Chapter 2

Linear Equations in One Variable

Introduction

What is an Equation?

An equation is a statement in which there is an equality sign between two algebraic expressions.

What is a Linear Equation?

An equation having only one variable and the highest degree of the variable is 1 is called Linear Equation.

$3x + 4 =$ Left Hand Side(LHS)

$10 =$ Right Hand Side(RHS)

For $x = 2$,
LHS$= 3(2) + 4 = 6 + 4 = 10$
RHS $= 10$
LHS $=$ RHS

$x = 2$ is the solution of the equation.

Solution of an equation is the value of the variable which when substituted in the equation makes LHS = RHS

Balancing Method

Methods for Solving Linear equations in one Variable

There are many methods let us start via the balancing method.

i) Balancing Method

The value of the left-hand side remains equal to the value of the right-hand side if,

1. The same number is added to both sides of the equation.

2. The same number is subtracted from both sides of the equation.

3. Both sides of the equation are multiplied by the same number.
4. Both sides of the equation are divided by the same number.

Solve:

$2x - 4 = 6$

Add 4 to both sides

$2x - 4 + 4 = 6 + 4$

$2x = 10$

Divide both sides by 2,

$x = 5$

Solve:

$\frac{1}{3}x + 1 = \frac{5}{3}$

Subtracting 1 from both the sides:-

$\frac{1}{3}x + 1 - 1 = \frac{5}{3} - 1$

$\frac{1}{3}x = \frac{2}{3}$ => $x = 2$

Transposing Method

ii) Transposing Method

Transposing means moving to the other side. When a number is transposed from one side to the other side, its sign changes. Transposition of a number has the same effect as adding same number to (or subtracting same number from) both the sides.

Solve:

$2x - 4 = 6$

Transposing 4 to right hand side

$2x = 6 + 4$

$2x = 10$

Divide both sides by 2,

$x = 5$

Cross-Multiplication Method

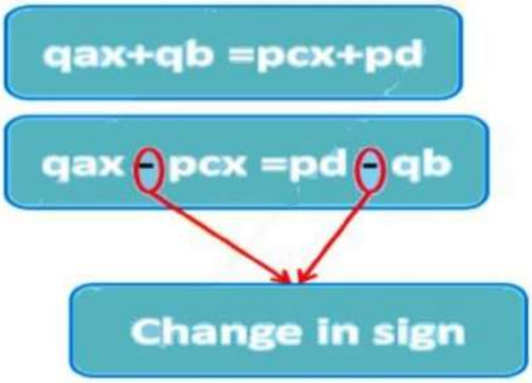
iii) Cross-Multiplication Method

When we multiply the numerator on the LHS with the denominator on the RHS and equate it to the product of denominator on the LHS with the numerator on the RHS, then it is called Cross Multiplication Method.
Let us consider the equation:

$$\frac{ax+b}{cx+d} \times \frac{p}{q}$$

$$q(ax+b) = p(cx+d)$$

When we transpose any term from LHS to the RHS or any term from RHS to the LHS the sign of that term changes.



Solve:

$$\frac{9x}{7-6x} = 15$$

$9x = 15(7-6x)$ (By Cross Multiplication)

$9x = 105 - 90x$

$9x + 90x = 105$ (Transposing $90x$ to LHS)

$99x = 105$

$\frac{99x}{99} = \frac{105}{99}$

Dividing both sides by 99

$x = \frac{35}{33}$

Solve: $\frac{2x+1}{2x+3} = \frac{2}{5}$

5(2x+1) = 2(2x+3) (By Cross Multiplication)

10x+5 = 4x + 6 (Transposing 4x to LHS and 5 to RHS)

(Sign changes as 4x moves to LHS and 5 moves to RHS)

10x – 4x = 6 – 5

6x = 1

x = $\frac{1}{6}$

Solve: $\frac{2x-1}{2x+3} = \frac{2}{5}$

5(2x-1) = 2(2x+3)   (By Cross Multiplication)

10x-5 = 4x + 6   (Transposing 4x to LHS and 5 to RHS)

10x – 4x = 6 + 5

(Sign changes as 4x moves to LHS and 5 moves to RHS)

6x = 11

$x = \frac{11}{6}$

Linear Equation with variable on one side

Linear Equation with Variable on One Side

Equations such as 2x-3 = 7, 4y = 2 or 4+3y = 7 that has linear expressions on the one side and numbers on the other side can be solved very easily using any of the methods that we have learned earlier.

