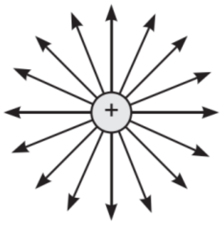
Electric Charges and Fields

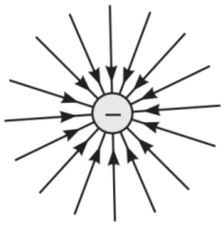
Case Study Based Questions

Case Study 1

A charge is a property associated with the matter due to which it experiences and produces an electric and magnetic fields. Charges are scalar in nature and they add up like real numbers. Also, the total charge of an isolated system is always conserved. When the objects rub against each other, charges acquired by them must be equal and opposite.



Electric field lines of a positive point charge



Electric field lines of a negative point charge

Read the given passage carefully and give the answer of the following questions:

Q1. The cause of charging is:

a. the actual transfer of protons

b. the actual transfer of electrons

c. the actual transfer of neutrons

d. None of the above

Q2. When a glass rod is rubbed with silk, then:

a. negative charge is produced on silk but no charge on glass rod

b. equal but opposite charges are produced on both

c. equal and similar charges are produced on both

d. positive charge is produced on glass rod but no charge on silk

Q3. If an object is positively charged theoretically, the mass of the object:

a. remains the same

b. increases slightly by a factor of 9.11×10^{-31} kg.

c. may increase or decrease

d. decreases slightly by a factor of 9.11 x 10^{-31} kg.

Q4. We have two bodies with charges q_1 and q_2 on them, then $q_1 + q_2 = 0$ signify:

a. q_1 and q_2 are equal charges with opposite signs

b. q_1 and q_2 are equal charges with same signs

c. q_1 and q_2 are not equal charges

d. q_1 and q_2 are equal charges.

Q5. The cause of quantisation of electric charges is:

a. transfer of an integral number of neutrons

b. transfer of an integral number of protons

c. transfer of an integral number of electrons

d. None of the above.

Solutions

1. (b) the actual transfer of electrons

The cause of charging is the actual transfer of electrons from one body to the other.

2. (b) equal but opposite charges are produced on both

When glass rod is rubbed with silk, glass rod loses electrons and silk grabs them. So, after rubbing, glass becomes positively charged and silk becomes negatively charged.

Thus, equal but opposite charges are produced on both.

3. (d) Decreases slightly by a factor of 9.11 × 10^{-31} kg.

When an object is positively charged, it loses some of its electrons. The mass of an electron is 9.11 x 10^{-31} kg, so the positively charged body loses electrons and its mass decreases by a factor of 9.11 × 10^{-31} kg.

4. (a) q_1 and q_2 are equal charges with opposite signs

$q_1 + q_2 = 0$, signifies that the net charge on the system is zero. This is possible only if q_1 and q_2 are equal but opposite in signs.

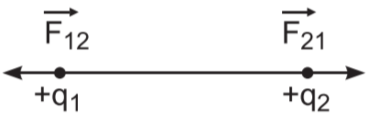
5. (c) transfer of an integral number of electrons.

The electric charges are said to be quantised when they exist in discrete amount rather than continuous value.

Case Study 2

Coulomb's law states that the electrostatic force of attraction or repulsion acting between two stationary points charges is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$



where F denotes the force between two charges q_1 and q_2 separated by a distance r in free space, ε_0 is a constant known as permittivity of free space. Free space is vacuum and may be taken to be air practically.

If free space is replaced by a medium, then ε_0 is replaced by $(\varepsilon_0 k)$ or $(\varepsilon_0 \varepsilon_r)$, where k is known as dielectric constant or relative permittivity.

Read the given passage carefully and give the answer of the following questions:

Q 1. In Coulomb's law, $F = k\frac{q_1 q_2}{r^2}$, then on which of the following factors does the proportionality constant *k* depends?

a. Electrostatic force acting between the two charges
b. Nature of the medium between the two charges
c. Magnitude of the two charges
d. Distance between the two charges

Q 2. Dimensional formula for the permittivity constant ε_0 of free space is:

a. $[ML^{-3}T^4A^2]$
b. $[M^{-1}L^3T^2A^2]$
c. $[M^{-1}L^{-3}T^4A^2]$
d. $[ML^{-3}T^4A^{-2}]$

Q 3. The force of repulsion between two charges of 1 C each, kept 1 m apart in vacuum is:

a. $\frac{1}{9\times10^{9}}$ N
b. 9×10^{9} N
c. 9×10^{7} N
d. $\frac{1}{9\times10^{12}}$ N

Q 4. Two identical charges repel each other with a force equal to 10 mg wt when they are 0.6 m apart in air. ($g = 10\ ms^{-2}$). The value of each charge is:

a. 2 mC
b. 2×10^{-7} mC
c. 2 nC
d. 2 μC

Q 5. Coulomb's law for the force between electric charges most closely resembles with:

a. law of conservation of energy
b. Newton's law of gravitation
c. Newton's 2nd law of motion
d. law of conservation of charge

Solutions

1. (b) Nature of the medium between the two charges.

2. (c) $[M^{-1}L^{-3}T^{4}A^{2}]$

As, we know that,

$$[\varepsilon_0] = \frac{1}{4\pi F}\cdot\frac{q_1q_2}{r^2} = \frac{(AT)^2}{[MLT^{-2}][L^2]}$$

$$= [M^{-1}L^{-3}T^{4}A^{2}]$$

3. (b) 9×10^{9} N

4. (d) 2 μC

Given that, $F = 10 \text{ mg wt} = 10 \times 10^{-3} \times 10$ N

$\because$ $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2}$$

$\therefore$ $$(10\times10^{-3})\times10 = \frac{(9\times10^{9})\times q^2}{(0.6)^2}$$

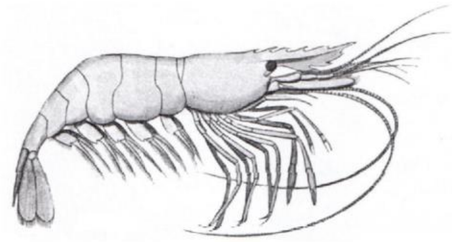
or $$q^2 = \frac{10^{-1} \times 0.36}{9 \times 10^9} = 4 \times 10^{-12}$$

or $$q = 2 \times 10^{-6}\ C = 2\ \mu C$$

5. (b) Newton's law of gravitation.

