\sin\frac{1}{x} = 1}$

Chapter 2 Derivatives Exercise 2.4 53E

(A) We have $\tan x = \frac{\sin x}{\cos x}$

Differentiating both sides of the equation with respect to x

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$\Rightarrow \quad \sec^2 x = \frac{(\cos x)\frac{d}{dx}(\sin x) - (\sin x)\frac{d}{dx}(\cos x)}{(\cos x)^2} \quad \text{[Quotient rule]}$$

$$\Rightarrow \quad \sec^2 x = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$\Rightarrow \quad \sec^2 x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

Thus, $\boxed{\sec^2 x = \frac{1}{\cos^2 x}}$ $[\cos^2 x + \sin^2 x = 1]$

(B) $\sec x = \frac{1}{\cos x}$

Or $\sec x = (\cos x)^{-1}$

Differentiating both sides of the equation with respect to x

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}(\cos x)^{-1}$$

$$\Rightarrow \quad \sec x \tan x = -(\cos x)^{-2}\frac{d}{dx}(\cos x) \quad \text{[Chain rule]}$$

$$\Rightarrow \quad \sec x \tan x = -(\cos x)^{-2}(-\sin x)$$

$$\Rightarrow \quad \boxed{\sec x \tan x = \frac{\sin x}{\cos^2 x}}$$

(C) $$\sin x+\cos x=\frac{1+\cot x}{\csc x}$$

Differentiating both sides with respect to x

$$\frac{d}{dx}(\sin x+\cos x)=\frac{d}{dx}\left(\frac{1+\cot x}{\csc x}\right)$$

$$\Rightarrow \quad (\cos x-\sin x)=\frac{(\csc x)\frac{d}{dx}(1+\cot x)-(1+\cot x)\frac{d}{dx}(\csc x)}{\csc^2 x} \quad \text{[Quotient rule]}$$

$$\Rightarrow \quad (\cos x-\sin x)=\frac{(\csc x)(-\csc^2 x)-(1+\cot x)(-\csc x\cot x)}{\csc^2 x}$$

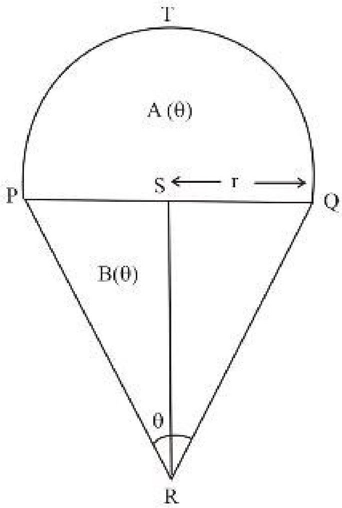
$$\Rightarrow \quad (\cos x-\sin x)=\frac{(-\csc^2 x)-(1+\cot x)(-\cot x)}{\csc x}$$

$$\Rightarrow \quad (\cos x-\sin x)=\frac{-\csc^2 x+\cot x+\cot^2 x}{\csc x}$$

$$\Rightarrow \quad (\cos x-\sin x)=\frac{-\csc^2 x+\cot x+\csc^2 x-1}{\csc x} \qquad [\text{Since} \quad \cot^2 x=\csc^2 x-1]$$

Thus, $$\boxed{\cos x-\sin x=\frac{\cot x-1}{\csc x}}$$

Chapter 2 Derivatives Exercise 2.4 54E



Let r be the radius of the semicircle PTQ and h be the height of the isosceles triangle.
Then by the property of isosceles triangle, we have "That bisector of an angle of an isosceles triangle bisects the opposite side and is perpendicular to this side"

In the figure RS is the perpendicular to PQ and PS = SQ = r (radius of semicircle),

$\angle SRQ=\angle SRP=\frac{\theta}{2}$ and RS = h

Since the triangle RSQ is a right angled triangle, we have

$$\tan\frac{\theta}{2}=\frac{QS}{SR}$$
$$=\frac{r}{h}$$
$$\Rightarrow r=h\tan\frac{\theta}{2} \text{ --- (1)}$$

Now area of triangle $PQR=B(\theta)=\frac{1}{2}$ base $\times$ height

i.e. $$B(\theta)=\frac{1}{2}PQ\times RS$$
$$=\frac{1}{2}(2r)\times h$$
$$=hr$$

i.e. $B(\theta)=h^2\tan\frac{\theta}{2}$ From equation (1)

And the area of semicircle

$$A(\theta)=\frac{1}{2}\pi r^2$$
$$=\frac{1}{2}\pi\left(h\tan\frac{\theta}{2}\right)^2$$
$$=\frac{\pi}{2}h^2\tan^2\frac{\theta}{2}$$

