Sample Question Paper 2020-21

Max. Marks: 80 | **Duration:3 hours**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:
1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

PART – A

Section – I

1. If α, β are the zeroes of a quadratic polynomial $f(x) = ax^2 + bx + c$, then calculate $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.
2. If $x = \sqrt{p} + \sqrt{q}$, where p and q are distinct primes then which of the following is a rational number?
   A. $(\sqrt{p} + \sqrt{q})^2$ B. $(\sqrt{p} - \sqrt{q})^2$
   C. $(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$ D. None of these

OR

Write three irrational numbers between √2 and √3

3. If the system of equations kx – 5y = 2, 6x + 2y = 7 has no solution, then find the value of k.

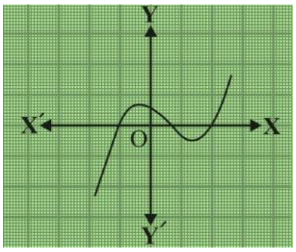
4. If angles A, B and C of a ΔABC form an AP, then ∠B =

OR

Show that the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

5. If HCF (6, a) = 2 and LCM (6, a) = 60, then calculate the value of a.

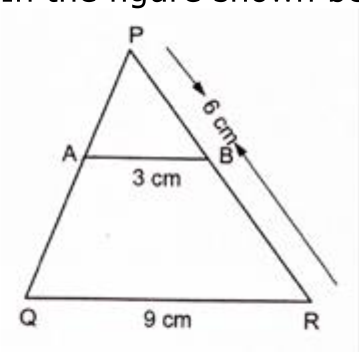
6. If the number of zeroes of the polynomial f(x) shown below is t, then find the value of $t^3 - 3t^2 + 9$.



7. If $\cos\theta = \frac{2}{3}$, then $2\sec^2\theta + 2\tan^2\theta - 7$ is equal to _____.

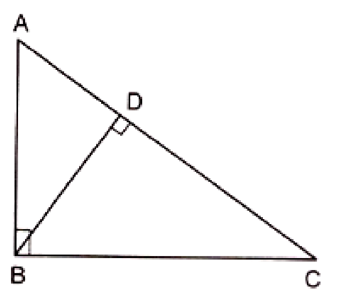
8. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is _____.

9. In the figure shown below, $AB \parallel QR$. Find the length of PB.



OR

In the figure shown below, $\angle ABC$ = 90° and $BD \perp AC$. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.



10. A die is thrown once, calculate the probability of getting an odd number.

11. The maximum number of roots possible for the polynomial $a_{100}x^{100} + a_{99}x^{99} + \dots + a_1x + a_0$ are _____.

12. The mid-point of line segment joining the points (3, 2) and (-7, -4) is ______.

OR

The distance between the points (a + b, b + c) and (a - b, c - b) is _____ units.

13. If sec θ + tan θ = 4, then prove that sec θ − tan θ = ______.

14. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠BOA is equal to _______.

15. The height of a cylinder whose volume is 296 cm^3 and area of base is 37 cm^2 is ______.

16. Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms?

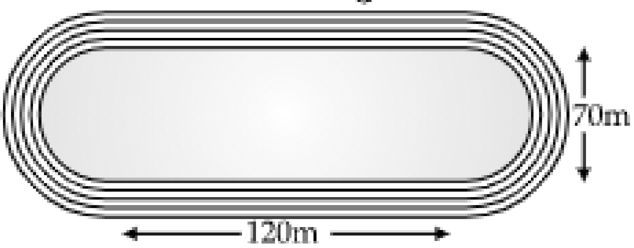
OR

The length of a line segment is of 10 units and the coordinates of one end-point are (2,-3). If the abscissa of the other end is 10, find the ordinate of the other end.

Section-II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark

17. Following figure depicts a park where two opposite sides are parallel and left and right ends are semi-circular in shape. It has a 7 m wide track for walking. Two friends Seema and Meena went to the park.



(i) Find the area of outer rectangular portion.
- (a) 8004 m^2
- (b) 8040 m^2
- (c) 8400 m^2
- (d) 8044 m^2

(ii) What is the area of inner rectangular portion?
- (a) 6720 m^2
- (b) 6027 m^2
- (c) 6270 m^2
- (d) 6207 m^2

(iii) Find the area of outer semi-circular portion.
- (a) 1950 m^2
- (b) 1920 m^2
- (c) 1925 m^2
- (d) 1955 m^2

(iv) Find the area of inner-semi-circular portion.
  (a) 1200 m^2
  (b) 1230 m^2
  (c) 1232 m^2
  (d) 1223 m^2

(v) Find the area of track.
  (a) 3066 m^2
  (b) 3600 m^2
  (c) 3006 m^2
  (d) 3060 m^2

18. Look at the picture given below and answer the questions that follows.

(i) How much cloth material will be required to cover 1 big central dome of radius 4.5 metres? (Take : π = 22/7)
  (a) 127.28 m^2
  (b) 172.17 m^2
  (c) 170.17 m^2
  (d) 120.17 m^2

