ISC 2025 EXAMINATION

Sample Question Paper - 3

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is: **[1]**

a) 6 b) -6

c) -7 d) 0

(b) $\int (3 - 5x)^7 \, dx = ?$ **[1]**

a) $\frac{-5(3-5x)^8}{8} + C$ b) $5(3 - 5x)^6 + C$

c) $4(3 + 5x)^6 + C$ d) $\frac{(3-5x)^8}{-40} + C$

(c) If $\cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2} = \frac{\theta}{2}$, then $4x^2 - 12xy \cos\frac{\theta}{2} + 9y^2 =$ **[1]**

a) $18 - 18\cos\theta$ b) $18 + 18\cos\theta$

c) $36 - 36\cos\theta$ d) 36

(d) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is **[1]**

a) Not defined b) 3

c) 1 d) 2

(e) Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Then **[1]**

$P(A|B) \cdot P(A'/B)$ is equal to

a) $\frac{2}{5}$
b) $\frac{6}{25}$
c) $\frac{3}{10}$
d) $\frac{3}{8}$

(f) The relation R in the set of natural numbers N defined as R = {(x, y) : x > y} is **[1]**

a) reflexive, transitive but not symmetric
b) transitive but neither reflexive nor symmetric
c) reflexive, symmetric but not transitive
d) an equivalence relation

(g) $\underset{x \to 0}{Lt} \frac{x(e^{\sin x}-1)}{1-\cos x}$ is equal to **[1]**

a) 2
b) 1
c) $\frac{1}{2}$
d) 0

(h) If y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) 1
b) $\frac{1}{2}$
c) -1
d) $\frac{-1}{2}$

(i) Which of the following is correct **[1]**

a) Determinant is a square matrix
b) Determinant is a number not associated to a square matrix.
c) Determinant is a number associated to a square matrix.
d) Determinant is a number associated to a matrix.

(j) **Assertion (A):** Scalar matrix A = $[a_{ij}] = \begin{cases} k, & i = j \\ 0, & i \neq j \end{cases}$, where k is a scalar, is an identity matrix when k = 1. **[1]**

Reason (R): Every identity matrix is not a scalar matrix.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

(k) Let f, g be two real functions defined by f(x) $= \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$. Then, describe the function f / g. **[1]**

(l) Express A = $\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix. **[1]**

(m) State with reason whether the function has inverse: **[1]**

h : {2, 3, 4, 5}$\rightarrow$ {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4,11), (5, 13)}

(n) A die is thrown. If E is the event **the number appearing is a multiple of 3** and F be the event **the number appearing is even** then find whether E and F are independent? **[1]**

(o) The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi? **[1]**

2. If $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$. **[2]**

OR

If the tangent to the curve y = x^3 + ax+ b at (1,-6) is parallel to the line x - y + 5 = 0, find a and b.

3. Evaluate: $\int \frac{2x+1}{(x+2)(x-3)^2} dx$ **[2]**

4. Find the absolute maximum value and the absolute minimum value of the function: **[2]**

f(x) = x^3, x ∈ [-2, 2]

5. Evaluate: $\int \sec^4 x \, dx$ **[2]**

OR

Evaluate the integral: $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$

6. Let A = [–1, 1]. Then, discuss whether the function defined on A by: g(x) = |x| **[2]**

is one-one, onto or bijective.

7. Find the value of the following: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ **[4]**

8. Evaluate the integral: $\int \frac{\log(1-x)}{x^2} dx$ **[4]**

9. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a\left(x^3 - y^3\right)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$, where -1 x < 1 and -1 < y < 1. **[4]**

OR

If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$

10. **Read the text carefully and answer the questions:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) Find the probability that she gets grade A in all subjects.

(b) Find the probability that she gets grade A in no subjects.

(c) Find the probability that she gets grade A in two subjects.

(d) Find the probability that she gets grade A in at least one subject.

OR

Read the text carefully and answer the questions: **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

(c) A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

(d) If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

11. **Read the text carefully and answer the questions:** **[6]**

In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories type I, type II and type III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(a) Write the matrix P, if P represents the matrix of number of units of each type produced by factory A for both boys and girls.