Case Study 3

Animals emit low frequency electric fields due to a process known as osmoregulation. This process allows the concentration of ions (charged atoms or molecules) to flow between the inside of our bodies and the outside. In order for our cells to stay intact, the flow of ions needs to be balanced. But balanced doesn't necessarily mean equal. The concentration of ions within a shrimp's body is much lower than that of the sea water it swims in. Their voltage or potential difference generated between the two concentrations across 'leaky' surfaces, can then be measured.



Read the given passage carefully and give the answer of the following questions:

Q1. The Gaussian surface for ions in the body of animals:

a. can pass through a continuous charge distribution

b. cannot pass through a continuous charge distribution

c. can pass through any system of discrete charges

d. can pass through a continuous charge distribution as well as any system of discrete charges

Q2. Gauss's law is valid for:

a. any closed surface

b. only regular close surfaces

c. any open surface

d. only irregular open surfaces

Q3. The electric field inside a shrimp's body of uniform charge density is:

a. zero

b. constant different from zero

c. proportional to the distance from the curve

d. None of the above

Q4. If a small piece of linear isotropic dielectric is swallowed by a shrimp and inside the body it is influenced by an electric field E, then the polarisation P is:

a. independent of E
b. inversely proportional to E
c. directly proportional to $\sqrt{E}$
d. directly proportional to E

Q5. Field due to multiple charges/ions inside shrimp's body at a point is found by using:

(i) superposition principle

(ii) Coulomb's law

(iii) law of conservation of charges

Choose the correct answer:

a. (i) and (ii)

b. (ii) and (iii)

c. (i) and (iii)

d. (i), (ii) and (iii)

Solutions

1. (a) can pass through a continuous charge distribution

2. (a) any closed surface

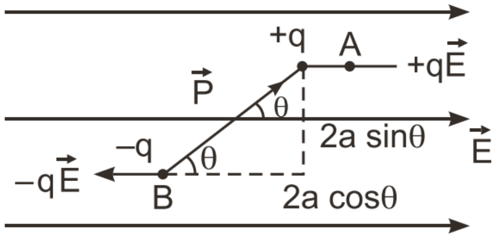
3. (a) zero

4. (d) directly proportional to E

5. (a) (i) and (ii)

Case Study 4

When electric dipole is placed in uniform electric field, its two charges experience equal and opposites forces, which cancel each other and hence net force on electric dipole in uniform electric field is zero. However, these forces are not collinear, so they give rise to some torque on the dipole. Since, net force on electric dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field. However, some work is done in rotating the dipole against the torque acting on it.



Read the given passage carefully and give the answer of the following questions:

Q 1. The dipole moment of a dipole in a uniform external field $\vec{E}$ is $\vec{P}$. What will be the torque $\vec{\tau}$ acting on the dipole?

Q 2. An electric dipole consists of two opposite charges, each of magnitude 1.0 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What is the maximum torque on the dipole?

Q 3. Torque on a dipole in uniform electric field is minimum when θ is equal what angle?

Q 4. When an electric dipole is held at an angle in a uniform electric field, what are the net force *F* and torque τ on the dipole?

Q 5. What is the net force on electric dipole in uniform electric field?

Solutions

1. As, $\tau =$ either force $\times$ perpendicular distance between the two forces

$$\tau = qE \times 2a \sin\theta \quad \text{or} \quad \tau = PE \sin\theta$$

or $$\tau = \vec{P} \times \vec{E} \qquad [\because q(2a) = P]$$

2. The maximum torque on the dipole in an external electric field is given by

$$\tau = PE = q(2a) \times E$$

Here, $q = 1\,\mu C = 10^{-6}$ C, $2a = 2\text{ cm} = 2 \times 10^{-2}$m

$$E = 10^5\ NC^{-1},\ \tau = ?$$

$\therefore$ $$\tau = 10^{-6} \times 2 \times 10^{-2} \times 10^5 = 2 \times 10^{-3}\text{Nm}$$

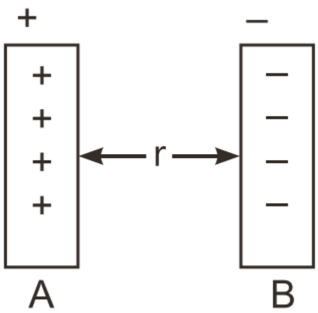
3. When θ is 0° or 180°, τ is minimum, which means the dipole moment should be parallel to the direction of the uniform electric field.

4. Net force is zero and torque acts on the dipole, trying to align p with E.

5. Zero.

Case Study 5

Surface charge density is defined as charge per unit surface area of surface charge distribution *i.e.,* $\sigma = \dfrac{dq}{dS}$. Two large thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs having magnitude of 17.0×10^{-22} C/m^2 as shown. The intensity of electric field at a point is $E = \dfrac{\sigma}{\varepsilon_0}$, where $\varepsilon_0 =$ permittivity of free space.



Read the given passage carefully and give the answer of the following questions:

Q1. What is the value of E in the outer region of the first plate ?

Q2. What is the value of E in the outer region of the second plate ?

