ICSE SEMESTER 2 EXAMINATION

SAMPLE PAPER - 3

MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hours

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any three** questions from **Section B.***

SECTION A

*(Attempt **all** questions from this Section.)*

Section-A (Attempt all questions)

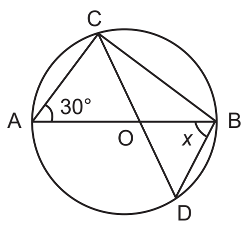
Question 1.

Choose the correct answers to the questions from the given options. (Do not copy the question, write the correct answer only.)

(i) The centroid of a triangle whose angular points are (4, – 3), (5, 6) and (3, – 3) respectively is:

(a) (0, 5) (b) (4, 0) (c) (3, – 3) (d) None of these

(ii) AB and CD be the diameters of a circle with centre O. If $\angle CAB = 30°$, find the value of x.



(a) 30° (b) 45° (c) 60° (d) 90°

(iii) If the base radius of two cylinders are in the ratio 4 : 5 and their heights are in the ratio 5 : 2, then the ratio of their volume is:

(a) 8 : 5 (b) 2 : 1 (c) 1 : 2 (d) 5 : 8

(iv) If the height of a tower and distance from the foot of the tower to a point is increased by 30%, then the elevation angle on the top of the tower:

(a) Increases (c) Remains unchanged

(b) Decreases (d) Can't say

(v) For a symmetrical distribution, which condition meets correctly:

(a) Mean = Mode < Median (c) Mean > Mode > Median

(b) Mean = Mode = Median (d) Mean = Median + Mode

(vi) Which of the following is not a form of probability?

(a) $\frac{2}{3}$ (b) $-\frac{1}{7}$ (c) 25% (d) 0.27

(vii) Determine the slope (m) and y-intercept (c) of the following line: $3y = \sqrt{3}x + 6$.

(a) $m = \sqrt{3}$, $c = \frac{1}{3}$

(b) $m = \frac{1}{\sqrt{3}}$, $c = \sqrt{3}$

(c) $m = \frac{1}{\sqrt{3}}$, $c = 2$

(d) $m = 1$, $c = -1$

(viii) The ratio of the base radii of two right circular cones of the same height is 2 : 1. Find the ratio of their volumes.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 5 (d) 4 : 1

(ix) Evaluate: $1 + \frac{\tan^2\theta}{1+\sec\theta}$:

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\sec\theta$ (d) $\tan\theta$

(x) Find the value of p, if:

Average = 42, $\Sigma fx = 16p + 40$ and $\Sigma f = 20$.

(a) 30 (b) 40 (c) 50 (d) 70

Section B (Attempt any three questions from this Section.)

Question 2.

(i) If sec A + tan A = p, show that : $\sin A = \frac{p^2-1}{p^2+1}$.

(ii) The coordinates of two point A and B are (– 3, 3) and (2, – 7) respectively. P is a point on the line segment AB such that AP : PB = 2 : 3. Find the coordinates of P without using square paper.

(iii) The table shows the distribution of the scores obtained by 120 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution.

Scores	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of shooters	5	9	16	22	26	18	11	6	4	3

Use your ogive to estimate the following:

(a) The median,

(b) The inter-quartile range,

(c) The number of shooters who obtained a score of more than 75%.

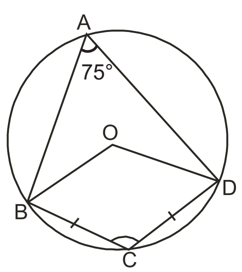
(iv) A game of numbers has cards marked with 11, 12, 13,........., 40. A card is drawn at random. Find the probability that the number on the card drawn is:

(a) A perfect square, (b) Divisible by 7.

Question 3.

(i) In the given figure, O is the centre of the circle, ∠BAD = 75° and chord BC = chord CD.

Find: (a) ∠BOC, (b) ∠OBD



(ii) Prove that : $\frac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$.