(b) Write the matrix Q, if Q represents the matrix of number of units of each type produced by factory B for both boys are girls.

(c) Find the total production of sports clothes of each type for boys.

12. Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right)$, given that y = 1 when x = 0. **[6]**

OR

Solve $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} xdy$

13. Find the local maxima and local minima of the function. Find also the local maximum and the local minimum value: f(x) = sinx - cosx, 0 < x < 2π. **[6]**

OR

Find the point on the curve $x^2 = 8y$ which is nearest to the point (2, 4).

14. **Read the text carefully and answer the questions:** **[6]**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(a) Find the probability that daughter is at one end, given that father and mother are in the middle.

(b) Find the probability that mother is at right end, given that son and daughter are together.

(c) Find the probability that father and mother are in the middle, given that son is at right end.

(d) Find the probability that father and son are standing together, given that mother and daughter are standing together.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is **[1]**

a) $\frac{1}{3}$ b) $\frac{1}{9}$

c) 9 d) 3

(b) Find the angle between the lines whose direction ratios are proportional to 4, -3,5 and 3,4,5. **[1]**

(c) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle. **[1]**

(d) The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$ **[1]**

The direction cosines of the line is

a) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ b) $\frac{3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

c) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ d) $\frac{-3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

(e) The equation of a line is 5x-3= 15y+7=3-10z. Write the direction cosines of the line. **[1]**

16. Find the altitude of a parallelopiped whose adjacent sides are determined by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, if the base is taken as the parallelogram determined by $\vec{a}$ and $\vec{b}$, and if $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$. **[2]**

OR

Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{c}.\vec{a}$ if $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$

17. Find the direction cosines of the perpendicular from the origin to the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 3 = 0$. **[4]**

OR

Find the angles between the lines $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - 5\hat{k}\right) + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right)$

18. Let f(x)= $Max\{x^2, (1-x)^2, 2x(1-x)\}, where\ 0 \leq x \leq 1$. **[4]**

Determine the area of the region bounded by the curves y=f(x), X-axis, x=0 and x=1.

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The cost function of a firm is given by C = $2x^2$ + x - 4. Find the average cost. **[1]**

a) $2x^2 + 4$ b) 2x + 3

c) $2x + 1 - \frac{4}{x}$ d) $2x - 1 - \frac{5}{x}$

(b) Region represented by x $\geq$ 0, y $\geq$ 0 is: **[1]**

a) forth quadrant b) second quadrant

c) first quadrant d) third quadran

(c) For 5 observations of pairs (x, y) of variables X and Y, the following results are obtained: **[1]**

Σx = 15, Σy = 25, Σx^2 = 55, Σy^2 = 135, Σxy = 83

Calculate the value of b_{xy} and b_{yx}.

(d) A company finds that total revenue from sale of q units of its product is 2q rupees whereas the cost is $500 + \frac{1}{2}\left(\frac{q}{20}\right)^2$. Find the rate of change of profit when q = 500. **[1]**

(e) The fixed cost of a new product is ₹ 35000 and the variable cost per unit is ₹500. If the demand function is p = 5000 - 100x, find the breakeven value(s). **[1]**

20. For manufacturing a certain item, the fixed cost is ₹6000 and the cost of producing each unit is ₹20. **[2]**

i. What is the cost function?
ii. What is the total cost and average cost of producing 15 units?
iii. What is the total cost and average cost of producing 100 units?

OR

The demand function for a monopolist is given by x = 100 - 4p. Find:

i. total revenue function
ii. average revenue function
iii. marginal revenue function
iv. price and quantity at which MR = 0

21. By using the data $\bar{x}$ = 25, $\bar{y}$ = 30, b_{yx} = 1.6 and b_{xy} = 0.4, find: **[4]**

i. The regression equation y on x.
ii. What is the most likely value of y when x = 60?
iii. What is the coefficient of correlation between x and y?

