S.O.M.

CHAPTER - Loading

- $\delta_{impact} = \delta_{static} \times I.F.$

  $\sigma_{impact} = \sigma_{static} \times I.F.$

  $$I.F. = 1 + \sqrt{1 + \frac{2h}{\delta_{static}}}$$

- EI_{NA} = flexural Rigidity of x-s/c

  AE = Axial rigidity of x-s/c

  GJ = Torsional rigidity of x-s/c

- $$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

  $$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l} \qquad \left[\phi = \gamma = \frac{R\theta}{l} \right]$$

CHAPTER - STRESS

- For ductile material

  S_{yc} or $S_{yt} > S_{ys}$

  $$(\delta_{total})_{Ductile} = \delta_{elastic} + \delta_{elasto\ plastic} + \delta_{plastic}$$

  very large

  $$\delta_{PD} >>> \delta_{ED} > \delta_{EPD}$$

- For Brittle material

  $S_{uc} > S_{us} >> S_{yt}$

  $$(\delta_{total})_{Brittle} = \delta_{elastic} + \delta_{elasto\ plastic}$$

- Mild steel fracture [Ductile material]

  under, tension $\longrightarrow$ Cup & cone fracture

  Torsion $\longrightarrow$ Plane transverse fracture (smooth)

- Cast iron fracture [Brittle material]

  under, tension $\longrightarrow$ Granular transverse fracture

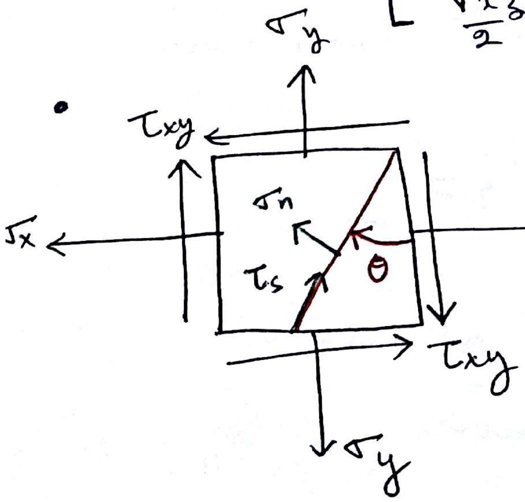
  torsion $\longrightarrow$ Granular helicoidal fracture

- Stress tensor

$$[\sigma]_{3D} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

- Strain tensor

$$[\varepsilon]_{3D} = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \varepsilon_z \end{bmatrix}$$

- 

Sign convension

θ = clockwise from x-face (+ve)

τ_{xy} = clockwise on x-face (+ve)

τ_s = anticlockwise (+ve)

$$(\sigma_n)_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$(\tau_s)_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$(\varepsilon_n)_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta)$$

$$\left(\frac{\gamma_s}{2}\right)_\theta = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin(2\theta) + \frac{\gamma_{xy}}{2}\cos(2\theta)$$

- $\varepsilon_v = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$

- For cylindrical bar

$$\varepsilon_v = \frac{\delta V}{V} = \frac{\delta l}{l} + 2\frac{\delta d}{d} = \varepsilon_l + 2\varepsilon_d$$

- For sphere

$$\varepsilon_v = \frac{\delta V}{V} = 3 \times \frac{\delta d}{d}$$

Here, $V = \frac{\pi}{4} d^2 l$ for cylinder

$V = \frac{4}{3}\pi r^3$ for sphere

$V \Rightarrow$ used in strain calculations

- In elastic region

$$\sigma_e \simeq \sigma_T$$

- In plastic region

$$\sigma_T = \sigma_e (1 + \varepsilon_e)$$

$$\varepsilon_T = \ln(1 + \varepsilon_e)$$

In plastic region $\mu = 0.5 \Rightarrow \delta V = 0$

- $E = 2G(1+\mu)$

  $E = 3K(1-2\mu)$

  $E = \frac{9KG}{3K+G}$

- $G = \frac{\tau}{\gamma} = \frac{\text{Shear stress}}{\text{Shear strain}}$

- $0 \le \mu \le \frac{1}{2}$

-

material	Ind. EC
isotropic	2
orthotropic	9
Anisotropic	21

- for, cork $\mu = 0$

  Rubber, Parafin wax $\Big\}$ $\mu = 0.5$ (i.e. $\delta V = 0$

- $E = \frac{\sigma}{\varepsilon_{long}}$.

- for metals

  $E > K > G$

- $K = \frac{\sigma}{\varepsilon_v}$

- $\mu = -\left(\frac{\text{lateral strain}}{\text{longitudinal strain}}\right) \Rightarrow \varepsilon_{lateral} = -\mu\, \varepsilon_{long}$.

- $\delta_{ms} < \delta_{CI}$ in elastic region

  $\delta_{ms} > \delta_{CI}$ upto fracture bcz $(\delta_{ms})_{Plastic}$ is very high.

- $$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \mu(\sigma_y + \sigma_z)\right) = \frac{\delta x}{x}$$

  $$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \mu(\sigma_x + \sigma_z)\right) = \frac{\delta y}{y}$$

  $$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \mu(\sigma_x + \sigma_y)\right) = \frac{\delta z}{z}$$

  $$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{(1-2\mu)(\sigma_x + \sigma_y + \sigma_z)}{E}$$

- Prismatic bar, under PAL

  $$\delta l = \frac{Pl}{AE}$$

- tapered bar under Axial loading

  $$\delta l = \frac{4Pl}{\pi d_1 d_2 E}$$

- Elongation of P.B. under self wt.

  $$\delta l = \frac{Pl}{2AE} = \frac{\gamma l^2}{2E} \qquad \left[P = \gamma A l = W\right]$$

- Elongation of conical bar under self wt.

  $$\delta l = \frac{Pl}{2AE} = \frac{\gamma l^2}{6E} \qquad \left[P = W = \frac{\gamma A l}{3}\right]$$

CHAPTER- Strain energy

- **Resilience** : strain energy absorbed by member within elastic region

  = Area of P-δ curve in elastic region

- **Proof Resilience** : max. strain energy absorbed by a member upto elastic point

  = Area of P-δ curve upto elastic region

Proof Resilience = $\frac{\sigma_{EL}^2}{2E} \times$ Vol.