(iii) Find the equation of the line parallel to line $3x + 2y = 8$ and passing through the point (0, 1).

(iv) Draw a histogram from the following frequency distribution and find the mode.

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Frequency	2	5	18	14	8	5

Question 4.

(i) The triangle ABC where A (1, 2), B (4, 8), C (6, 8) is reflected in the x-axis to triangle A′B′C′. The triangle A′B′C′ is then reflected in the origin to triangle A″ B″C″. Write down the coordinates of A″B″C″. Write down a single transformation that maps ABC onto A″B″C″.

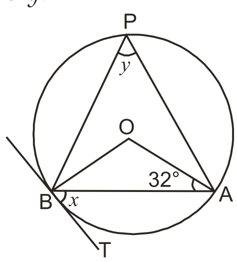
(ii) The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer, correct to one decimal place.

(iii) The marks obtained by a group of students in an examination are given below :

Marks	5	10	15	20	25	30
No. of students	6	4	6	12	x	4

Given that the mean of the group is 18, calculate the numerical value of x.

(iv) In the following figure, AB is a chord of the circle with centre O and BT is a tangent to the circle at B. If ∠OAB = 32°, find the values of x and y.

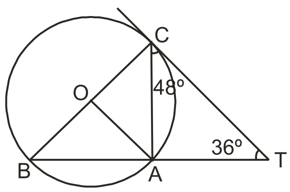


Question 5.

(i) Find the equation of a straight line which passes through the point (2, 3) and makes equal intercepts on the axes.

(ii) A box contains 7 blue, 8 white and 5 black marbles. If a marble is drawn at random from the box, what is the probability that it will be : (a) Black, (b) Blue or black, (c) Not black, (d) Green ?

(iii) A, B and C are three points on a circle. The tangent at C meets BA produced at T. Given that ∠ATC = 36° and ∠ACT = 48°. Calculate the angle subtended by AB at the centre of the circle.



(iv) An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching. Give your answer to the nearest m^2 .

Question 6.

(i) If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p.

(ii) The following table gives the wages of workers in a factory :

Wages in ₹	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
No. of workers	5	8	30	25	14	12	6

Calculate the mean by short cut method.

(iii) Prove : $\dfrac{1+\cos A}{1-\cos A} = (\text{cosec } A + \cot A)^2$.

(iv) A vessel in the shape of an inverted cone surmounted by a cylinder having common radius of 7 cm. It was filled with liquid till it covered 1/3rd height of the cylinder. If height of each part (conical and cylindrical) be 9 cm and the vessel is turned upside down, find the volume of the liquid and to what height will it reach in the cylindrical part.

Answers

Section-A

Answer 1.

(i) (b) (4, 0)

Explanation :

The co-ordinates of the centroid of a triangle are:

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{4+5+3}{3}, \frac{-3+6-3}{3}\right) \text{ i.e. } (4, 0).$$

(ii) (a) 30°

Explanation :

Since, angle in a semi-circle is a right angle.

$\therefore$ $\angle ACB = \angle CBD = 90°$

So, $\angle ABC = 90° - \angle CAB$

$\Rightarrow$ $\angle ABC = 90° - 30° = 60°$

Now, $\angle CBD = 90°$

$\angle ABC + \angle ABD = 90°$

$\Rightarrow$ $60° + x = 90°$

$\Rightarrow$ $x = 30°$.