22. Maximize Z = $3x_1 + 4x_2$, if possible, **[4]**

Subject to the constraints

$x_1 - x_2 \leq -1$

$-x_1 + x_2 \leq 0$

$x_1, x_2 \geq 0$

OR

Solve the Linear Programming Problem graphically:

Maximize Z = 15x + 10y

Subject to

$3x + 2y \leq 80$

$2x + 3y \leq 70$

$x, y \geq 0$

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) **(b)** -6

Explanation: {

-6

(ii) **(d)** $\frac{(3-5x)^8}{-40}+C$

Explanation: {

Given = $\int$(3-5 x)^7

Let, 3 – 5x = z

⇒ -5dx = dz

So,

$\int$(3-5 x)^7 dx

$=-\int \frac{z^7}{5} dz$

$=-\frac{1}{5}\frac{z^8}{8}+c$ where c is the integrating constant.

$=-\frac{z^8}{40}+c$

$=-\frac{(3-5x)^8}{40}+c$

(iii) **(a)** 18 - 18 cos θ

Explanation: {

$\cos^{-1}$ x + $\cos^{-1}$ y = $\cos^{-1}$ $(xy-\sqrt{1-x^2}\sqrt{1-y^2})$

$\Rightarrow \cos^{-1}\frac{x}{3}+\cos^{-1}\frac{y}{2}=\frac{\theta}{2}$

$\Rightarrow \cos^{-1}\left(\frac{x}{3}\times\frac{y}{2}-\sqrt{1-\left(\frac{x}{3}\right)^2}\sqrt{1-\left(\frac{y}{2}\right)^2}\right)=\frac{6}{2}$

$\Rightarrow \frac{xy}{6}-\sqrt{1-\left(\frac{x^2}{9}\right)}\sqrt{1-\left(\frac{y^2}{4}\right)}=\cos\frac{\theta}{2}$

$\Rightarrow \frac{xy-6\cos\frac{\theta}{2}}{6}=\frac{\sqrt{9-x^2}\sqrt{4-y^2}}{6}$

$\Rightarrow xy-6\cos\frac{\theta}{2}=\sqrt{9-x^2}\sqrt{4-y^2}$

Taking square on both sides,

⇒ x^2y^2 - 12xy cos $\frac{\theta}{2}$ + 369 $\cos^2$ $\frac{\theta}{2}$ = (9 - x^2) (4 - y^2)

⇒ x^2y^2 - 12xy cos $\frac{\theta}{2}$ + 36 $\cos^2$ $\frac{\theta}{2}$ 36 - 9y^2 - 4x^2 + x^2y^2

⇒ 4x^2 + 9y^2 - 12xy cos ^{2} $\frac{\theta}{2}$ = 36 $\left(1-\cos^2\frac{\theta}{2}\right)$

⇒ 4x^2 + 9y^2 - 12xy $\cos^2$ $\frac{\theta}{2}$ = 36 $\left(1-\frac{1+\cos\theta}{2}\right)$

⇒ 4x^2 + 9y^2 - 12xy $\cos^2$ $\frac{\theta}{2}$ = 18 - 18 cos θ

(iv) **(a)** Not defined

Explanation: {

It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

The given differential equation is not a polynomial equation in its derivative

Therefore, its degree is not defined.

(v) **(b)** $\frac{6}{25}$

Explanation: {

Here, P(A) = $\frac{3}{8}$, P(B) = $\frac{5}{8}$ and $P(A\cup B)=\frac{3}{4}$

∵ P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$\Rightarrow P(A\cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{3+5-6}{8}=\frac{2}{8}=\frac{1}{4}$

$\because P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{1/4}{5/8}=\frac{8}{20}=\frac{2}{5}$

And $P\left(A'/B\right) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B)-P(A\cap B)}{P(B)}$

$= \frac{\frac{5}{8}-\frac{1}{4}}{\frac{5}{8}} = \frac{\frac{5-2}{8}}{\frac{5}{8}} = \frac{3}{5}$

$\therefore P(\frac{A}{B})\cdot P\left(\frac{A'}{B}\right) = \frac{2}{5}\cdot\frac{3}{5} = \frac{6}{25}$

(vi) **(b)** transitive but neither reflexive nor symmetric

Explanation: {

Since, x is greater than y ∀ x, y ∈ N

Let (x, x) ∈ R

For x R x, x > x is not true for any x ∈ N.