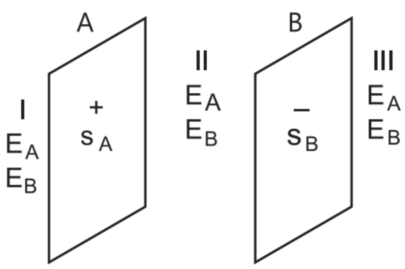
Q3. What will be the value of E between the plates ?

Q4. What is the ratio of E from right side of B at distances of 2 cm and 4 cm, respectively ?

Q5. In order to estimate the electric field due to a thin finite plane metal plate, what is the shape of the Gaussian surface ?

Solutions

1. There are two plates A and B having surface charge densities, $\sigma_A = 17.0 \times 10^{-22}$ C/m^2 on A and $\sigma_B = -17.0 \times 10^{-22}$ C/m^2 on B, respectively.



According to Gauss's theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$E_I = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

2. The electric field in region III is also zero.

$$E_{III} = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

3. In region II or between the plates, the electric field

$$E_{II} = E_A - E_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{\sigma(\sigma_A \text{ or } \sigma_B)}{\varepsilon_0} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$

$$\therefore \quad E = 1.9 \times 10^{-10} \text{ NC}^{-1}$$

4. Electric field due to an infinite plane sheet of charge does not depend on the distance of observation point from the plane sheet of charge.

So, ratio of E will be 1 : 1.

5. We take a cylindrical cross-sectional area A and length $2r$ as Gaussian surface.

Solutions for Questions 6 to 15 are Given Below

Case Study 6

Coulomb's Law

Coulomb's law states that the electrostatic force of attraction or repulsion acting between two stationary point charges is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

where F denotes the force between two charges q_1 and q_2 separated by a distance r in free space, ε_0 is a constant known as permittivity of free space. Free space is vacuum and may be taken to be air practically.
If free space is replaced by a medium, then ε_0 is replaced by $(\varepsilon_0 k)$ or $(\varepsilon_0\varepsilon_r)$ where k is known as dielectric constant or relative permittivity.

(i) In coulomb's law, $F = k\frac{q_1q_2}{r^2}$, then on which of the following factors does the proportionality constant k depends?
- (a) Electrostatic force acting between the two charges
- (b) Nature of the medium between the two charges
- (c) Magnitude of the two charges
- (d) Distance between the two charges.

(ii) Dimensional formula for the permittivity constant ε_0 of free space is
- (a) $[M L^{-3} T^4 A^2]$
- (b) $[M^{-1} L^3 T^2 A^2]$
- (c) $[M^{-1} L^{-3} T^4 A^2]$
- (d) $[M L^{-3} T^4 A^{-2}]$

(iii) The force of repulsion between two charges of 1 C each, kept 1 m apart in vaccum is
- (a) $\frac{1}{9\times10^9}$ N
- (b) 9×10^9 N
- (c) 9×10^7 N
- (d) $\frac{1}{9\times10^{12}}$ N

(iv) Two identical charges repel each other with a force equal to 10 mgwt when they are 0.6 m apart in air. ($g = 10$ m s^{-2}). The value of each charge is

(a) 2 mC (b) 2×10^{-7} mC (c) 2 nC (d) 2 μC

(v) Coulomb's law for the force between electric charges most closely resembles with

(a) law of conservation of energy (b) Newton's law of gravitation

(c) Newton's 2nd law of motion (d) law of conservation of charge

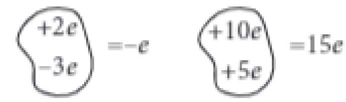
Case Study 7

Quantization of Electric Charge

Smallest charge that can exist in nature is the charge of an electron. During friction it is only the transfer of electrons which makes the body charged. Hence net charge on any body is an integral multiple of charge of an electron [1.6×10^{-19} C] *i.e.*

$$q = \pm ne$$

where $n = 1, 2, 3, 4, \ldots$



Hence no body can have a charge represented as $1.1e$, $2.7e$, $\frac{3}{5}e$, etc.

Recently, it has been discovered that elementary particles such as protons or neutrons are composed of more elemental units called quarks.

(i) Which of the following properties is not satisfied by an electric charge?

(a) Total charge conservation. (b) Quantization of charge.

(c) Two types of charge. (d) Circular line of force.

(ii) Which one of the following charges is possible?

(a) 5.8×10^{-18} C (b) 3.2×10^{-18} C

(c) 4.5×10^{-19} C (d) 8.6×10^{-19} C

(iii) If a charge on a body is 1 nC, then how many electrons are present on the body ?

(a) 6.25×10^{27} (b) 1.6×10^{19}

(c) 6.25×10^{28} (d) 6.25×10^{9}

(iv) If a body gives out 10^9 electrons every second, how much time is required to get a total charge of 1 C from it?

(a) 190.19 years (b) 150.12 years (c) 198.19 years (d) 188.21 years

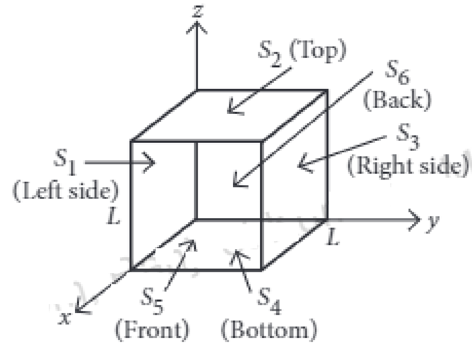
(v) A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7}C. Calculate the number of electrons transferred.

