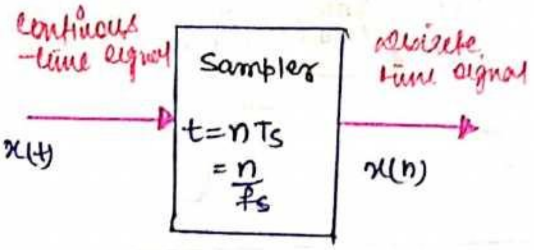
Signal and system

[Only important concepts and formulae]

Conversion of continuous to Discrete :



T_s : Sampling interval.

f_s : Sampling frequency.

$x(t)$ ke function me $t = \frac{n}{f_s}$ put kardena hai.

Concept :

If $x(n) = \{ \overrightarrow{A \quad B} \quad C \quad D \}$

then $x(-n) = \{ D \quad C \quad \overleftarrow{B \quad A} \}$

Eg : $x(n) = \{ 1, -1, 2, 3 \}$

$\Rightarrow x(-n) = \{ 3 \quad 2 \quad -1 \quad 1 \}$

Concept:

Eg $x(n) = \{ 1, -1, 2, 3 \}$

$\quad n=-1 \quad \underset{n=0}{\uparrow} \quad n=1 \quad n=2$

(1) agar $x(n-3)$ banana hai to $x(n)$ ke arrow ko 3 unit left me shift kardo.

$x(n-3) = \{ 0, 0, 1, -1, 2, 3 \}$

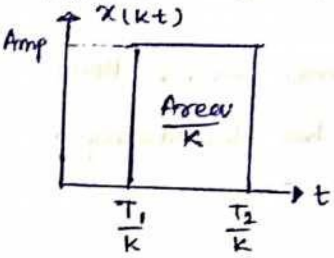
$\underset{n=0}{\uparrow} \quad n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5$

(2) agar $x(n+2)$ banana hai to $x(n)$ ke arrow ko 2 unit right me shift kardo

$x(n+2) = \{ 1 \quad -1 \quad 2 \quad 3 \}$

Note : aisy time scaling property me hi time width me change ata hai.

General Rule for time scaling :



$x(kt)$

- $k > 1$ ↓ Area ↓ ↓ Time compression ↓ Speed up
- $0 < K < 1$ ↓ Area ↑ ↓ Time Expan ↓ Speed down

$x[kn]$

- $K > 1$ ↓ $x[Mn]$ M ∈ integer $M > 1$ ↓ Samples ↓ ↓ Down Sampling ↓ ** Decimation [Time compression]
- $0 < k < 1$ ↓ $x[\frac{n}{L}]$ L ∈ integer $L > 1$ ↓ Samples ↑ ↓ Up sampling ↓ ** Interpolation [Time Expansion]

Note : Samples Lost in Decimation cannot be recovered.

Note :

(1) $x(t) \xrightarrow[\text{Time expansion}]{t \to \frac{t}{2}} x\left(\frac{t}{2}\right) \xrightarrow[\text{Time compression}]{t \to 2t} x(t)$

(2) $x(t) \xrightarrow[\text{Time compression}]{t \to 2t} x(2t) \xrightarrow[\text{Time Expansion}]{t \to \frac{t}{2}} x(t)$

(3)

$$x[n] \xrightarrow[\text{Interpolati}]{n \to \frac{n}{2}} x\left[\frac{n}{2}\right] \xrightarrow[\text{Decimation}]{n \to 2n} x[n]$$

(4)

$$x[n] \xrightarrow[\text{Decimate}]{n \to 2n} x[2n] \xrightarrow[\text{Interpolat}]{n \to \frac{n}{2}} \neq x[n]$$

→ samples that are lost during Decimat cannot be recovered with Interpolation.

$x(n) \to x(3-4n)$ Banane ke liye pehle Shifting use karna hai phir scaling.

Concept:

① $$x(t) \xrightarrow[\text{Time shifting}]{t \to t-t_0} x(t-t_0) \xrightarrow[\text{Time scaling}]{t \to kt} x(kt-t_0)$$
$$= y(t) \qquad = y(kt)$$

② $$x(t) \xrightarrow[\text{Time scaling}]{t \to kt} x(kt) \xrightarrow[\text{Time shifting}]{t \to t-\frac{t_0}{k}} x\left[k\left(t-\frac{t_0}{k}\right)\right]$$
$$= z(t)$$
$$= x[kt-t_0]$$
$$= z\left(t-\frac{t_0}{k}\right).$$

Agar time shifting, time scaling ke bad hora hai to shifting amount (t_0) ko divide by k karenge.

Area Concept:

$$-A \cdot x[-kt \pm t_0]$$

Amplitude Inversion → -ve Area

Time Inversion → No Change in Area

Amplitude Scaling → A x area

Time Scaling → $\frac{\text{Area}}{K}$

Time shifting → no change in area

If $x(t) \to$ Area

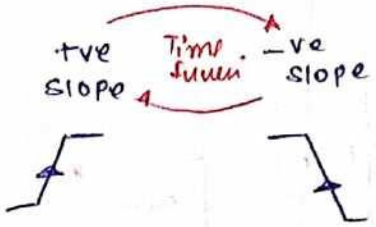
then

Area of $\left[-A \cdot x[-kt \pm t_0]\right]$

$$= -\frac{A}{K} \times \text{Area}_{\text{old}}.$$

Concept:

Agar Time Inversion hua to +ve slope -ve slope banenge aur -ve slope, +ve slope banenge.



Unit step signal:

① $u(kt) = u(t)$

for all +ve k i.e $k > 0$.

② $u(k(t \pm t_0)) = u(t \pm t_0)$

Unit step signal pe time scaling ka koi asar nai hota hai.

③ $u(-kt) = u(-t)$

$k > 0$

④ $u(-k(t \pm t_0)) = u(-(t \pm t_0))$

$k > 0$

Step signal waveform:

$U[\pm kt \pm t_0]$

- put term inside bracket = 0
- find the t from above step
  ∴ t = constant term
- Amplitude = 1
  - +ve sign with t → constant to ∞
  - -ve sign with t → constant to -∞.

① $u(t) + u(-t) = 1$

② $u(t \pm t_0) + u(-(t \pm t_0)) = 1$

Infinite duration signal :

Sum of coefficient (sum of amplitude with sign) of all the steps term $\neq 0$

for eg $u(t) + u(t-2)$

Sum of coefficient $= 1+1$
$= 2 \neq 0$

Finite duration signal :

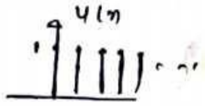
Sum of coefficient of all step term $= 0$

for eg : $u(t) - u(t-2)$

Sum of coefficient $= 1-1$
$= 0.$

Unit step sequence :

$$U[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



→ u(t) is not defined at t = 0 but U(n) is defined at n = 0 .

① $U[kn] = U[n]$

Decimation $k > 1$ $k \in$ integer greater than 1 .

② $U\left[\frac{n}{k}\right] \neq U[n]$

Interpolation $k > 1$

③ $U[-kn] = U[-n]$

Decimation

$U\left[-\frac{n}{k}\right] \neq U[-n]$

Interpolation

Except Interpolation , all properties of x(t) are valid for x(n).

Note : Amplitude = 1

- $U[n - \text{constant}] \rightarrow$ from constant to $+\infty$
- $U[\text{constant} - n] \rightarrow$ from constant to $-\infty$
- $U[\pm n + n_0] \Rightarrow n_0 =$ constant

$\pm n \rightarrow$ integer

$+n \rightarrow$ constant to $+\infty$

$-n \rightarrow$ constant to $-\infty$

Same

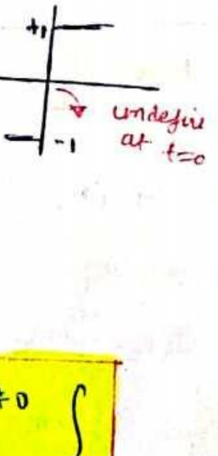
Decimation + Shifting ke liye valid nahi hai.

General Rule for finding No. of samples :

$$U[n-a] - U[n-b] = \begin{cases} 1 & a \leq n \leq b-1 \\ 0 & \text{else} \end{cases}$$

No. of samples = $b - a$

SIGNUM Function :



By Gibb's phenomena

$$\text{sgn}(0) = \frac{\text{sgn}(0^-) + \text{sgn}(0^+)}{2} = \frac{-1+1}{2} = 0.$$

$$\boxed{\text{sgn}(t) = \begin{cases} \frac{t}{|t|} & t \neq 0 \\ 0 & t = 0 \end{cases}}$$

① $\boxed{\text{sgn}(t) = -\text{sgn}(-t)}$

Reflects odd function property so sgn is odd function.

$f(-t) = -f(t)$.

② $\boxed{\operatorname{sgn}(t \pm t_0) = -\operatorname{sgn}(-(t \pm t_0))}$

③

$$\operatorname{sgn}(t) = u(t) - u(-t)$$

④

$$\boxed{u(t \pm t_0) = \frac{\operatorname{sgn}(t \pm t_0) + 1}{2}}$$

⑤ $\operatorname{sgn}[Kt] = \operatorname{sgn}(t)$

$K > 0$

Scaling does not affect sgn function.

General rule:

$\operatorname{sgn}[\pm Kt \pm t_0] \Rightarrow t = \text{constant}$

If $-t$

RHS $= -1$

LHS $= +1$

$\operatorname{sgn}(-t)$

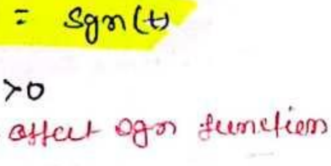
If $+t$

RHS $= +1$

LHS $= -1$

$\operatorname{sgn}(t)$.

Signum Sequence:



$$\operatorname{sgn}[n] = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$

Relationship b/n step sequence and signum sequence:

① $\operatorname{sgn}(n) = u(n) - u(-n)$

② $\overset{*}{\operatorname{sgn}}(n) = u(n-1) - u(-n-1)$

Real Exponential Signal:

* Continuous time signal ·

① $x(t) = Ae^{\alpha t}, \alpha > 0$ Growing Exponential

② $x(t) = Ae^{\alpha t}, \alpha < 0$

or $Ae^{-\beta t}, \beta > 0$

Decaying exponentially

* Discrete time signal :-

(1) $x[n] = a^n, a > 1$

unilateral

Growing exponential sequence

(2) $x[n] = a^n, \quad 0 < a < 1$

unilateral

Decaying Exponential sequence

(3) $x[n] = a^n, -1 < a < 0$

Bilateral

Decaying exponential sequence

(4) $x[n] = a^n \cdot a < -1$

Bilateral

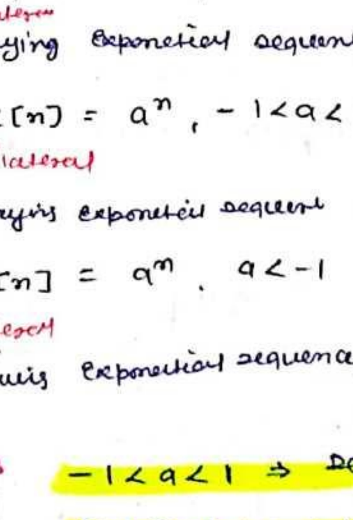
Growing exponential sequence.

Note: $-1 < a < 1 \Rightarrow$ Decaying

$a > 1$ or $a < -1 \Rightarrow$ Growing

Rectangular or Gate function·

Rect :



$$\underset{\text{Amplitude}}{A} \operatorname{rect}\left(\frac{t}{\underset{\text{Total width}}{\tau}}\right) = \begin{cases} A & |t| < \frac{\tau}{2} \\ \frac{A}{2} & |t| = \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$

Area $= A\tau$

* Rect is an Even signal,

$$\boxed{A \operatorname{rect}\left(\frac{t}{\tau}\right) = A \operatorname{rect}\left(\frac{-t}{\tau}\right)}$$

$A\,\text{rect}\left(\frac{t \pm t_0}{\tau}\right) = A\,\text{rect}\left[-\left(\frac{-t \pm t_0}{\tau}\right)\right]$

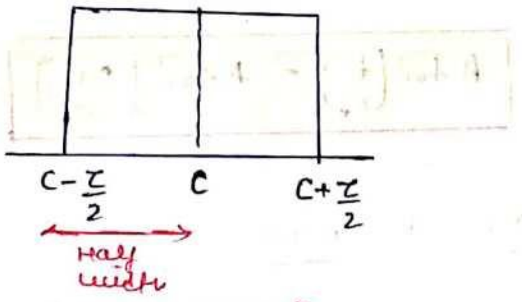
Relation b/m rectangular function and step signal :-

$$\boxed{A\,\text{rect}\left(\frac{t}{\tau}\right) = A\,u\left(t+\frac{\tau}{2}\right) - A\,u\left(t-\frac{\tau}{2}\right)}$$

Relationship b/m Rect function and sgn function :

$$A\,\text{rect}\left(\frac{t}{\tau}\right) = \frac{A}{2}\,\text{sgn}\left(t+\frac{\tau}{2}\right) - \frac{A}{2}\,\text{sgn}\left(t-\frac{\tau}{2}\right)$$

rect ka waveform plot karte samaj phle scaling karege phir shifting .



eg plot . rect [5-2t]

step ① $5-2t = 0$

$t = 2.5$

centre = 2.5

step ② $\text{rect}(-2t) = \text{rect}(2t)$

$\tau = \frac{1}{2}$

$\therefore \frac{\tau}{2} = \frac{1}{4}$

$\therefore$ RHS = $2.5 + \frac{1}{4}$

$= 2.75$

LHS = $2.5 - \frac{1}{4}$

$= 2.25$.

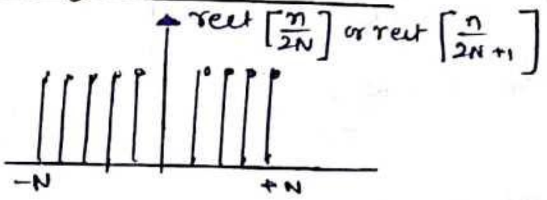
$$x(t) = A \cdot \text{rect}\left[\frac{t - \overset{\frac{RHS+LHS}{2}}{\text{centre}}}{\underset{\substack{\text{Total width} \\ = RHS - LHS}}{\tau}}\right]$$

$\downarrow$ Amplitude change = upper-lower

method for write rect function from graph .

→ rect function likhne ke do tarike hote hai , Horizontally aur vertically to option ke hisab se exam me jayge ki Horizontally likhne ya vertically .

Rectangular sequence :



$$\text{rect}\left[\frac{n}{2N}\right] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{Else} \end{cases}$$

No. of samples $= N-(-N)+1$

$= 2N+1$.

rect $\left[\frac{n}{4}\right]$	rect $\left[\frac{n}{5}\right]$
Even agar even hai to 2N se equate karna hai	Odd raha to 2N+1 se equat karna hai .
$2N = 4$ $N = 2$ $\therefore -2 \le N \le +2$	$2N+1 = 5$ $N = 2$ $\therefore$ In $-2 \le N \le +2$.

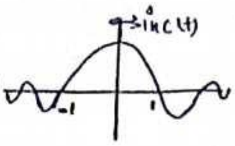
Relation b/m Rectangular sequence and step sequence :

$\text{rect}\left[\frac{n}{2N}\right] = u[n+N] - u[n-(N$

No. of sample = 2N+1

* Sinc function :-

(1) $\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$



* (2) $\text{sinc}(0) = 1$

* (3) $\text{sinc}(\pm\infty) = 0$

(4) Zero crossing points $t = \pm n = \pm 1, \pm 2$.

(5) $\text{sinc}(t) = \text{sinc}(-t)$ → Even signal.

(6) Non-periodic signal

(7) monotonically decreasing function.

** (8) $\text{Area}[\text{sinc}(t)] = 1$

* (9) $\text{Energy}[\text{sinc}(t)] = 1$

(9) $\text{Area}[\text{sinc}(Kt)] = \frac{1}{K}$

(10) zero crossing points : $t = \frac{\pm n}{K} = \frac{\pm 1}{K}, \frac{\pm 2}{K}$..

** Sampling function :

① $Sa(t) = \frac{\sin t}{t}$

② $Sa(0) = 1$

③ $Sa(\pm\infty) = 0$

(4) zero crossing Point $t = \pm n\pi$,
$n = 1, 2, \ldots$

(5) $Sa(t) = Sa(-t)$ → Even signal.

(6) Non periodic signal

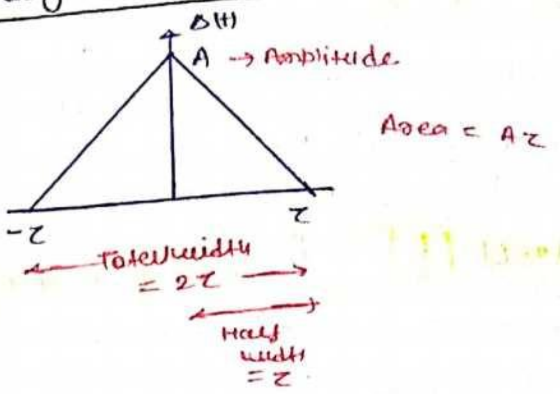
(7) Monotonically decreasing function

(8) $\text{Area}[Sa(t)] = \pi$

Relation-ship sinc(t) and Sa(t) :

$$\boxed{\begin{aligned} \text{sinc}(t) &= Sa(\pi t) \\ Sa(t) &= \text{sinc}\left(\frac{t}{\pi}\right) \end{aligned}}$$

* Triangular function :



$$A\,\text{tri}\left(\frac{t}{\tau}\right) = A\left[1 - \frac{|t|}{\tau}\right] \quad |t| < \tau \text{ or } -\tau < t < \tau$$

↓

Half width hai subki reet me full width hota hai 'τ',

$$= A\left[1 - \frac{t}{\tau}\right] \quad 0 < t < \tau$$

$$= A\left[1 + \frac{t}{\tau}\right] \quad -\tau < t <$$

$$= 0 \quad \text{else}$$

$$\boxed{A\,\text{tri}\left(\frac{t}{\tau}\right) = A\,\text{tri}\left(\frac{-t}{\tau}\right)} \rightarrow \text{Even signal}$$

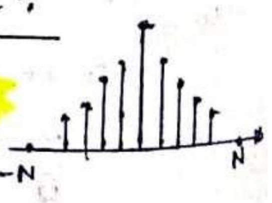
Triangular waveform :

$A\,\text{tri}\left[\frac{t - \text{centre}}{\tau}\right]$

- Bracket term = 0, then we centre
- RHS shift = constant +
- LHS time = constant

Triangular Sequence :

$$\text{tri}\left(\frac{n}{N}\right) = 1 - \frac{|n|}{N}$$



Impulse signal: (Most Imp)

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$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t\neq 0 \end{cases} \quad \text{an} \quad \int_{-\infty}^{\infty}\delta(t)\,dt = 1$$

- $\delta(t) = \lim_{a\to 0} \frac{1}{a}\,\text{rect}\left(\frac{t}{a}\right)$
- $\delta(t) = \lim_{a\to 0} \frac{1}{a}\,\text{tri}\left(\frac{t}{a}\right)$
- $\delta(t) = \lim_{a\to 0} \frac{1}{a}\,\text{sinc}\left(\frac{t}{a}\right)$

→ δ(t) is an Even signal.

$$\boxed{\delta(t) = \delta(-t)}$$

$$\delta(t \pm t_0) = \delta(-(t \pm t_0))$$

→ $\delta(t) = \frac{d}{dt}u(t)$.