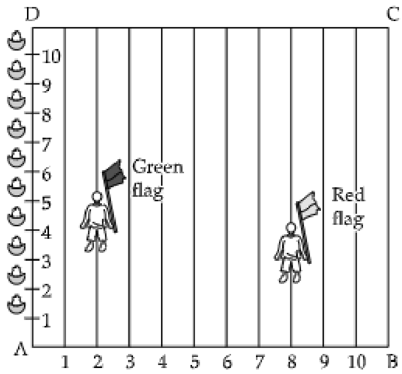
(ii) Write the formula to find the volume of a cylindrical pillar.
  (a) $\pi r^2 h$
  (b) $\pi r l$
  (c) $\pi r (l + r)$
  (d) $2\pi r$

(iii) Find the lateral surface area of two pillars of height of the pillar is 6 m and radius of the base is 1.2 m
  (a) 89.5 m^2
  (b) 90.5 m^2
  (c) 91.5 m^2
  (d) 88.5 m^2

(iv) How much is the volume of a hemisphere if the radius of the base is 2.5 m?
  (a) 30.7 m^3
  (b) 32.0 m^3
  (c) 32.73 m^3
  (d) 37.3 m^2

(v) Write the formula to find the volume of a hemispherical domes
  (a) $2/3\ \pi r^3$
  (b) $4/3\ \pi r^3$
  (c) $2\pi r^3$
  (d) $2/5\ \pi r^3$

19. To conduct sports day activities in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m form each other along AD, as shown in figure



(i) What is the distance between both the flags?
  (a) $\sqrt{51}$
  (b) $\sqrt{71}$
  (c) $\sqrt{261}$
  (d) $\sqrt{41}$

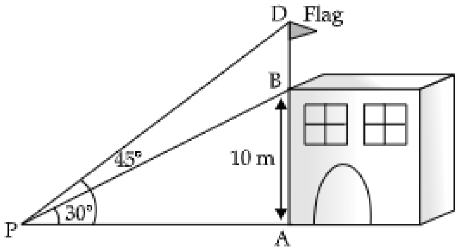
(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post her flags?
  (a) 4^{th} line
  (b) 3^{rd} line
  (c) 6^{th} line
  (d) 5^{th} line

(iii) Write the distance formula between points (2, 5) and (5, 9).
  (a) 5 units
  (b) 4 units
  (c) 7 units
  (d) 3 units

(iv) Write the mid points for points (a, b) and (3a, 3b).
  (a) (a, b)
  (b) (2a, 2b)
  (c) (3a, 3b)
  (d) (4a, 4b)

(v) If Niharika runs $1/10^{th}$ the distance of AD on the 3^{rd} line, and post the yellow flag. What will the coordinates of flag posted?
  (a) (2, 10)
  (b) (1, 10)
  (c) (3, 10)
  (d) (3, 25)

20. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and angle of elevation of the top of the flagstaff from P is 45°.



(i) Find the length of flagstaff.
- (a) 6.32 m
- (b) 7.32 m
- (c) 8.32 m
- (d) 9.32 m

(ii) Find the distance of the building from the point P
- (a) 17.32 m
- (b) 18.32 m
- (c) 19.32 m
- (d) 20.32 m

(iii) What is the value of tan 30°?
- (a) 1
- (b) $\sqrt{3}/2$
- (c) $\sqrt{3}$
- (d) $1/\sqrt{3}$

(iv) What is the value of tan 45°?
- (a) 2
- (b) 0
- (c) 1
- (d) $\sqrt{3}/2$

(v) Write the Pythagoras theorem for ΔAPB.
- (a) $BP^2 = AB^2 + AP^2$
- (b) $AB^2 = AP^2 + BP^2$
- (c) $AP^2 = AB^2 + BP^2$
- (d) None of these

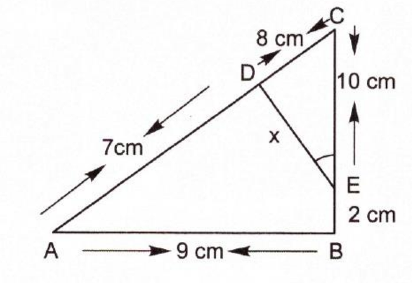
Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Write the following rational numbers in $\frac{p}{q}$ form such that p and q are co-prime.

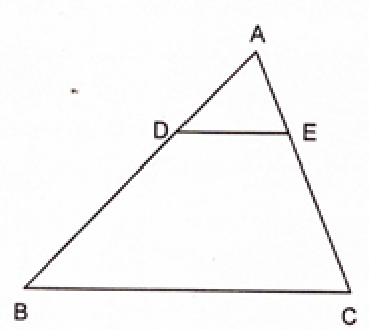
(i) 3.14 (ii) $2.\overline{32}$

22. In Figure, ∠A = ∠CED, find the value of x.



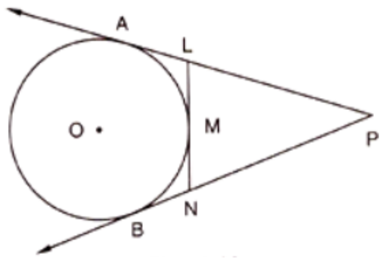
OR

In the figure shown below $DE \parallel BC$ such that AE = (1/4) AC. If AB = 6 cm, find AD.



23. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.

24. In the given figure, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at A. Prove that PL + LM = PN + MN.



25. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

26. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag.
What is the probability that the ball drawn is
(i) Red?
(ii) Not red?

OR

If a number x is chosen from the numbers 1, 2, 3, and a number y is selected from the numbers 1, 4, 9. Then, find P(xy < 9).

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

27. Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

OR

Define composite numbers with an example. Explain why (7×11×13 + 13) and (7×6×5×4×3×2×1 + 5) are composite numbers.

28. What must be added to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$.

29. If the square of the difference of the zeroes of a quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

30. Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.

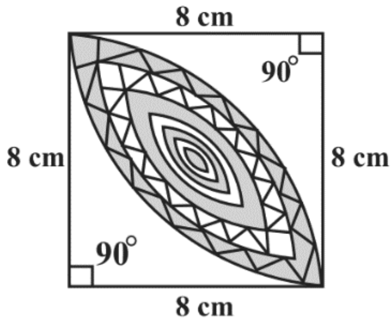
31. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

32. Prove that $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

OR

Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

33. Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each. (use π = 3.14).



Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

34. An aeroplane left 50 minutes later than its schedule time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed b 250 km / hr from its usual speed. Find its usual speed.

OR

The sum of two numbers "a" and "b" is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the numbers "a" and "b".

35. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

36. The following frequency distribution gives the monthly consumption of electricity of the consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units):	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

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Hints & Solutions

PART – A

Section – I

1. Solution: We know that, if α and β are the roots of a quadratic polynomial $ax^2 + bx + c$ then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Here, we have to find

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right)}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)}{\frac{c^2}{a^2}}$$

$$= \frac{b^2 - 2ac}{c^2}$$

2. Solution: We have

$(\sqrt{p} + \sqrt{q})^2 = p + q + 2\sqrt{p}\sqrt{q}$

$= p + q + 2\sqrt{pq}$

Now 'p' and 'q' are distinct primes therefore $\sqrt{pq}$ can't be a perfect square and it's an irrational number.

Similarly, $(\sqrt{p} - \sqrt{q})^2$ is an irrational number.

But, $(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$ is a rational number

OR

Solution:

There are countless irrational numbers between any two real numbers.

Now, we have

$\sqrt{2} = 1.41....$

$\sqrt{3} = 1.73...$

So, we can write any three rational numbers for example

1.42420420042000...

1.50500500050000..

1.71711711171111...

3. Solution: We know, if the pair of linear equations

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

has no solution, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

In this case,

$\Rightarrow \frac{k}{6} = -\frac{5}{2} \neq \frac{2}{7}$

$\Rightarrow$ k = -15 and k $\neq \frac{12}{7}$

###TOPIC###Maths||Pair of Linear Equations in Two Variables||Pair of Linear Equations in Two Variables###

4. Solution: Let the angles A, B and C be 'a – d', 'a' and 'a + d' respectively
[In AP]
By Angle sum property of triangle, we have
$\angle A + \angle B + \angle C = 180°$
$\Rightarrow a - d + a + a + d = 180$
$\Rightarrow 3a = 180$
$\Rightarrow a = 60°$
$\Rightarrow \angle B = 60°$

OR

Solution:
Put n = 1
$a_1 = 3(1)^2 - 5 = 3 - 5 = -2$
Put n = 2
$a_2 = 3(2)^2 - 5 = 12 - 5 = 7$
Put n = 3
$a_3 = 3(3)^2 - 5 = 27 - 5 = 22$
Common difference, $d_1 = a_2 - a_1 = 7 - (-2) = 9$
Common difference, $d_2 = a_3 - a_2 = 22 - 9 = 13$
Since, $d_1 \neq d_2$
Therefore, it's not an A.P.

5. Solution: We know that,
LCM(a, b) × HCF(a, b) = a × b
$\Rightarrow$ HCF(6, a) × LCM(6, a) = 6 × a
$\Rightarrow 2 \times 60 = 6 \times a$
$\Rightarrow a = 20$

6. Solution: Since, Curve intersects the x-axis at three points, there are three zeroes and t = 3
Hence,
$t^3 - 3t^2 + 9$
$= (3)^3 - 3(3)^2 + 9 = 9$

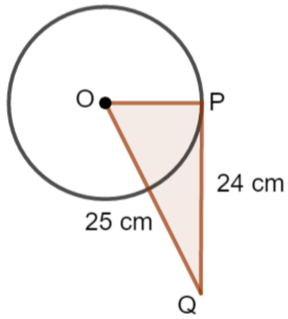
7. Solution: Given,

$\cos\theta = \frac{1}{\sec\theta} = \frac{2}{3} \Rightarrow \sec\theta = \frac{3}{2}$

We know that, $\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow 2\sec^2\theta + 2(\sec^2\theta - 1) - 7$
$= 4\sec^2\theta - 2 - 7$
$= 4\left(\frac{3}{2}\right)^3 - 9$
$= 4 \times \frac{9}{4} - 9$
$= 9 - 9 = 0$