Let us look at some of the examples.

Example: Find the solution of 5x – 9 = 11.
Sol.

- Step 1: Add 9 to both sides.
  5x – 9 + 9 = 11 + 9
  5x = 20

- Step 2: Next divide both sides by 5.

$$\frac{5x}{5} = \frac{20}{5}$$

x = 4 is the required solution.

Example: Find the solution of 2x – 4 = 6.
Sol.

- Step 1: Add 4 to both sides.

$2x - 4 + 4 = 6 + 4$
$2x = 10$

- Step 2: Next divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

x = 5 is the required solution.

Solving equation having variable on both the sides

We have seen equations such as 2x-3 = 7, 4y = 2 or 4+3y = 7 that have linear expressions on the one side and numbers on the other ide but this might not be the case always.

Both sides could have expressions with variables.

Let us look at some of the examples.

Example: Solve 2x – 4 = x + 2
Sol.
We have 2x – 4 = x + 2
Transposing 4 to the other side, we get
2x = x + 2 + 4
2x = x + 6
2x – x = x + 6 – x (subtracting x from both sides)
x = 6 is the required answer.

Example: Solve 3x – 3 = 2x + 2
Sol.
We have 3x – 3 = 2x + 2
Transposing 3 to the other side, we get
3x = 2x + 2 + 3
3x = 2 + 5
3x – 2x = 2x + 5 – 2x (subtracting 2x from both sides)
x = 5 is the required answer.

Some Applications of linear equations

Linear equations are used in our daily life. There are many ways in which the applications of linear equations are useful to us.

Let us look at some of the examples.

Example: The perimeter of a rectangle is 20 cm and its width is 6 cm. Find its length.
Sol.
Let length of the rectangle be ‘l’
And it is given that breadth of the rectangle = 6 cm
Perimeter of the rectangle = 2 (length + breadth)
Perimeter of the rectangle = 2 (l + 6)
It is given that perimeter of the rectangle = 20
So,
2(l+6) = 20
2l + 12 = 20

- Step 1 Subtract 12 to both sides.
  2l +12-12 = 20 -12
  2l = 8
- Step 2 Next divide both sides by 2.

$$\frac{2l}{2} = \frac{8}{2}$$

l= 4 is the required length of the rectangle.

Example: The present age of Sahil’s mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages. (REFERENCE: NCERT)
Sol.

Let Sahil’s present age be x years.
Then, Mother’s present age = 3x

	Sahil	Mother	Sum of ages
Present age	x	3x	4x
Age after 5 yrs	x + 5	3x + 5	4x+10

Therefore,

4x + 10 = 66

This equation determines Sahil’s present age which is x years.

To solve the equation, we transpose 10 to RHS,
$4x = 66 - 10$
$4x = 56$
$x = \frac{56}{4} = 14$
Thus, Sahil's present age is 14 years and his mother's age is 42 years.

Example: Karan is twice as old as Priya. Five years ago his age was three times Priya's age. Find their present ages.

Sol.
Let us take Priya's present age to be x years.
Then Karan's present age would be 2x years.
Priya's age five years ago was (x – 5) years.
Karan's age five years ago was (2x – 5) years.
It is given that Karan's age five years ago was three times Priya's age.
Thus, $2x - 5 = 3(x - 5)$
or $2x - 5 = 3x - 15$
or $15 - 5 = 3x - 2x$
or $10 = x$
So, Priya's present age = x = 10 years.
Therefore, Karan's present age = $2x = 2 \times 10 = 20$ years.