→ $u(t) = \int_{-\infty}^{t}\delta(\tau)\,d\tau$ Normal Integration not area

Properties of Impulse signal:-

Area is $\int_{-\infty}^{+\infty}\delta(t)$

① **Area:** $\int_{-\infty}^{\infty}\delta(t)\,dt = 1$ ↓ at t=0 it exist

Note: If existence of Impulse is in Between the limits of Integration, then its area is 1.

Ex-1 $\int_{-0}^{+0}\delta(t)\,dt = 1$
↓ at t=0 pe Impulse exist karta aur 0 +0 aur -0 ke Bich Exist karta hai.

Ex-2 $\int_{0^+}^{\infty}\delta(t)\,dt = 0$
↓ at t=0 pe Impulse exist karta aur 0 0^+ to ∞ ke Bahar ata hai.

Ex-3 ** $\int_{0}^{\infty}\delta(t)\,dt$ = undefined.

If any one of limit of Integration is same as the point of existence of δ(t), the value is undefined.

② Time scaling with Inversion :

$$\delta(Kt) = \frac{1}{|K|}\delta(t)$$

K = any value other than Zero(0).

$$\delta(K(t\pm t_0)) = \frac{1}{|K|}\delta(t \pm t_0)$$

③ Product Property or Equivalence property or sampling property :

- $f(t)\,\delta(t) = f(0)\cdot\delta(t)$

f(t) must be defined or continuous at t=0.

- $f(t)\,\delta(t-a) = f(a)\cdot\delta(t)$
- $f(t)\,\delta(t+a) = f(-a)\cdot\delta(t)$.

④ Modified product property or Integration property :-

- $\int_{-\infty}^{\infty} f(t)\cdot\delta(t)\,dt = f(0)$
- $\int_{-\infty}^{\infty} f(t)\cdot\delta(t-a)\,dt = f(a)$
- $\int_{-\infty}^{\infty} f(t)\cdot\delta(t+a)\,dt = f(-a)$.

→ Yahan bhi wo between the limits aur at limit wala rule applicable rahega.

⑤ Convolution property :-

$$x(t)\otimes h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\,d\tau$$

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- $x(t)\otimes\delta(t) = x(t)$
- $x(t)\otimes\delta(t-a) = x(t-a)$
- $x(t)\otimes\delta(t+a) = x(t+a)$

Unit step function act as a limit changer when comes in Integration.

Impulse function properties :

$$\delta[g(t)] = \sum_i \frac{\delta(t-t_i)}{|g'(t_i)|} = \sum \frac{\delta(t-t_i)}{\left|\frac{d}{dt}(g(t))\right|_{t=t_i}}$$

mod i.e. only magnitude lena hai.

→ t_i are roots (values) of $g(t)$.

$$\int_{-\infty}^{\infty} f(t)\cdot\delta(g(t))\,dt = \sum_i \frac{f(t_i)}{|g'(t_i)|}$$

Note :

* $$\delta[t^2-a^2] = \frac{1}{2|a|}\left[\delta(t+a) + \delta(t-a)\right]$$

* $$\int_{-\infty}^{\infty} f(t)\cdot\delta[t^2-a^2]\,dt = \frac{1}{2|a|}\left[f(-a) + f(a)\right]$$

* $$\delta[A\sin kt] = \frac{1}{A\times k}\sum_{n=-\infty}^{n=\infty}\delta\left(t-\frac{n\pi}{k}\right)$$

* $$\delta[A\cos kt] = \frac{1}{A\times k}\sum_{n=-\infty}^{n=\infty}\delta\left(t-(2n+1)\frac{\pi}{2k}\right)$$

* $$u[\sin t] = \sum_{n=-\infty}^{\infty} u[t-2n\pi] - \sum_{n=-\infty}^{\infty} u(t-(2n+1)\pi)$$

Doublet function :

$$\delta'(t) = \frac{d}{dt}\,\delta(t)$$

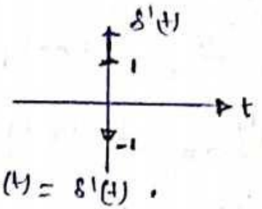
→ It is odd signal

$$\delta'(t) = -\delta'(-t)$$

Area of odd signal = 0

$$\int_{-\infty}^{\infty}\delta'(t) = 0 .$$

$$\delta'(t) = \begin{Bmatrix} +\infty \text{ and } -\infty & t=0 \\ 0 & t\neq 0 \end{Bmatrix}$$



$$\lim_{a\to 0} f(t) = \delta'(t) .$$

Properties of doublet functions ;

① Convolution property :

$$\delta'(t) \xrightarrow[\text{trans}]{\text{Laplace}} s$$

$$x(t) \otimes \delta'(t) = x'(t)$$
$$x(t) \otimes \delta'(t-a) = x'(t-a)$$
$$x(t) \otimes \delta'(t+a) = x'(t+a)$$

② Sampling property :

$$f(t)\cdot\delta'(t) = f(0)\cdot\delta'(t) - f'(0)\cdot\delta(t)$$

$$f(t)\cdot\delta'(t-a) = f(a)\cdot\delta'(t-a) - f'(a)\cdot\delta(t-a)$$

$$f(t)\cdot\delta'(t+a) = f(-a)\cdot\delta'(t+a) - f'(-a)\,\delta(t+a)$$

f · doublet − f' impulse

ese yad rakh skte hai.

\# $$t\cdot\delta'(t) = -\delta(t)$$

\# $$t^2\cdot\delta'(t) = 0$$

\# $$t^n\cdot\delta^n(t) = (-1)^n n!\,\delta(t)$$

\# $$t^{n+1}\cdot\delta^n(t) = (-1)^n n!\,t\cdot\delta(t) =$$

③ generalised derivative :

$$\int_{-\infty}^{\infty} f(t)\ \delta'(t)\ dt = -f'(0)$$

$$\int_{-\infty}^{\infty} f(t)\ \delta'(t-a)\ dt = -f'(a)$$

$$\int_{-\infty}^{\infty} f(t)\ \delta'(t+a)\ dt = -f'(-a)$$

$$\int_{-\infty}^{\infty} f(t)\cdot\delta^{n}(t)\,dt = (-1)^{n} f^{n}(0)$$

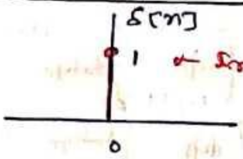
$$\int_{-\infty}^{\infty} f(t)\cdot\delta^{n}(t-a)\,dt = (-1)^{n} f^{n}(a)$$

$$\int_{-\infty}^{\infty} f(t)\cdot\delta^{n}(t+a)\,dt = (-1)^{n} f^{n}(-a)$$

④ Time scaling with Time Inversion ;

$$\boxed{\delta'(Kt) = \frac{1}{K|K|}\delta'(t)}$$

Unit Impulse sequence or Sample sequence



← In n domain, it amplitude = Area

$$\boxed{\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}}$$

$$\boxed{\text{Area}\,[\delta(n)] = \delta[0] = 1}$$

Properties :

① Even signal :

$$\boxed{\delta[n] = \delta[-n]}$$

$$\delta[n \pm n_0] = \delta(-(n \pm n_0)]$$

② Time scaling ;

$$\boxed{\delta[Kn] = \delta[n]}$$

where as $\delta[Kt] = \frac{1}{|K|}\delta(t)$

$\delta(2n) = \delta(n]$ ← decimation

$\delta(\frac{n}{2}) = \delta(n)$ ← interpolation

} → Both valid

$$\delta[K(n \pm n_0)] = \delta(n \pm n_0)$$

$$\delta[-Kn] = \delta[-n] = \delta(n).$$

③ Convolution property :

$$x(n) \otimes \delta[n] = x(n)$$

$$x(n) \otimes \delta[n-n_0] = x[n-n_0]$$

$$x(n) \otimes \delta[n+n_0] = x(n+n_0)$$

④ Product Property :

$$x[n]\cdot\delta[n] = x(0)\cdot\delta[n]$$

$$x[n]\cdot\delta[n-a] = x(a)\cdot\delta[n-a]$$

$$x[n]\cdot\delta[n+a] = x(-a)\cdot\delta[n+a]$$

⑤ Summation property :

$$\sum_{n=-\infty}^{\infty} x[n]\cdot\delta[n] = x(0)$$

$$\sum_{n=-\infty}^{\infty} x[n]\cdot\delta[n-a] = x(a)$$

$$\sum_{n=-\infty}^{\infty} x[n]\cdot\delta[n+a] = x[-a].$$

Yahan bhi outside the limit aur inside the limit ka care karna hai.

Relationship B/w Impulse Sequence ($\delta(n)$) and ~~unit~~ Step sequ ($u(n)$) :-

① $\boxed{\delta[n] = u(n) - u(n-1)}$ —Ⓐ

$$\delta[n \pm n_0] = u[n \pm n_0] - u(n \pm n_0 - 1)$$

② $\boxed{\delta[n] = \delta[-n] = u[-n] - u(-n-1)}$

Replace n by −n in Ⓐ

③ $\boxed{1 + \delta[n] = u[n] + u[-n]}$

④ $\boxed{u[n] = \sum_{K=0}^{\infty} \delta(n-K) = \sum_{K=-\infty}^{\infty} u(K)\delta(n-K)}$

$$\boxed{u[n \pm n_0] = \sum_{k=0}^{\infty} \delta[n \pm n_0 - k] = \sum_{k=\pm n_0}^{\infty} \delta[n-k]}$$

Jahan Jaha u aega series ke andar wo limit change karjaega.

⑤ $$\boxed{u[n] = \sum_{k=-\infty}^{n} \delta[k]}$$ **.

$u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \cdots$

Series Expansion Formulae :

① $$\sum_{n=0}^{\infty} \alpha^n = 1 + \alpha + \alpha^2 + \cdots \infty = \frac{1}{1-\alpha}$$

$|\alpha| < 1$ or $-1 < \alpha < 1$

If $|\alpha| < 1$, then series converges

② $$\sum_{n=k}^{\infty} \alpha^n = \alpha^K + \alpha^{K+1} + \alpha^{K+2} + \cdots \infty$$

$$= \frac{\alpha^K}{1-\alpha} \quad |\alpha| < 1$$

③. $$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & \alpha \neq 0 \\ N & \alpha = 1 \end{cases}$$

④ $$\sum_{n=0}^{\infty} n \cdot \alpha^n = \alpha + 2\alpha^2 + 3\alpha^3 + \cdots$$

$$= \frac{\alpha}{(1-\alpha)^2} \quad |\alpha| < 1.$$

Note : $$\boxed{\sum_{n=1}^{\infty} \frac{1}{n} = \infty}$$ = Harmonic divergent series

Not absolutely Summable.

Unit Ramp signal :

$$r(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$$

$$\boxed{r(t) = t\,u(t).}$$

Relationship B/w ramp, step and Impulse :

$\delta(t) \rightarrow 1$

$u(t) \rightarrow \frac{1}{S}$

$r(t) \rightarrow \frac{1}{S^2}$

i.e $\int \delta(t) = u(t)$.

$\int u(t) = r(t)$.

$\frac{d}{dt}\delta(t) = \delta'(t)$ $\quad \delta'(t)$ $\quad \delta(t) = \int_{-\infty}^{t} \delta'(z)\,dz$

diff ↑ Integr ↓

$\frac{d}{dt}u(t) = \delta(t)$ $\quad \delta(t)$ $\quad u(t) = \int_{-\infty}^{t} \delta(z)\,dz$

diff ↑ Integr ↓

$\frac{d}{dt}r(t) = u(t)$ $\quad u(t)$ $\quad r(t) = \int_{-\infty}^{t} u(z)\,dz$.

diff ↑ Integr ↓

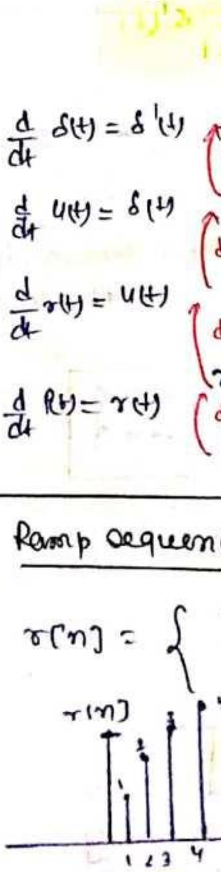
$\frac{d}{dt}R(t) = r(t)$ $\quad r(t)$ $\quad R(t) = \int_{-\infty}^{t} r(z)\,dz$

diff ↑ Integrate ↓

$P(t)$

Ramp sequence :

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



① $$\boxed{r[n] = n \cdot u[n]}$$ **

$r[n \pm n_0] = [n \pm n_0]\,u[n \pm n_0]$

② $$\boxed{u[n] = r[n+1] - r[n]}$$ **

$u[n \pm n_0] = r[n \pm n_0 + 1] - r[n \pm n_0]$

t-domain	n-domain
① Integration : $y(t) = \int_{-\infty}^{t} x(z)\,dz$	① Summation : $y[n] = \sum_{k=-\infty}^{n} x[k]$
② differentiation : $y(t) = \frac{d}{dt} x(t)$	② difference equation $y[n] = x(n) - x(n-1)$ or $y(n) = x(n+1) - x(n)$

Parabolic function:

$$P(t) = \frac{t^2}{2} u(t) = \begin{cases} \frac{t^2}{2} & t \geq 0 \\ 0 & t \leq 0 \end{cases}$$

$$P(t \pm t_0) = \frac{(t \pm t_0)^2}{2} u(t \pm t_0)$$

$$P(n) = \frac{n^2}{2} u(n) = \begin{cases} \frac{n^2}{2} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

★★★. causal, Non-Causal, and Anti causal signals :-

causal signal	Anti-causal signals	Non-Causal signal
$t \geq 0$ or $n \geq 0$	$t \leq 0$ or $n \leq 0$	$-\infty \leq t \leq \infty$ or $-\infty \leq n \leq \infty$
time ⇒ 0 to +∞	time → 0 to −∞	time → both +ve and −ve value of time
Right sided signal.	left sided signal	→ two sided or double sided signal
Ex : $u(t)$, $r(t)$, $e^{-at}u(t)$, $u(n-2)$, $\delta(n-3)$	Ex : $u(-t)$, $r(-t)$, e^{-at} $u(-t)$, $\delta(n+3)$	Ex : rect(t), tri(t), sgn(t), sin(t), cos(t).

(1)

$$\text{causal signal} \xleftrightarrow[\text{Inversion}]{\text{Time}} \text{Anti causal Signal}$$

$u(t)$ $u(-t)$

② Non causal Signal × causal signal $= $ causal signal

e^{-at} $u(t)$ $e^{-at}\,u(t)$

③ Non causal signal × Anti causal signal $=$ Anti causal signal

$\sin(t)$ $u(-t)$ $\sin t\, u(-t)$

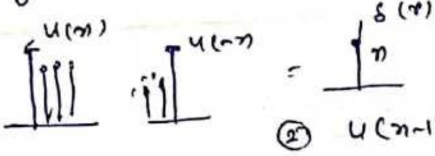
④ causal signal + Anticausal signal $=$ Non causal signal

$u(t)$ $u(-t)$

⑤ causal signal × Anti causal signal

$$= \begin{cases} \text{signal at } n = 0 \\ 0 \quad n \neq 0 \end{cases}$$

Eg. $u[n] \cdot u[-n] = \delta(n)$



② $u(n-1) \cdot u(-n) = 0$.

Note : Non causal signal jis signal se multiply hoga result other signal (jisse multiply hua) hoga. Point ② and ③.

Even signal :

(1) An even signal is symmetrical about y-axis.

(2) for even signal, RHS of y-axis = LHS of y-axis

Mathematically,

$$\boxed{\begin{aligned} x(t) &= x(-t) \\ x(n) &= x(-n) \end{aligned}}$$

Examples :

(1) Impulse (2) Rect (3) Tri

(4) Sinc (5) Sa (6) cosine

Note :

(1) constants (1, 2, . . . n), t^2, t^4, $t^{\text{even power}}$, $|t|$ are even signals

(2) Area of even signal may or may not be equal to zero.

For Even signal

Total Area = 2× RHS Area
= 2× LHS Area

$$\text{Area}[x_e(t)] = \int_{-\infty}^{\infty} x_e(t)\,dt$$
$$= 2\int_{0}^{\infty} x_e(t)\,dt = 2\times \text{RHS Area}$$
$$= 2\int_{-\infty}^{0} x_e(t)\,dt = 2\times \text{LHS Area}$$

(2). Odd signals :

(1) An odd signal is an Anti-symmetrical about y-axis
(symmetrical about origin).

(2) For an odd signal, opposite Quadrants are always same.

mathematical $x(t) = -x(-t)$
$x(n) = -x(-n)$

Examples :

(1) Doublet function (2) Signum function

(3) Sine function

→ Area of odd signal is always Zero.

→ $t^{\text{odd power}}$ are odd signals.

(3) Neither Even Nor Odd [NENO]

$$x(t) \neq x(-t)$$
$$-x(-t)$$

Examples :

$u(t)$, $r(t)$, $p(t)$, e^{-at},
e^{at}, $e^{-at}u(t)$, $u(t)$, $e^{at}u(t)$

Even signal or Odd signal	+ − ×	NENO signal	= NENO signal

Properties of Even and Odd signal:

① Uniqueness property :

If a function is both Even and odd then it is an all zero signal.
[It is equal to zero anywhere it is defined].

② ***

$x_1(t)$ or $x_1(n)$	$x_2(t)$, $x_2(n)$	$x_1(t) \pm x_2(t)$ Addition	$x_1(t)\cdot x_2(t)$ multiplication
Even	Even	Even	Even
Odd	Odd	Odd	Even
Even Odd	Odd Even	NENO*	Odd**

Dhyan rakhna hai do opp. nature ke function add hue to NENO aur multiply hue to Odd. aur donon same nature ke multiply honge to Even hi aega.

③ Multiplying by a constant :

Constant = K = Even function

K . Even = Even → E × E = E
K . odd = odd → E × O = O

④ Energy property :

$$E(t) = \int_{-\infty}^{\infty} |\underset{\text{real}}{x(t)}|^2\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$\boxed{x^2(t) = x_e^2(t) + x_o^2(t) + 2x_e(t)\,x_o(t)}$$

very useful.