(iii) (a) 8 : 5

Explanation :

$\because$ Volume of cylinder $= \pi r^2 h$

Here, $r_1 : r_2 = 4 : 5$ and $h_1 : h_2 = 5 : 2$

$\therefore$ $V_1 : V_2 = \pi r_1^2 h_1 : \pi r_2^2 h_2$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{4}{5}\right)^2 \times \frac{5}{2}$$

$$= \frac{16}{25} \times \frac{5}{2} = \frac{8}{5} \text{ or } 8:5.$$

(iv) (c) Remains unchanged

Explanation:

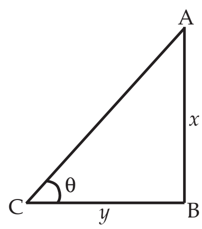
Let AB = x, BC = y

then, $\tan\theta = \frac{AB}{BC} = \frac{x}{y}$

When AB = x + 30% of x

$$= x + \frac{30}{100} \times x = \frac{130}{100}x$$

Similarly, $BC = \frac{130}{100}y$



Then, $\tan\theta = \dfrac{AB}{BC} = \dfrac{(130/100)x}{(130/100)y} = \dfrac{x}{y}$

(v) (b) Mean = Mode = Median.

(vi) (b) $-\dfrac{1}{7}$

Explanation :

$\because$ The probability lies between 0 and 1.

$\therefore$ The probability can not be negative.

(vii) (c) $m = \dfrac{1}{\sqrt{3}}, c = 2$

Explanation :

Equation of line, $3y = \sqrt{3}x + 6$

or $$y = \frac{\sqrt{3}x}{3} + \frac{6}{3}$$

$\Rightarrow$ $$y = \frac{1}{\sqrt{3}} + 2$$

Comparing with $y = mx + c$, we get

$$m = \frac{1}{\sqrt{3}} \text{ and } c = 2$$

(viii) (d) 4 : 1

Explanation :

Given, $r_1 : r_2 = 2 : 1$ and $h_1 = h_2 = h$

$\therefore$ Ratio of volumes $= V_1 : V_2$

$$= \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$= r_1^2 h : r_2^2 h \qquad (\because h_1 = h_2 = h)$$

$$= r_1^2 : r_2^2$$

$$= \left(\frac{r_1}{r_2}\right)^2 \text{ or } \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

$\therefore$ $V_1 : V_2 = 4 : 1$.

(ix) (c) $\sec\theta$

Explanation :

$$1 + \frac{\tan^2\theta}{1+\sec\theta} = 1 + \frac{\sec^2\theta - 1}{(1+\sec\theta)}$$

$$= 1 + \frac{(\sec\theta+1)(\sec\theta-1)}{(1+\sec\theta)}$$

$$= 1 + \sec\theta - 1$$

$$= \sec\theta$$

(x) (c) 50

Explanation :

$$\text{Average} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow \qquad 42 = \frac{16p+40}{20}$$

$$\Rightarrow \qquad 16p + 40 = 840$$

$$\Rightarrow \qquad 16p = 800$$

or $\qquad p = 50.$

Section B (Attempt any three questions from this Section.)

Answer 2.

(i) Given, $\qquad \sec A + \tan A = p$

Now,
$$\frac{p^2-1}{p^2+1} = \frac{(\sec A+\tan A)^2-1}{(\sec A+\tan A)^2+1}$$

$$= \frac{\sec^2 A+\tan^2 A+2\sec A\tan A-1}{\sec^2 A+\tan^2 A+2\sec A\tan A+1}$$

$$= \frac{(\tan^2 A+1)+\tan^2 A+2\sec A\tan A-1}{\sec^2 A+(\sec^2 A-1)+2\sec A\tan A+1} \qquad \left[\begin{array}{l}\because \sec^2 A=\tan^2 A+1 \\ \tan^2 A=\sec^2 A-1\end{array}\right]$$

$$= \frac{2\tan^2 A+2\sec A\,\tan A}{2\sec^2 A+2\sec A\,\tan A}$$

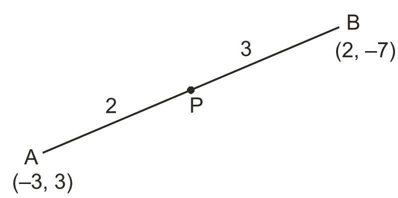
$$= \frac{2\tan A(\tan A+\sec A)}{2\sec A(\sec A+\tan A)}$$

$$= \frac{\tan A}{\sec A} = \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A}} = \sin A$$

Hence Proved.