- modulus of Resilience = $\frac{\text{Proof resilience}}{\text{Vol.}}$

$= \frac{\sigma^2}{2E}$

= Area of $\sigma - \varepsilon$ curve upto elastic region

- Toughness :- Strain energy absorbed by member upto fracture point

= Area of $P-\delta$ curve upto fracture

- modulus of toughness = $\frac{\text{Toughness}}{\text{Vol.}}$

= Ar. $\sigma - \varepsilon$ curve upto fracture

- Young's modulus (E) is independent of % C in steels

- Strain energy of bar under Axial load

$$U = \int_a^b \frac{P_{x-x}^2\, dx}{2(AE)_{x-x}} = \frac{P^2 l}{KAE}$$

K = 2 ⟶ P.B. under PAL

K = 6 ⟶ P.B. under Self wt. ($P = W = \gamma AL$)

K = 10 ⟶ C.B. under self wt. ($P = W = \frac{\gamma AL}{3}$)

$U = \frac{\sigma^2}{2E} \times$ Vol. for PAL

- Strain energy of tapered bar under PAL

$$U = \frac{4P^2 l}{2\pi d_1 d_2 E}$$

- $U \propto \frac{L}{A}$ ⟶ P.B. under P A L

  $U \propto A L^3$ ⟶ P.B. under self wt.

- Strain energy of a beam under bending

$$U = \int_a^b \frac{M_{x-x}^2 \, dx}{2(EI)_{x-x}} = \frac{M^2 L}{2EI} \text{ for Pure bending of P.B.}$$

- Strain energy of Prismatic shaft under Pure torsion

$$U = \int_a^b \frac{T_{x-x}^2 \, dx}{2(GJ)_{x-x}} = \frac{T^2 l}{2GJ} \text{ for Pure torsion}$$

$$U = \frac{(\tau_{max})^2}{4G} \times \text{vol.} \longrightarrow \text{for solid shaft}$$

$$= \frac{(\tau_{max})^2}{4G} \times \text{vol.} \times (1+K^2) \longrightarrow \text{for hollow shaft}$$

- Strain energy of rectangular block under pure shear

$$U = \frac{\tau^2}{2G} \times \text{Vol.}$$

- for composite bar under Axial load

$$\delta l = \frac{Pl}{A_1E_1 + A_2E_2} \quad [\text{when } l_1 = l_2]$$

CHAPTER – THERMAL STRESSES

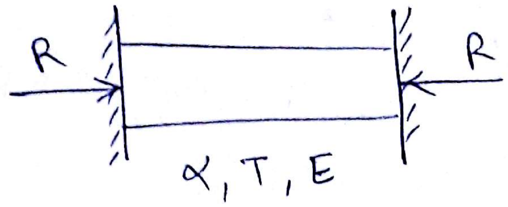
- $\sigma_{total} = \sigma_{mech.} + \sigma_{thermal}$
- In case of free expansion

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha T$$

$$\sigma_x = \sigma_y = \sigma_z = 0$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\alpha T$$

- INVAR is having lowest (next to diamond) coefficient of thermal expansion.
- Completely restricted expansion



$$R = \alpha T E A$$

$$\sigma_{th} = \alpha T E$$

$$\delta d = \alpha T d(1+\mu)$$

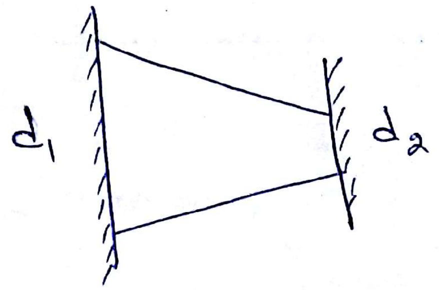
- Partially restricted expansion (λ = Expansion permitted)

$$\delta_{th} - \delta_a = \lambda$$

$$\alpha T l - \frac{Rl}{AE} = \lambda$$

- Completely restricted tapered bar

$$\delta_{th} - \delta_a = 0$$



$$\alpha T l - \frac{4Rl}{\pi d_1 d_2 E} = 0$$

$$\Rightarrow R = \alpha T E \left(\frac{\pi}{4} d_1 d_2\right)$$

$$\left.\begin{aligned} \Rightarrow \sigma_{max} &= \alpha T E \left(\frac{d_1}{d_2}\right) \\ \sigma_{min} &= \alpha T E \left(\frac{d_2}{d_1}\right) \end{aligned}\right\} \Rightarrow \frac{\sigma_{max}}{\sigma_{min}} = \left(\frac{d_1}{d_2}\right)^2$$

- Thermal stresses in compound bars in series

$$\underbrace{\alpha_1 T l_1 + \alpha_2 T l_2}_{\text{Sum of thermal deformation}} = \underbrace{\frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2}}_{\text{Sum of axial deformation}}$$

$$\sigma_1 A_1 = \sigma_2 A_2$$

- Compound bars in Parallel

$$\underbrace{\alpha_1 T l - \alpha_2 T l}_{\text{diff. of thermal deformation}} = \underbrace{\frac{\sigma_1 l}{E_1} + \frac{\sigma_2 l}{E_2}}_{\text{Sum of Axial deformation}}$$

$$\sigma_1 A_1 = \sigma_2 A_2$$

CHAPTER - SFD, BMD

- Bending moment is maximum or minimum where shear force changes its sign.
- Point of contraflacture ⇒ Point where BM = 0 (i.e. BM changes its sign)

