Heron's Formula

Heron's formula for area of a triangle

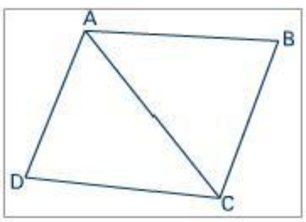
Let a, b, c denotes the lengths of the sides of a triangle. Let 's' denote the semi-perimeter i.e., $s = \frac{a+b+c}{2}$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

This formula is also known as Heron's formula.

Area of Quadrilaterals using Heron's formula

We can divide a quadrilateral into two triangles and find the area of the quadrilateral as the sum of the areas of these two triangles.



Surface Areas and Volumes

Solids are three dimensional and they occupy some space which is called their volume. We also measure their surface areas. Examples of solids are cuboid, cylinder, cone and sphere. In day to day life we see water tanks which are sometimes cylindrical in shape. Sometimes they have the shape of a cuboid. Their volumes give us their capacity.

A cylinder is formed by revolving a rectangle about its axis. It is bound by a curved surface and two flat surfaces which are circles when the curved surface is opened out, it has the shape of a rectangle.

By hollow cylinder, we mean a tube.

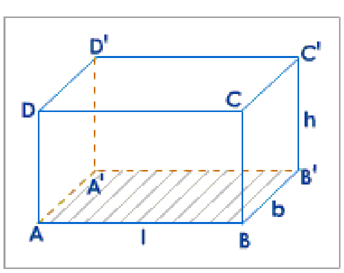
A cone is formed by revolving a right angled triangle about one of its sides (other than the hypotenuse). It is bound by a curved surface and a flat surface which is a circle.

When the curved surface is opened out, it has the shape of the sector of a circle.

A sphere is a solid obtained by revolving a circle about any of its diameters. A spherical shell is the solid enclosed between two concentric spheres.

Cuboid

A solid all of whose faces are rectangles is called a Cuboid or a rectangular solid.

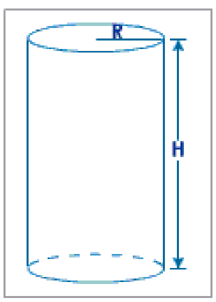


It has six faces as the figure above shows.
Area of 4 side faces = $2 \times l \times h + 2 \times b \times h$
$= 2 (l + b) \times h$
= perimeter of base $\times$height

Cube
A cuboid having length, breadth and height all equal is called a cube. Each face is a square.
Area of 4 side faces = $4a^2$
Total surface area = $6a^2$
Volume of a cube = $(\text{length})^3 = a^3$
(where a = length of side)
$V = l^3$
Longest diagonal of a cube
$= \sqrt{l^2 + b^2 + h^2}$
$= l\sqrt{3}$, as $l = b = h$.

Uniform Cross-Section
If two opposite faces of a regular solid have the same shape and the same size, the solid is said to be of uniform cross-section.

Cylinder
The base and top of a cylinder are in the shape of circles. The line joining the centers is perpendicular to the planes of the circles. The length of this line is called the height of the cylinder.



The radius of each circle is called the radius of the cylinder. We also call such a cylinder as a right circular cylinder.
(1) Volume = area of base $\times$ height
$V = \pi R^2 H$
(2) Area of curved surface
= perimeter of base $\times$ height
Curved surface area
$= 2\pi RH$
(3) Total surface area of solid cylinder

= Area of two circles + curved surface area

$= 2\pi R^2 + 2\pi RH$

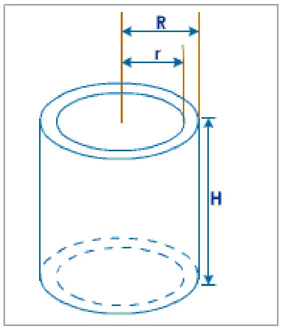
Total surface area = $2\pi R (R + H)$

(4) Volume of hollow cylinder (tube),

Let outer radius = R

Inner radius = r

height = H



∴ Volume of hollow cylinder = Area of ring × height

Volume of tube = $\pi (R^2 - r^2) \times H$

(5) Total surface area of a hollow cylinder

= Outer curved surface area + inner curved surface area + 2 rings

$= 2\pi RH + 2\pi rH + 2\pi (R^2 - r^2)$

Total surface area of a tube

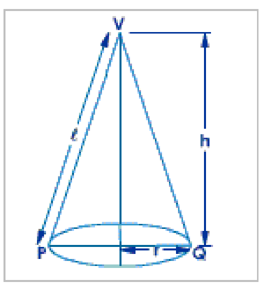
$= 2\pi [RH + rH + (R^2 - r^2)$

Cone

The vertex V of a right circular cone is equidistant from all points in the circumference of the circular base. The perpendicular from V meets the base at the center of the circle.

Let us draw sector VPQ on a paper and cut it out. A right circular cone is formed when radii VP and VQ are brought together.

The radius of the sector (made into a cone) is called the slant height (l).



(1) Volume = $\frac{1}{3}$ Volume of a cylinder.

$$V = \frac{1}{3}\pi r^2 h$$

(2) Curved surface area $= \pi r l$

where l = slant height $\left[l = \sqrt{h^2 + r^2}\right]$

(3) Total surface area

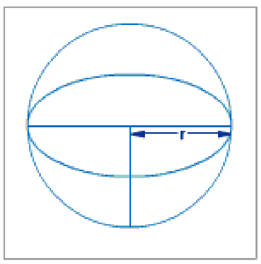
= Area of circle + Curved surface area

$= \pi r^2 + \pi r l$

Total surface area $= \pi r (r + l)$

Sphere

A sphere has been defined earlier as the set of points in space that are equidistant from a fixed point called the center. If a plane cuts a sphere through the center, the circle so obtained is called the great circle. In our study, we deal with the great circle of the sphere. We may also obtain the small circle if a plane cuts the sphere and it does not contain the center of the sphere.

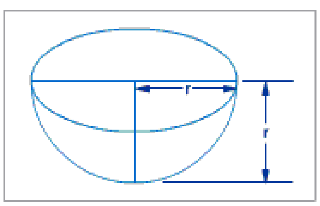


(1) Volume $= \frac{4}{3} \pi r^3$

(2) Total surface area = 4 times the area of a circle

Total surface area $= 4\pi r^2$

(3) Total surface area of a hemisphere



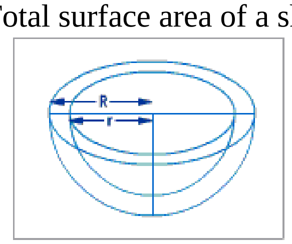
= Area of $\frac{1}{2}$sphere + Area of circle

$= \frac{1}{2} \times 4 \pi r^2 + \pi r^2$

$= 2 \pi r^2 + \pi r^2$

Total surface area of hemisphere $= 3\pi R^2$

(4) Total surface area of a shell,



let outer radius = R,
inner radius = r

$= \frac{1}{2} \times$ Outer surface area + $\frac{1}{2} \times$ Inner surface area + Area of the ring

$$= \frac{1}{2} \times 4\pi R^2 + \frac{1}{2} \times 4\pi r^2 + \pi(R^2 - r^2)$$

$$= 2\pi R^2 + 2\pi r^2 + \pi R^2 - \pi r^2$$

$$= 3\pi R^2 + \pi r^2$$

Total surface area of shell = $\pi(3R^2 + r^2)$