(ii) Given, $\qquad AP : PB = 2 : 3$

$$A(x_1, y_1) = (-3, 3)$$
$$B(x_2, y_2) = (2, -7)$$



By section formula,

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

$$= \frac{2\times 2+3(-3)}{2+3} = \frac{4-9}{5} = \frac{-5}{5} = -1$$

$$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$= \frac{2(-7)+3(3)}{2+3} = \frac{-14+9}{5} = \frac{-5}{5} = -1$$

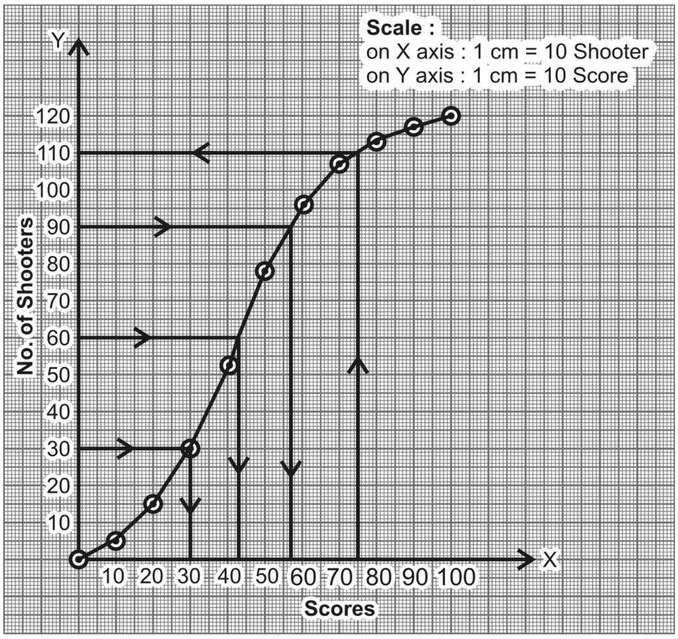
Hence, coordinates of the point P is $(-1, -1)$.

(iii)

Scores	No. of Shooters	Less than	No. of shooters
0 – 10	5	10	5
10 – 20	9	20	14
20 – 30	16	30	30
30 – 40	22	40	52
40 – 50	26	50	78
50 – 60	18	60	96
60 – 70	11	70	107
70 – 80	6	80	113
80 – 90	4	90	117
90 – 100	3	100	120

(a) $$\text{Median} = \frac{N}{2}\text{th term} = 60\text{th} = 43 \text{ (from ogive)}$$

(b)
$$\begin{aligned}\text{Interquartile range} &= Q_3 - Q_1 \\ &= \left(\frac{3N}{4}\right)\text{th term} - \left(\frac{N}{4}\right)\text{th term} \\ &= 90\text{th term} - 30\text{th term} \\ &= 57 - 30 \text{ (from ogive)} \\ &= 27\end{aligned}$$



(c) Number of shooters who score more than 75% *i.e.,* more than 75 score.

$$= 120 - 110 = 10$$

(iv) Sample space *i.e.,* total events = {11, 12, 13, ... 40}

$\therefore$ $$n(S) = 30$$

(a) Perfect square number = {16, 25, 36}

$\Rightarrow$ $$n(E) = 3$$

Probability that the number on the card is a perfect square

$$= \frac{\text{Favourable events } n(E)}{\text{Total no. of possible events } n(S)}$$

$$= \frac{3}{30} = \frac{1}{10}$$

(b) Card number divisible by 7= {14, 21, 28, 35}

$\Rightarrow$ $n(E) = 4$

Probability that the number on the card is divisibile by 7

$$= \frac{\text{Favourable events } n(E)}{\text{Total no. of possible events } n(S)}$$

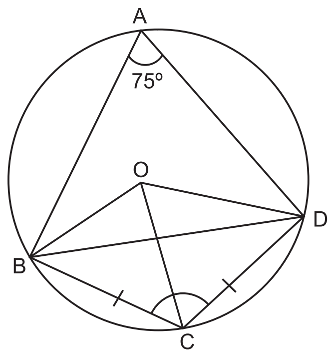
$$= \frac{4}{30} = \frac{2}{15}$$

Answer 3.