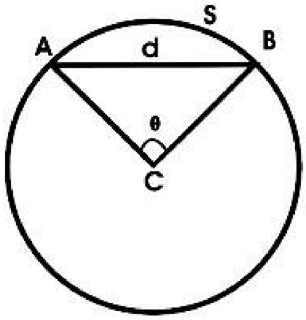
Then

$$\frac{A(\theta)}{B(\theta)}=\frac{\frac{\pi}{2}h^2\tan^2\frac{\theta}{2}}{h^2\tan\frac{\theta}{2}}$$
$$=\frac{\pi}{2}\tan\frac{\theta}{2}$$

$$\lim_{\theta\to0^+}\frac{A(\theta)}{B(\theta)}=\lim_{\theta\to0^+}\frac{\pi}{2}\tan\frac{\theta}{2}$$
$$=\frac{\pi}{2}\lim_{\theta\to0^+}\tan\frac{\theta}{2}$$
$$=0$$

Hence $\boxed{\lim_{\theta\to0^+}\frac{A(\theta)}{B(\theta)}=0}$

Chapter 2 Derivatives Exercise 2.4 55E



We know that formula

Length of the arc $=\frac{\theta}{360^0}\times 2\pi r$

Length of the arc is defined as

$$S=r\theta \qquad \text{--- (1)}$$

D is the midpoint of AB.

$\triangle CDB$ is rightangled triangle.

by apply trigonometric sinangle formula

Where r is the radius of the circle and length of the chord is defined as

$$d=2.r\sin\frac{\theta}{2} \qquad \text{--- (2)}$$

Let

$$\frac{\theta}{2}=t$$
$$\theta\to0^+ as\ t\to0^+$$

$$\lim_{\theta\to0^+}\frac{S}{d}=\lim_{t\to0^+}\frac{t}{\sin(t)}$$
$$=\lim_{t\to0^+}\frac{1}{\frac{\sin t}{t}}$$
$$=\frac{1}{1}=1 \qquad \left[\lim_{\theta\to0}\frac{\sin\theta}{\theta}=1\right]$$

$$\boxed{\lim_{\theta\to0^+}\frac{S}{d}=1}$$

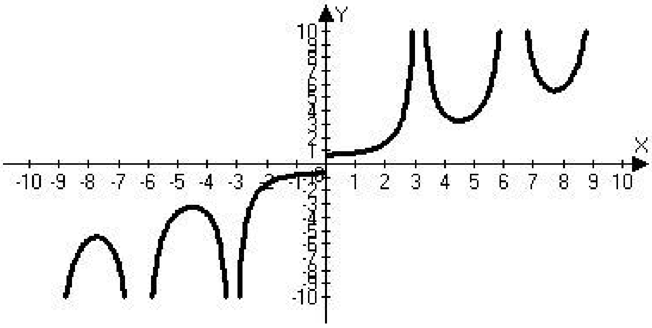
Chapter 2 Derivatives Exercise 2.4 56E

Let $f(x)=\dfrac{x}{\sqrt{1-\cos 2x}}$

Formula: $1-\cos 2x = 2\sin^2 x$

$\therefore\ f(x)=\dfrac{x}{\sqrt{1-\cos 2x}}=\dfrac{x}{\sqrt{2\sin^2 x}}=\dfrac{x}{\sqrt{2}\sin x}$

$\therefore\ f(x)=\pm\dfrac{x}{\sqrt{2}\sin x}$



(a)

From the graph left hand limit 0^- and right hand limit 0^+ exist and are finite, but not equal

$\boxed{\therefore\ 0 \text{ is a jump discontinuty or step discontinuty}}$

(b) $\displaystyle \lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{x}{\sqrt{2}\sin x}$

$\displaystyle =\lim_{h\to 0}\frac{(0+h)}{\sqrt{2}\sin(0+h)}$

$\displaystyle =\lim_{h\to 0}\frac{h}{\sqrt{2}\sin h}=\frac{1}{\sqrt{2}}$ $\qquad \left[\because \displaystyle\lim_{h\to 0}\frac{h}{\sin h}=1\right]$

$\boxed{\therefore\ \text{Right limit of } f \text{ at } 0 \text{ is } \dfrac{1}{\sqrt{2}}}$

$\displaystyle \lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{-x}{\sqrt{2}\sin x}$

$\displaystyle =\lim_{h\to 0}\frac{-(0-h)}{\sqrt{2}\sin(0-h)}$

$\displaystyle =\lim_{h\to 0}\frac{-1}{\sqrt{2}}\frac{h}{\sin h}$

$\displaystyle =\frac{-1}{\sqrt{2}}$ $\qquad \left[\because \displaystyle\lim_{h\to 0}\frac{h}{\sin h}=1\right]$

$\boxed{\begin{array}{l}\therefore\ \text{Right hand limit at } 0 \text{ is } \dfrac{1}{\sqrt{2}} \\ \text{left hand limit at } 0 \text{ is } -\dfrac{1}{\sqrt{2}}\end{array}}$

$\boxed{\text{yes, these values confirm our answer to part (a)}}$