(a) 2×10^{12} (b) 3×10^{12} (c) 2×10^{14} (d) 3×10^{14}

Case Study 8

Electric Flux through a Cube

Net electric flux through a cube is the sum of fluxes through its six faces. Consider a cube as shown in figure, having sides of length $L = 10.0$ cm. The electric field is uniform, has a magnitude $E = 4.00 \times 10^3$ N C^{-1} and is parallel to the xy plane at an angle of 37° measured from the $+x$-axis towards the $+y$-axis.



(i) Electric flux passing through surface S_6 is
(a) $-24\ \text{N m}^2\ \text{C}^{-1}$ (b) $24\ \text{N m}^2\ \text{C}^{-1}$ (c) $32\ \text{N m}^2\ \text{C}^{-1}$ (d) $-32\ \text{N m}^2\ \text{C}^{-1}$

(ii) Electric flux passing through surface S_1 is
(a) $-24\ \text{N m}^2\ \text{C}^{-1}$ (b) $24\ \text{N m}^2\ \text{C}^{-1}$ (c) $32\ \text{N m}^2\ \text{C}^{-1}$ (d) $-32\ \text{N m}^2\ \text{C}^{-1}$

(iii) The surfaces that have zero flux are
(a) S_1 and S_3 (b) S_5 and S_6 (c) S_2 and S_4 (d) S_1 and S_2

(iv) The total net electric flux through all faces of the cube is
(a) $8\ \text{N m}^2\ \text{C}^{-1}$ (b) $-8\ \text{N m}^2\ \text{C}^{-1}$ (c) $24\ \text{N m}^2\ \text{C}^{-1}$ (d) zero

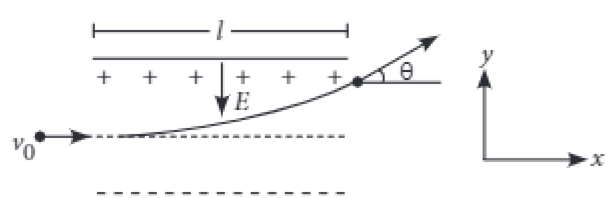
(v) The dimensional formula of surface integral $\oint \vec{E}\cdot d\vec{S}$ of an electric field is
(a) $[\text{M L}^2\ \text{T}^{-2}\ \text{A}^{-1}]$ (b) $[\text{M L}^3\ \text{T}^{-3}\ \text{A}^{-1}]$
(c) $[\text{M}^{-1}\ \text{L}^3\ \text{T}^{-3}\ \text{A}]$ (d) $[\text{M L}^{-3}\ \text{T}^{-3}\ \text{A}^{-1}]$

Case Study 9

Motion of Charged Particle in Uniform Electric Field

When a charged particle is placed in an electric field, it experiences an electrical force. If this is the only force on the particle, it must be the net force. The net force will cause the particle to accelerate according to Newton's second law. So

$$\vec{F}_e = q\vec{E} = m\vec{a}$$



If $\vec{E}$ is uniform, then $\vec{a}$ is constant and $\vec{a} = q\vec{E}/m$. If the particle has a positive charge, its acceleration is in the direction of the field. If the particle has a negative charge, its acceleration is in the direction opposite to the electric field. Since the acceleration is constant, the kinematic equations can be used.

(i) An electron of mass m, charge e falls through a distance h metre in a uniform electric field E. Then time of fall,
(a) $t = \sqrt{\dfrac{2hm}{eE}}$ (b) $t = \dfrac{2hm}{eE}$ (c) $t = \sqrt{\dfrac{2eE}{hm}}$ (d) $t = \dfrac{2eE}{hm}$

(ii) An electron moving with a constant velocity v along X-axis enters a uniform electric field applied along Y-axis. Then the electron moves

(a) with uniform acceleration along Y-axis
(b) without any acceleration along Y-axis
(c) in a trajectory represented as $y = ax^2$
(d) in a trajectory represented as $y = ax$

(iii) Two equal and opposite charges of masses m_1 and m_2 are accelerated in an uniform electric field through the same distance. What is the ratio of their accelerations if their ratio of masses is $\frac{m_1}{m_2} = 0.5$?

(a) $\frac{a_1}{a_2} = 2$
(b) $\frac{a_1}{a_2} = 0.5$
(c) $\frac{a_1}{a_2} = 3$
(d) $\frac{a_1}{a_2} = 1$

(iv) A particle of mass m carrying charge q is kept at rest in a uniform electric field E and then released. The kinetic energy gained by the particle, when it moves through a distance y is

(a) $\frac{1}{2}qEy^2$
(b) qEy
(c) qEy^2
(d) qE^2y

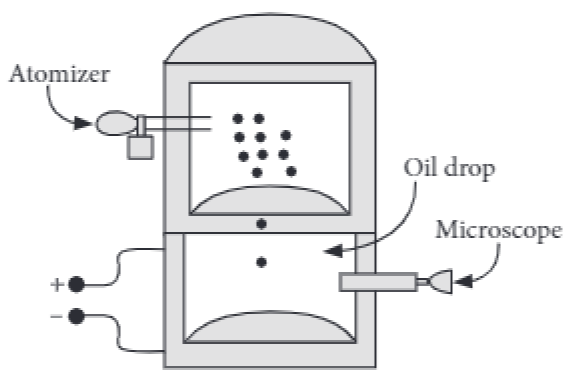
(v) A charged particle is free to move in an electric field. It will travel

(a) always along a line of force
(b) along a line of force, if its initial velocity is zero
(c) along a line of force, if it has some initial velocity in the direction of an acute angle with the line of force
(d) none of these.

Case Study 10

Millikan's Oil-drop Experiment

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equaled Mg. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C the charge of the electron. For this, he won the Nobel prize.