(i) Given, $\angle BAD = 75°$

(a) $\therefore$ $\angle BOD = 2\angle BAD$

[Angle subtended by a chord at centre is twice the angle subtended by the chord at circumference of same arc]



$\therefore$ $\angle BOD = 2 \times 75° = 150°$

$\therefore$ $\angle BOC = \frac{1}{2}\angle BOD = 75°$

[$\because$ $\Delta BOC \cong \Delta COD$]

(b) $OB = OD$ [Radius of circle]

$\therefore$ $\angle OBD = \angle ODB$

In ΔOBD,

$\angle OBD + \angle ODB + \angle BOD = 180°$

$\Rightarrow$ $2\angle OBD + 150° = 180°$

$\Rightarrow$ $2\angle OBD = 180° - 150°$

$\Rightarrow$ $\angle OBD = \frac{30º}{2} = 15º$

$\Rightarrow$ $\angle OBD = 15°$

(ii) $$\text{L.H.S.} = \frac{1 - \sin A}{1 + \sin A}$$

$$= \frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$ [$\because$ $1 - \sin^2 A = \cos^2 A$]

$$= \left(\frac{1 - \sin A}{\cos A}\right)^2$$

$$= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$

$= (\sec A - \tan A)^2$

= R.H.S. **Hence Proved.**

(iii) Given equation of line is, $3x + 2y = 8$

$$\text{Slope of the given line} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$$

$$= -\frac{3}{2}$$

$\therefore$ Slope of parallel line $(m) = -\frac{3}{2}$

Equation of the line passing through (x_1, y_1) having slope m is given as

$$y - y_1 = m\,(x - x_1)$$

$\Rightarrow$ $$y - 1 = -\frac{3}{2}(x - 0)$$

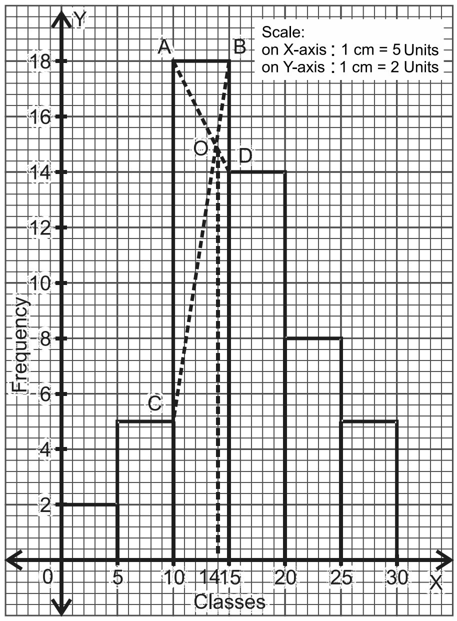
$\Rightarrow$ $$2y - 2 = -3x$$

$\Rightarrow$ $$3x + 2y - 2 = 0$$

$\therefore$ Equation of line parallel to the given line is $3x + 2y - 2 = 0$.

(iv)

Class	Frequency
0 – 5	2
5 – 10	5
10 – 15	18
15 – 20	14
20 – 25	8
25 – 30	5



$\therefore$ Mode = 14 (from histogram)

Answer 4.

(i) Given, vertices of ΔABC are A (1, 2), B (4, 8) and C (6, 8).