Example: The sum of three consecutive multiples of 3 is 342. Find these multiples.

Sol.
Let the first multiple of 3 be x.
Then, second multiple of 3 = x +3
And third multiple of 3 = x +3+3 = x+6

Now,
Sum of three consecutive multiple of 3 = $x + (x+3) + (x+6) = 3x + 9$
It is given that Sum of three consecutive multiple of 3 = 342
So,
$3x+9 = 342$
Transposing 9 to the RHS, we get
$3x = 342 - 9$
$3x = 333$
Dividing both sides by 3, we get

$\frac{3x}{3} = \frac{333}{3}$

x = 111

So,

The first multiple of 3 is 111.

Then, second multiple of 3 = 111 +3 = 114

And third multiple of 3 = 111+6 = 117

Example: The difference between two whole numbers is 36. The ratio of the two numbers is 2 : 5. What are the two numbers?

Sol.

Since the ratio of the two numbers is 2: 5, we may take one number to be 2x and the other to be 5x. (As 2x: 5x is same as 2: 5.)

The difference between the two numbers is (5x – 2x) = 3x

It is given that the difference is 36.

Therefore,

3x = 36

x = 12

Since the numbers are 2x and 5x,

They are 2 × 12 or 24 and 5 × 12 or 60, respectively.

Example: The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

(REFERENCE: NCERT)

Sol.

Let us take the two-digit number such that the digit in the units place is b.

The digit in the tens place differs from b by 3.

Let us take it as b + 3.

So the two-digit number is 10 (b + 3) + b = 10b + 30 + b = 11b + 30.

With interchange of digits, the resulting two-digit number will be

10b + (b + 3) = 11b + 3

If we add these two two-digit numbers, their sum is

(11b + 30) + (11b + 3) = 11b + 11b + 30 + 3 = 22b + 33

It is given that the sum is 143.

Therefore,

22b + 33 = 143

22b = 143 – 33

$22b = 110$

$b = \frac{110}{22}$

$b = 5$

The units digit is 5 and therefore the tens digit is 5 + 3 which is 8.
The number is 85.

Reducible linear equations

Reducing Equations to a simpler form

There are some equations that are complex but can be reduced to a simpler form.

For example: Solve $\frac{6x+1}{3} + 1 = \frac{x+1}{3}$

(REFERENCE: NCERT)

Sol.

Multiplying both sides by 6, we get

$(\frac{6x+1}{3} + 1) \times 6 = (\frac{x+1}{3}) \times 6$

$2 (6x + 1) + 6 = x - 3$

$12x + 2 + 6 = x - 3$

$12x + 8 = x - 3$

$12x - x + 8 = - 3$

$11x + 8 = - 3$

$11x = -3 - 8$

$11x = -11$

$x = - 1$

Example: Solve $5x - 2 (2x - 7) = 2 (3x - 1) + \frac{9}{2}$

Sol.

$LHS = 5x - 4x + 14 = x + 14$

$RHS = 6x - 2 + \frac{9}{2} = 6x - \frac{4}{2} + \frac{9}{2} = 6x + \frac{5}{2}$

Now,

x + 14= 6x + $\frac{5}{2}$

14 - $\frac{5}{2}$ = 6x −x

$\frac{28-5}{2}$ = 5x

$\frac{23}{2}$ = 5x

$\frac{23}{10}$ = x

So, x = 2.3

Equations reducible to linear form

There are some equations that are not linear but can be reduced to linear form.

For example, Solve $\frac{x+1}{2x+3}$ = $\frac{3}{10}$

Sol.

Here, the given equation is not in linear forms.

On cross multiplying, we get

10(x+1) = 3(2x + 3)

So it gets converted into the linear form.

Now, 10x + 10 = 6x + 9

Transposing 10 and 6x to the other side, we get

10x – 6x = 9 – 10

4x = -1

Dividing both sides by 4, we get

x = - $\frac{1}{4}$