8. Solution: Consider a circle with center O, PQ is a tangent from point Q such that PQ = 24 cm and OQ = 25 cm i.e. distance of Q from center.
To find: Radius = OP



Now, ΔOPQ is a right-angled triangle at P as OP ⊥ PQ
[Tangent through a point on the circle is perpendicular to the radius through point of contact]
By Pythagoras theorem, we have
$OP^2 + PQ^2 = OQ^2$
$\Rightarrow OP^2 + 24^2 = 25^2$
$\Rightarrow OP^2 + 576 = 625$
$\Rightarrow OP^2 = 625 - 576 = 49$
$\Rightarrow$ OP = 7 cm

9. Solution: We have ΔPAB and ΔPQR
∠P = ∠P (Common)
∠PAB = ∠PQR (Corresponding angles)
Then, ΔPAB ~ ΔPQR (BY AA similarity)
So, $\frac{PB}{PR} = \frac{AB}{QR}$ (Corresponding parts of similar triangle area proportion)
Or , $\frac{PB}{6} = \frac{3}{9}$
Or PB = $\frac{3}{9}$ x 6
Or PB= 2cm

OR

Solution:
We have , <ABC = 90° and BD Perpendicular AC
In Δ ABY and ΔBDC
<C = <C (Common)
<ABC = <BDC (Each 90° angles)
Then, Δ ABC ~ΔBDC (By AA Similarity)

So, $\frac{AB}{BD} = \frac{BC}{DC}$ (Corresponding parts of similar triangle area proportion)
Or $\frac{5.7}{3.8} = \frac{BC}{5.4}$
Or BC = 5.7/3.8 x 8.1
Or BC = 12.15cm

10. Solution: When a dice is thrown, total possible outcomes are {1, 2, 3, 4, 5, 6}
⇒ Total number of outcomes = 6
Odd numbers are {1, 3, 5}
⇒ favourable number of outcomes = 3
E is getting an odd number
$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}$

11. Solution: For a polynomial, the maximum number of roots equals the degree of the polynomial
Here, degree of given polynomial is 100
∴ maximum number of roots possible are 100.

12. Solution: We know, the mid-point of line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Putting values, we get mid-point is

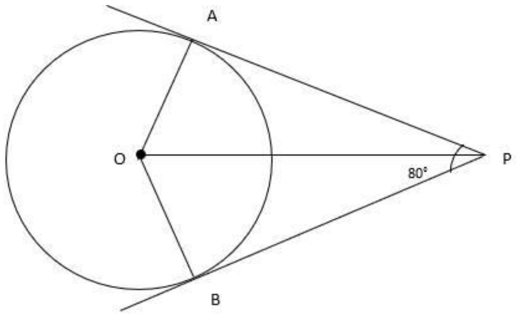
$$\left(\frac{3-7}{2}, \frac{2-4}{2}\right) = (-2, -1)$$

OR

Solution: We know, distance between two points (x_1, y_1) and (x_2, y_2) is
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Here points are (a + b, b + c) and (a - b, c - b)
Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distance $= \sqrt{\{(a-b) - (a+b)\}^2 + \{(c-b) - (b+c)\}^2}$
Distance $= \sqrt{\{a - b - a - b\}^2 + \{c - b - b - c\}^2}$
Distance $= \sqrt{\{-2b\}^2 + \{-2b\}^2}$
Distance $= \sqrt{8b^2} = 2\sqrt{2}b$ units
###TOPIC###Maths||Coordinate Geometry||Distance Formula###

13. Solution: We know that,
$\sec^2\theta - \tan^2\theta = 1$
$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$
$\Rightarrow (\sec\theta - \tan\theta)4 = 1$
$\Rightarrow \sec\theta - \tan\theta = \frac{1}{4}$

14. Solution: Given: ∠APB = 80°



Property 1: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*
Property 2: *Sum of all angles of a quadrilateral = 360°.*
By property 1,
∠PAO = 90°
∠PBO = 90°
By property 2,
∠APB + ∠PAO + ∠PBO + ∠AOB = 360°
⇒ ∠AOB = 360° - ∠APB + ∠PAO + ∠PBO
⇒ ∠AOB = 360° - (80° + 90° + 90°)
⇒ ∠AOB = 360° - 260°
⇒ ∠AOB = 100°

15. Solution: We know that,
Volume of a cylinder = $\pi r^2 h$
⇒ Volume of a cylinder = Area of base × height
⇒ 296 = 37 × h
⇒ h = 8 cm

16. Solution: Two arithmetic progressions have the same common difference.
Let, common difference = d
First term of first A.P. = a
First term of second A.P. = a'
Given that difference between their 100th term is 100
100th term of first A.P., a_{100} = a + 99d
100th term of second A.P., a'_{100} = a' + 99d
Acc. To question,
$a_{100} - a'_{100} = 100$
a + 99d – a' – 99d = 100
a – a' = 100 (i)
1000th term of first A.P., a_{1000} = a + 999d
1000th term of second A.P., a'_{1000} = a' + 999d
Acc. To question,
$a_{1000} - a'_{1000}$ = a + 999d – a' – 999d
= a – a'
$a_{1000} - a'_{1000}$ = 100 [from (i)]
Hence, difference between 1000th term of two A.P. is 100.