Now, Reflection in the x-axis Vertex A (1, 2) is A′ (1, – 2)
Vertex B (4, 8) is B′ (4, – 8)
Vertex C (6, 8) is C′ (6, – 8)

Reflection in the origin Vertex A′ (1, – 2) is A″ (– 1, 2)
Vertex B′ (4, – 8) is B″ (– 4, 8)
Vertex C′(6, – 8) is C″ (– 6, 8)

Transformation of the Vertex A (1, 2) is A″ (– 1, 2)
Vertex B (4, 8) is B″ (– 4, 8)
Vertex C (6, 8) is C″ (– 6, 8)

So, the single transformation that maps ABC onto A″B″C″ is y-axis.

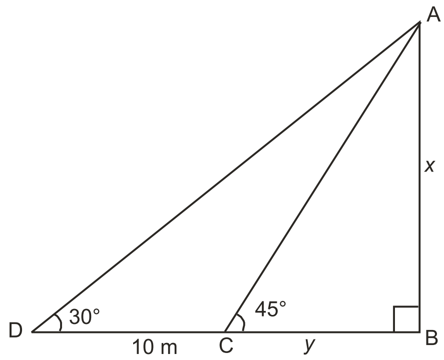
(ii) Let, AB be the height of the tower and CD be the increase in length of the shadow.

$\therefore$ CD = 10 m (Given)

Let, AB = x m and BC = y m

In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$



$$\Rightarrow \quad 1 = \frac{x}{y}$$

$$\Rightarrow \quad y = x \quad ...(i)$$

In ΔABD, $\tan 30° = \frac{AB}{BD}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{10+y}$$

$$\Rightarrow \quad \sqrt{3}x = 10 + y \quad ...(ii)$$

$\therefore$ From equations (i) and (ii), we have

$$\sqrt{3}x = 10 + x$$

$$\Rightarrow \quad \sqrt{3}x - x = 10$$

$$\Rightarrow \quad x(\sqrt{3}-1) = 10$$

$$\Rightarrow \quad x = \frac{10}{\sqrt{3}-1}$$

$$\Rightarrow \quad x = \frac{10}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$\Rightarrow \quad x = \frac{10 \times (1.732+1)}{3-1}$$

$$= \frac{10 \times 2.732}{2}$$

$$= 5 \times 2.732 = 13.66$$

$\therefore$ The required height 13.7 m (correct to 1 decimal place).

(iii)

Marks (x)	No. of Students (f)	$f.x$
5	6	30
10	4	40
15	6	90
20	12	240
25	x	$25x$
30	4	120
	$\Sigma f = 32 + x$	$\Sigma fx = 520 + 25x$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow \quad 18 = \frac{520+25x}{32+x}$$

$$\Rightarrow \quad 18(32+x) = 520 + 25x$$

$$\Rightarrow \quad 576 + 18x = 520 + 25x$$

$$\Rightarrow \quad 25x - 18x = 576 - 520$$

$$\Rightarrow \quad 7x = 56$$

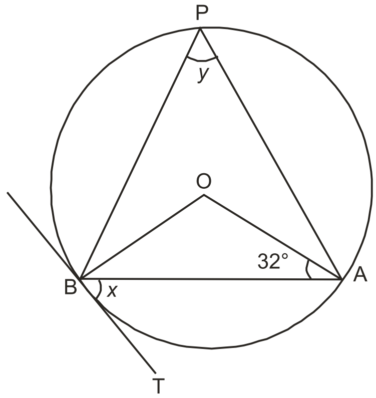
$$\Rightarrow \quad x = \frac{56}{7} = 8$$

$$\therefore \quad x = 8.$$

(iv) Given, $\angle OAB = 32°$

$\angle ABT = \angle BPA$ [Alternate segment angle]

$\Rightarrow \quad x = y$ ...(i)



Also, $OB = OA$ [Radius of circle]

$\therefore \quad \angle OBA = \angle OAB$

$\Rightarrow \quad \angle OBA = 32°$ ...(ii)