(i) If a drop of mass 1.08×10^{-14} kg remains stationary in an electric field of 1.68×10^5 N C^{-1}, then the charge of this drop is

(a) 6.40×10^{-19} C
(b) 3.2×10^{-19} C
(c) 1.6×10^{-19} C
(d) 4.8×10^{-19} C

(ii) Extra electrons on this particular oil drop (given the presently known charge of the electron) are

(a) 4
(b) 3
(c) 5
(d) 8

(iii) A negatively charged oil drop is prevented from falling under gravity by applying a vertical electric field 100 V m^{-1}. If the mass of the drop is 1.6×10^{-3} g, the number of electrons carried by the drop is ($g = 10$ m s^{-2})

(a) 10^{18} (b) 10^{15} (c) 10^{12} (d) 10^{9}

(iv) The important conclusion given by Millikan's experiment about the charge is

(a) charge is never quantized (b) charge has no definite value

(c) charge is quantized (d) charge on oil drop always increases.

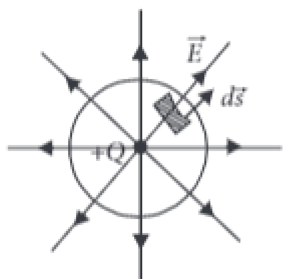
(v) If in Millikan's oil drop experiment, charges on drops are found to be 8μC, 12μC, 20μC, then quanta of charge is

(a) 8μC (b) 20μC (c) 12μC (d) 4μC

Case Study 11

Guass's Law and Its Significance

Gauss's law and Coulomb's law, although expressed in different forms, are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is $\varepsilon_0\phi = q_{encl}$, when q_{encl} is the net charge inside an imaginary closed surface called Gaussian surface. $\phi = \oint \vec{E} \cdot d\vec{A}$ gives the electric flux through the Gaussian surface. The two equations hold only when the net charge is in vacuum or air.



Gaussian
spherical surfaces

(i) If there is only one type of charge in the universe, then ($\vec{E} \rightarrow$ Electric field, $d\vec{s} \rightarrow$ Area vector)

(a) $\oint \vec{E} \cdot d\vec{s} \neq 0$ on any surface

(b) $\oint \vec{E} \cdot d\vec{s}$ could not be defined

(c) $\oint \vec{E} \cdot d\vec{s} = \infty$ if charge is inside

(d) $\oint \vec{E} \cdot d\vec{s} = 0$ if charge is outside, $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$ if charge is inside

(ii) What is the nature of Gaussian surface involved in Gauss law of electrostatic ?

(a) Magnetic (b) Scalar (c) Vector (d) Electrical

(iii) A charge 10 μC is placed at the centre of a hemisphere of radius $R = 10$ cm as shown. The electric flux through the hemisphere (in MKS units) is

(a) 20×10^5 (b) 10×10^5 (c) 6×10^5 (d) 2×10^5

(iv) The electric flux through a closed surface area S enclosing charge Q is ϕ. If the surface area is doubled, then the flux is

(a) 2ϕ (b) $\phi/2$ (c) $\phi/4$ (d) ϕ

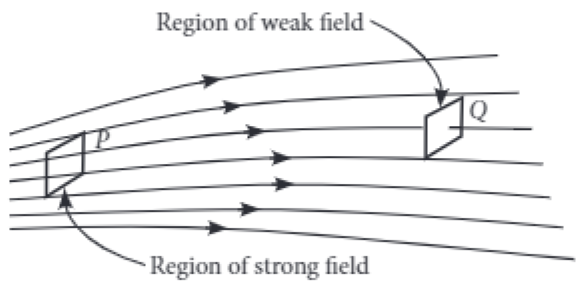
(v) A Gaussian surface encloses a dipole. The electric flux through this surface is

(a) $\dfrac{q}{\varepsilon_0}$ (b) $\dfrac{2q}{\varepsilon_0}$ (c) $\dfrac{q}{2\varepsilon_0}$ (d) zero

Case Study 12

Relation between Strength of Electric Field and Density of Lines of Force

Electric field strength is proportional to the density of lines of force *i.e.*, electric field strength at a point is proportional to the number of lines of force cutting a unit area element placed normal to the field at that point. As illustrated in the given figure, the electric field at P is stronger that at Q.



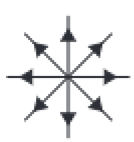
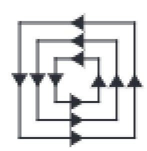
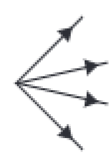
(i) Electric lines of force about a positive point charge are

(a) radially outwards (b) circular clockwise
(c) radially inwards (d) parallel straight lines.

(ii) Which of the following is false for electric lines of force?

(a) They always start from positive charges and terminate on negative charges.
(b) They are always perpendicular to the surface of a charged conductor.
(c) They always form closed loops.
(d) They are parallel and equally spaced in a region of uniform electric field.

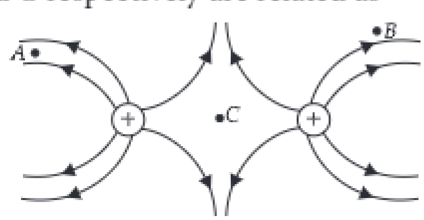
(iii) Which one of the following pattern of electric line of force in not possible in filed due to stationary charges ?

(a)  (b)  (c)  (d) 

(iv) Electric lines of force are curved

(a) in the field of a single positive or negative charge (b) in the field of two equal and opposite charges
(c) in the field of two like charges (d) both (b) and (c).