Therefore, R is not reflexive.

Let (x, y) ∈ R ⇒ xRy

x > y

but y > x is not true for any x, y ∈ N

Thus, R is not symmetric.

Let xRy and yRz

x > y and y > z

⇒ x > z

⇒ xRz

So, R is transitive.

(vii) **(a)** 2

Explanation: {

$\lim\limits_{x\to0}\frac{x(e^{\sin x}-1)}{1-\cos x}$

$= \lim\limits_{x\to0}\frac{\frac{(e^{\sin x}-1)}{x}}{\frac{1-\cos x}{x^2}} = \lim\limits_{x\to0}\frac{(e^{\sin x}-1)}{\sin x}.\frac{\sin x}{x}.2 = 2\left(\because \lim\limits_{x\to0}\frac{1-\cos x}{x^2} = \frac{1}{2}\right)$

(viii) **(b)** $\frac{1}{2}$

Explanation: {

Given that y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using 1 - cos x = 2sin^2 $\frac{x}{2}$ and Using sin x = 2 sin x $\frac{x}{2}$cos $\frac{x}{2}$, we obtain

y = $\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or y = $\tan^{-1}\tan\frac{x}{2}$

$y = \frac{x}{2}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{1}{2}$

(ix) **(c)** Determinant is a number associated to a square matrix.

Explanation: {

The determinant is an operation that we perform on arranged numbers. A square matrix is a set of arranged numbers. We perform some operations on a matrix and we get a value that value is called as a determinant of that matrix hence a determinant is a number associated to a square matrix.

(x) **(c)** A is true but R is false.

Explanation: {

A scalar matrix A = $[a_{ij}] = \begin{cases} k; & i=j \\ 0; & i\neq j \end{cases}$ is an identity matrix when k = 1.

But every identity matrix is clearly a scalar matrix.

(xi) We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\Rightarrow \left(\frac{\mathrm{f}}{\mathrm{g}}\right)(\mathrm{x}) = \frac{\sqrt{\mathrm{x}+1}}{\sqrt{9-\mathrm{x}^2}}$

$\therefore \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x+1}{9-x^2}}$

as earlier, domain of $\frac{f}{g} = [-1, 3]$

However, f/g (x) is defined for all real values of $x \in$ (1-3) except for the case when 9 -x^2 = 0 or x = + 3

When $x = \pm 3$, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeteminate

=> domain of $\frac{f}{g} = [-1, 3] - \{-3, 3\}$

=> domain of f/g = [-1,3)

Thus, $\frac{f}{g} : [-1, 3) \rightarrow R$ is given by $\left(\frac{f}{g}\right)(x) = \sqrt{\frac{x+1}{9-x^2}}$

(xii) A = $\begin{bmatrix} 4 - 3 \\ 2 - 1 \end{bmatrix} \Rightarrow$ A^T = $\begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$

P = $\frac{A+A^T}{2}$ = $\frac{1}{2}\begin{bmatrix} 8 - 1 \\ -1 - 2 \end{bmatrix}$

Q = $\frac{A-A^T}{2}$ = $\frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Now, A = P + Q

P + Q = $\frac{1}{2}\begin{bmatrix} 8 - 6 \\ 4 - 2 \end{bmatrix} = \begin{bmatrix} 4 - 3 \\ 2 - 1 \end{bmatrix}$ = A

(xiii) : {2, 3, 4, 5) $\rightarrow$ {7, 9, 11, 13} given by

h {(2, 7), (3, 9), (4, 11), (5, 13)}

we can observe that different element of domain have different image in co-domain.

Thus, h is a bijection and is invertible.