$\because \quad OB \perp BT$ [Radius $\perp$ tangent]

$\therefore \quad \angle OBA + \angle ABT = 90°$

$\Rightarrow \quad 32° + x = 90°$

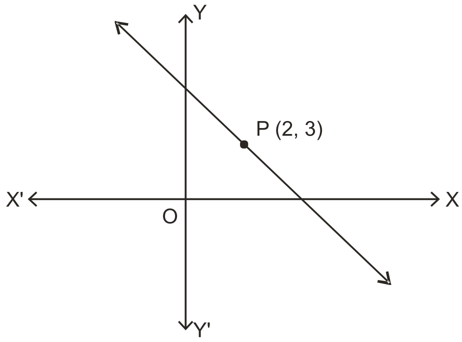
$\Rightarrow \quad x = 90° - 32 = 58°$

$\therefore \quad x = y = 58°$

Answer 5.

(i) Equation of the line which makes intercept a unit and b unit on X-axis and Y-axis, respectively is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad ...(i)$$



Given, line makes equal intercepts on both axes *i.e.*,

$$a = b$$

$\therefore \quad \frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad x + y = a$

Line passes through the point P (2, 3)

$\therefore \quad 2 + 3 = a$

$\Rightarrow \quad a = 5$

Hence, equation of line is $x + y = 5$.

(ii) Total number of marbles = 7 + 8 + 5 = 20.

$\therefore \quad n(S) = 20.$

(a) Let, A be the event of drawing a black marble.

$\therefore \quad n(A) = 5$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{5}{20} = \frac{1}{4}.$

(b) Let, B be the event of drawing blue or black marble.

$\therefore \quad n(B) = 7 + 5 = 12$

$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{12}{20} = \frac{3}{5}.$

(c) Let, C be the event of drawing a marble which is not black.

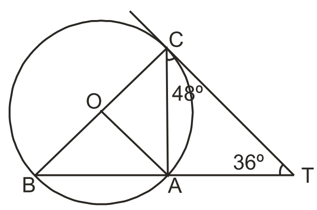
$\therefore \quad P(C) = 1 - P(A) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}.$

(d) Let, D be the event of drawing a green marble.

$\therefore \quad n(D) = 0$ [As there is no green marble]

$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{0}{20} = 0.$

(iii) Given, $\angle ATC = 36°$ and $\angle ACT = 48°$.



$\therefore \quad \angle OCT = 90° \; (\because CT \perp OC)$

$\Rightarrow \quad \angle OCA + \angle ACT = 90°$

$\Rightarrow \quad \angle OCA + 48° = 90°$

$\Rightarrow \quad \angle OCA = 90° - 48°$

$= 42°.$

$\therefore \quad \angle BCA = 42°$

Now, $\angle AOB = 2\,\angle BCA$ [Angle subtended by chord at the centre is twice the angle subtended by the same chord at the circumference.]

$= 2 \times 42°$

$= 84°.$

(iv) Given, diameter of cylinder $= 168$ m

$$\text{Radius } (r) = \frac{168}{2}$$

$= 84$ m

Height of cylindrical part $(h_2) = 50$ m

$\therefore$ Height of conical part $(h_1) = 85 - 50$

$= 35$ m

$\therefore$ Slant height of conical part $(l) = \sqrt{r^2 + h_1^2}$

$$= \sqrt{84^2 + 35^2}$$

$$= \sqrt{7056 + 1225}$$

$$= \sqrt{8281}$$

$= 91$ m.

$\therefore$ Area of canvas required for tent = Curved surface areas of conical part and cylindrical part.