(v) The figure below shows the electric field lines due to two positive charges. The magnitudes E_A, E_B and E_C of the electric fields at points A, B and C respectively are related as

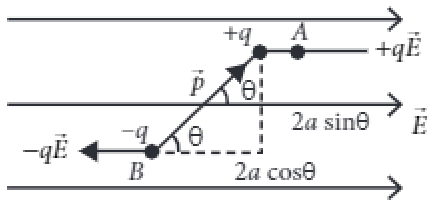


(a) $E_A > E_B > E_C$ (b) $E_B > E_A > E_C$ (c) $E_A = E_B > E_C$ (d) $E_A > E_B = E_C$

Case Study 13

Torque on a Dipole in a Uniform Electric Field

When electric dipole is placed in uniform electric field, its two charges experience equal and opposite forces, which cancel each other and hence net force on electric dipole in uniform electric field is zero. However these forces are not collinear, so they give rise to some torque on the dipole. Since net force on electric dipole in uniform electric field is zero. so no work is done in moving the electric dipole in uniform electric field. However some work is done in rotating the dipole against the torque acting on it.



(i) The dipole moment of a dipole in a uniform external field $\vec{E}$ is $\vec{P}$. Then the torque $\vec{\tau}$ acting on the dipole is

(a) $\vec{\tau} = \vec{P} \times \vec{E}$ (b) $\vec{\tau} = \vec{P} \cdot \vec{E}$ (c) $\vec{\tau} = 2(\vec{P} + \vec{E})$ (d) $\vec{\tau} = (\vec{P} + \vec{E})$

(ii) An electric dipole consists of two opposite charges, each of magnitude 1.0 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 N C^{-1}. The maximum torque on the dipole is

(a) 0.2×10^{-3} Nm (b) 1×10^{-3} Nm (c) 2×10^{-3} Nm (d) 4×10^{-3} Nm

(iii) Torque on a dipole in uniform electric field is minimum when θ is equal to

(a) 0° (b) 90° (c) 180° (d) Both (a) and (c)

(iv) When an electric dipole is held at an angle in a uniform electric field, the net force F and torque τ on the dipole are

(a) $F = 0, \tau = 0$ (b) $F \neq 0, \tau \neq 0$ (c) $F = 0, \tau \neq 0$ (d) $F \neq 0, \tau = 0$

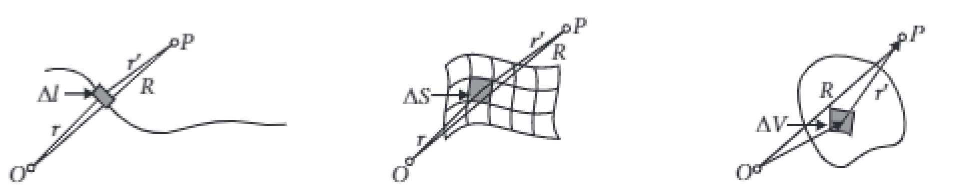
(v) An electric dipole of moment p is placed in an electric field of intensity E. The dipole acquires a position such that the axis of the dipole makes an angle θ with the direction of the field. Assuming that the potential energy of the dipole to be zero when θ = 90°, the torque and the potential energy of the dipole will respectively be

(a) $pE\sin\theta, -pE\cos\theta$ (b) $pE\sin\theta, -2pE\cos\theta$ (c) $pE\sin\theta, 2pE\cos\theta$ (d) $pE\cos\theta, -pE\sin\theta$

Case Study 14

Continuous Charge Distribution

In practice, we deal with charges much greater in magnitude than the charge on an electron, so we can ignore the quantum nature of charges and imagine that the charge is spread in a region in a continuous manner. Such a charge distribution is known as continuous charge distribution. There are three types of continuous charge distribution : (i) Line charge distribution (ii) Surface charge distribution (iii) Volume charge distribution as shown in figure.

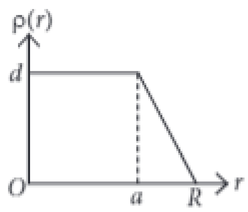


Line charge $\Delta Q = \lambda \Delta l$ Surface charge $\Delta Q = \sigma \Delta S$ Volume charge $\Delta Q = \rho \Delta V$

(i) **Statement 1 :** Gauss's law can't be used to calculate electric field near an electric dipole.
Statement 2 : Electric dipole don't have symmetrical charge distribution.
(a) Statement 1 and statement 2 are true
(b) Statement 1 is false but statement 2 is true.
(c) Statement 1 is true but statement 2 is false.
(d) Both statements are false.

(ii) An electric charge of 8.85×10^{-13} C is placed at the centre of a sphere of radius 1 m. The electric flux through the sphere is
(a) $0.2\ \mathrm{N\,C^{-1}\,m^2}$
(b) $0.1\ \mathrm{N\,C^{-1}\,m^2}$
(c) $0.3\ \mathrm{N\,C^{-1}\,m^2}$
(d) $0.01\ \mathrm{N\,C^{-1}\,m^2}$

(iii) The electric field within the nucleus is generally observed to be linearly dependent on r. So,



(a) $a = 0$
(b) $a = \frac{R}{2}$
(c) $a = R$
(d) $a = \frac{2R}{3}$

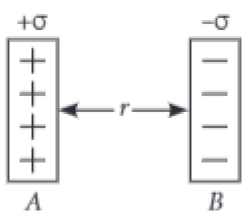
(iv) What charge would be required to electrify a sphere of radius 25 cm so as to get a surface charge density of $\frac{3}{\pi}\ \mathrm{C\,m^{-2}}$?
(a) 0.75 C
(b) 7.5 C
(c) 75 C
(d) zero

(v) The SI unit of linear charge density is
(a) C m
(b) $\mathrm{C\,m^{-1}}$
(c) $\mathrm{C\,m^{-2}}$
(d) $\mathrm{C\,m^{-3}}$

Case Study 15

Parallel Sheet of Charge

Surface charge density is defined as charge per unit surface area of surface charge distribution. *i.e.*, $\sigma = \frac{dq}{dS}$. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs having magnitude of $17.0 \times 10^{-22}\ \mathrm{C\,m^{-2}}$ as shown. The intensity of electric field at a point is $E = \frac{\sigma}{\varepsilon_0}$, where ε_0 = permittivity of free space.