- $(SF)_{xx} = \left(\frac{dM}{dx}\right)_{x-x}$ = slope of BMD

- $-w_{xx} = \left(\frac{d(SF)}{dx}\right)_{xx}$ = slope of SFD

- $M_B - M_A = \int_a^b (SF)_{xx}\, dx$ = Area of SFD

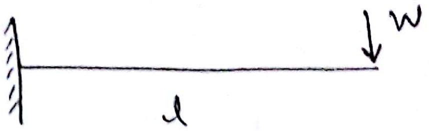
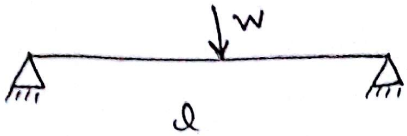
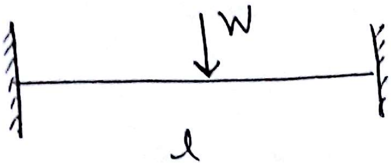
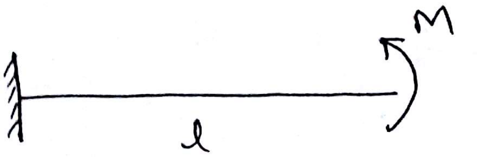
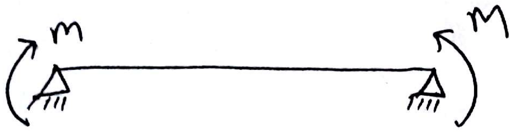
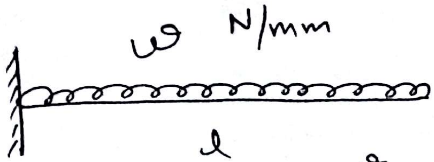
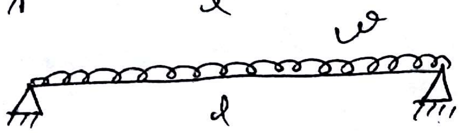
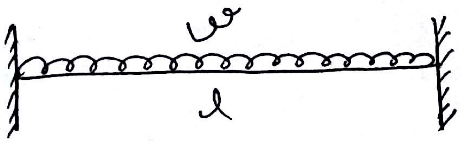
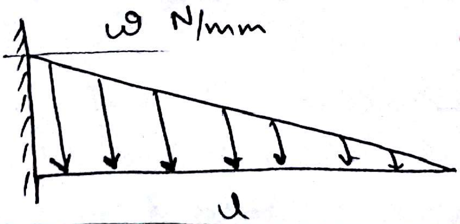
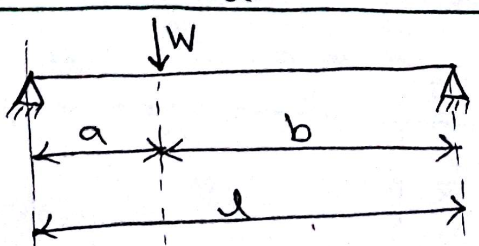
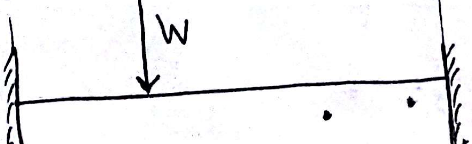
- $(SF_B - SF_A) = \int_a^b -w_{x-x}\, dx$ = Area of loading diagram

- SF is one degree higher than loading digram & BMD is one degree higher than SFD diagram.

CHAPTER - DEFLECTION OF BEAM

- Under unsymmetrical loaded SSB, deflection is maximum in the region b/w Point of application of load & mid span

SF	BM	$R = \frac{EI}{M}$	shape of elastic curve
0	0	∞	st. line
0	const.	const.	circular arc
$\neq 0$	Varies	varies continously	Parabola

S. no.	Type of Beam [EI = const]	$\theta = \frac{Wl^2}{K_1 EI}$	$Y = \frac{Wl^3}{K_2 EI}$
		2	3
		16	48
		—	192
		$\theta = \frac{Ml}{K_1 EI}$	$Y = \frac{Ml^2}{K_2 EI}$
		1	2
		2	8
		$\theta = \frac{wl^3}{K_1 EI}$	$Y = \frac{wl^4}{K_2 EI}$
		6	8
		24	$\frac{384}{5}$
		—	384
		24	30
		$\theta_B = \frac{Wab(a-b)}{3EI \cdot l}$	$Y_B = \frac{Wa^2b^2}{3EI \cdot l}$
			$Y_B = \frac{Wa^3b^3}{3EI \cdot l^3}$

- Double integration method

$$(EI)_{x-x} \cdot \frac{d^2y}{dx^2} = M_{x-x}$$

$\left[\frac{dy}{dx} = 0 \right.$

$y =$ deflection

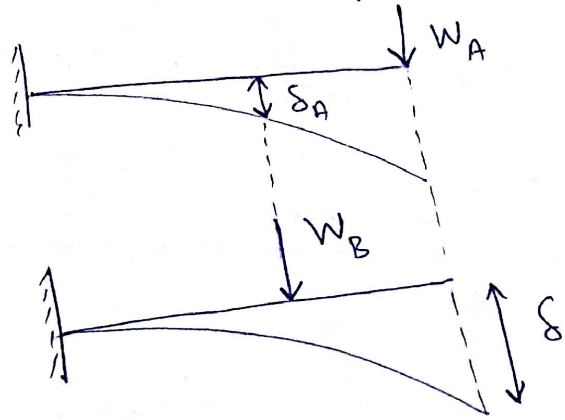
- Moment Area method

$\theta_B - \theta_A =$ Area of $\frac{M}{EI}$ diagram b/w A & B

$y_B - y_A =$ Moment of Area $(A\bar{x})$ of $\frac{M}{EI}$ diagram b/w A&B

A & B are selected such that
- one point is point of zero slope
- other point is non-zero slope point (i.e. origin, where $x=0$)