OR

Solution:

Let the ordinate of other end is k

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$10 = \sqrt{(10-2)^2 + (k+3)^2}$

On squaring both sides, we get

$100 = (10-2)^2 + (k+3)^2$

$100 = 64 + k^2 + 6k + 9$

$k^2 + 6k - 27 = 0$

$k^2 + 9k - 3k - 27 = 0$

$k(k+9) - 3(k+9) = 0$

$(k-3)(k+9) = 0$

$k = 3;\ k = -9;$

Therefore, ordinate is 3 or -9.

Section-II

17. Solution:
   - (i) Answer: 8400 m^2
   - (ii) Answer: 6720 m^2
   - (iii) Answer: 1925 m^2
   - (iv) Answer: 1232 m^2
   - (v) Answer: 3066 m^2

18. Solution:
   - (i) Answer: 127.28 m^2
   - (ii) Answer: $\pi r^2 h$
   - (iii) Answer: 90.5 m^2
   - (iv) Answer: 32.73 m^2
   - (v) Answer: 2/3 πr^3

19. Solution:
   - (i) Answer: $\sqrt{261}$
   - (ii) Answer: 5^{th} line
   - (iii) Answer: 5 units
   - (iv) Answer: (2a, 2b)
   - (v) Answer: (3, 10)

20.
   - (i) Answer: 7.32 m
   - (ii) Answer: 17.32 m
   - (iii) Answer: $1/\sqrt{3}$
   - (iv) Answer: 1
   - (v) Answer: $BP^2 = AB^2 + AP^2$

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Solution:

(i) We have,

$$3.14 = \frac{314}{100} = \frac{157}{50}$$

(ii) Let $x = 2.\overline{32}$

⇒ x = 2.3232... [1]

⇒ 100x = 232.3232... [2]

Now [2] – [1] gives

⇒ 100x – x = 232.3232... - 2.3232...

⇒ 99x = 230

$$\Rightarrow x = \frac{230}{99}$$

22. Solution:

Given: ∠A = ∠CED

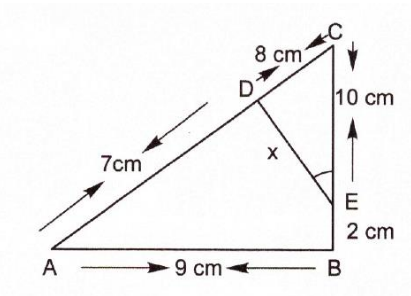
To prove: Δ CAB ~ Δ CED

To find: The value of x.

Theorem Used:

If two triangles are similar, then the ratio of their corresponding sides are equal.

Explanation:



We have, ∠A = ∠CED

In ΔCAB and ΔCED

∠C = ∠C (Common)

∠A = ∠CED(Given)

Then, ΔCAB ~ ΔCED(By AA similarity)

As corresponding parts of similar triangle are proportional.

So,

$$\frac{CA}{CE} = \frac{AB}{ED}$$

Substituting the given values, we get,

$$\frac{15}{9} = \frac{9}{x}$$

⇒ 15x = 90

⇒ x = 90/15

⇒ x = 6 cm

OR

Solution: We have, DE||BC, AB = 6cm and AE = 1/4 AC

In ΔADE and ΔABC

∠A = ∠A (Common)

∠ADE = ∠ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (By AA similarity)

So, $\frac{AD}{AB} = \frac{AE}{AC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ (AE = 1/4 AC Given)

Or , $\frac{AD}{6} = \frac{1}{4}$

Or, AD = 6/4

Or, AD = 1.5cm

23. Solution:

Given:

$a_4 = 3a_1$ ...(i)

$a_7 = 2a_3 + 1$ ...(ii)

Using nth term formula i.e.

nth term of an AP, $a_n = a + (n - 1)d$

where ‘a’ and ‘d’ are first term and common difference respectively.

Here,

$a_3 = a + 2d$

$a_4 = a + 3d$

$a_7 = a + 6d$

Put all the values of a_1 and a_4 in (i),

a + 3d = 3a

3d = 2a

$d = \frac{2a}{3}$

Put all the values of a_7 and a_3 in (ii),

a + 6d= 2(a +2d) +1

2d = a +1

Put d = $\frac{2a}{3}$,

$2(\frac{2a}{3}) = a + 1$

$\frac{4a}{3} - a = 1$

a= 3

Now, difference, $d = \frac{2a}{3}$

⇒ d= 2

Hence, first term is 3 and the common difference is 2.

24. Solution: Theorem: Tangents drawn from an external point to a circle are equal.

Here, PA and PB are tangents from an external point P

⇒ PA = PB

⇒ PL + LA = PN + NB [1]

Also, LA and LM are tangents from an external point L
⇒ LA = LM [2]
MN and NB are tangents from an external point N
⇒ NB = MN [3]
Using [2] and [3] in [1], we get
⇒ PL + LM = PN + MN
Hence Proved!