(i) E in the outer region of the first plate is
(a) 17×10^{-22} N/C
(b) 1.5×10^{-25} N/C
(c) 1.9×10^{-10} N/C
(d) zero

(ii) E in the outer region of the second plate is
(a) 17×10^{-22} N/C
(b) 1.5×10^{-15} N/C
(c) 1.9×10^{-10} N/C
(d) zero

(iii) E between the plates is
(a) 17×10^{-22} N/C
(b) 1.5×10^{-15} N/C
(c) 1.9×10^{-10} N/C
(d) zero

(iv) The ratio of E from right side of B at distances 2 cm and 4 cm, respectively is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) $1:\sqrt{2}$

(v) In order to estimate the electric field due to a thin finite plane metal plate, the Gaussian surface considered is

(a) spherical (b) cylindrical (c) straight line (d) none of these

HINTS & EXPLANATIONS

6. **(i) (b):** The proportionality constant k depends on the nature of the medium between the two charges.

(ii) (c): As, $[\varepsilon_0] = \frac{1}{4\pi F}\cdot\frac{q_1 q_2}{r^2} = \frac{[\text{AT}]^2}{[\text{M L T}^{-2}][\text{L}^2]}$

$= [\text{M}^{-1}\,\text{L}^{-3}\,\text{T}^4\,\text{A}^2]$

(iii) (b)

(iv) (d): $F = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{d^2}$

$\therefore\ (10\times10^{-3})\times10 = \frac{(9\times10^9)\times q^2}{(0.6)^2}$

or $q^2 = \frac{10^{-1}\times0.36}{9\times10^9} = 4\times10^{-12}$

or $q = 2\times10^{-6}\ \text{C} = 2\ \mu\text{C}$

(v) (b)

7. **(i) (d)**

(ii) (b): From, $q = ne$, $n = \frac{q}{e} = \frac{3.2\times10^{-18}}{1.6\times10^{-19}} = 20$

As n is an integer, hence this value of charge is possible.

(iii) (d): Charge on the body is $q = ne$

$\therefore$ No. of electrons present on the body is

$$n=\frac{q}{e}=\frac{1\times10^{-9}\,C}{1.6\times10^{-19}\,C}=6.25\times10^{9}$$

(iv) (c): Here, $n = 10^9$ electrons per second

Charge given per second,

$q = ne = 10^9 \times 1.6 \times 10^{-19}$ C

$q = 1.6 \times 10^{-10}$ C

Total charge, $Q = 1$ C (given)

$\therefore$ Time required $=\frac{Q}{q}=\frac{1}{1.6\times10^{-10}}$ s $= 6.25 \times 10^9$s

$\therefore \frac{6.25\times10^{9}}{3600\times24\times365}$ year = 198.19 years.

(v) (a): As $q = ne$, $n=\frac{3.2\times10^{-7}}{1.6\times10^{-19}}$

$\Rightarrow n = 2 \times 10^{12}$ electrons.

8. **(i) (d):** Electric flux, $\phi = \vec{E}\cdot\vec{A} = EA\cos\theta$, where $\vec{A} = A\hat{n}$

For electric flux passing through S_6, $\hat{n}_{S_6} = -\hat{i}$ (Back)

$\therefore \phi_{S_6} = -(4 \times 10^3\ N\,C^{-1})\ (0.1\ m)^2 \cos 37°$

$= -32\ N\ m^2\ C^{-1}$

(ii) (a): For electric flux passing through S_1,

$\hat{n}_{S_1} = -\hat{j}$ (Left)

$\therefore \phi_{S_1} = -(4 \times 10^3\ N\,C^{-1})\ (0.1\ m)^2 \cos (90° - 37°)$

$= -24\ N\ m^2\ C^{-1}$

(iii) (c): Here, $\hat{n}_{S_2} = +\hat{k}$ (Top)

$\therefore \phi_{S_2} = -(4 \times 10^3\ N\,C^{-1})\ (0.1\ m)^2 \cos 90° = 0$

$\hat{n}_{S_3} = +\hat{j}$ (Right)

$\hat{n}_{S_4} = -\hat{k}$ (Bottom)

$\therefore \phi_{S_4} = (4 \times 10^3\ N\,C^{-1})\ (0.1\ m)^2 \cos 90° = 0$

And, $\hat{n}_{S_5} = +\hat{i}$ (Front)

$\therefore \phi_{S_5} = +(4 \times 10^3\ N\,C^{-1})\ (0.1\ m)^2 \cos 37°$

$= 32\ N\ m^2\ C^{-1}$

S_2 and S_4 surface have zero flux.

(iv) (d): As the field is uniform, the total flux through the cube must be zero, *i.e.*, any flux entering the cube must leave it.

(v) (b): Surface integral $\oint \vec{E}\cdot d\vec{S}$ is the net electric flux over a closed surface S.

$\therefore [\phi_E] = [M\,L^3\,T^{-3}\,A^{-1}]$

9. **(i) (a):** From Newton's law

$F = m\vec{a}$ or $qE = m\vec{a} \Rightarrow a = \frac{qE}{m} = \frac{eE}{m}$

Using, $s = ut + \frac{1}{2}at^2$

$\therefore h = 0 + \frac{1}{2}\times\frac{eE}{m}t^2 \Rightarrow t=\sqrt{\frac{2hm}{eE}}$

(ii) (c)

(iii) (b): Force is same in magnitude for both.