- Maxwell's reciprocal theorem



$$W_A \, \delta_B = W_B \, \delta_A$$

- Castigliano's theorem

$$U = \text{S.E. due to Bending} = \int_a^b \frac{M_{xx}^2 \, dx}{2(EI)_{xx}}$$

$$y_A = \frac{\partial U}{\partial W_A} \quad \& \quad \theta_A = \frac{\partial U}{\partial M_A}$$

$\left[\begin{array}{l} W_A = \text{Conc. load at A} \\ M_A = \text{conc. moment at A} \end{array} \right.$

CHAPTER - BENDING STRESSES

- $$\frac{M_R}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}$$

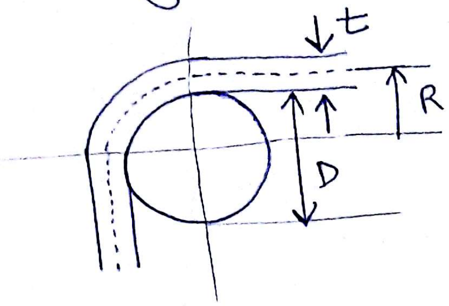
- $$\sigma_b = \frac{My}{I_{NA}} \Rightarrow (\sigma_b)_{max} = \frac{My_{max}}{I_{NA}} = \frac{M}{Z_{NA}}$$

- $M_R = Z_{NA} \times \sigma_{per}$

- $\dfrac{\sigma_b}{y} = \dfrac{E}{R} \Rightarrow (\sigma_b)_{max} = \dfrac{E\, y_{max}}{R}$

- $R = \dfrac{E\, I_{NA}}{M_R}$

- Bending stress in flat belt



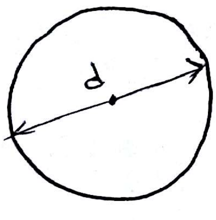
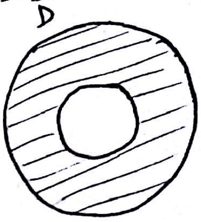
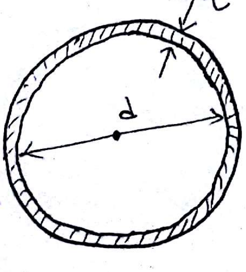
$$y_{max} = \frac{t}{2}$$

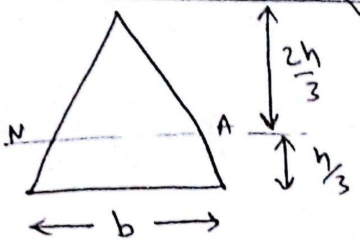
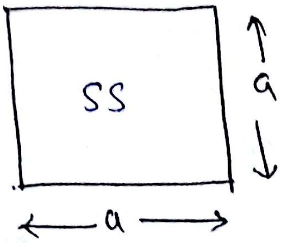
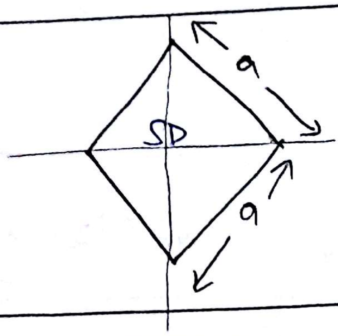
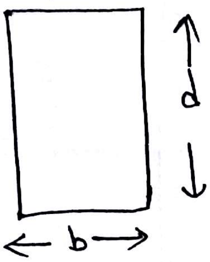
$$\sigma = \frac{E\, y_{max}}{R} = \frac{Et}{2R}$$

$$= \frac{E\, t/2}{\frac{1}{2}(D+t)} = \frac{Et}{D+t} = \frac{Et}{D}$$

- For a given x-s/c area

$Z_I > Z_T > Z_{▯} > Z_{▨} > Z_{●} > [Z_{▬} \approx Z_{◆}]$

x-s/c	Area (A)	MOI (I)	Polar MOI $J = 2I_{xx}$	$Z_{NA} = \dfrac{I_{NA}}{y_{max}}$	$Z_p = \dfrac{J}{R}$
 d	$\frac{\pi}{4} d^2$	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{32}$	$\frac{\pi d^3}{32}$	$\frac{\pi d^3}{16}$
$K = \frac{d}{D}$ 	$\frac{\pi}{4} D^2(1-K^2)$	$\frac{\pi D^4}{64}(1-K^4)$	$\frac{\pi D^4}{32}(1-K^4)$	$\frac{\pi D^3}{32}(1-K^4)$	$\frac{\pi D^3}{16}(1-K^4)$
t  d	$\pi D t$	$\pi r^3 t$	$2\pi r^3 t$	$\pi r^2 t$	$2\pi r^2 t$

X-S/C	Area (A)	MOI (I_{NA})	$Z_{NA} = \frac{I_{NA}}{Y_{max}}$
	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$
	a^2	$\frac{a^4}{12}$	$\frac{a^3}{6}$
	a^2	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}}$
	bd	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$

- Beam of uniform strength (σ_b = const.]

method-1 by varrying width & keeping depth as constant

$$b_x = b \cdot \frac{x}{l}$$

method-2 by varrying depth & keeping width as constant

$$d_x = d \cdot \sqrt{\frac{x}{l}}$$

CHAPTER- SHEAR STESSES IN BEAMS

- $$\tau = \frac{P}{I_{NA}} \times \left(\frac{A\bar{Y}}{b}\right)$$

P = shear force on that x-s/c

A = area above/ below the fibre at which τ is needed.