25. Solution:
Formula:
Volume of cylinder = $\pi r^2 h$
Where r is base radius and h is height of the cylinder.
Let the time taken be 't' hours.
Observe that, volume of water flowing through pipe will be equal to volume of water in cylindrical tank.
For pipe, we can consider
Radius = ½ × diameter = 10 cm = 0.1 m
Height = water flown in 't' hours
⇒ Height = speed of water × time taken = '3t' km = '3000t' meters
For tank,
Radius = ½ × diameter = 5 m
Height = 2 m
∴ $\pi(0.1)^2(3000t) = \pi(5)^2 2$
$\Rightarrow t = \frac{50}{3 \times 0.01} = \frac{5000}{3000} = \frac{5}{3}$ hours
$\Rightarrow t = \frac{5}{3} \times 60 = 100$ minutes

26. Solution: (i) Total number of balls in the bag = 8
No of red balls = 3
Let E be the event of getting a red ball, then
No of favourable outcomes = No of red balls = 3
No of possible outcomes = No of total balls = 8
The probability of the ball drawn to be red:
$= \frac{3}{8}$
Hence, the probability of getting a red ball is 3/8.
(ii) We know that:
$P(\bar{E}) = 1 - P(E)$
where E and $(\bar{E})$ are complementary events, as getting a red ball and not getting a red ball are complementary events
Hence,
The probability of not getting the red ball
= 1 - Probability of getting a red ball
= 1 – 3/8
= 5/8
Hence, the probability of getting a ball not red is 5/8.

OR

Solution: Total numbers of elementary events are: 3 × 3 = 9
Let E be the event of (xy < 9)
Favourable outcomes are: 1 × 1 = 1, 1 × 4 = 4, 2 × 1 = 2, 2 × 4 = 8, 3 × 1 = 3
Numbers of favourable outcomes are: 5
P(xy < 9) = 5/9

Part –B

27. Solution: Since the number leaves the remainder 9 and 7 when divides 285 and 1249 respectively.
⇒ number completely divides 285 – 9 = 276 and 1249 – 7 = 1242
And greatest of that number will be HCF(276, 1242)
Now, 1242 = 276 × 4 + 138
276 138 × 2 + 0
So, HCF(276, 1242) = 138 and that is required number.

OR

Solution: Composite numbers are those numbers which have a factor other than 1 and itself.
For example: 28 is a composite number as 28 = 2 × 2 × 7 and has factors other than 1 and 28.
Now,
(7 × 11 × 13 + 13)
= 13(7 × 11 + 1)
= 13 × 78
Since this number has two factors other than 1 and itself it is a composite number.
And
(7×6×5×4×3×2×1 + 5)
= 5(7 × 6 × 4 × 3 × 2 + 5)
= 5(1008 + 5)
= 5 × 1013

28. Solution: By division algorithm for polynomials, we have
f(x) = g(x)q(x) + r(x)
If f(x) has to be divisible by g(x), then r(x) = 0
i.e. -r(x) should be added to f(x).

$$\begin{array}{l} x^2+2x-3\overline{)4x^4+2x^3-2x^2+x-1}(4x^2-6x+22 \\ \qquad\qquad 4x^4+8x^3-12x^2 \\ \qquad\qquad\quad \overline{-6x^3+10x^2+x-1} \\ \qquad\qquad\quad -6x^3-12x^2+18x \\ \qquad\qquad\qquad \overline{22x^2-17x-1} \\ \qquad\qquad\qquad 22x^2+44x-66 \\ \qquad\qquad\qquad\quad \overline{-61x+65} \end{array}$$

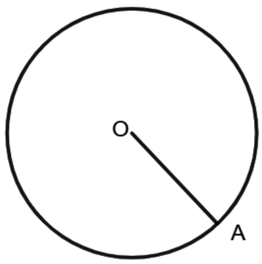
Here, r(x) = -61x + 65
-r(x) = 61x - 65

Hence, 61x – 65 should be added.

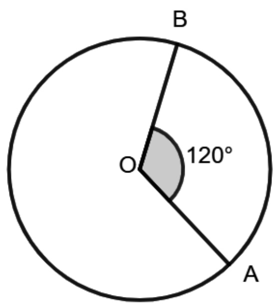
29. Solution: We have, $f(x) = x^2 + px + 45$
Let zeroes be α and β
We know that, if α and β are the roots of a quadratic polynomial $ax^2 + bx + c$ then
$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$
Here, α + β = -p and αβ = 45
Given,
$(\alpha - \beta)^2 = 144$
$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 144$
$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta = 144$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 144$
$\Rightarrow (-p)^2 - 4(45) = 144$
$\Rightarrow p^2 - 180 = 144$
$\Rightarrow p^2 = 324$
$\Rightarrow p = \pm 18$

30. Solution: Vertices of a parallelogram ABCD are: A (1,- 2), B (3, 6), C (5, 10) and D (3, 2)
Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Length of side AB = $\sqrt{(3-1)^2 + (6+2)^2}$ = $\sqrt{4+64}$ = $\sqrt{68}$ units
Length of side BC = $\sqrt{(5-3)^2 + (10-6)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ units
Length of side CD = $\sqrt{(3-5)^2 + (2-10)^2}$ = $\sqrt{4+64}$ = $\sqrt{68}$ units
Length of side DA = $\sqrt{(3-1)^2 + (2+2)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ units
Length of diagonal BD = $\sqrt{(3-3)^2 + (2-6)^2}$ = $\sqrt{16}$ = 4 units
Length of diagonal AC = $\sqrt{(5-1)^2 + (10+2)^2}$ = $\sqrt{16+144}$ = $\sqrt{160}$ units
Since opposite sides are equal but diagonals are not equal. Therefore, given vertices are the vertices of a parallelogram.