Area Integration:

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \underbrace{\int_{-\infty}^{\infty} x_e^2(t)\,dt}_{E} + \underbrace{\int_{-\infty}^{\infty} x_o^2(t)\,dt}_{E}$$

$$+ \int_{-\infty}^{\infty} 2 \cdot \underbrace{x_e(t) \cdot x_o(t)}_{E \times 0 \,=\, 0}\,dt$$

$$\int_{-\infty}^{\infty} \text{Even}\,dt = \text{Area}$$

$$-\int_{-\infty}^{\infty} \text{odd}\,dt.$$

$$\boxed{\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} x_e^2(t)\,dt + \int_{-\infty}^{\infty} x_o^2(t)\,dt}$$

Energy of signal $x(t)$ = Energy of even part $x_e(t)$ + energy of odd part $x_o(t)$.

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{n=\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Integration and differentiation property:

→ Integration or differentiation of an odd signal is always even signall.

$x(t)$	Differentiation $y(t) = \frac{d}{dt}x(t)$	Integration $z(t) = \int_{-\infty}^{t} x(z)\,dz$
Odd	Even	Even
Even	Odd	Odd + constant If Area $[x(t)] = 0$, then constant $= 0$, $z(t) =$ odd signal. Area $[z(t)] = 0$ If Area $[x(t)] \neq 0$ then constant $\neq 0$, $z(t) =$ Odd + con $=$ NENO. Area $[z(t)] = \infty$

Even and odd Parts of Signal:

(1) $x_e(t) = \dfrac{x(t) + x(-t)}{2}$ (2) $x_o(t) = \dfrac{x(t) - x(-t)}{2}$

(2) If $x(t) \rightarrow$ even signal

$x(t) = x_e(t)$ and $x_o(t) = 0$

(3) If $x(t) \rightarrow$ odd signal

$x(t) = x_o(t)$ and $x_e(t) = 0$

(4) If $x(t)$ is NENO then $x_e(t) \neq 0$ and $x_o(t) \neq 0$.

Conjugate Symmetric and Conjugate Anti-Symmetric:

(1) conjugate symmetric signals:

$$\boxed{\begin{aligned} x_{cs}(t) &= x^{*}(-t) \\ x_{cs}(n) &= x^{*}(-n) \end{aligned}}$$

(2) conjugate Anti symmetric signals:

$$\boxed{\begin{aligned} x_{CAS}(t) &= -x^{*}(-t) \\ x_{CAS}[n] &= -x^{*}(-n) \end{aligned}}$$

Even se relate karo C·S ko aur odd se relate karo C·A·S ko Bus conjugate lena hai alag se.

$$\rightarrow \boxed{[A+B]^* = A^* + B^*}$$

conjugate symmetric part:

$$\boxed{x_{cs}(t) = \frac{x(t) + x^*(-t)}{2}}$$

conjugate Anti symmetric part:

$$\boxed{x_{CAS}(t) = \frac{x(t) - x^*(-t)}{2}}$$

For Conjugate symmetric signal:

Real part – Even
Imaginary part – Odd

For conjugate Anti symmetric signal:

Real part – odd
Imaginary – Even

Just opp. to C.S

→ Function check karte time agar kisi function me sirf Real part hai to bus Real part se judge karege aur agar sirf Imaginary part hai to Imaginary part se judge karege.

Eg. $x(t) = t^4$

$x(t) = t^4 + j0$

Real → Even

It is C.S

Eg. $x(t) = j+3$

$x(t) = \underbrace{j+3}_{\text{Odd}}$ Img → odd

It C.S.

Note:

→ Real → Even, Img → Even neither C.S nor C.A.S

→ Real → odd, Img → odd neither C.S nor C.A.S

Trick:

C.S match with Even signal, hence its Real Part is Even.
C.A.S match with odd signal, as its Real Part is odd.

Continuously Time periodic & Non Periodic signals:

→ A periodic signal is an everlasting signal.

→ For periodic signals,

$$x(t \pm T_0) = x(t)$$

where $T_0 = \frac{2\pi}{\omega_0}$

Representation of Continuous time periodic signal:

① $A \sin\omega_0 t$ ② $A \cos\omega_0 t$

③ $A \sin(\omega_0 t + \phi)$ ④ $A \cos(\omega_0 t + \phi)$

⑤ $A e^{j\omega_0 t}$ ⑥ $A e^{j(\omega_0 t + \phi)}$

Note:
Also

Periodic Signal $\pm$ $\times$ Non-periodic Signal = Non Periodic Signal

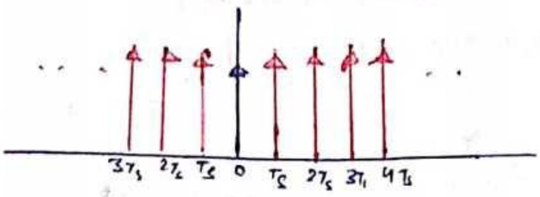
↓

[$u(t)$, $r(t)$, $p(t)$,
$sgn(t)$, $u(-t)$,
$rect(t)$, $tri(t)$,
$Sa(t)$, $sinc(t)$,
e^{at}, $e^{at}u(t)$,
e^{at}, $e^{at}u(t)$]

DC term + Periodic signal (T_0) = Periodic signal with Amplitude shifted in y-axis with Period T_0

Periodic Impulse Train :

$$\boxed{x(t) = \sum_{k=-\infty}^{k=\infty} \delta(t - kT_s)}$$



→ Periodic Impulse Train.

or

→ Dirac Comb Function :

$$\boxed{\begin{array}{c}\text{Time period } T_0 = T_s \\ f_0 = \frac{1}{T_s} \text{ Hz}\end{array}}$$

Periodicity and Time Scaling :

$$\boxed{\begin{array}{l} x(t) \longrightarrow T_0 \\ x(kt) \longrightarrow \frac{T_0}{k} \\ k>0. \end{array}}$$

** → Time period of x(t).

→ Time period is never affected by Amplitude Inversion, Amplitude Scaling, Time Inversion, and time Shifting.

Periodicity of multiple Continuous time Periodic Signals :

(1). $x_1(t) \rightarrow T_1$ → $\frac{T_1}{T_2}$ = Rational for Periodicity
$x_2(t) \rightarrow T_2$

(2). Fundamental time period will be least common multiple (LCM) of T_1 and T_2

(3) Fundamental Angular Frequency will be Highest Common factor (HCF) Or Greatest common factor of ω_1 and ω_2.

Step 1 :

$\omega_1 \Rightarrow T_1 = \frac{2\pi}{\omega_1}$

$\omega_2 \Rightarrow T_2 = \frac{2\pi}{\omega_2}$

Step 2 :

$\frac{T_1}{T_2} = \frac{P}{Q}$ = Rational no.
= Ratio of Integers
→ then Periodic.

$\frac{T_1}{T_2} \neq$ Ratio of Integers → Non periodic

Step 3 :

If Periodic

T_0 = LCM (T_1 and T_2)

W_0 = HCF (ω_1, ω_2)

f_0 = HCF (f_1, f_2)

Concept of Ratio of Integers :

① Rational Number = Ratio of Integers

$\frac{T_1}{T_2} = \frac{6}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{3}, \frac{4}{3}, \frac{22}{7}$

$\frac{1}{3} \rightarrow 0.\overline{3}$ repeat

$\frac{22}{7} \rightarrow 3.\overline{142857}$ repeat

② Irrational Number ≠ Ratio of Integers.

$\frac{T_1}{T_2} = \sqrt{2}, \frac{1}{\sqrt{2}}, \pi, \frac{1}{\pi}, \sqrt{3}$

$\sqrt{2} \rightarrow 1.41421326$

$\pi \rightarrow 3.141592654$

$\frac{1}{\pi} \rightarrow 0.3180$

No repeated pattern

Note $\frac{22}{7} > \pi$.

Concept of Lcm of fraction :

$$\text{LCM}\left(\frac{x_1}{x_2}, \frac{x_3}{x_4}\right) \Rightarrow \frac{\text{LCM of } x_1 \text{ and } x_3}{\text{HCF of } x_2 \text{ and } x_5}$$

- Kabhi bhi finite graph diya hai with periodic time period to usko extend karenge fir dono side phir expression likhenge.

Discrete Time Periodic and Non Periodic Signals :

- In discrete time, Fundamental time period must be an integer and for discrete time periodicity :-

① $x(n \pm N_0) = x(n)$

N_0 : Fundamental discrete time period in secs'

② $$\boxed{\frac{m}{N_0} = \frac{\omega_0}{2\pi} = \frac{p}{q} = \text{ratio of integers}}$$

(rational → Periodic)

m : order of Harmonics, p and q are integers.

If Periodic, then

$$\boxed{m = p \text{ and } N_0 = q}$$

+ Jabki continuous mein $T = \frac{2\pi}{\omega_0}$ tha, isliye confuse nai hona.

Discrete Domain :

$\cos\omega_0 n$, $\sin\omega_0 m$, $\cos(\omega_0 n + \phi)$, $\sin(\omega_0 n + \phi)$, $e^{j\omega_0 n}$, $e^{j(\omega_0 n + \phi)}$

→ Periodic if $\omega_0 = f(\pi')$

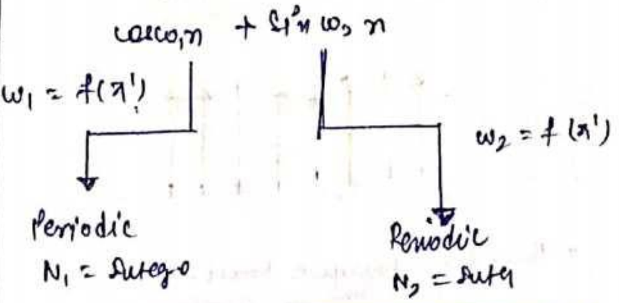
→ Non Periodic if $\omega_0 \neq f(\pi')$

For ; $\pi^0, \pi^2, \frac{1}{\pi}$).

→ In Exam if two option matches for period then we will have to select lower value as it will be Fundamental time-period.

Periodicity of multiple Discrete Time Signals :

$\cos\omega_1 n + \sin\omega_2 n$



$$\boxed{\frac{N_1}{N_2} = \text{Ratio of Integer} \Rightarrow \text{Periodic}}$$

$$\boxed{N_0 = LCM(N_1, N_2)}$$

Heart ✱

Continuous Time Energy and Power signals :

Energy signals :

① Energy signal are signals which have finite energy and zero power.

② They are mostly defined for non periodic signals, finite duration signals and stable signals.

Ar

$$\boxed{\begin{aligned} E &= \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2\, dt \\ E &= \int_{-\infty}^{\infty} |x(t)|^2\, dt \end{aligned}}$$

Power signals :

① Power signals are signals which have finite power and infinite energy.

② They are mostly defined for periodic signals, infinitely duration signals.

$$\boxed{P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt}$$

$$\boxed{P = \lim_{T\to\infty} \frac{E}{T}}$$ ***

Energy Signal	Power signals	Neither Energy nor Power signals
$0 < E < \infty$ $P = 0$	$0 < P < \infty$ $E = \infty$	$P = \frac{\infty}{\infty}$ Indeterm form

Properties of Energy Signals:

(1) $E[x(t)] = A[x^2(t)]$

→ Energy of $x(t)$ = Area of $x^2(t)$.

Time Inversion
A, E

Time Scaling
A/K, E/K

$$-C\cdot x[-Kt \pm t_0] \longrightarrow \frac{-C}{K}\cdot A,\ \frac{C^2}{K}\cdot E$$

Amplitude Inversion
$-A$, E

Amplitude Scaling
$C\cdot A$, $C^2 E$

Time Shift
A, E

A = Area

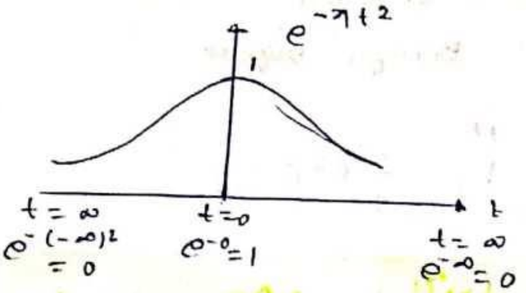
E = Energy

(1) Sinc Function:

- Non Causal • ∞ duration
- Non periodic • Even
- Bounded (and stable) and amplitude → 0 when $t \to \pm\infty$.

$\text{Sinc}(t) \to A = 1,\ E = 1$

② Gaussian Function:



- Bell shaped • ∞ duration
- Non periodic • Non causal
- Bounded • Even signal.

$$\boxed{\text{Gamma function} = \int_0^\infty e^{-x}\cdot x^{n-1}\,dx = \Gamma n}$$

- Area $[e^{-\pi t^2}] = 1$
- Energy $[e^{-\pi t^2}] = \frac{1}{\sqrt{2}}$

$e^{-\pi(tK)^2} \xrightarrow[K]{\text{Time scale}} A' = \frac{A}{K} = \frac{1}{K}$

$E' = \frac{E}{K} = \frac{1}{\sqrt{2}K}$.

③ Rectangular Function:

- Non causal • Even • Finite duration
- Bounded • Stable

→ Area $= A\tau$

→ Energy $= A^2\tau$

$\Big\}\to A\,\text{rect}\left(\frac{t}{\tau}\right)$

- $A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot u(t) \longrightarrow \text{Area} = \frac{A\tau}{2},\ \frac{A^2\tau}{2}$

④ Triangular Function :

- Non causal
- Finite duration
- Bounded
- Even
- Energy signal

**

- $E = \frac{2}{3}A^2\tau$

$$\boxed{A\ \mathrm{tri}\left(\frac{t}{\tau}\right) \longrightarrow \text{Area} = A\tau,\ \text{Energy} = \frac{2}{3}A^2\tau}$$

→ Two Half Triangle of Tri' functi ka Energy $\frac{2}{3}A^2\tau$ But one half ka $\frac{1}{3}A^2\tau$ because Tri' is Even funct.

$$E_T(\mathrm{tri}') = E_{LHS} + E_{RHS}$$
$$= \frac{1}{3}A^2\tau + \frac{1}{3}A^2\tau$$

Similar is the case with Rect. Functi Also.

$$\boxed{A\ \mathrm{tri}\left(\frac{t}{\tau}\right)\cdot u(t) \longrightarrow \text{Area} = \frac{A\tau}{2},\ E = \frac{1}{3}A^2\tau}$$

Property of Integration :

$$\int (at+b)^n = \frac{(at+b)^{n+1}}{a(n+1)}$$

**

⑤ Impulse function [$\delta(t)$] :

$$\boxed{\begin{aligned} E &= \infty \\ P &= \frac{\infty}{\infty} \end{aligned}}$$

→ Neither Energy nor power signal.

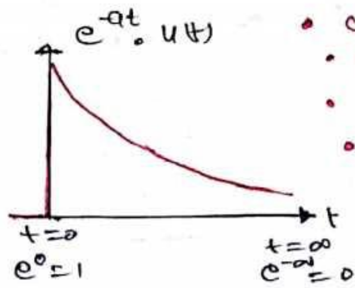
⑥ Ramp signal :

$$\boxed{\begin{aligned} E &= \infty \\ P &= \frac{\infty}{\infty} \end{aligned}}$$

→ Neither Energy nor power signal.

⑦ Real exponential signal :

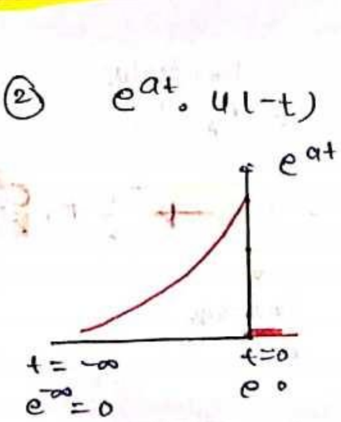
① $e^{-at}\cdot u(t)$



- causal
- ∞ duration
- Bounded
- Non periodic
- NENP
- Exponentially decreasing

$$\boxed{e^{-at}\ u(t) \longrightarrow \text{Area} = \frac{1}{a},\ \text{Energy} = \frac{1}{2a}}$$

② $e^{at}\cdot u(-t)$

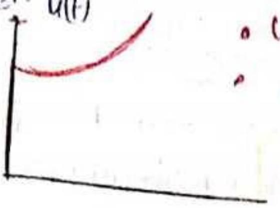


- Anti causal
- Bounded

$$\boxed{e^{at}\ u(-t) \longrightarrow \text{Area} = \frac{1}{a},\ \text{Energy} = \frac{1}{2a}}$$

→ Kabhi Bhi kisi signal ka amplitude ∞ ki taraf jaa raha hoga to wo hamesa Neither Energy nor power signal hoga.

③



- Causal
- Unbounded

→ It is Neither energy nor Power Signal.

Concept:

In exponential function, the function to be stable and Bounded

e ki power aur U(t) ke sign opp. hona chahi, same sign hue to unstable, NENP, Unbounded

all have same sign.

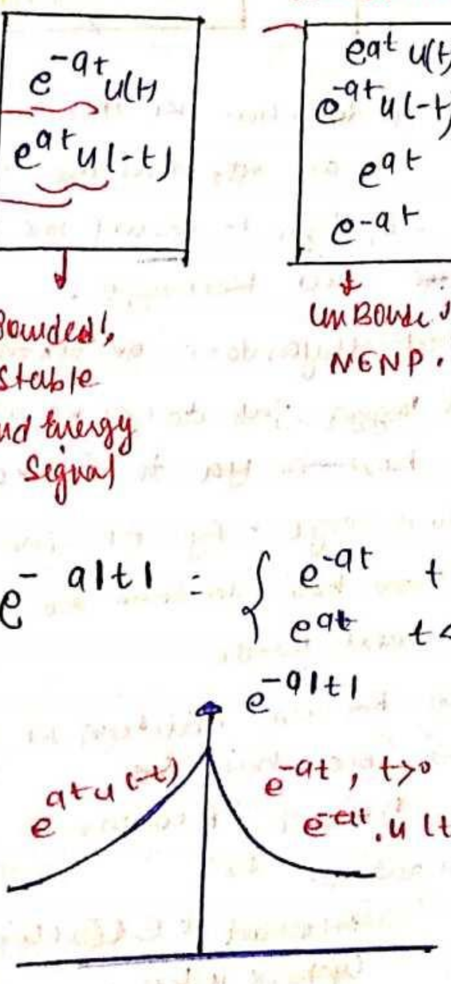
$e^{-at}u(t)$ $e^{at}u(-t)$	$e^{at}u(t)$ $e^{-at}u(-t)$ e^{at} e^{-at}
Bounded, Stable And Energy Signal	UnBounded, NENP.

opp Sign

opp Sign

④ $e^{-a|t|} = \begin{cases} e^{-at} & t>0 \\ e^{at} & t<0 \end{cases}$

★★

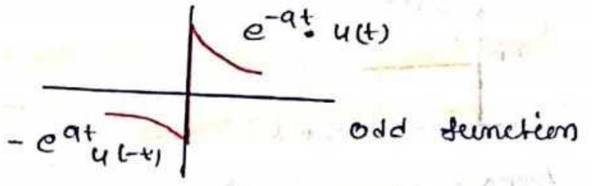


$$e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$$

- Non causal
- Double sided Exponentially decrease function
- Bounded
- Even function
- ∞ derivatives

$$\boxed{e^{-a|t|} \longrightarrow \text{Area} = \frac{2}{a} \quad E = \frac{1}{a}}$$

⑤ $e^{-a|t|}.\text{sgn}(t)$



odd function

$$\boxed{E = \frac{1}{a} \qquad A = 0}$$

If $|x(t)|$ = Even signal → Energy = E → Power = P

Then

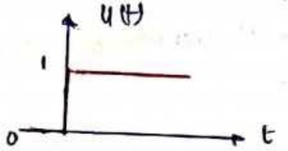
$|x(t).u(t)|$ → Energy $= \frac{E}{2}$ → Power $= \frac{P}{2}$

→ agar u(t) se multiply kardenge to right hand side graph banega to original energy power ka half hojaega । vese hi u(−t) se kardie to RHS ka Energy Bachega aur RHS = 0 (concept valid for Even function).