$\bar{y}$ = distance of centroid of Area from N.A.

b = width of X-S/c where τ is to be calculated

- $\tau = \frac{P}{I_{NA}} \times \left(\frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{b} \right)$ [when non symmetric areas are present above the fibre

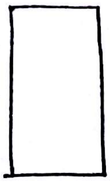
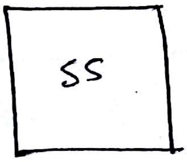
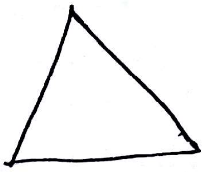
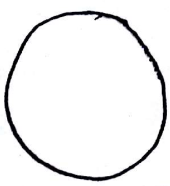
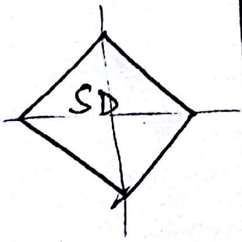
- $\frac{(\tau_s)_f}{(\tau_s)_w} = \frac{b_w}{b_f}$

- Generalised eqn for τ on rectangular X-S/c

$\tau = \frac{6P}{bd^3} \left[\left(\frac{d}{2} \right)^2 - y^2 \right]$

y = distance of fibre from N.A.

d = depth.

X-S/c	$\tau_{max} = K_1 \tau_{avg}$	Distance of τ_{max} from NA	$\tau_{NA} = K_2 \tau_{avg}$
	$\frac{3}{2}$	0	$\frac{3}{2}$
	$\frac{3}{2}$	0	$\frac{3}{2}$
	$\frac{3}{2}$	$\frac{h}{6}$	$\frac{4}{3}$
	$\frac{4}{3}$	0	$\frac{4}{3}$
	$\frac{9}{8}$	$\frac{h}{8}$	1

CHAPTER- TORSIONAL SHEAR STRESS IN SHAFT

- $\frac{T_R}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$

- $T_R = Z_p \cdot \tau_{per}$

- ϕ = shear angle
  θ = angle of twist $\Big] \Rightarrow \phi l = R\theta$
  $\Rightarrow \phi = \gamma = \frac{R\theta}{l}$

- $\theta \propto l$
  $\phi \propto \gamma$

- $\tau = \frac{T}{Z_p} = \frac{16T}{\pi d^3}$ → for solid shaft
  $= \frac{16T}{\pi d^3(1-K^4)}$ → for hollow shaft

- $\theta = \frac{Tl}{GJ} \Rightarrow \frac{T}{\theta} = S_\theta = \frac{GJ}{l}$ = torsional stiffness

- for a given outer dia

$$\frac{I_{●}}{I_{\odot}} = \frac{J_{●}}{J_{\odot}} = \frac{Z_{●}}{Z_{\odot}} = \frac{(Z_p)_{●}}{(Z_p)_{\odot}} = \frac{1}{1-K^4}$$

- for a given X-s/c area

$$\frac{I_{●}}{I_{\odot}} = \frac{J_{●}}{J_{\odot}} = \frac{1-K^2}{1+K^2}$$

$$\frac{Z_{●}}{Z_{\odot}} = \frac{Z_{p●}}{Z_{p\odot}} = \frac{\sqrt{1-K^2}}{1+K^2}$$

- Shafts in series

$$\theta_{total} = \theta_1 + \theta_2 + \theta_3 ---- \theta_n$$

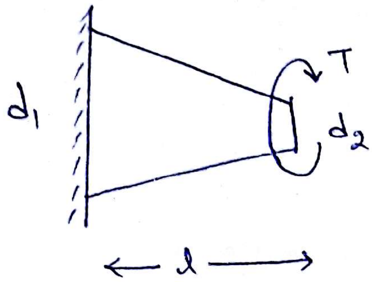
$$T = T_1 = T_2 = T_3 = --- T_n$$

- Shafts in Parallel

$$T = T_1 + T_2$$

$$\theta = \theta_1 = \theta_2 \Rightarrow \theta = \frac{Tl}{G_1J_1 + G_2J_2} = \frac{T_1 l}{G_1 J_1} = \frac{T_2 l}{G_2 J_2}$$

- Tapered shaft



$$\theta_{total} = \frac{32 Tl}{G\pi}\left[\frac{d_1^2 + d_1 d_2 + d_2^2}{3 d_1^3 d_2^3}\right]$$

$$U = \frac{32 T^2 l}{2G\pi}\left[\frac{d_1^2 + d_1 d_2 + d_2^2}{3 d_1^3 d_2^3}\right]$$

$$\tau_{max} = \frac{16T}{\pi d^3_{smaller}} \; ; \; \tau_{min} = \frac{16T}{\pi d^3_{larger}}$$

CHAPTER- PRINCIPAL STRESSES & STRAINS

- $\sigma_1 + \sigma_2 + \sigma_3 = \sigma_x + \sigma_y + \sigma_z$ → for triaxial

  $\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$ → for biaxial

- $$\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

  location of Principal Planes ⇒ $\tan(2\theta) = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

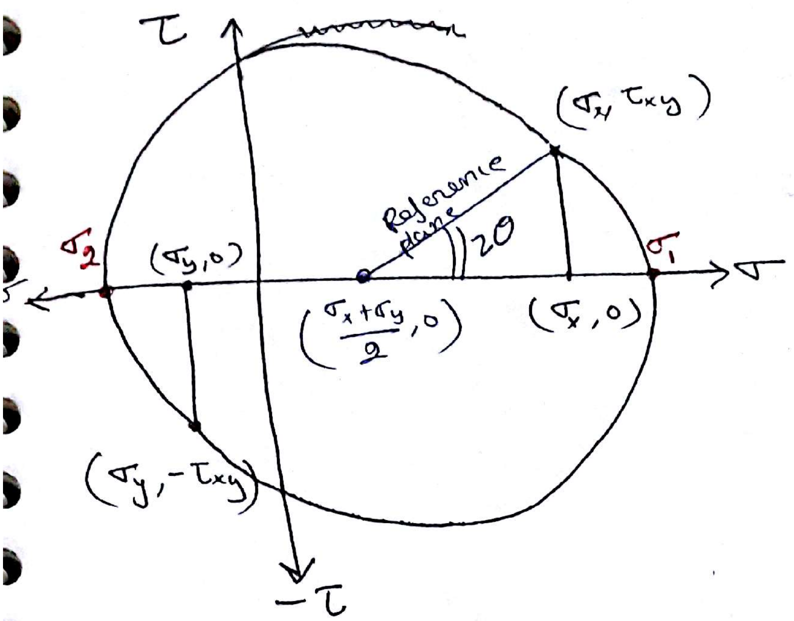
- σ_n^* = normal stress on τ_{max} plane $= \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{\sigma_1 + \sigma_2}{2}$