$\therefore m_1a_1 = m_2a_2$;

$\frac{a_1}{a_2}=\frac{m_2}{m_1}=\frac{1}{0.5}=2$

(iv) (b): Here, $u = 0$; $a=\frac{qE}{m}$; $s = y$

Using, $v^2 - u^2 = 2as \Rightarrow v^2 = 2\frac{qE}{m}y$

$\therefore$ K.E. $=\frac{1}{2}mv^2 = qEy$

(v) (b): If charge particle is put at rest in electric field, then it will move along line of force.

10. **(i) (a):** As, $qE = mg \Rightarrow q=\frac{1.08\times10^{-14}\times9.8}{1.68\times10^{5}}$

$= 6.4 \times 10^{-19}$ C

(ii) (a): $q = ne$ or $\Rightarrow n=\frac{6.4\times10^{-19}}{1.6\times10^{-19}}=4$

(iii) (c): For the drop to be stationary,

Force on the drop due to electric field = Weight of the drop

$qE = mg$

$$q=\frac{mg}{E}=\frac{1.6\times10^{-6}\times10}{100}=1.6\times10^{-7}\ C$$

Number of electrons carried by the drop is

$$n=\frac{q}{e}=\frac{1.6\times10^{-7}\ C}{1.6\times10^{-19}\ C}=10^{12}$$

(iv) (c)

(v) (d): Millikan's experiment confirmed that the charges are quantized, *i.e.*, charges are small integer multiples of the base value which is charge on electron. The charges on the drops are found to be multiple of 4. Hence, the quanta of charge is 4 μC.

11. **(i) (d):** If there is only one type of charge in the universe then it will produce electric field somehow. Hence Gauss's law is valid.

(ii) (c)

(iii) (c): According to Gauss's theorem,

Electric flux through the sphere $=\frac{q}{\varepsilon_0}$

$\therefore$ Electric flux through the hemisphere $= \frac{1}{2}\frac{q}{\varepsilon_0}$

$$= \frac{10 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}} = 0.56 \times 10^6 \text{ N m}^2 \text{ C}^{-1}$$

$$\approx 0.6 \times 10^6 \text{ Nm}^2 \text{ C}^{-1} = 6 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$$

(iv) (d): As flux is the total number of lines passing through the surface, for a given charge, it is always the charge enclosed Q/ε_0. If area is doubled, the flux remains the same.

(v) (d): As net charge on a dipole is
$(-q + q) = 0$
Thus, when a gaussian surface encloses a dipole, as per Gauss's theorem, electric flux through the surface,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0} = 0$$

12. **(i)** **(a)**

(ii) **(c)**: Electric lines of force do not form any closed loops.

(iii) (c): Electric field lines can't be closed.

(iv) **(d)**

(v) **(a)**

13. **(i) (a):** As τ = either force $\times$ perpendicular distance between the two forces

$= qaE\sin\theta$ or $\tau = PE\sin\theta$

or $\tau = \vec{P} \times \vec{E}$ $\quad (\because qa = P)$

(ii) (c) : The maximum torque on the dipole in an external electric field is given by

$\tau = pE = q(2a) \times E$

Here, $q = 1\ \mu\text{C} = 10^{-6}$ C, $2a = 2$ cm $= 2 \times 10^{-2}$ m,

$E = 10^5 \text{ N C}^{-1}, \tau = ?$

$\therefore \quad \tau = 10^{-6} \times 2 \times 10^{-2} \times 10^5 = 2 \times 10^{-3}$ N m

(iii) (d): When θ is 0 or 180°, the τ is minimum, which means the dipole moment should be parallel to the direction of the uniform electric field.

(iv) (c): Net force is zero and torque acts on the dipole, trying to align p with E.

(v) (a): Torque, $\tau = pE\sin\theta$ and potential energy, $U = -pE\cos\theta$

14. **(i) (a):** Gauss's law is applicable for any closed surface. Gauss's law is most useful in situation where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over the plane. Whereas electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.

So both statements are correct.

(ii) (b): According to Gauss's law, the electric flux through the sphere is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{8.85 \times 10^{-13} \text{C}}{8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}} = 0.1 \text{ N C}^{-1} \text{ m}^2$$

(iii) (c): For uniformly volume charge density,

$$E = \frac{\rho r}{3\varepsilon_0}$$

$E \propto r$

(iv) (a): $r = 25$ cm $= 0.25$ m, $\sigma = \frac{3}{\pi}$ C/m^2

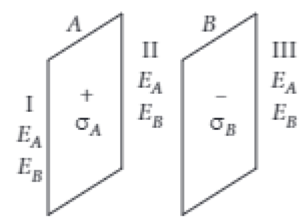
As, $\sigma = \frac{q}{4\pi r^2} \Rightarrow q = 4\pi \times (0.25)^2 \times \frac{3}{\pi} = 0.75$ C

(v) (b): The line charge density at a point on a line is the charge per unit length of the line at that point

$$\lambda = \frac{dq}{dL}$$

Thus, the SI unit for λ is C m^{-1}.

15. **(i) (d):** There are two plates A and B having surface charge densities,



$\sigma_A = 17.0 \times 10^{-22}$ C/m^2

on A and $\sigma_B = -17.0 \times 10^{-22}$ C/m^2 on B, respectively. According to Gauss' theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$E_I = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(ii) (d): The electric field in region III is also zero.

$$E_{III} = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(iii) (c) : In region II or between the plates , the electric field

$$E_{II} = E_A - E_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{\sigma(\sigma_A \text{ or } \sigma_B)}{\varepsilon_0} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$

$E = 1.9 \times 10^{-10}$ N C^{-1}

(iv) (c): Since, electric field due to an infinite-plane sheet of charge does not depend on the distance of observation point from the plane sheet of charge. So, for the given distances, the ratio of E will be 1 : 1.

(v) (b): In order to estimate the electric field due to a thin finite plane metal plate, we take a cylindrical cross-sectional area A and length $2r$ as the gaussian surface.