Note: Constant signal is a power signal.

Signal x(t)	Power	Energy
A	A^2	∞
$A\,u(t)$	$\frac{A^2}{2}$	∞
$A\cdot e^{j\omega t}$	A^2	∞
$A\cdot e^{j\omega t}u(t)$	$\frac{A^2}{2}$	∞
$A\cdot e^{j(\omega t+\phi)}$	A^2	∞
$A\cdot e^{j(\omega t+\phi)}\cdot u(t)$	$\frac{A2}{2}$	∞

⑥ Step signal :



Area = ∞

Energy = ∞

$$P = \frac{1}{T}\int_0^{T/2} 1\,dt$$

$$= \frac{1}{T}\times\frac{T}{2}$$

$$\boxed{P = \frac{1}{2}}$$

→ step signal is power signal where as impulse signal and ramp signal are NENP.

→ Power of a signal is affected only by amplitude scaling and independent of time shifting, time scaling, time inversion and Amplitude inversion. Imp.

$u(t) \to P = \frac{1}{2}$

$A\cdot u(t) \to \frac{A2}{2} = P$

$x(t) \to P$

$C\cdot x(t) \to C^2\cdot P$

Sinusoidal signals :

$A\cos\omega t$, $A\sin\omega t$, $A\cos(\omega t+\phi)$, $A\sin(\omega t+\phi)$.	$P = \frac{A^2}{2}$	$E = \infty$
$A\cdot\cos(\omega t)\cdot u(t)$, $A\sin(\omega t)\cdot u(t)$, $A\cos(\omega t+\phi)\cdot u(t)$, $A\sin(\omega t+\phi)\cdot u(t)$	$P = \frac{A^2}{4}$	$E = \infty$

$$A\cos(\omega t+\phi_1) + B\cos(\omega_2 t+\phi_2)$$

or

$$A\sin(\omega t+\phi_1) + B\sin(\omega_3 t+\phi_2)$$

$\omega_1 \neq \omega_2$

$$\boxed{P = \frac{A^2}{2} + \frac{B^2}{2}}$$

$\omega_1 = \omega_2$

$$\boxed{P = \frac{A^2}{2} + \frac{B^2}{2} + AB\cos(\phi_1-}$$

(1) Dhyan ye dena hai ki upor formula me $+\phi_1$ aur $+\phi_2$ hai agar questn me $-\phi_1$ diya to result me negatv value put karenge.

(2) Aur dhyan dena ye formula tab hi lagega jab do cos ke term add hore ho ya do sin ke term add hore honge. Agar ek sine aur ek cos me hai to dono ko ek me convert kardo

(3) Ye formula addition ke liye hi applicable hai i.e

$A\cos\omega t - B\cos\omega t$

dia rahega to

$A\cos\omega t + B\cos(\omega t+180°)$

likhege usko.

Note :

Power [DC + sinusoidal] = P[DC] + P[sinu]

= Ampl.^{2} + Amp^2

Concept :

calculate the Power and RMS value.

$x(t) = 2\sin\omega_0 t + 3\cos(\omega_0 t + \frac{\pi}{3})$.

$= 2\cos(\omega_0 t - 90^\circ) + 3\cos(\omega_0 t + \frac{\pi}{3})$

$\Rightarrow \omega_1 = \omega_2$

$\therefore P = \frac{A^2}{2} + \frac{B^2}{2} + AB\cos(90 - \frac{\pi}{3})$

$P = 1.303$ watts.

Now

$$\boxed{\text{RMS value} = \sqrt{\text{Power}}}$$ **

Power of Periodic signal ;

$$\boxed{P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 \, dt}$$

Time period of signal

Not value limit But batana hai limit- will the Period

Note :

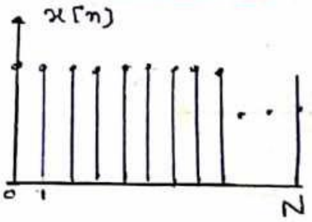
(1) ** All the periodic signals are power signals but all power signals are not periodic. Mostly all power signals are periodic signals.

(2) ** Energy signals are absolutely integrable signals but all absolutely integrable signals are not Energy signals.

→ $\delta(t)$ is absolutely integrable ($\because \delta(t)$ = Area = 1) but it is NENP.

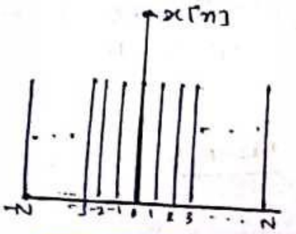
- Periodic signals
  - → Power signal (eg $A\cos\omega t$)
  - → NENP (eg : Impulse train)
- Non-periodic signal
  - → Energy (Rect
  - → Power ($u(t)$
  - → NENP $\delta(t)$
- Finite Duration signal
  - → Energy (Area
  - → NENP ($\delta(t)+\delta(t-$
- Infinite Duration signal
  - → Energy ($e^{-at}u(t)$
  - → Power ($u(t)$)
  - → NENP ($r(t)$)

Energy and power of Discrete Time signals :



$$\boxed{\sum_{n=0}^{N} x[n] = A\cdot[N+1]}$$

↓ No. of samples = N+1



$$\boxed{\sum_{n=-N}^{+N} x[N] = A[2N+}$$

↓ No. of sample = 2N+1

Energy :

$$\boxed{E = \sum_{n=-\infty}^{n=\infty} |x(n)|^2}$$ **

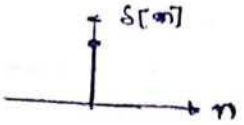
Power :

$$\boxed{P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{n=+N} |x(n)|^2}$$

$$\therefore \boxed{P = \lim_{N\to\infty} \frac{E}{2N+1}}$$

Just to ne
n and t
Remains
$T_0 = 2N$

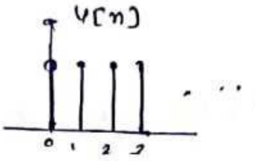
① $\delta[n]$



$$\boxed{\delta[n] \xrightarrow[\text{signal}]{\text{Energy}} E=1,\ P=0}$$

Note: continuous time me 'δ' is NENP, but in Discrete domain, δ(n) is Energy signal.

② $U[n]$



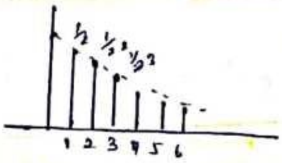
PQ

$$\boxed{u[n] \xrightarrow[\text{signal}]{\text{Power}} P=\frac{1}{2},\ E=\infty}$$

→ Step function in continuous domain as well as in discrete domain is power signal, and its power is $\frac{1}{2}$ at both domain.

Note: In discrete time system, energy is independent of amplitude inversion, time inversion, time shifting and zero interpolation.

③ $\left(\frac{1}{2}\right)^n u(n)$



Jaise Area, Energy, Power, formulae se nikalen

Energy $E = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{2n} u^2(n)$

$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n.$

[illegible]

Note: $a^n u(n) \Rightarrow$ Stable $\Rightarrow |a|<1$

$\downarrow$

$A = \frac{1}{1-a},\ E = \frac{1}{1-a^2} \leftarrow$ Energy signal

$a^{-n}u(-n) \Rightarrow$ unstable $\Rightarrow |a|>1 \Rightarrow$ NENP

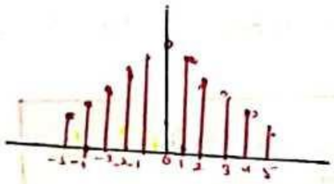
AA

④ $\left(\frac{1}{2}\right)^{|n|}$

- Non causal
- even
- bounded
- doubly sided exponential decaying sequence

$\left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^n \quad n>0$

$= \left(\frac{1}{2}\right)^{-n} \quad n<0$



n : domain

$$\sum_{n=-\infty}^{n=\infty} (x(n)) = \underbrace{\sum_{n=-\infty}^{-1} x(n) + \sum_{n=0}^{n=\infty} x(n)}$$

Ese hi limit break karenge agar $-\infty$ to 0 aur 0 to ∞ kiye to value of $x(0)$ 2 bar ajayega.

If $x(n)$ is Even →

$$\text{Area}[x(n)] = 2\sum_{n=1}^{\infty} x[n] + x[(0)]$$

$$= 2\sum_{n=0}^{\infty} x[n] - x[0]$$

$$\text{Area}[a^{|n|}] = \frac{1+a}{1-a} \quad \text{"}|a|<1\text{"}$$

$$\text{Energy}[a^{|n|}] = \frac{1+a^2}{1-a^2} \quad \text{'}|a|<1\text{'}$$

Note:

Hamesa, Time domain and n-domain me phle time shifting perform karo aur phir time scaling and inversion.

→ n-domain me to khaskarke bhi shifting hi perform karna hai.

Note:

kabhi $\delta(t)$ ke sath function jo dia hai usme $\delta(t)$ ki property lagane se $\frac{0}{0}$ form ya $\frac{\infty}{\infty}$ form ya indeterminate form banra ho to us function ki limit nikalenge aur wahi Answer hoga.

Eg. $\left(\frac{\sin kt}{t}\right)\delta(t)$.

$\Rightarrow \frac{0}{0} \rightarrow$ form

$$\therefore \overbrace{\left(\frac{\sin kt}{kt}\right)}^{\text{limit}=1} K\,\delta(t)$$

$\therefore$ $K\delta(t)$ is Answer.

Note: kabhi bhi even function ke ander negative term hoga to usko +ve banake solve karenge.

Eg. $\text{tri}(-1/4) = \text{tri}(1/4)$

$\text{rect}\left(\frac{-2}{3}\right) = \text{rect}\left(\frac{2}{3}\right)$.

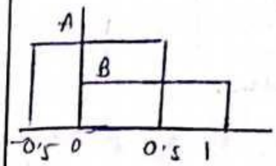
Note:

agar $\frac{m_1}{N_1} = \frac{3}{2}$ $\quad \frac{m_3}{N_3} = \frac{5}{4}$

$\frac{m_2}{N_2} = \frac{7}{2}$

agar Harmonic related to m_2 bola to sabka 'N' common banalenge aur phir jo m_2 aega wo hi Harmonic related to 2nd signal hoga.

Concept:



$A\,\text{rect}(t) + B\,\text{rect}(t - 0.$

$$\boxed{E = A^2 + B^2 + 2AB \times \text{common } \tau}$$

-: System :-

→ A system is a physical device that gives output or response for a given input signal.

1. Static and Dynamic system:

A system is called static if its output at any instant of time depends on input at that instant of time.

→ In static system O/P depends on present input only. So it is called memoryless system.

→ In Dynamic system, O/P depends on past or future values of input in addition to present values.

→ Differentiator, Integrator and summation are dynamic system so they depend on past values of input.

$$\boxed{\text{Future} \xleftarrow[t_0]{\text{LHS to}} \text{Position of time "}t_0\text{" (input)} \xrightarrow[\text{to } t_0]{\text{RHS}} \text{Past}}$$

Eg: $x(t) = y(t - t_0)$ → Past o/p wrt i/p

$x(t) = y(t + t_0)$ → Future o/p wrt i/p

$$\boxed{\text{Past} \xleftarrow[t_0]{\text{LHS of}} \text{instant of the '}t_0\text{'} \xrightarrow[\text{of } t_0]{\text{RHS}} \text{future}}$$

Eg $y(t) = x(t - t_0)$ Past i/p wrt o/p

$y(t) = x(t + t_0)$ Future i/p wrt o/p

Checking of static system :

Isme ye check karna hai ki $y(*) = x(*)$ matlab y ke bracket ka ander ka chiz x ke bracket ke ander ke chiz se match hona chahi same to same baki kuch bhi add ho ya multiply ho bracket ke bahar se.

Some Examples :

(1) $y(n+1) = x(n+1)$ ✓ static

$$\underbrace{y(n+1) = x(n+1)}_{\text{same}}$$

(2) $y(t) = \sqrt{x(t)}$

(3) $y[n] = \frac{1}{x[n]}$

(4) $y[n] = \text{sng}[x(n)] = \frac{x(n) - x^*(n)}{2}$

Some examples of Dynamic ;

(1) $[y(n)] = x(n-1)$

$n=0,\ y(0) = x(-1) \rightarrow$ past

$n=1,\ y(1) = x(0) \rightarrow$ past

$n=-1,\ y(-1) = x(-2)$ past

$y = x(t)$

matlab ~~$y = x(t)$~~ ~~$y(t) = x(t \pm t_0)$~~ form me agar to dabbe me value to se kam ati hai $x(*)$ ke ander to past aur jada ati hai to future aur barabar ata hai to present.

(2) $y(t) = x(1-t)$

$t=0,\ y(0) = x(1)$ fut

$t=-1,\ y(-1) = x(2)$ fut

$t=(0.5),\ y(0.5) = x(0.5)$ pres.

Trick :

(1) for static system :

$$\boxed{y[t] = \text{Basic signal}[x(t)]}$$

(2) for Dynamic system :

$$\boxed{y(t) = x\,[\text{Basic signal}]}$$

Basic signals : $u(t)$, $r(t)$, $\delta(t)$, rect(t), tri(t), sgn(t), exponentials, sinc(t), Sa(t), sin(t), cos(t).

---

2. Causal and Non Causal System :

Causal system :

→ Isme o/p kisi bhi instant pe present and past input pe depend karega but not on future input.

→ causal system are also know as physical or non anticipative system.

Non Causal system :

→ Isme most important or necessary condition hai ki o/p, future input pe depend kare, aur chahe to future input ke sath sath present or past pe bhi depend karsakta hai

→ These are also called acausal system or anticipative system.

3. Anti causal system :

1st Priority

→ Depends on future input and present input but independent of past inputs.

2nd priority.

→ Anti causal system are also non-causal but converse is not true.

→ A non causal system that has any dependency on past input is not Anti causal.

Note :

(1) All static systems are causal system but all causal system are not static.

eg) $y(t) = 2x(t)$ → static, causal

$y(t) = x\underbrace{[n-2]}_{\text{Past}}$, dynamic, causal.

(2) All non-causal system are dynamic but all dynamic system are not non-causal.

$y[n] = x[-n]$ Dynamic, non causal

$y[n] = x[n-2]$ Dynamic, causal.

Signals →	Causal	Non causal	Anti causal
Dependency →	Only on Present and past input → Independent of Future	Present, Past, Future	Future and present → Independent of Past

Checking of causal, Non causal, Anti causal system :

→ Abhi bhi bas $y(t)$ aur $x(t)$ ke bracket ke andar ko check karenge. Baher kuch bhi multiply ho, add ho matlab nai.

→ Differentiator is not always causal system.

→ $\frac{d}{dt}\underbrace{x(t)}$ – Past

→ $\frac{d}{dt}\underbrace{x(t^2)}$ future

Note :

(1)

when [Linear, Static, Causal, Time invariant, Stable] system are connected in cascade then the overall system S is → [Linear, static, Causal, Time invariant, Stable]

(2) when [Non Linear, Dynamic, Non-Causal, Time variant, Unstable] system are connected in cascade then the overall system may or may not be – [Linear, Static, Causal, Time inv, Stable]

→ For a non LTI system, transfer function and ZIR are cannot be calculated, so stability is decided by ZSR.

If ZSR → stable, then system is stable,

If ZSR → unstable, then system is unstable,

→

Fourier Series

I. Continuous time fourier series :

- Representation of periodic signal in term of sinusoids.

Existence of fourier series :

Dirichlet's condition :

- The function x(t) is absolutely integrable over one period i.e

$$\int_{T_0} |x(t)|\,dt < \infty$$

- x should consist of finite no. of maxima and minima in one period
- x(t) should consist of finite no. of discontinuities in one period and each of these discontinuities should be finite.
- x(t) should a single valued function in a given interval.

** Trigonometric fourier series :

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\omega_0 n + b_n \sin\omega_0 t}$$

a_0 = DC value of Avg. value

a_0 = Area of one time period

Time period = T_0

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\boxed{a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt} \quad **$$

$$\boxed{\begin{aligned} a_n &= \frac{2}{T_0}\int_{T_0} x(t)\,(\cos\omega_0 n t)\,dt \\ b_n &= \frac{2}{T_0}\int_{T_0} x(t)\,\sin\omega_0 n t)\,dt \end{aligned}}$$

a_0, a_n, b_n are trigonometric fourier series coefficient.

Polar form fourier series :

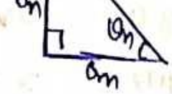
$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(\omega_0 n t - \theta_n)}$$

$$A_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$a_n = A_n \cos\theta_n$$

$$b_n : A_n \sin\theta_n$$



$$A_n \cos(\omega_0 n t - \theta_n) = A_n \cos\theta_n \cos\omega_0 n t + A_n \sin\omega_0 n t \cdot \sin$$

*** Exponential fourier series

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{+j\omega_0 n t}}$$

$$\boxed{C_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 n t}\, .\, dt}$$

C_n = Exponential fourier series coefficient.

Relationship b/n Exponential Fourier series and Trignometric Fourier series ;

(1) $$C_n = \frac{a_n - j b_n}{2}$$

(2) $$C_{-n} = \frac{a_n + j b_n}{2}$$

(3) $$\underbrace{a_0 = C_0}_{\downarrow}$$

dc value .