- For plane stress problems in plane τ_{max} is considered abso. τ_{max} = Larger of $\left\{\dfrac{(\sigma_1 - \sigma_2)}{2}, \dfrac{(\sigma_2 - \sigma_3)}{2}, \dfrac{(\sigma_3 - \sigma_1)}{2}\right\}$

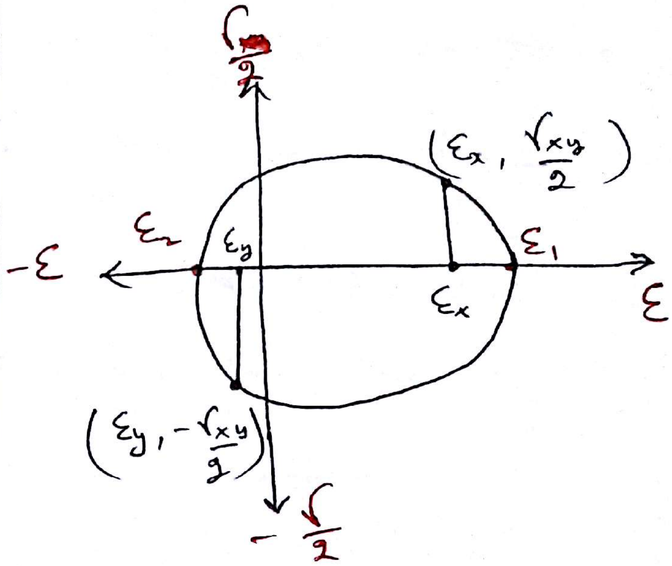
- τ^* = plane of pure shear $= \sqrt{-\sigma_1 \sigma_2}$

- All angles are to be measured from reference plane. reference plane is the stat of stress on x-face

- mohr's circle for stress



mohr's circle for strain



- $$\varepsilon_{1,2} = \frac{1}{2}\left[(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon_x - \varepsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\,\right]$$

$$\tan(2\theta) = \frac{2 \times \left(\frac{\gamma_{xy}}{2}\right)}{\varepsilon_x - \varepsilon_y}$$

- Relationship b/w Principal stress & Principal strain

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2 + \sigma_3)\right)$$

$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \mu(\sigma_1 + \sigma_3)\right)$$

$$\varepsilon_3 = \frac{1}{E}\left(\sigma_3 - \mu(\sigma_1 + \sigma_2)\right)$$

- Expression for $\sigma_{1,2}$ in terms of ε_1 & ε_2

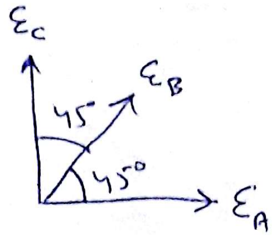
$$\sigma_1 = \frac{E}{(1-\mu^2)} \times (\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_2 = \frac{E}{(1-\mu^2)} \times (\varepsilon_2 + \mu\varepsilon_1)$$

- Strain Rosette

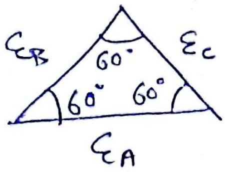
1. Rectangular strain rosette

   $[\alpha = 45°, \quad \theta = 0°, 45°, 90°]$



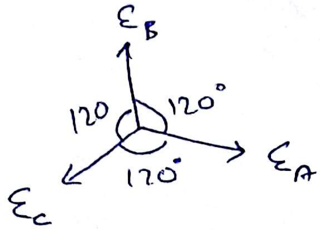
2. Delta strain rosette

   $[\alpha = 60°, \quad \theta = 0°, 60°, 120°]$



3. Star strain rosette

   $[\alpha = 120°, \quad \theta = 0°, 120°, 240°]$



⇒ In all strain rosette cases use below equations

$$(\varepsilon_n)_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta)$$

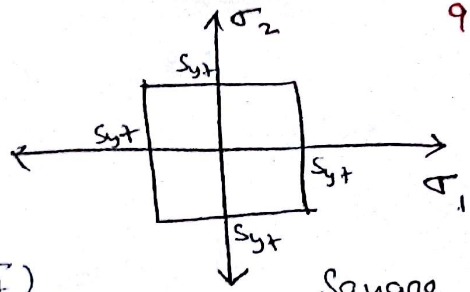
above eqn is used for calculating $\varepsilon_x, \varepsilon_y, \tau_{xy}$

⇒ For strain analysis strain in any 3 arbitrary dirn are to be Known.