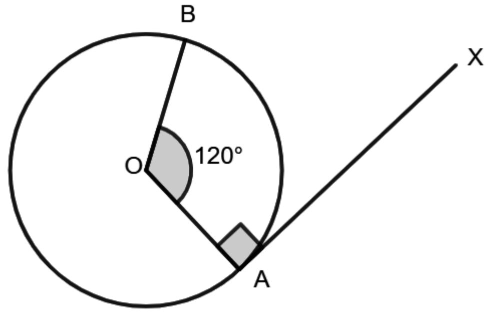
31. Solution: **Steps of construction:**
1) Draw a circle of radius 5 cm, and draw a radius OA anywhere in the circle.



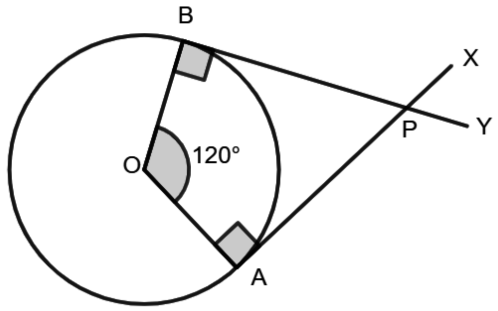
2) Taking OA as base, draw an angle AOB such that ∠AOB = 120°.



3) At A, Draw a line AX such that AX ⊥ OA.



4) At B, Draw a line BY such that BY ⊥ OB.



5) AX and BY intersect at P; AP and BP are required tangents.

32. Solution: L.H.S: $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha}$

$\left[\because \cot^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} \text{ and } \operatorname{cosec} \alpha = \frac{1}{\sin \alpha}\right]$

$= 1 + \frac{\frac{\cos^2 \alpha}{\sin^2 \alpha}}{1 + \frac{1}{\sin \alpha}}$

$= 1 + \frac{\frac{\cos^2 \alpha}{\sin^2 \alpha}}{\frac{\sin \alpha + 1}{\sin \alpha}} = 1 + \frac{\cos^2 \alpha}{\sin \alpha(1 + \sin \alpha)}$

$= \frac{\sin \alpha + \sin^2 \alpha + \cos^2 \alpha}{\sin \alpha + \sin^2 \alpha}$ $[\because \sin^2 \alpha + \cos^2 \alpha = 1]$

$= \frac{1 + \sin \alpha}{\sin \alpha(1 + \sin \alpha)}$ $\left[\because \frac{1}{\sin \alpha} = \operatorname{cosec} \alpha\right]$

$= \frac{1}{\sin \alpha} = \operatorname{cosec} \alpha$

Hence, Proved!

OR

Solution:

Taking LHS

(1 + cot θ - cosec θ) (1 + tan θ + sec θ)

Change all the trigonometric ratios in terms of sin and cos

We know, $\text{cosec}\,\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

[∵ (a − b) (a + b) = $a^2 - b^2$]

$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

[∵$\sin^2\theta + \cos^2\theta = 1$]

$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$ = 2 = RHS

Hence, proved.

33. Solution: Area of unshaded region = 2(Area of square − Area of quadrant)

Also,

Area of shaded region = Area of square − Area of unshaded region

= Area of square − 2(Area of square − Area of quadrant)

= 2(Area of quadrant) − Area of square

Area of quadrant $= \frac{1}{4}\pi(\text{radius})^2$

Area of square = $(\text{side})^2$

We have, radius of quadrant = side of square = 8 cm

Putting values, we get

Area of shaded region $= 2 \times \frac{1}{4}\pi(8)^2 - 8^2$

= 32π − 64

= 32(3.14) − 64

= 36.48 cm^2

Part –B

34. Solution: Given, aeroplane left 50 minutes later than its schedule time,

and in order to reach the destination, 1250 km away, in time,

it had to increase its speed to 250 km / hr. from its usual speed.

Let the usual speed be 'a'.