Important Results :

(1) $C_n = C_{-n}^*$

or

$C_{-n} = C_n^*$

(2) $|C_n| = |C_{-n}| = \frac{\sqrt{a_n^2 + b_n^2}}{2}$

magnitude spectrum → even function

(3) $\angle C_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

$\angle C_{-n} = \tan^{-1}\left(\frac{b_n}{a_n}\right)$ $\Rightarrow \angle C_n = -\angle C_{-n}$

phase spectrum (odd function) .

(4) $Re[C_n] = Re[C_{-n}] = \frac{a_n}{2}$

(5) $Img[C_n] = -Img[C_{-n}] = -\frac{b_n}{2}$.

Exponential Fourier Series :

(1) E.F.S coefficient

$$C_n = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-j\omega_0 n t}\, dt$$

(2) EFS Representation :

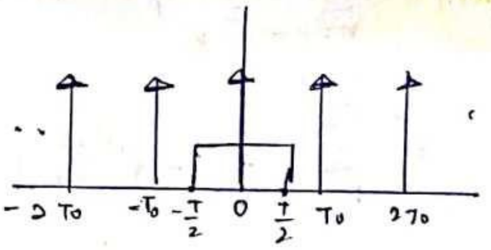
$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 n t}$$

$$X(f) = \sum_{n=-\infty}^{\infty} C_n \cdot \delta(f - n f_0)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} 2\pi C_n \cdot \delta(\omega - \omega_0 n)$$

Periodic Impulse Train :

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - K T_0)$$



Time period $= T_0$

$\omega_0 = \frac{2\pi}{T_0}$

Results :

(1) $a_0 = \frac{1}{T_0}$ → Area in one time period → Time period

(2) $b_n = 0$ ∵ x(t) is even function

(3) $a_n = \frac{2}{T_0}$

T.F.S representation ;

$$x(t) = \frac{1}{T_0} + \sum_{n=1}^{\infty} \frac{2}{T_0} \cos n\omega_0 t$$

*yad rakh ai

E.F.S Results:

$\boxed{C_0 = \frac{1}{T_0}}$ (∵ $C_0 = a_0$)

$$C_n = \frac{a_n - jb_n}{2}$$

$$b_n = 0$$

∴ $C_n = \frac{a_n}{2}$

$\boxed{C_n = \frac{1}{T_0}}$

E.F.S Representation:

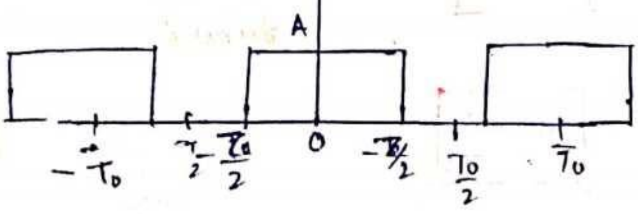
(1) $$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{j\omega_0 n t}$$

$$= \sum_{n=-\infty}^{\infty} f_0 \cdot e^{j\frac{2\pi}{T_0} \cdot n t}$$

(2) $$x(f) = \sum_{n=-\infty}^{\infty} f_0 \, \delta(f - nf_0)$$

(3) $$= \sum_{n=\infty}^{\infty} f_0 \cdot \delta\left(f - \frac{n}{T_0}\right)$$

Periodic Pulse train:



$\boxed{a_0 = \frac{A\tau}{T_0}}$ *

$\boxed{b_n = 0}$ (∵ x(t) is even)

$$\boxed{a_n = \frac{2A\tau}{T_0} \text{sinc}(f_0 n \tau)}$$

T.F.S representation:

$$x(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{2A\tau}{T_0} \text{sinc}(f_0 n t) \cos\omega_0 n t$$

E.F.S coefficient:

$$\boxed{C_n = \frac{A\tau}{T_0} \cdot Sa\left(\frac{\omega_0 n \tau}{2}\right) \qquad C_n = \frac{A\tau}{T_0} \text{sinc}(f_0 n \tau)}$$

$$\omega_0 = 2\pi f_0$$

$$\boxed{C_0 = \frac{A\tau}{T_0}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{A\tau}{T_0} \text{sinc}(f_0 n \tau) \cdot e^{j\omega_0 n t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{A\tau}{T_0} \cdot Sa\left(\frac{\omega_0 n \tau}{2}\right) e^{j\omega_0 n t}$$

Relationship b/m T.F.S and E.F.S

(1) $$A_n = \sqrt{a_n^2 + b_n^2}$$

$$|C_n| = \frac{\sqrt{a_n^2 + b_n^2}}{2}$$

∴ $\boxed{A_n = 2|C_n|}$ * $\qquad \theta_n = \tan^{-1}\left(\frac{Img}{Real}\right)$

$$x(t) = C_n e^{j(\omega_0 t - \phi_n)}$$

$$x(t) = A_n \cos(\omega_0 t - \theta_n)$$

then $\boxed{\phi_n = \theta_n}$

Note : (1)

Kabhi bhi Amplitude of fundamental Bole to humlog $\sqrt{a_1^2+b_1^2}$ nikalenge i.e A_1 or $2C_1$

$\because |A_n| = 2|C_n|$

(2)

$$\boxed{\text{Rms value} = \sqrt{\text{Rms}^2_{DC} + \text{Rms}^2_{AC}}}$$

$$\boxed{\text{Rms AC} = \sqrt{\text{Rms}^2 - \text{Rms}^2_{DC}}}$$

Conjugate Properties and Relations :-

$x(t) \longleftrightarrow C_n$

$x(-t) \longrightarrow C_{-n}$ (time reversible)

$x^*(t) \longrightarrow C^*_{-n}$ [conjugate]

*Imp

$x(t)$	C_n
Real part $x(t) = x^*(t)$	Conjugate symmet $C_n = C^*_{m}$ $\begin{bmatrix} Re(C_n) = \text{even} \\ Im(C_n) = \text{odd} \end{bmatrix}$
Imaginery Port $x(t) = -x^*(t)$	Conjugate Anti-sym $C_n = -C^*_{-n}$ $\begin{bmatrix} Re(C_n) \to \text{odd} \\ Im(C_n) = \text{even} \end{bmatrix}$

Half wave Symmetry :

$$\boxed{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$$

T_0 = Time period

for periodicity, $x(t) = x(t \pm T_0)$.

A periodic signal with half wave symmetry consist of only odd Harmonics of cosine and sine terms.

$$\boxed{\begin{aligned} &a_n \rightarrow a_1, a_3, a_5, a_7 \ldots \\ &b_n \rightarrow b_1, b_3, b_5, \ldots \\ &a_0 = 0 \end{aligned}}$$

Symmetry	T.F.S coefficient
Even	$a_0 = 0$ $a_n \neq 0$ $b_n = 0$ $\neq 0$
odd	$a_0 = 0$ $a_n = 0$ $b_n \neq 0$
Even Half wave symm	$a_0 = 0$ $b_n = 0$ $a_n = \begin{cases} = 0 & n \to \text{even} \\ \neq 0 & n \to \text{odd} \end{cases}$
Odd H.W.S	$a_0 = 0$ $a_n = 0$ $b_n \begin{cases} = 0 & n \to \text{even} \\ \neq 0 & n \to \text{odd} \end{cases}$

Real periodic signal :

$* \; x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos w_0 nt + b_n \sin w_0 nt)$

$= \sum_{n=-\infty}^{n=\infty} C_n e^{jw_0 nt}$

$* \; C_n = \frac{a_n - jb_n}{2}, \; C_{-n} = \frac{a_n + jb_n}{2}$

① $x(t) = \cos w_0 t$

→ Real $x(t) = x^*(t)$

→ Even $x(t) = x(-t)$

→ H.W.S → $x(t) = -x\left(t \pm \frac{T_0}{2}\right)$

It is Real Even H.W.S

$\therefore a_0 = 0$ $b_n = 0$. $a_1 \cos w_0 t = \cos w_0 t$

$\boxed{a_1 = 1}$.

E.F.S :

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jw_0nt}$$

$$x(t) = \cos w_0 t = \frac{e^{jw_0t} + e^{-jw_0t}}{2}$$

$$c_0 = 0$$

$$c_1 = \frac{1}{2} = \frac{a_1}{2} \quad \text{due to Real part}$$

$$c_{-1} = \frac{1}{2} = \frac{a_1}{2}$$

Even

$$c_n = c_{-n}^{*}$$

$$A\cos(Kw_0t) = \frac{A}{2}\left[\delta(n-K) + \delta(n+K)\right]$$

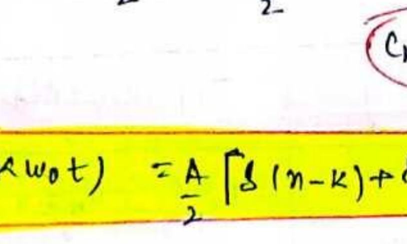
c_n → Real

→ Even ($c_n = c_{-n}$)

→ conjugate symmetric

$(c_n = c_n^{*})$.

$$c_n = \frac{1}{2}\delta(n-1) + \frac{1}{2}\delta(n+1)$$



→ Discrete magnitude spectrum.

F.S → $x(t)$ → continuous and periodic

c_n [spectrum] → Discrete and Non-periodic.

② $x(t) = \sin w_0 t$

(1) Real $x(t) = x^{*}(t)$

(2) Odd $x(t) = -x(-t)$

$$a_0 = 0, \quad a_n = 0$$

$$b_n \neq 0$$

(3) symmetry (H.W.S)

$$x(t) = -x\left(t \pm \frac{T_0}{2}\right)$$

T.F.S :

$$x(t) = b_1 \sin w_0 t + b_2 \sin 2w_0 t + \ldots$$

$$\sin w_0 t = b_1 \sin w_0 t$$

$$\boxed{b_1 = 1}$$

E.F.S :

$$x(t) = \frac{e^{jw_0t} - e^{-jw_0t}}{2j}$$

$$c_n e^{jw_0nt} = \frac{1}{2j} e^{jw_0t}$$

$$c_1 = \frac{1}{2j}.$$

$$c_{-n} = -\frac{1}{2j} e^{-jw_0t}$$

$$c_1 = -\frac{j}{2} = -j\frac{b_1}{2}$$

$$c_{-1} = \frac{j}{2} = j\frac{b_1}{2}$$

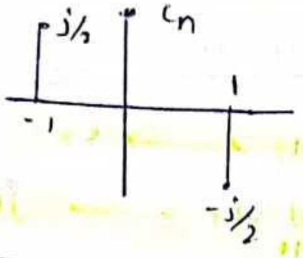
c_n → Imaginary

Odd [$c_n = -c_{-n}$]

Conjugate symmetry $c_n = c_{-n}^{*}$

$$c_n = \frac{j}{2}\delta(n-1) + \frac{j}{2}\delta(n+1)$$

$$A\sin(Kw_0t) = \frac{A}{2j}\delta(n-K) - \frac{A}{2j}\delta(n+K$$



x (t)	a_0, a_n, b_n	$C_n = \frac{a - j b_n}{2}$ or $C_{-n} = \frac{a + j b_n}{2}$
Real and even $x(t) = x^*(t)$ (Real) $x(t) = x(-t)$ (even)	$a_0 = 0$ $a_0 \neq 0$ $a_n \neq 0$ $b_n = 0$	$C_n = \frac{a_n}{2}$ & $C_{-n} = \frac{a_n}{2}$ Real $C_n = C_{-n}^*$ Conjugate symt. $C_n = C_{-n}$ even Real even conjugate symmetric
Real and odd $x(t) = x^*(t)$ Real $x(t) = -x(-t)$ odd	$a_0 = 0$ $a_n = 0$ $b_n \neq 0$	$C_n = -j\frac{b_n}{2}$ $C_{-n} = j\frac{b_n}{2}$ (Img) $C_n = C_{-n}^*$ (conjugate symt. $C_n = -C_{-n}$ (odd) Imaginary odd conjugate symmetric
Imaginary and even $y(t) = -y^*(t)$ Imag. $y(t) = y(-t)$ even	$a_0 = 0$ $a_0 \neq 0$ $a_n \neq 0$ $b_n = 0$	$d_n = j\frac{a_n}{2}$ $d_{-n} = j\frac{a_n}{2}$ (Img) $d_n = -d_{-n}^*$ Conjugate anti symm $d_n = d_{-n}$ even Imaginary even conjugate anti symmetric
Imaginary and Odd $y(t) = -y^*(t)$ Img. $y(t) = -y(-t)$ odd	$a_0 = 0$ $a_n = 0$ $b_n \neq 0$	$d_n = \frac{b_n}{2}$ $d_{-n} = -\frac{b_n}{2}$ (Real) $d_n = -d_{-n}^*$ (conjugate anti sym) $d_n = -d_{-n}$ (odd) Real odd conjugate anti symmetric

$$\boxed{\begin{aligned} d_n &= j c_n = j\left[\frac{a_n - j b_n}{2}\right] \\ &= \frac{j a_n + b_n}{2} \\ d_{-n} &= \frac{j a_n - b_n}{2} \end{aligned}}$$

→ agar a_0 exist kio ya phir C_0 to H.W.S nai hoga.

→ aur agar even Harmonic term aye tobhi H.W.S nai hoga.

→ agar C_n deke DC value puche to $n=0$ put karenge Expression me aur agar $\frac{0}{0}$ ya $\frac{\infty}{\infty}$ Form bana to L-Hopitals lagaenge.

→ agar x(t) NENO and Real hai to C_n will not purely real

→ agar x(t) Real + even hai to C_n will be real.

→ $\boxed{C_n = |C_n| e^{j\angle C_n}}$

→ Sum of finite GP series:

$$\boxed{\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + \dots r^{N-1} = \frac{1 - r^N}{1 - r}}$$

Properties of Cn:

① Linearity:

$$x(t) \xleftrightarrow{\text{CTFS}} C_n$$
$$y(t) \longleftrightarrow d_n$$

$$\boxed{a x_1(t) + b y(t) \xleftrightarrow{\text{CTFS}} a C_n + b d_n}$$

② Time shifting:

$$x(t) \xleftrightarrow{\text{CTFS}} C_n$$
$$x(t - t_0) \xleftrightarrow{\text{CTFS}} e^{-j\omega_0 n t_0} C_n$$

$$x(t+t_0) \xleftrightarrow{CTFS} e^{+j\omega_0 n t_0} \cdot C_n$$

$$T_0 = \frac{2\pi}{\omega_0}$$

③ Time Reversal :

$$\boxed{\begin{array}{l} x(t) \xleftrightarrow{CTFS} C_n \\ x(-t) \xleftrightarrow{CTFS} C_{-n} \end{array}}$$

$|C_n| = |C_{-n}|$

magnitude same

Even: $x(t) = x(-t)$
$C_n = C_{-n}$

odd: $x(t) = -x(-t)$
$C_n = -C_{-n}$.

④ Time Scaling :

$$\boxed{\begin{array}{l} x(t) \xleftrightarrow{CTFS} C_n \\ [T_0, \omega_0] \\ x(Kt) \xleftrightarrow{CTFS} C_n \\ T_0' = \frac{T_0}{K}, \quad \omega_0' = K\omega_0 \end{array}}$$

Effect of Time scaling :

$C_n \rightarrow$ same.

frequency → change.

Time period → changes.

⑤ multiplication :

$x(t) \longleftrightarrow C_n$

$y(t) \longleftrightarrow d_n$

**.

$$\boxed{x(t) \cdot y(t) \longleftrightarrow C_n \otimes d_n = \sum_{k=-\infty}^{\infty} C_n \cdot d_{n-k} = \sum_{k=-\infty}^{\infty} d_k \cdot C_{n-k}}$$

⑥ Conjugate property :

$$\boxed{\begin{array}{l} x(t) \longleftrightarrow C_n \\ x^*(t) \longleftrightarrow C^*_{-n} \end{array}}$$

Real conjugate Symmetric

$x(t) = x^*(t)$
$C_n = C^*_{-n}$

Conj. $x(t) = -x^*(-t)$
C.A.S $C_n = -C^*_{-n}$

⑦ Time Differentiation :

$$\boxed{\begin{array}{l} x(t) \longleftrightarrow C_n \\ \frac{d}{dt} x(t) \longleftrightarrow j\omega_0 n \cdot C_n \end{array}}$$

⑧ Time integration ;

$$\boxed{\begin{array}{l} x(t) \longleftrightarrow C_n \quad \text{with } C_0 = 0,\ a_0 = 0 \\ \int_{-\infty}^{t} x(\tau) \cdot d\tau \longleftrightarrow \frac{C_n}{j\omega_0 n} \end{array}}$$

x(t) must be odd or even with zero Avg in 1 period

**

⑨ Frequency shift :

$$\boxed{\begin{array}{l} x(t) \longleftrightarrow c_n \\ e^{jK\omega_0 t} \longleftrightarrow C_{n-K} \\ e^{-jK\omega_0 t} \longleftrightarrow C_{n+K} \end{array}}$$

⑩ Convolution :

$x(t) \longleftrightarrow c_n, \quad y(t) \longleftrightarrow d_n$

$$\boxed{x(t) \otimes y(t) \longleftrightarrow T_0 \cdot C_n \cdot d_n}$$

Fundamental time Period

Note:

$$\boxed{e_n = C_n + d_n}$$

Parseval's Theorem :

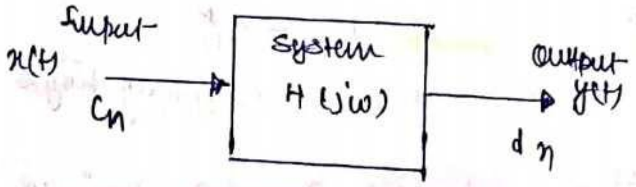
It gives Power for periodic signal

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2 = |C_0|^2 + 2\sum_{n=1}^{\infty} |C_n|^2$$

$$= |a_0|^2 + \sum_{n=1}^{\infty} \left(\frac{a_n^2 + b_n^2}{4} \right)$$

Steady state output :-



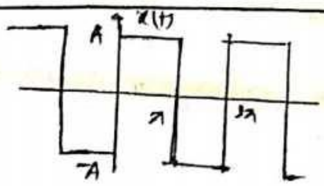
$$d_n = C_n \left[H(j\omega) \right]_{\omega = n\omega_0}$$

Time domain	Frequency domain
continuous	Non-periodic eg: rect(t) → sinc(f)
Non- periodic	Continuous rect(t) → sinc(f)
Periodic	Discrete $\cos\omega_0 t = \frac{1}{2}\delta(\gamma - 1) + \frac{1}{2}\delta(\gamma + 1)$
Discrete	Periodic $\delta(n+1) = z = e^{j\omega}$

Trignometric F.S and concept

9 Half wave symmetry :-

①

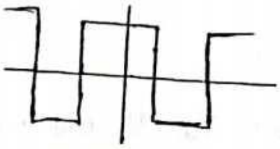


x(t) → odd

$a_0 = 0$, $a_n = 0$ H.W.S

$b_n \neq 0$.