CHAPTER - THEORIES OF FAILURE

S. no.	TOF	Short form	Scientist
1.	Maximum Principal Stress theory	M.P.S.T	Rankine's theory
2.	Maximum shear stress theory	M.S.S.T.	Guest & Tresca's theory
3.	Maximum Principal strain theory	M.P.St.T.	St. Venant's theory
4.	Total strain energy theory	T.S.E.T.	Haigh's theory
5.	Maximum Distortion energy theory or Maximum shear strain energy theory	M.D.E.T.	Von-mise's & Hencky's theory

1. MPST [Rankine's theory]



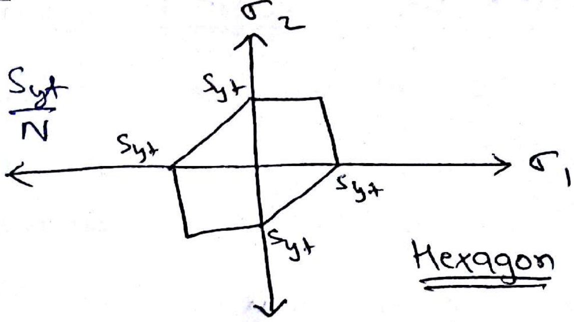
Square

- $\sigma_1 \le \frac{S_{yt}}{N} \longrightarrow$ 3D, 2D

- $M_e = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right] \le \frac{\pi d^3}{32} (\sigma_t)_{per}$

- $\frac{S_{ys}}{S_{yt}} = 1$

2. MSST [Guest & Tresca's theory]



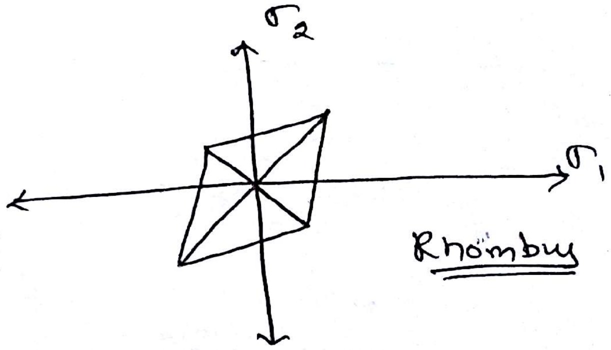
Hexagon

- larger of $\{(\sigma_1-\sigma_2), (\sigma_2-\sigma_3), (\sigma_3-\sigma_1)\} \le \frac{S_{yt}}{N}$

- $T_e = \sqrt{M^2+T^2} \le \frac{\pi d^3}{16} (\tau)_{per}$

- $\frac{S_{ys}}{S_{yt}} = \frac{1}{2} = 0.5$

3. M.P.St.T [St. venant theory]

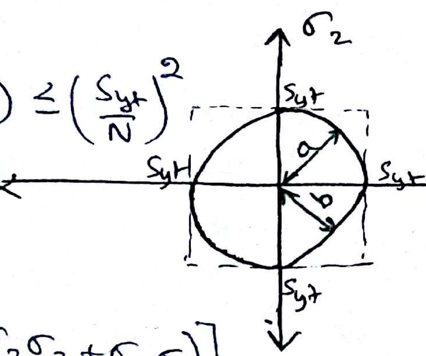


Rhombus

- $\sigma_1 - \mu(\sigma_2+\sigma_3) \le \frac{S_{yt}}{N}$

- $\frac{S_{ys}}{S_{yt}} = \frac{1}{1+\mu} = 0.77$ for $\mu = 0.3$

4) T.S.E.T. [Haigh's theory]



$a = \frac{S_{yt}}{\sqrt{1-\mu}}$

$b = \frac{S_{yt}}{\sqrt{1+\mu}}$

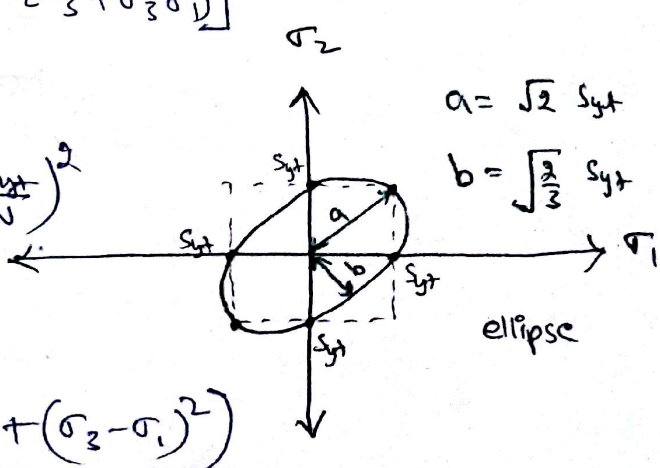
ellipse

- $\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \le \left(\frac{S_{yt}}{N}\right)^2$

- $\frac{S_{ys}}{S_{yt}} = \frac{1}{\sqrt{2(1+\mu)}} = 0.62$ for $\mu = 0.3$

- $\frac{S.E.}{Vol.} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$

5) MD.E.T. [Von-mises's theory]



$a = \sqrt{2}\, S_{yt}$

$b = \sqrt{\frac{2}{3}}\, S_{yt}$

ellipse

- $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \le 2\left(\frac{S_{yt}}{N}\right)^2$

- $\frac{S_{ys}}{S_{yt}} = \frac{1}{\sqrt{3}} = 0.577$

- $\frac{D.E}{Vol.} = \left(\frac{1+\mu}{6E}\right)\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)$

- Put $\sigma_1 = S_{ys}$ & $\sigma_2 = -S_{ys}$ in 2D design eqn to get $\frac{S_{ys}}{S_{yt}}$
- Shape of safe boundary are valid for ductile material only.
- MPST → best for brittle material under every state of stress condition

MDET → Best for Ductile material because it gives safe and economic design (MSST gives over safe design)

MSST & MDET → not suitable for hydrostatic state of stress

CHAPTER- COLUMNS [only GATE]

- long column → fails due to buckling → Euler's eqn
- medium column → fails due to both buckling & crushing → Rankine's eqn
- Short column → fails due to crushing
- Johnson's eqn consider all type of columns

