Algebraic Identities

An identity is an equality, which is true for all values of the variables.

Chart

Identities

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

Problem Based on Identity

Q. Expand $(2x - 5y)^2$

Sol. $(2x - 5y)^2$

$= (2x)^2 - 2(2x)(5y) + (5y)^2$

$= 4x^2 - 20xy + 25y^2$

Q. Find the value of $\frac{107^2 - 103^2}{210}$

Sol. $\frac{107^2 - 103^2}{210} = \frac{(107 + 103)(107 - 103)}{210}$

$= \frac{210 \times 4}{210} = 4$

Q. If $x + \frac{1}{x} = 3$ find $x^2 + \frac{1}{x^2}$

Sol. $x + \frac{1}{x} = 3$

$\left(x + \frac{1}{x}\right)^2 = 3^2$

$x^2 + 2x \times \frac{1}{x} + \frac{1}{x^2} = 9$

$x^2 + 2 + \frac{1}{x^2} = 9$

$x^2 + \frac{1}{x^2} = 9 - 2 = 7$

Factorization

The process of finding two or more expression whose product is the given expression is called factorization.

I. Factorization by taking out the common factor.

Ex. $8x^3y^2 - 4yx = 4xy(2x^2y - 1)$

Ex. $x(x + 3) + 2(x + 3) = (x + 3)(x + 2)$

II. Factorization by grouping.

Ex. $ax + by + ay + bx = ax + ay + bx + by$

$= a(x + y) + b(x + y) = (x + y)(a + b)$

III. Factorization the difference of two squares.

$a^2 - b^2 = (a + b)(a - b)$

Ex. $9x^2 - 16y^2 = (3x)^2 - (4y)^2$

$= (3x - 4y)(3x + 4y)$

IV. Factorization of quadratic trinomial

Ex. Factorize $x^2 + 9x + 18$

Sol. $x^2 + 9x + 18 = x^2 + 6x + 3x + 18$

$= x(x + 6) + 3(x + 6)$

$= (x + 6)(x + 3)$

Ex. Factorize $9x^2 - 22x + 8$

Sol. $9x^2 - 22x + 8 = 9x^2 - 18x - 4x + 8$

$= 9x(x - 2) - 4(x - 2)$

$= (x - 2)(9x - 4)$