②



x(t) → even

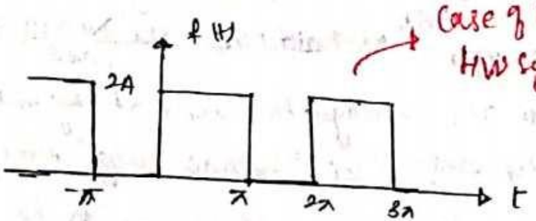
$a_0 = 0$

$a_n \neq 0$

Its H.W.S, $b_n = 0$.

$$a_n = \begin{cases} \neq 0 & n \to odd \\ 0 & n \to even \end{cases}.$$

③



Case of Hidden HW symmetry.

$a_0 \neq 0$

$a_n = 0$

$b_n \neq 0$.

Sabse pahle a_0 nikalenge phir x(t) me a_0 minus karenge aur given wave phir left ya right shift karenge. aur phir Half wave symmetry check karenge.

★ → Fourier Series me Periodic signal Power signal hote hai unka energy infinite hota hai.

But remember All periodic signals are not Power signal.

Magnitude of n^{th} F.S coefficient

of square wave $\propto \frac{1}{n}$

and that of triangular wave $\propto \frac{1}{n^2}$.

→ agar question me real value likh diya matlab $C_n = C_{-n}^*$ hona chahie ~~aur Cn~~

Trick(2) ;

→ C.S → Real ko darsata hai

→ C.A.S → Imaginary ko darsata h

→ $x(t)$ → Real to C_n 100% conjugate Symmetric

→ $x(t)$ → Imaginary to C_n 100% conjugate Anti Symmetric

→ Real aur Even Bhai hai aur Imaginary aur Odd hai bhai Islie Bhai sath aenge to C_n hamesa Real Banega.

→ Real aur odd dushman hai aur Imaginary aur Even dushman hai Islie dushman sath aenge to C_n → Imaginary Banega.

Trick : (1)

(1) agar $\{C_n\}$ → real hai (pure)

to $x(t)$ → can be pure real or can be pure imaginary

(2) agar C_n → odd ya Even hai

to $x(t)$ → 100% odd ya Even hoga respectively.

(3) agar C_n → conjugate symmetric hai i.e

$$C_n = C_{-n}^*$$

to $x(t)$ → 100% Real hoga

(4) agar C_n → conjugate Anti symmetric hai i.e

$$C_n = -C_{-n}^*$$

to $x(t)$ → 100% Imaginary hoga

(5) agar C_n → Imaginary hai (pure)

to $x(t)$ → Can be Real or Imaginary (Pure).

Laplace Transform :

- ⊙ Laplace transform is valid for both stable and unstable.

- ⊙ Fourier transform is only valid for stable signals.

Bilateral or Two sided Laplace Transform :

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \, dt}$$

$$s = \underset{\text{real}}{\sigma} + \underset{\text{Imag.}}{j\omega}$$

→ BLT is used for causal signals, non-causal signals and anti-causal signals.

→ BLT is used to describe frequency response, causality and stability directly from transfer function of pole zero plot.

Unilateral or Right-sided Laplace transform :

$$\boxed{X(s) = \int_{0^-}^{\infty} x(t) e^{-st} \, dt}$$

→ In signal and system Laplace transform is generally BLT, ULT hoga to likh ke aega.

$0^{\ominus}$ to include and define $\delta(t)$.

for causal signal , BLT = ULT

for Non-causal signal , BLT ≠ ULT

Convergence of Laplace Transform,

$$\boxed{\int_{-\infty}^{\infty} | x(t) \cdot e^{-\sigma t} | \, dt < \infty}$$

↓ Absolute Integrability

If x(t) is unstable, $e^{-\sigma t}$ will make it stable.

Region of convergence (ROC) of BLT :

The value of σ for which Laplace transform converge is known as region of convergence.

ROC = σ = Re(s) = Real part of s

Properties of ROC :

① The ROC of X(s) consist of strip which are parallel to jω axis in the s-plane

② If the L.T X(s) of x(t) is rational then ROC is bounded by poles and/or Extends to ∞

③ ROC does not contain any poles.

④ If x(t) is of finite duration and is absolutely Integrable then ROC is Entire s-plane.

ex – δ(t), rect (t), tri(t).

⑤ If L.T of (x(t)) is Rational , and x(t) is Right sided Infinite duration signal, then ROC is the Region in the s-plane to Right of Right-most pole.

⑥ If L.T of x(t) is rational and x(t) is left sided and then ROC is the region in s-plane to left of the left most pole.

⑦. If Laplace x^{form} of x(t) is rational and x(t) is two sided and then ROC is a strip in the s-plane bounded by poles.

- If ROC consist of jω axis, then signal is stable and its Fourier transform can be calculated directly.

Properties:

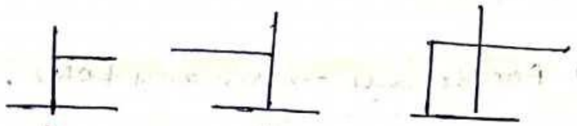
(1) Time shifting property :

If $x(t) \xrightarrow{L\cdot T} X(s)$ ROC : R

Then $x(t-t_0) \xrightarrow{L\cdot T} e^{-st_0} X(s)$ ROC : R

$x(t+t_0) \xrightarrow{L\cdot T} e^{st_0} X(s)$ ROC : R

By shifting property, ROC doesnot change.



Causal → Right sided

Anticausal → left sided

Non-Causal → Right sided.

(1) $u(t) \xrightarrow{L\cdot T} \frac{1}{s}$ ROC : Re(s) > 0

∵ ~~[illegible]~~ t = +ve.

(2) $u(-t) \xrightarrow{L\cdot T} -\frac{1}{s}$ ROC : Re(s) < 0

(3) $-u(-t) \xrightarrow{L\cdot T} \frac{1}{s}$ ROC : Re(s) < 0 ∵ t = -ve.

when performing inversion, replace s by -s in existing L.T

Eg. Find BLT of $u(-t-1)$

- $u(t) \xleftrightarrow{L\cdot T} \frac{1}{s}$ ROC : Re(s) > 0

$u(t-1) \xleftrightarrow{L\cdot T} \frac{e^{-s}}{s}$ ROC : Re(s) > 0

$u(-t-1) \xrightarrow{L\cdot T} \frac{e^{-s}}{s}\Big|_{\text{put } s=-s}$

$u(-t-1) \xleftrightarrow{L\cdot T} -\frac{e^{s}}{s}$ Re(s) < 0

(2) Time scaling property :

If $x(t) \xleftrightarrow{L\cdot T} X(s)$ ROC : R

then, $x(at) \xleftrightarrow{L\cdot T} \frac{1}{|a|} X\left(\frac{s}{a}\right)$; Roc : $\textcircled{aR}$

Here ROC gets multiplied by the factor with which scaling is done.

(3) Frequency shifting property :

If $x(t) \longleftrightarrow X(s)$ ROC : R

The

$e^{+s_0t} \cdot x(t) \longleftrightarrow X(s-s_0)$ ROC : R+Re(s)

$e^{-s_0t} \cdot x(t) \longleftrightarrow X(s+s_0)$ ROC : R−Re(s)

Stability :

Causal signal (Iske control ke stability samajh kar yad rakhna hai)	LHS pole → stable RHS pole → unstable	Re(s) > Pole
Anti causal signal (Same causal ke just opp. kaha hoga).	RHS Pole → unstable LHS Pole → stable	Re(s) < Pole

Q. Find L.T of $e^{-a(t-b)} \cdot u(t-b)$

→ $u(t-b) \longleftrightarrow \frac{e^{-bs}}{s}$

Frequency shifting :

$e^{-at} u(t-b) \xleftrightarrow[s \to s+a]{} \frac{e^{-b(s+a)}}{s+a}$

$e^{-at} u(t-b) \longleftrightarrow \frac{e^{-ab} e^{-bs}}{s+a}$

$e^{-at} e^{ab} u(t-b) \longleftrightarrow \frac{e^{-bs}}{s+a}$

$$e^{-a(t-b)}u(t-b) \longleftrightarrow \frac{e^{-bs}}{s+a}$$

→ Jahan frequency shift chahiye, to phle se present hai 's' me shifting chahiye.

Linearity Property :

If $x_1(t) \longleftrightarrow X_1(s)$ ROC : R_1

$x_2(t) \longleftrightarrow X_2(s)$ ROC : R_2

$$\boxed{a x_1(t) + b x_2(t) \xrightarrow{L\cdot T} a X_1(s) + b X_2(s)}$$

$$\boxed{\text{ROC} : R_1 \cap R_2}$$

↓ also valid for $a x_1(t) - b x_2(t)$.

→ amplitude scaling (+ve and -ve) does not change the ROC.

→ Laplace transform exist only if there is a common ROC b/w R_1 and R_2. If there is no common ROC then L·T does not exist even if it possible to calculate its value.

① $\text{Sgn}(t)$

B·L·T of sgn(t) does not exist.

U·L·T exist, i.e U·L·T $= \frac{1}{s}$

But

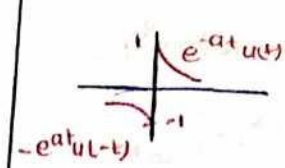
~~For sgn(t),~~ B··

$$\boxed{\text{U·L·T of Sgn}(t) = \text{B·L·T } u(t)}$$

B·L·T of 1 also does not exist

U·L·T of 1 $= \frac{1}{s}$.

② $e^{-a|t|}\text{sgn}(t)$



$$e^{-a|t|}\text{sgn}(t) = e^{-at}u(t) - e^{at}u(-t)$$

③ $\cos \omega t \cdot u(t)$

$$\cos\omega t \cdot u(t) \xleftrightarrow{L\cdot T} \frac{s}{s^2+\omega^2} \; ; \text{ROC} : \text{Re}(s) > 0$$

$$\cosh\omega t \cdot u(t) \xleftrightarrow{L\cdot T} \frac{s}{s^2-\omega^2} \; ; \text{ROC} : \text{Re}(s) > \omega$$

→ Hyperbolic me $\text{Re}(s) > \omega$ hota hai jabki Regular functions me $\text{Re}(s) > 0$ hota hai.

→ Exam me $\cos\omega t$ deke aur $t > 0$ likh dia matlab usko $\cos\omega t \; u(t)$ dia hai.

→ $\cos\omega t = \cos\omega t \times 1 \longrightarrow$ NO B·L·T.

Note:

For time shifting there should be equal change in all 't'.

Eg. $\cos\omega(t-t_0) \cdot u(t-t_0)$

$$f(t) = \cos\omega t \cdot u(t) \xleftrightarrow{L\cdot T} \frac{s}{s^2+\omega^2} = F(s)$$

Time shifting $t \to t_0$

$$f(t-t_0) = \cos\omega(t-t_0) \cdot u(t-t_0)$$

$\Updownarrow$ L·T

$$e^{-st_0} \cdot F(s) = X(s) = \frac{e^{-st_0} \cdot s}{s^2+\omega^2}.$$

Q. $x(t) = \sin\omega_0 t \cdot u(t-t_0)$

agar ek t me shifting hai aur dusre nai to same shifting dusre t me create karo.

Q. $e^{-at}\cos\omega t \cdot u(t)$

Isme nikhenge ki denominator se ROC kese nikalte hai.

$\cos\omega t \cdot u(t) \longrightarrow \frac{s}{s^2+\omega^2}$

$e^{-at}\cos\omega t \cdot u(t) \xrightarrow[s \to s+a]{} \frac{s+a}{(s+a)^2+\omega^2}$

denominator = 0

$(s+a)^2 + \omega^2 = 0$

$(s+a)^2 = (j\omega)^2$

$s+a = \pm j\omega$

$s = -a \pm j\omega$

Real part = $-a$ – Roc

∴ Roc : Real of s i.e $Re(s) > -a$

↓

due to +t.

Multiplication by t property

Frequency differentiation property :

If $x(t) \xleftrightarrow{L\cdot T} X(s)$ ROC : R

Then

$$\boxed{\begin{aligned} & t\cdot x(t) \xrightarrow{L\cdot T} -\frac{d}{ds}X(s) \quad \text{ROC : R} \\ & t^n\cdot x(t) \longrightarrow (-1)^n \frac{d^n X(s)}{ds^n} \quad \text{Roc : R} \end{aligned}}$$

multiplicatn by t^n does not change the ROC.

$$\boxed{t^n u(t) \xrightarrow{L\cdot T} \frac{n!}{s^{n+1}}} \quad Re(s) > 0$$

Note : $\boxed{|F_1(s)|^2 = F_1^*(s)\, F_1(s)}$

Time Integration property :

If $x(t) \longleftrightarrow X(s)$ ROC : R

$$\boxed{\int_0^t x(\tau)\, d\tau \xleftrightarrow{U\cdot L\cdot T} \frac{X(s)}{s}}$$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{B\cdot L\cdot T} \frac{X(s)}{s} + \frac{1}{s}\int_{-\infty}^{0} x(\tau)\, d\tau$$

Integration me +t hai isliye Roc > 0.

L.H.S Area of x(t)

ROC : $R \cap Re(s) > 0$

Time Differentiation Property :

If $x(t) \longleftrightarrow X(s)$ ROC : R

U.L.T

$$\boxed{\begin{aligned} & \frac{d}{dt}x(t) \longleftrightarrow sX(s) - x(0) \\ & \frac{d^2}{dt^2}x(t) \longleftrightarrow s^2X(s) - sx(0) - x'(0) \end{aligned}}$$

B.L.T

$$\boxed{\begin{aligned} & \frac{d}{dt}x(t) \longrightarrow s\cdot X(s) \\ & \frac{d^n}{dt^n}x(t) \longleftrightarrow s^n X(s) \end{aligned}}$$

ROC : R condition.

↓

qki u(t) ko diff karenge to δ(t) banega aur Roc badlega.

Division by 't' property :

If $x(t) \longleftrightarrow X(s)$

then $\boxed{\frac{x(t)}{t} \longleftrightarrow \int_s^{\infty} X(s)\, ds}$

Note : for inverse replace transform of $\cot^{-1}s$ or $\log_e$ (function of s), search options related to division by 't' property.

$L[\delta(t)] = \frac{d}{dt}\delta(t) = s \cdot 1 = s.$

Initial value theorem:

$$\boxed{[x(t)]_{t=0} = x(0) = \lim_{s \to \infty} s \cdot X(s)}$$

→ Initial value theorem can be applied only if degree of Nr is less than degree of Dr i.e. function is proper.

→ If degree of N° is greater than equal to degree of Dr then signal x(t) has δ(t) or δ'(t) whose value at t = 0 is ∞. Hence initial value theorem cannot be applied.

Final value theorem:

$$\boxed{[x(t)]_{t=\infty} = x(\infty) = \lim_{s \to 0} s \cdot X(s)}$$

→ F.V.T is only valid if all the poles have negative real part (or lie on left of s-plane). except a simple pole at s = 0.
(single)

Note: Do not consider discontinuity and Gibbs phenomena while applying Initial value theorem.

Convolution Integral:

$$\boxed{y(t) = x(t) \otimes h(t)}$$

→ $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$

$= \text{Area}\, [x(\tau) \cdot h(t-\tau)]$

→ $y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)\, dt$

$= \text{Area}\, [h(\tau) \cdot x(t-\tau)]$

Properties and Imp. Result of Convolution Integral:

① Laplace Transform:

$$\boxed{x(t) \otimes h(t) \xleftrightarrow{L.T} X(s) \cdot H(s)}$$

② Commutative:

$$\boxed{x(t) \otimes h(t) = h(t) \otimes x(t)}$$

③ Associative:

$$\boxed{x(t) \otimes [h(t) \otimes g(t)] = [x(t) \otimes h(t)] \otimes g(t)}$$

④ Distributive:

$$\boxed{x(t) \otimes [h(t) + g(t)] = x(t) \otimes h(t) + x(t) \otimes g(t)}$$

⑤ Convolution with Step:

$$\boxed{x(t) \otimes u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

$$\boxed{x(t) \otimes u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau}$$

⑥ Differentiation property:

If $y(t) = x(t) \otimes h(t)$

$\frac{d}{dt} y(t) = \left[\frac{d}{dt} x(t)\right] \otimes h(t)$

$$\boxed{\frac{d}{dt} y(t) = x(t) \otimes \left[\frac{d}{dt} h(t)\right]}$$

Note:

$$\boxed{x(t-t_1) \otimes h(t-t_2) = \underbrace{y(t-t_1-t_2)}_{\text{Same sign leke aya jo i/p me di thi}}}$$

⑦ $y(t) = x(t) \otimes h(t)$, then

$$\boxed{x(kt) \otimes h(kt) = \frac{1}{|k|}\, y(kt)}$$

⑧ If $y(t) = x(t) \otimes h(t)$, then

$$\boxed{\text{Area}[y(t)] = \text{Area}[x(t)] \cdot \text{Area}[h(t)]}$$

Eg: $u(t-1) \otimes u(t-2)$

$$\Rightarrow \int_{-\infty}^{t-1} u(\tau-2)\, d\tau$$

$$= r(t-1-2) - \cancel{u(-\infty)}^{\,0}$$

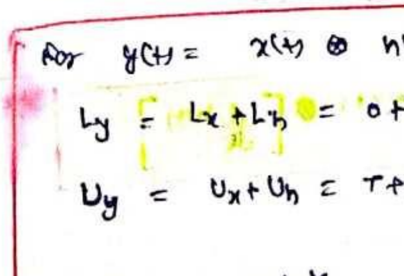
$$= r(t-3)$$

or

$$u(t-1) \otimes u(t-2)$$

$$= r(t-1-2) = r(t-3).$$

Concept 1 : Convolution of Two Step or Rectangular wave of Same time duration T.



Area [h(t)] = B T

Area [x(t)] = A T

Time duration = U − L

Imp Part

For $y(t) = x(t) \otimes h(t)$

$L_y = L_x + L_h = 0 + T = T$.

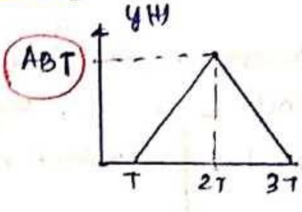
$U_y = U_x + U_h = T + 2T = 3T$.

Time duration of y(t)

$= U_y - L_y = 3T - T$

$= 2T$

Note : when two rectangular waveform of equal time duration are convolved, then the shape of resulting waveform is triangular.



Area y(t) = ABT²

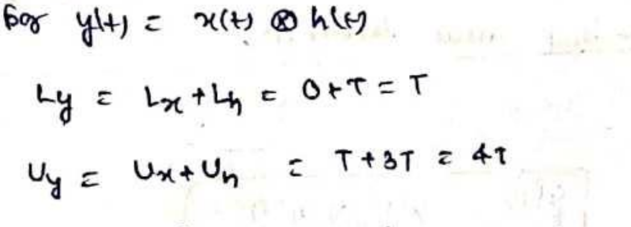
Dhyan dena hai Amplitude me T bhi multiply hua hai

$$\text{Slope} = \frac{\text{Amplitude change}}{\text{Time change}}$$

$$AB = \frac{\text{Amplitude} - 0}{2T - T}$$

Amplitude = ABT.