Euler's formula

- $$P_e = \frac{n\pi^2 E I_{min}}{l^2} = \frac{\pi^2 E I_{min}}{l_e^2}$$

P_e = Euler's buckling load

$n = \frac{1}{\alpha^2}$ = end fixity coefficient

α = length fixity coefficient

$l_e = \alpha l$

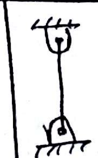
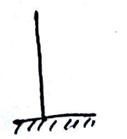
- $$\frac{P_{e\,●}}{P_{e\,◎}} = \frac{I_{●}}{I_{◎}} = \frac{1}{1-K^4} \longrightarrow \text{when outer dia is same}$$

$$= \frac{1-K^2}{1+K^2} \longrightarrow \text{when Area is same}$$

- Slenderness ratio (S)

$$S = \frac{l_e}{K}$$

$$K = \sqrt{\frac{I_{min}}{A}}$$

End condition				
n	4	2	1	$\frac{1}{4}$
α	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	1	2

- $\frac{P_e}{A} = \frac{\pi^2 E I_{min}}{l_e^2 \; A} \Rightarrow \sigma = \frac{\pi^2 E}{S^2}$

- short column → $S \le 30$
  medium column → $30 < S \le 100$
  long column → $S > 100$
  ⟹ for steels

Rankine's formula

$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$

$P_R = \frac{\sigma_c . A}{1 + cS^2}$

$P_c = A\sigma_c$ = crushing load

c = Rankine's const.

$= \frac{1}{1600}$ for CI

CHAPTER – PRESSURE VESSELS

- $\frac{D}{t} \ge 20$ → thin vessel
- $\frac{D}{t} < 20$ → thic vessel

Thin cylindrical Pressure vessel

$\varepsilon_1 = \varepsilon_h = \frac{PD}{4tE}(2-\mu) = \frac{\delta d}{d}$

$\varepsilon_2 = \varepsilon_l = \frac{PD}{4tE}(1-2\mu) = \frac{\delta l}{l}$

$\sigma_h = \frac{PD}{2t}$ or $\frac{PD}{2t\,\eta_l}$

$\sigma_l = \frac{PD}{4t}$ or $\frac{PD}{4t\,\eta_c}$

$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_l + 2\varepsilon_h = \frac{PD}{4tE}(5-4\mu)$

Thin spherical Pressure vessel

$\sigma_1 = \sigma_2 = \sigma_h = \frac{PD}{4t}$

$\varepsilon_1 = \varepsilon_2 = \frac{\delta d}{d} = \frac{PD}{4tE}(1-\mu)$

$\varepsilon_v = \frac{\delta V}{V} = 3\frac{PD}{4tE}(1-\mu)$

whenever $\frac{\delta V}{V} = \varepsilon_v$ is calculated

$V = \frac{\pi}{4} d^2 l$ → for cylinder

$V = \frac{4}{3}\pi r^3$ → for sphere

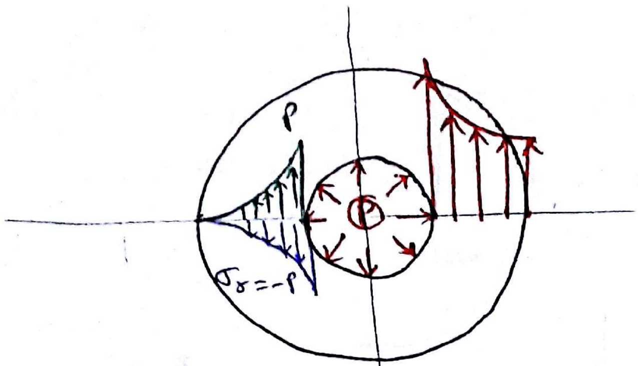
Thick cylindrical Pressure vessel (only ESE)

$-(\sigma_r)_x = P_x = -a + \frac{b}{x^2}$

$(\sigma_h)_x = a + \frac{b}{x^2}$

⟹ Lame's eqn
constants are calculated by boundary condition

Thick CPV under IP



σ_h = tensile

σ_r = compressive

at $x = R_i$

$P_x = P$

$\sigma_{r_x} = -P$

$\sigma_h = (\sigma_h)_{max}$

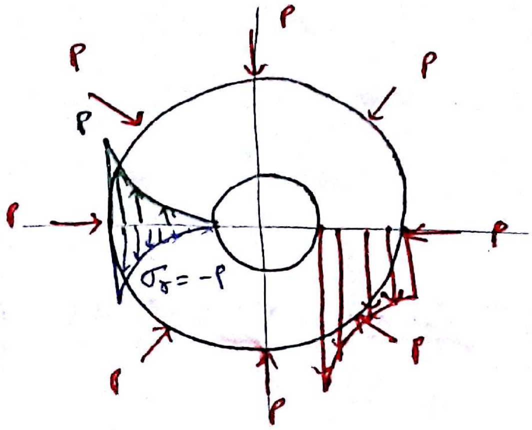
at $x = R_o$

$P_x = 0$

$\sigma_{r_x} = 0$

$\sigma_{h_x} = (\sigma_{min})_h$

Thick CPV under EP



σ_h = compressive

σ_r = compressive

at $x = R_i$

$P_x = 0$

$\sigma_{r_x} = 0$

$\sigma_{h_x} = \sigma_{h\,max}$

at $x = R_o$

$P_x = P$

$\sigma_{r_x} = -P$

$\sigma_{h_x} = \sigma_{h\,min}$

- σ_h is always maximum at inner surface
- $\boxed{(\sigma_h)_{max} = (\sigma_h)_{min} + P}$
- if $P_i = P_o = P$ then $(\sigma_h)_x = -P$ = constant

Thick Spherical Pressure vessel

$$-\sigma_{r_x} = P_x = -a + \frac{2b}{x^3}$$

$$\sigma_{h_x} = a + \frac{b}{x^3}$$