Increased speed = a + 25

As,

$Time = \frac{distance}{speed}$

So usual time is $\frac{1250}{a}$

When speed is increased time will be $\frac{1250}{a+250}$

As speed increases time decreases

So according to question,

$$\Rightarrow \frac{1250}{a} - \frac{1250}{a+250} = \frac{50}{60}$$

$$\Rightarrow \frac{1250}{a} - \frac{1250}{a+250} = \frac{5}{6}$$

$$\Rightarrow \frac{1250(a+250) - 1250a}{a(a+250)} = \frac{5}{6}$$

$$\Rightarrow \frac{1250(a+250-a)}{a(a+250)} = \frac{5}{6}$$

$\Rightarrow 6 \times 1250 \times (a + 250 - a) = 5(a^2 + 250a)$

$\Rightarrow a^2 + 250a - 375000 = 0$

$\Rightarrow a^2 + 750a - 500a - 375000 = 0$

$\Rightarrow a(a + 750) - 500(a + 750) = 0$

$\Rightarrow (a + 750)(a - 500) = 0$

$\Rightarrow a = 500$ or $a = -750$

$\because$ speed can't be negative $\therefore a = 500$

Hence the usual speed is **500 km/hr**.

OR

Solution: Let the numbers are 'a' and 'b'

According to given conditions:

$a + b = 15$

$\Rightarrow b = 15 - a$ .... (1)

Also, $\frac{1}{a} + \frac{1}{b} = \frac{3}{10}$

From (1),

$$\Rightarrow \frac{1}{a} + \frac{1}{15-a} = \frac{3}{10}$$

$$\Rightarrow \frac{15-a+a}{a(15-a)} = \frac{3}{10}$$

$$\Rightarrow \frac{15}{a(15-a)} = \frac{3}{10}$$

$\Rightarrow 15 \times 10 = 3(15a - a^2)$

$\Rightarrow 15 \times 10 = 45a - 3a^2$

$\Rightarrow 3a^2 - 45a + 150 = 0$

$\Rightarrow a^2 - 15a + 50 = 0$

$\Rightarrow a^2 - 15a - 5a + 50 = 0$

$\Rightarrow a(a - 10) - 5(a - 10) = 0$

$\Rightarrow (a - 5)(a - 10) = 0$

$\Rightarrow a = 5, 10$

If $a = 5$, $b = 15 - 5 = 10$

If $a = 10$, $b = 15 - 10 = 5$

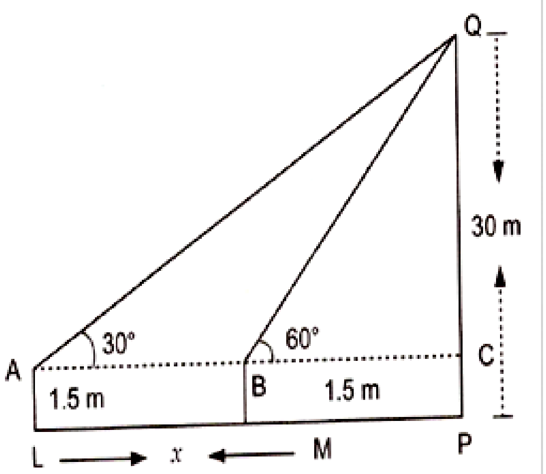
Hence, Numbers are 5,10 or 10, 5.

35. Solution: To find: Distance travelled

Formula Used:

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

Explanation:



Let the initial point of boy is L and final point is M and x is the distance he walked.

QC = QP – CP

QC = 30 - 1.5

= 28.5 m.

In ΔQCA,

$$\tan 30° = \frac{QC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{AC}$$

⇒ AC = 28.5√3 .... (1)

In ΔQCB,

$$\tan 60° = \frac{QC}{BC}$$

$$\sqrt{3} = \frac{28.5}{BC}$$

$$BC = \frac{28.5}{\sqrt{3}}$$

$$PB = \frac{28.5\sqrt{3}}{3}\text{m}$$

AB = AC – BC

$$\Rightarrow AB = 28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3}$$

$$\Rightarrow AB = 28.5\sqrt{3}\left(1 - \frac{1}{3}\right)$$

$$\Rightarrow AB = 28.5\sqrt{3} \times \frac{2}{3}$$

$$\Rightarrow AB = \frac{57\sqrt{3}}{3}$$

= 19√3 m

Therefore, the walking distance of boy is 19√3 m.

36. Solution:

Mean $= \frac{\sum f_i x_i}{N}$

$= \frac{9320}{68}$

$= 137.06$

Class interval	Mid value (x_i)	Frequency(f_i)	$f_i x_i$	Cumulative frequency
65-85	75	4	300	4
85-105	95	5	475	9
105-125	115	13	1495	22
125-145	135	20	2700	42
145-165	155	14	2170	56
165-185	175	8	1400	64
185-205	195	4	780	68
total		N=68	$\sum f_i x_i$ =9320	

We have, N= 68, N/2 = 34

Hence, medium class =125- 145, such that:

l = 125, f = 20, cf = 22, h = 20

Median $= l + \frac{\frac{n}{2} - cf}{f} \times h$

$= 125 + \frac{34-22}{20} \times 20 = 137$

Here, we may observe that maximum class frequency is 20 belonging to the class interval 125-145.

So, modal class= 125-145

Lower limit, l = 125

$f_0 = 13$, $f_2 = 14$, f = 20, h = 20

Mode $= l + \left(\frac{f - f_0}{2f - f_0 - f_2}\right) h$

$= 125 + \left(\frac{20-13}{40-13-14}\right) 20$

$= 125 + \frac{140}{13}$

$= 135.77$

***