Concept 2 : Convolution of two step or rectangular waveform of unequal time duration.



For $y(t) = x(t) \otimes h(t)$

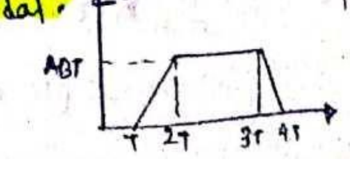
$L_y = L_x + L_h = 0 + T = T$

$U_y = U_x + U_h = T + 3T = 4T$

Time duration : $U_y - L_y = 4T - T$

$= 3T$.

Note : when two rectangular waveform of unequal time duration are convolved then the shape of resulting waveform is Trapezoidal.



T = ~~duration~~ duration of less duration pulse.

Note: priority constant function ko shift karne ka rehta hai par jab fig me multiple step wave rehte hai to kosis karte hai ki usse ko shift karde.

Causal Impulse Response :

① $h(t) = 0$ for $t < 0$
$h(t) \neq 0$ for $t > 0$

ROC : Right Hand side of Right most pole.

Anti-causal Impulse Response :

① $h(t) = 0$ for $t > 0$
$h(t) \neq 0$ for $t < 0$

ROC : left hand side of left most pole.

Stable Impulse Response :

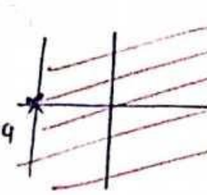
① $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

$h(t) \longrightarrow$ Absolutely integrable.

ROC : Includes jω axis.

Causal + stable Impulse Response :

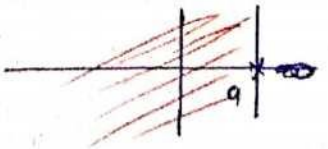
→ poles on LHS of s-plane



ROC : (1) RHS of pole
(2) jω axis is included.

Anticausal + stable Impulse Response :

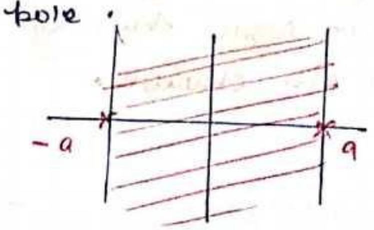
→ Poles on RHS of s-plane



ROC : (1) LHS of pole
(ii) Include jω axis.

Non causal + stable Impulse Response

→ ROC Bounded b/m L.H.S and R.H.S pole.



Minimum phase system :

Condition :

(1)* H(s) should be causal + stable impulse response.

(2)* $H_{inv}(s)$ should be causal + stable inverse LTI system.

H(s) and $H_{inv}(s)$ ke poles and zero opp. hote hai because $H(s) = \frac{1}{H_{inv}(s)}$.

In MPS, all poles and zeroes should be on LHS of s-plane.

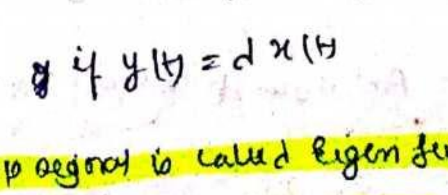
Non Minimum phase system :

Condition :

① At least one zero of H(s) should be on RHS.

② on taking H_{inv}, poles will be on RHS then H_{inv} will be causal + unstable.

Eigen function and Eigen value :



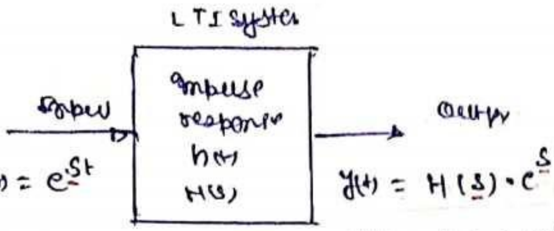
If o/p is scaler multiple of input

i.e if $y(t) = \alpha x(t)$

then

i/p signal is called Eigen function and α is called Eigen value.

- Jahan se Bhi O/P, Input ka scaler multiple hoga wo Eigen function concept based hoga, exam me hume pehchanna ana chahiye.



$y(t) = H(s) \cdot e^{st}$

$y(t) = H(s) \cdot x(t)$

$H(s) = \text{constant}$

kyi H me jab s daleg to ek constant value milegi.

$$\boxed{\begin{aligned} \therefore y(t) &= H(s) \cdot e^{st} = H(s) \cdot x(t) \\ &= \lambda \cdot x(t) \end{aligned}}$$

$x(t) = e^{st}$ = Eigen function

$d = H(s) = [\text{Transfer Function}]_{s=s}$

= Eigen value.

Note: $\cos\omega_0 t + \sin\omega_0 t$ are Eigen signals for $H(j\omega_0) = H(-j\omega_0)$ i.e. $h(t)$ Real and Even.

Steady state Response of LTI system with Sinusoidal Input:

$x(t) = A\cos(\omega_0 t + \phi)$ or $A\sin(\omega_0 t + \phi)$

$y(t) = |H(j\omega_0)| \angle H(j\omega_0) \cdot A\cos(\omega_0 t + \phi)$

$$y(t) = \underbrace{A \cdot |H(j\omega_0)|}_{\text{magnitude}} \cdot \underbrace{\cos(\omega_0 t + \phi + \angle H(j\omega_0))}_{\text{Angle}}$$

Concept:

If $x(t) \neq 0 \quad t_1 < t < t_2$

and $h(t) \neq 0 \quad t_3 < t < t_4$

then $\boxed{y(t) \neq 0 \text{ for } t_1 + t_3 < t < t_2 + t_4}$

$y(t) = x(t) \circledast h(t)$

Note:

(1) Zero Input response ko free response or Natural response Bhi bolte hai

(2) Zero State response ko Forced response bolte hai.

- Agar kabhi T.F dete aur pole ki steady state pe value (For Eg lets say 8 hai) par given T.F se 8 S.S nai ara hoga to usme System gain ka concept lagega i.e K ka role hoga.

Z-Transform

Bilateral Z-transfer or Double side Z-Tramfform :

$$\boxed{X(z) = \sum_{n=-\infty}^{n=\infty} x(n) \cdot z^{-n}}$$

- → Agar Z ka power -ve hai series me to wo Causal signal hai
- → Agar Z ka power +ve hai to wo Anticausal signal hay
- → Agar Z ka power dono hai +ve and -ve to Non-causal hai

Power of Z	signal
+ve	Anticausal
-ve	causal
+ve and -ve	Non Causal

Unilateral Z-Transform :

$$\boxed{X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}} \ast$$

U.L.T is used to solve Difference equation.

Note :

Here also for causal signal,

B.L.T = U.L.T

Relationship B/W Laplace transform and Z-transform :

$$z^{-n} = e^{-snT_s}$$

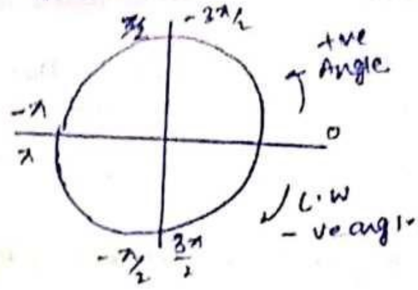
$$\boxed{z = e^{sT_s}}$$

where, T_s → sampling frequency.

Z plane :

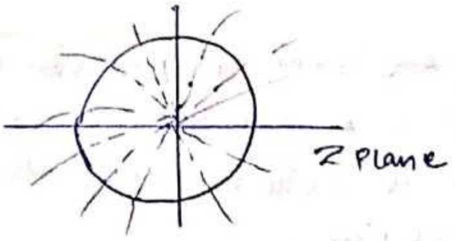
$$\boxed{z = \underset{\text{magnitude}}{r} \cdot e^{\overset{\text{phase}}{j\omega}}}$$

Complex variable z.



$$z \xrightarrow{r=1} z = e^{j\omega}$$

If ROC includes unit circle (circle of radius 1), then stable Discrete time Fourier transform.



Convergence or Existence :

→ Due to ROC, L.T and Z.T are valid for both stable and unstable signal,

L.T $\int_{-\infty}^{\infty} |x(t) e^{-\sigma t}| \, dt \leq \infty$

Z.T $\sum_{n=-\infty}^{\infty} |x(n) r^{-n}| < \infty$

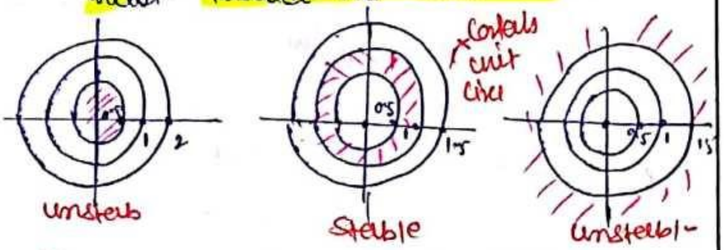
∴ $x(n) r^{-n}$ → Absolutely summable.

- → Z-plane is set of concentric circles with radii varying from R=0 to R=∞.
- → jω axis in s-plane transform to a circle with unit radius on z-plane called unit circle.

S-plane	z-plane
Imaginary-axis	unit circle
RHS pole	Circle with radii >1 outside the unit circle
LHS pole	circle with radius<1 inside unit circle.

Note points :

① For a signal to be stable, ROC must include the unit circle.



Pink colour represent common roc.

② If ROC is not specified then assume signal to be causal and poles must lie inside the unit circle for stability.

③ Signal will be marginal stable for single pole at $z=1$ and unstable for multiple pole at $z=1$

Region of Convergence (ROC):

ROC represent different value of $|z|$ for which $X(z)$ is finite

$u(n)$ ROC : $|z|>$
$u(-n)$ ROC : $|z|<$

→ ROC does not contain any poles

→ The ROC is a concentric ring (or) circle in the z-plane centred about origin.

→ If x(n) is of finite duration and x(n) is a causal sequence then ROC is Entire z-plane except "$z = 0$"

→ If x(n) is of finite duration and x(n) is anti causal sequence then ROC is Entire z-plane except "$z = \infty$".

→ If x(n) is of finite duration and x(n) is non-causal sequence then ROC is entire z-plane Except $z = \infty$ and $z = 0$.

Note :

(1) $\delta(n)$ is the only signal whose ROC is Entire z-plane.

(2) ROC of causal signal nikalte samay $z = \infty$ put karke check karenge agar $z=\infty$ put karne pe $Z(n)$ infinite aya to ROC will be

$|z|>a$ but $|z|<\infty$

aur agar $z=\infty$ pe $Z(n)=$ finite aaya to $|z|>a$ hi rahega.

(3) ROC of Anti causal signal nikalte samay $z=0$ put karke check karenge agar $z(n) = 0$ aaya to $|z|<a$ rahega but agar $Z(n)=\infty$ aagaya to $|z|>0$ aur $|z|<a$ hojaega.

Some Basic Transform :

(1) $\delta(n)$

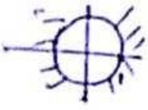
$$\boxed{\delta(n) \longleftrightarrow 1}$$ ROC : Entire z-plane.

(2) $u(n)$

$$\boxed{u(n) \xleftrightarrow{Z\cdot T} \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}}$$

ROC : $\underset{\text{Greater}}{\underset{\downarrow}{|z|>1}}$

Because in u(n) n is +ve.



← Marginable stable.

Time Shifting Property :

If $x(n) \longrightarrow X(z)$ ROC : R

$x(n-k) \longleftrightarrow z^{-k}\cdot X(z)$

$x(n+k) \longleftrightarrow z^{k}\cdot X(z)$

ROC : R except possible addition or deletion of poles at $\underbrace{z=0}_{\text{pole}}$ or $\underbrace{z=\infty}_{\text{Zero}}$.

Linearity property :

$x_1(n) \longleftrightarrow X_1(z)$ ROC : R_1

$x_2(n) \longleftrightarrow X_2(z)$ ROC : R_2

$a x_1(n) + b x_2(n) \longleftrightarrow a X_1(z) + b X_2(z)$

ROC : $R_1 \cap R_2$

Common : Intersection (Exception ROC bas diya tha na!) .

* NO Z.T. if no common ROC.

Exception of Linearity property :

$x(n) = u(n) - u(n-1)$

$$X(z) = \underbrace{\frac{z}{z-1}}_{|z|>1} = \frac{z^{-1}\cdot z}{z-1} = \frac{z-1}{z-1} = \underset{\text{Entire z-plane}}{\underset{\downarrow}{1}}$$

$|z|>1$ $|z|>1$

$= |z|>1$

But Answer has come Constant; we will take Entire z-plane as ROC.

**

Time Reversible Property :

If $x(n) \longleftrightarrow X(z)$ ROC : R ($|z|>R$)

$x(-n) \longleftrightarrow X(z^{-1})$ ROC : $\frac{1}{R}$ ($|z|<\frac{1}{R}$)

time reverse karne se $|z|>R$ se $|z|<\frac{1}{R}$ hojaega.

Time Reversible + time shifting :-

$x(-n-k) = x(-(n+k))$

~~Phle re common~~ |

Phle humlog time reversible perform karleg phir shifting karege.

$x(-n) \longrightarrow X(z^{-1})$

$x(-(n+k)) \longleftrightarrow z^{k} X(z^{-1})$.

Aur agar phle shifting karenge tab process hoga ·

$x(n-k) \longleftrightarrow z^{-k} X(z)$

$x(-n-k) \longleftrightarrow \left[z^{-k} X(z)\right]_{z=z^{-1}}$

Pehle shifting ke Equation le lo phir z ko z^{-1} se replace karege.

$x(-n-k) \longleftrightarrow$ [illegible]

Signal	X(z)	Roc	Beha	Stabil
$u(n)$	$\frac{z}{z-1}$	Roc: $\lvert z\rvert>1$	Causal seque	Marginal stable
$u(-n-1)$	$\frac{z}{z-1}$	Roc: $\lvert z\rvert<1$	Anticausal Sequen	Margin stable.

Remember :

(1) $u(-n) \longleftrightarrow \frac{-1}{z-1}$ | Roc : $|z|<1$

(2) $-u(-n) \longleftrightarrow \frac{1}{z-1}$ | Roc : $|z|<1$

Time scaling property :

$x(n) \longleftrightarrow X(z)$ Roc : R

$x\left(\frac{n}{k}\right) \longleftrightarrow X(z^k)$ Roc : $R^{1/k}$

$k \rightarrow$ Integer

Note :

(1) $u\left[\frac{n}{k}\right] \xleftrightarrow{Z.T} \frac{z^k}{z^k-1}$ | Roc! $|z|>1$

(2) If $x(n) \xleftrightarrow{Z.T} X(z)$

then $x(2n) \xleftrightarrow{Z.T} \frac{1}{2}\left[X(\sqrt{z})+X(-\sqrt{z})\right]$

(3) $u[2n] \xleftrightarrow{Z.T} \frac{z}{z-1}$ Roc : $|z|>1$

↓ no effect of Decimation in Z Transform of $u[n]$.

Scaling in Z :

$x(n) \longleftrightarrow X(z)$ Roc : R

$a^n x(n) \longleftrightarrow X\left(\frac{z}{a}\right)$ Roc : $|a|R$.

frequent use.

frequent use.

$a^n u(n) \longleftrightarrow \frac{z}{z-a}$ | Roc : $|z|>a$

Causal signal. Stable if $|a|<1$ Unstable if $|a|>1$

$-a^n . u(-n-1) \longleftrightarrow \frac{z}{z-a}$ Roc : $|z|<a$ or $|z|<|a|$

Anti Causal Sequence Stable if $|a|>1$ unstable $|a|<1$

$a^n . u(n)$	$\frac{z}{z-a}$	Roc : $\lvert z\rvert>a$	Causal sequen	Stable if $\lvert a\rvert<1$
$-a^n . u(-n-1)$	$\frac{z}{z-a}$	Roc $\lvert z\rvert<a$	Anticausal seque	stable if $\lvert a\rvert>1$

Note :

$a^{|n|} \longleftrightarrow \frac{z}{z-a} - \frac{z}{z-\frac{1}{a}}$ Roc : $a<|z|<\frac{1}{a}$

$|a|<1$ stable.

Note :

In discrete, two signals are said to be orthogonal when Area of product ~~should~~ are zero. ie $x(n) . y(n) = \{0\}$.