$= \pi r l + 2\pi r h_2$

$$= \frac{22}{7} \times 84 \times 91 + 2 \times \frac{22}{7} \times 84 \times 50$$

$$= \frac{22}{7} \times 84(91 + 100)$$

$= 22 \times 12 \times 191$

$= 50424 \text{ m}^2$

$\therefore$ Area of extra canvas required for stitching and folding

$$= 20\% \text{ of } 50424 = \frac{20}{100} \times 50424$$

$= 10084.8 \text{ m}^2$

$\therefore$ Total area of canvas required $= 50424 + 10084.8$

$= 60508.8$

$= 60509 \text{ m}^2$ (in nearest m^2)

Answer 6.

(i) Given, lines are $y = 3x + 7$ ...(i)

and $2y + px = 3$ ...(ii)

$\therefore$ From equation (i),

Slope $(m_1) = 3$ [Compare by $y = mx + c$]

From equation (ii), $2y = -px + 3$

$$\Rightarrow \quad y = -\frac{p}{2}x + \frac{3}{2}$$

$$\therefore \quad \text{Slope } (m_2) = -\frac{p}{2}$$

$\because$ Lines are perpendicular to each other,

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad 3 \times \left(-\frac{p}{2}\right) = -1$$

$$\Rightarrow \quad -p = -1 \times \frac{2}{3}$$

$$\Rightarrow \quad p = \frac{2}{3}$$

(ii)

Wages (₹)	**No. of workers (*f*)**	**Mid. value (*x*)**	***d* = *x* – A**	***fd***
45 – 50	5	47.5	– 15	– 75
50 – 55	8	52.5	– 10	–80
55 – 60	30	57.5	– 5	–150
60 – 65	25	62.5 = A	0	0
65 – 70	14	67.5	5	70
70 – 75	12	72.5	10	120
75 – 80	6	77.5	15	90
	$\Sigma f = 100$			$\Sigma fd = -25$

Let, A = 62.5

$$\therefore \quad \text{Mean} = A + \frac{\Sigma fd}{\Sigma f} = 62.5 + \frac{(-25)}{100}$$

$$= 62.5 - 0.25$$

$$= 62.25$$

(iii) To prove : $\dfrac{1+\cos A}{1-\cos A} = (\operatorname{cosec} A + \cot A)^2$

$$\text{L.H.S.} = \frac{1+\cos A}{1-\cos A}$$

$$= \frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$$

$$= \frac{(1+\cos A)^2}{1-\cos^2 A}$$

$$= \frac{(1+\cos A)^2}{\sin^2 A} \qquad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \left(\frac{1+\cos A}{\sin A}\right)^2$$

$$= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right)^2 \qquad \left[\because \cot A = \frac{\cos A}{\sin A}\right]$$

$$= (\operatorname{cosec} A + \cot A)^2$$

$$= \text{R.H.S.}$$

Hence Proved.

(iv) Given,

Base radius of cone (r) = Base radius of cylinder = 7 cm

Height of cone (h) = Height of cylinder = 9 cm

Volume of water = Volume of cone + Volume of cylinder upto $\left(\frac{1}{3}\right)^{\text{rd}}$ height

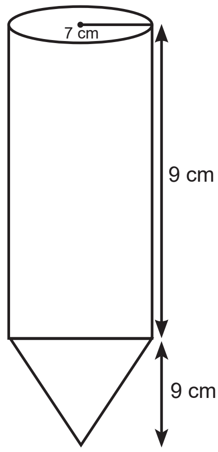
$$= \frac{1}{3}\pi r^2 h + \pi r^2 \frac{h}{3}$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 9$$

$$= 2 \times 22 \times 7 \times 3$$

$$= 924 \text{ cm}^3$$



Let, water level in cylindrical part be x cm.

$$\text{Volume of cylinder} = 924 \text{ cm}^3$$

$$\pi r^2 x = 924$$

$\Rightarrow$ $$\frac{22}{7} \times 7 \times 7 \times x = 924$$

$\Rightarrow$ $$x = \frac{924 \times 1}{22 \times 7}$$

$\Rightarrow$ $$x = 6 \text{ cm}$$

$\therefore$ Water level in cylinder = 6 cm.

❑❑