Signal	Z-transform	ROC	Causality	Stability
$\cos w_0 n \cdot u(n)$	$\frac{z^2 - z\cos w_0}{z^2 - 2z\cos w_0 + 1}$	$\lvert z\rvert>1$	Causal	Marginal Stable .
$\cos w_0 n \cdot u(-n-1)$	$\frac{z^2 - z\cos w_0}{z^2 - 2z\cos w_0 + 1}$	$\lvert z\rvert<a$	Anti Causal	Marginal Stable
$a^n \cos w_0 n . u(n)$	$\frac{z^2 - az\cos w_0}{z^2 - 2az\cos w_0 + a^2}$	$\lvert z\rvert>a$	Causal	Stable if $\lvert a\rvert<1$
$a^n \cos w_0 n \cdot u(-n-1)$	$\frac{z^2 - az\cos w_0}{z^2 - 2az\cos w_0 + a^2}$	$\lvert z\rvert<a$	Anticausal	Stable if $\lvert a\rvert>$

Signal	Z-transform	ROC	Causality	Stability
$\sin w_0 n \cdot u(n)$	$\frac{z\sin w_0}{z^2 - 2z\cos w_0 + 1}$	$\lvert z\rvert>1$	Causal	Marginal Stable
$-\sin w_0 n \cdot u(-n-1)$	$\frac{z\sin w_0}{z^2 - 2z\cos w_0 + 1}$	$\lvert z\rvert<1$	Anticausal	Marginal Stable
$a^n \sin w_0 n \cdot u(n)$	$\frac{az\sin w_0}{z^2 - 2za\cos w_0 + a^2}$	$\lvert z\rvert>a$	Causal	Stable if $\lvert a\rvert>1$.
$-a^n \sin w_0 n \cdot u(-n-1)$	$\frac{az\sin w_0}{z^2 - 2za\cos w_0 + a^2}$	$\lvert z\rvert<a$	Anticausal	Stable if $\lvert a\rvert<1$

Multiplication by n Property :

$$x(n) \longleftrightarrow X(z) \quad \text{ROC}: R$$

$$n \cdot x(n) \longleftrightarrow -z\frac{d}{dz}X(z) \quad \text{ROC}: R$$

$$n^2 x(n) \longleftrightarrow -z\frac{d}{dz}\left(-z\frac{d}{dz}X(z)\right)$$

$n \cdot u(n) = r(n)$

$$u(n) \longleftrightarrow \frac{z}{z-1}$$

$$n \cdot u(n) \longleftrightarrow -z\frac{d}{dz}\left[\frac{z}{z-1}\right] = \underset{\text{2 poles at } |z| -.\ \text{unstable .}}{\frac{z}{(z-1)^2}}$$

Signal	z-xform	ROC	Stability
$n \cdot u(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert>1$	unstable
$n^2 \cdot u(n)$	$\frac{z^2+z}{(z-1)^3}$	$\lvert z\rvert>1$	Unstable
$a^n \cdot n\, u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert>a$	Stable if $\lvert a\rvert<1$
$a^n \cdot n^2 u(n)$	$\frac{az^2+a^2z}{(z-a)^3}$	$\lvert z\rvert>a$	Stable if $\lvert a\rvert<1$

Note :

(1) $X(z) = \ln(1+az^{-1})$

$$\Rightarrow x(n) = -\frac{(-a^n)}{n}u(n-1)$$

$$= -\frac{(-a)^n}{n}\sum_{k=1}^{\infty}\delta(n-k)$$

(2) $X(z) = \ln\left(\frac{1}{1-az^{-1}}\right) = -\ln(1-az^{-1})$

$$\Rightarrow \boxed{x(n) = \sum_{k=1}^{\infty}\frac{a^n}{n}\delta(n-k)}$$

or $\boxed{x(n) = \frac{a^n}{n} \cdot u(n-1)}$

Prefer notes for derivation part :

(3) . vimp .

$$\frac{1}{n!}u(n) \longleftrightarrow e^{1/z}$$

$$a^n \cdot \frac{1}{n!}u(n) \xleftrightarrow[z \to \frac{z}{a}]{} e^{a/z}$$

(4) $\boxed{\frac{1}{(n-n_0)!}u(n-n_0) \longleftrightarrow z^{-n_0}e^{1/z}}$

Inverse z-transform :

Ans.

$$\boxed{x(n) = \frac{1}{2\pi j} \oint X(z)\, z^{n-1}\, dz}$$

Residue method :

For simple pole :

$$x(n) = \left[(z-p_i)\, X(z) \cdot z^{n-1} \right]_{z=p_i}$$

For multiple pole :

$$x(n) = \frac{1}{(m-1)!} \left[\frac{d^{m-1}}{dz^{m-1}} \left((z-p_i)^m \cdot X(z) \cdot z^{n-1} \right) \right]_{z=p_i}$$

Note : In long division method we can find causal sequence and Anticausal sequence But not non causal sequence.

→ Long division karte samay agar causal sequence chahiye to Divisor ke z^0 to z^{-ve} ki taraf likho

i.e $1-z^{-1}+z^{-2} \cdots$

aur agar Anti causal chahiye to Z ke -ve se z^0 taraf likh

$z^{-3}+z^{-2}+z^{-1}+1+\cdots$

Always prefer Partial method compare to residue method.

- Numerator me constant hoga to avoid Residue theorem.

Convolution property :-

$$\boxed{x_1(n) \otimes x_2(n) \longleftrightarrow X_1(z) \cdot X_2(z) \qquad ROC : R_1 \cap R_2}$$

Note :

$$X(z) = \sum_{n=-\infty}^{n=\infty} x(n) \cdot z^{-n}$$

Put $z=1$

$$\boxed{X(1) = \sum_{n=-\infty}^{n=\infty} x(n) = \text{Area of } x(n)}$$

X(1) represent Area of x(n) in z-transform.

Similarly X(s=0) i.e X(0) represents Area of x(t) in Laplace transform.

Convolution in n-domain (Discrete time convolution) :

$$\boxed{x(n) \otimes h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)}$$

Properties :

① commutative and Associative :-

$$x(n) \otimes h(n) = h(n) \otimes x(n)$$

$$x(n) \otimes [h(n) \otimes g(n)] = [x(n) \otimes h(n)] \otimes g(n)$$

② Distributive :

$$x(n) \otimes [h(n)+g(n)] = x(n) \otimes h(n) + x(n) \otimes g(n)$$

③ If $y(n) = x(n) \circledast h(n)$ then

$$\boxed{x(n-n_1) \circledast h(n-n_2) = y(n-n_1-n_2)}$$

④ Difference property :

$$y(n) = x(n) \circledast h(n)$$

$$y(n-1) = x(n-1) \circledast h(n)$$

$$= x(n) \circledast h(n-1)$$

I :

$$y(n)-y(n-1) = [x(n)-x(n-1)] \circledast h(n)$$

II :

$$y(n)-y(n-1) = x(n) \circledast [(h(n)-h(n-1))]$$

⑤ Step properties :

$$\boxed{\begin{aligned} x(n) \circledast u(n) &= \sum_{K=-\infty}^{\infty} x(K) \\ x(n) \circledast u(n-n_0) &= \sum_{K=-\infty}^{n-n_0} x(K) \end{aligned}}$$

Relationship B/w Impulse response and step response :

$$\underbrace{y(n)}_{\text{Step response}} = \underbrace{\boxed{\sum_{K=-\infty}^{n} h(K)}}_{\text{running Summation}} \Rightarrow \text{Impulse Response}$$

Impulse response :

$$\because y(n)-y(n-1) = [x(n)-x(n-1)] \circledast h(n)$$

$x(n) = u(n)$ $y(n) \rightarrow$ step response

$$y(n)-y(n-1) = [u(n)-u(n-1)] \circledast h(n)$$

$$y(n)-y(n-1) = \delta(n) \circledast h(n)$$

Step response :

$$\underbrace{\boxed{y(n)-y(n-1) = h(n)}}_{\text{Difference Equation}} \Rightarrow \text{Impulse Response}$$

Z-Transform with Initial condition :

$$x(n) \longleftrightarrow X(z)$$

$$x(n-1) \longleftrightarrow z^{-1}X(z) + x(-1)$$

$$x(n-2) \longleftrightarrow z^{-1}[z^{-1}X(z) + x(-1)] + x(-2)$$

(1) for frequency response ;

put $z = e^{j\omega}$ in $H(Z)$.

(2) phase response $\phi(\omega)$

If $\angle H(e^{j\omega})$.

(3) Time delay :

$\tau(\omega)$

$$\tau(\omega) = -\frac{d}{d\omega}\phi(\omega)$$

$\phi(\omega)$ = radian

ω = rad/sec

$$\tau(\omega) = -\frac{\text{rad}}{\text{rad/sec}} = \text{sec}$$

-ve sign with differential represent lag or delay.

★★★

Note :

$$\boxed{\frac{d}{d\omega}\tan^{-1}\left(\frac{N^r}{D^r}\right) = \frac{D^r\frac{d}{d\omega}N^r - N^r\frac{d}{d\omega}D^r}{N^2+D^2}}$$

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \Rightarrow \frac{d\phi(\omega)}{d\omega} = \frac{d}{d\omega}\phi_1(\omega) + \frac{d}{d\omega}\phi_2(\omega)$$

Causality and stability of discrete time LTI system from Impulse Response :-

I. Causality : $\boxed{h(n) = 0 \text{ for } n < 0}$

* H(z) is causal if ROC is the Exterior of the circle outside the outer most pole.

* with H(z) Expressed as ratio of polynomials or |H(z)| ratio the highest power of z in Nr must not exceed the highest power of z in Dr.

Eg :

Causal $h(n) = u(n)$ $H(z) = \frac{z}{z-1}$

Highest power of Z in $N^r = 1$
$D^r = 1$

Non causal $h(n) = u(n+1) = \frac{z^2}{z-1}$

$N^r = 2$ $N > D_r$
$D_r = 1$

II. Stability :

* h(n) must be Absolutely summable

$$\boxed{\sum_{n=-\infty}^{n=\infty} |h(n)| < \infty}$$

→ ROC of H(z) must lie inside the unit circle

→ Causal and stable :

all poles of H(z) must lie inside unit circle.

→ Anticausal and stable :

all poles of H(z) must lie outside the poles.

→ Non causal and stable :

ROC must be

$|a| < z | |b|$

where ~~|a| > 0 but~~

Eg $0.5 < |z| < 2$
$0.8 < |z| < \infty$
$0 < |z| < 2$.

	H(z)	$H^{o}_{inv}(z)$
Minimum phase	causal + stable	causal + stable
Non-minimum phase	causal + stable	causal + unstable

minimum phase system :

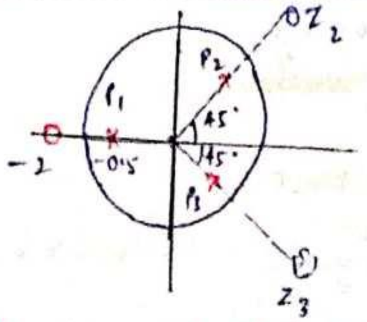
All poles and zeroes lie inside unit circle.

Non-minimum phase system :

(1) Mixed phase system :
Some but not all zeroes lie outside unit circle.

(2) Maximum phase system :
All zeroes lies outside the unit circle.

Pole - Zero pattern of All pass system :



Pole aur zero ek hi line me aaye.
then ek pole ke ek zero hoga.

Note : z-transform is only for non periodic signal.

Initial value theorem :

$$x(n)_{n=0} = x(0) = \lim_{z \to \infty} X(z)$$

Note : Initial value theorem tab nai lagega jab X(z) me N^r ka highest power of z aur D^r me highest power of z same hoga yki tab signal non-causal ban jayega.

Final value theorem.

$$(x(n))\Big|_{n=\infty} = x(\infty) = \lim_{z \to 1} (1-z^{-1}) \cdot X(z)$$

F.V.T not applicable to unstable system.

→ For F.V.T $(z-1) \cdot X(z)$ must be stable i.e All the poles of $(z-1) \cdot X(z)$ must lie inside unit ~~[illegible]~~ circle.

Periodic Convolution :

$$\underset{\underset{N_1}{\downarrow}}{x(n)} \circledast \underset{\underset{N_2}{\downarrow}}{h(n)} = \sum_{k=0}^{N-1} x(k) \cdot h(n-k)$$

$N_0 = LCM(N_1, N_2)$

$= N$

→ Sequences can be periodically extended upto fundamental period = length of sequence in one period

→ During convolution procedure both sequence will correspond to ~~same~~ same fundamental period and same length.

Eg :

$x(n) = \{\underset{\uparrow}{1}, 2, 3\}$ $N_1 = 3, L_1 = 3, A_1 = 1$

$h(n) = \{\underset{\uparrow}{1}, 1, 2\}$ $N_2 = 3$ $L_2 = 3$ $A_1 = 1$

$y(n) = x(n) \circledast h(n)$

$L_T = L_1 = L_2 = 3 \cdot A_T = A_1 = A_2$.

Correlation :

Cross correlation - r_{xy}

$x(n) \circledast y(-n)$

Auto correlation - r_{xx}

$x(n) \circledast x(-n)$

$r_{yy} = y(n) \circledast y(-n)$

Note :

$z = e^{j\omega}$ or $z = e^{s}$.

Digital filters :

① Distortionless system :

$y(n) = K.x(n-n_0)$

$$\boxed{\text{Freq. Response : } H(j\omega) = Ke^{-j\omega n_0}}$$

Condition for distortionless :

$$\boxed{\begin{aligned} |H(e^{j\omega})| &= k = \text{constant} \\ \angle H(e^{j\omega_0}) &= -\omega n_0 \end{aligned}}$$ → imp.

n_0.

② Ideal filters :

(1) Magnitude Response : constant.

→ Pass Band : Gain = 1 (K>1).

→ Stop Band : Gain = 0 (K=0).

(2) Phase Response : Linear

Group delay or time delay $= \tau_g(\omega)$

$= \frac{-d}{d\omega}(\phi(\omega))$

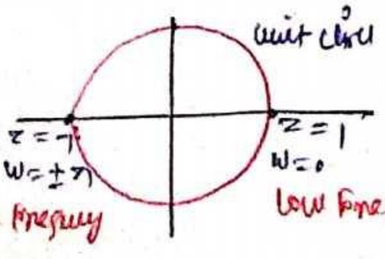
$\tau_g(\omega) = \frac{-d}{d\omega}(-\omega n_0) = n_0 = \text{constant}$

Constant group delay

$z = e^{j\omega} \Rightarrow |z| = 1 \rightarrow z = 1 \text{ or } -1$

$z = e^{j\omega} = 1 \Rightarrow \omega = 0 \rightarrow$ low frequency

$z = e^{j\omega} = -1 \Rightarrow \omega = \pi \ (\pm) \rightarrow$ high frequency.

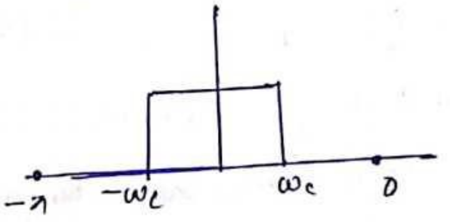


High frequency   Low frequency.

③

IIR	FIR
① Recursive or F/b	① Non Recursive or Non F/b
② Length of $h(n) = \infty$	② Length of $h(n)$ = finite
③ $h(n) = u(n)$ $H(z) = \frac{z}{z-1}$ ∞ duration	③ $h(n) = \delta(n)$ $\rightarrow H(z) = 1$ finite duration

④ Low pass filter :

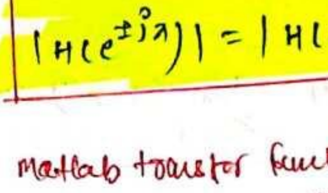


$$\boxed{\begin{aligned} |H(e^{j0})| &= |H(z=1)| = 1 \\ |H(e^{\pm j\pi})| &= |H(z=-1)| = 0 \end{aligned}}$$

Matlab transfer function pe agar z = 1 pe value aayi and z = −1 pe koi value nai aayi to low pass filter.

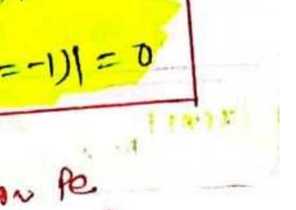
$$\boxed{H(z) = \frac{1-a}{2}\left(\frac{1+z^{-1}}{1-az^{-1}}\right)}$$ → T.F of L.P.F

$|a| < 1$ → for stability.



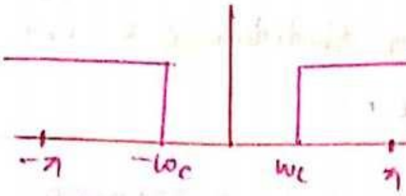
Zero on unit circle at high 'f'.

Pole close to unit circle near L.F



Poles can be in −ve side also.

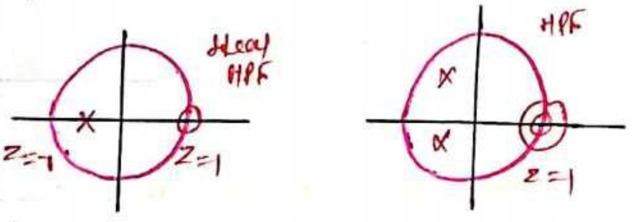
⑤ High Pass Filter



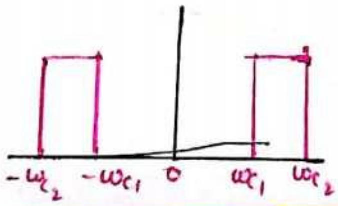
$$\boxed{\begin{array}{l} T.F = \\ H(z) \end{array} \quad \frac{1-a}{2}\left(\frac{1-z^{-1}}{1+az^{-1}}\right)} \quad |a|<1$$

$$\boxed{\begin{aligned} |H(e^{j0})| &= |H(z=1)| = 0 \\ |H(e^{\pm j\pi})| &= |H(z=-1)| = 1 \end{aligned}}$$

matlab $z=1$ pe 0
aur $z=-1$ pe 1 ya koi
value ani chahi.



⑥ Band Pass Filter :

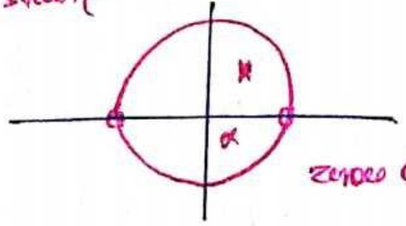


Condition to be B.P.F

$$\boxed{\begin{aligned} |H(e^{j0})| &= |H(z=1)| = 0 \\ H(e^{\pm j\pi}) &= |H(z=-1)| = 0 \end{aligned}}$$

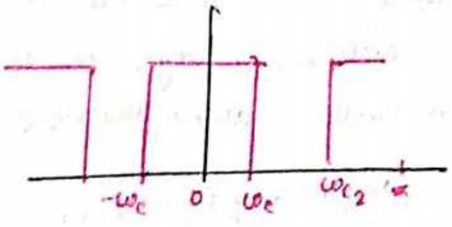
$$H(z) = \frac{(1-z^{-1})(1+z^{-1})}{(1-re^{j\omega_0}z^{-1})(1-re^{-j\omega_0}z^{-1})}$$

$|r|<1 \rightarrow$ for stable

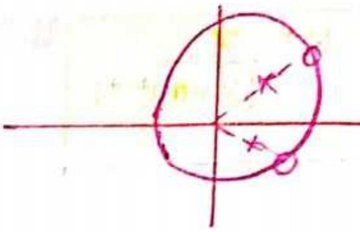


zeroes at $z = \pm 1$

⑦ Band stop or Notch filter :



$$\boxed{\begin{aligned} |H(e^{j0})| &= |H(z=1)| = 1 \\ |H(e^{\pm j\pi})| &= |H(z=-1)| = 1 \end{aligned}}$$



⑧ All pass filter :

$|H(e^{j\omega})| = 1$ for all ω.

Note :

Agar bol dia ki x(n) ~~ko~~ signal ki summable matlab uske ROC me unit circle aega hi.

→ agar ROC pucha hai aur option me Entire z-plane dia hai to ek bar pura X(z) ko solve karke dekh lenge kahi value 1 ~~0~~ ya koi constant to nai ara, agar aya to Entire z-plane hoga answer.

→ first difference of x(n) ka matlab,
$y(n) = x(n) - x(n-1)$.

→ for stable system, poles should lie inside unit circle ie $|P|<1$

agar transfer function deke stability puche to poles ke magnitude ko check karna agar less than 1 to stable warna unstable.

|magnitude of pole| < 1.

Note : Important Result :-

If $x(n) = n(n-1)\dots(n-k+1)\,a^{n-k}u(n)$

the

$$\boxed{X(z) = k!\,\frac{z}{(z-a)^{k+1}}}$$

N

- Kabhi bhi FVT lagate time Nr or Dr me power aane agya to ek bar divide dedenge fir FVT lagaenge.

- Kabhi bhi Initial condition dea rahega to focus karna hai $n, n-1, n-2$ wala form rahe agar $n, n+1 \dots$ wala form rahega to use adjust karenge ke $n, n-1, \dots$ wala banjaye.