(xiv) Two event A and B are independent if $P(A \cap B) = P(A). P(B)$

Sample space of the experiment is, S = {1, 2, 3, 4, 5, 6}

Now E = {3, 6}, F = { 2, 4, 6} and E ∩ F = {6}

Then P(E) = $\frac{2}{6} = \frac{1}{3}$, P(F) = $\frac{3}{6} = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{6}$

Clearly P(E ∩ F) = P(E). P(F) $= \frac{1}{6}$

Hence E and F are independent events.

(xv) One student is selected at random.

Let P(A) be the probability of students passing in English

Let P(B) be the probability of students passing in Hindi.Therefore,we have,

P(B) = $\frac{4}{5}$

Let P(A ∩ B) be the probability of students passing in both English and Hindi.Therefore,we have,

P(A ∩ B) = $\frac{1}{2}$

The probability that he will pass in English given that he passes in Hindi is given by,

P(A/B)

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/2}{4/5}$

$= \frac{5}{8}$

2. Let $u = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Putting x = tan θ, we have

u = $\tan^{-1}$(tan 3 θ) and v = $\tan^{-1}$(tan 2 θ)

$\Rightarrow u = 3\theta$ and $v = 2\theta$ $\left[\because -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \Rightarrow -\frac{1}{\sqrt{3}} < \tan\theta < \frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} < 3\theta < \frac{\pi}{2} \text{ and } -\frac{\pi}{3} < 2\theta < \frac{\pi}{3}\right]$

$\Rightarrow$ u = 3 $\tan^{-1}$x and v = 2 $\tan^{-1}$x

$\Rightarrow \frac{du}{dx} = \frac{3}{1+x^2}$ and $\frac{dv}{dx} = \frac{2}{1+x^2}$

$\Rightarrow \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{3}{1+x^2}}{\frac{2}{1+x^2}} = \frac{3}{2}$.

OR

We have,

y = x^3 + ax + b ...(i)

x - y + 5 = 0 ...(ii)

now,

Point (1, -6) lie on (i), so,

-6 = 1 + a + b

$\Rightarrow$ a + b = -7 ...(iii)

Also,

Slope of tangent to (i) is

$\frac{dy}{dx} = 3x^2 + a$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,-6)} = 3 + a$

And slope of the tangent to (ii) is

$\frac{dy}{dx} = 1$

According to the question slope of equation (i) and equation (ii) are parallel

$\therefore$ 3 + a = 1

$\Rightarrow$ a = -2

From (iii)

b = -5

3. Let I = $\int \frac{2x+1}{(x+2)(x-3)^2} dx$

   Using partial fractions,

   $\frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

   $\Rightarrow$ 2x + 1 = A (x - 3)^2 + B (x +2) (x - 3) + C (x + 2)

   = (A + B)x^2 + (-6A - B + C)x + (9A - 6B + 2C)

   Equating similar terms, we have

   A + B = 0 $\Rightarrow$ A = -B

   - 6A - B + C = 2 $\Rightarrow$ 5B + C = 2

   9A - 6B + 2C = 1 $\Rightarrow$ -15B + 2C = 1

   Solving, we get, B = $\frac{3}{25}$, C = $\frac{7}{5}$, A = -$\frac{3}{25}$

   Thus,

   I = - $\frac{3}{25} \int \frac{dx}{x+2} + \frac{3}{25} \int \frac{dx}{x-3} + \frac{7}{5} \int \frac{dx}{(x-3)^2}$

   I = $-\frac{3}{25} \log|x+2| + \frac{3}{25} \log|x-3| - \frac{7}{5(x-3)} + c$

4. It is given that f(x) = x^3, x $\in$ [-2, 2]

   $\Rightarrow$ f'(x) = 3x^2

   Now, f'(x) = 0

   $\Rightarrow$ x = 0

   Further, we evaluate the value of f at critical point x = 0 and at end points of the interval [-2, 2].

   f(0) = 0

   f(-2) = (-2)^3 = -8

   f(2) = (2)^3 = 8

   Therefore, the absolute maximum value of f on [-2, 2] is 8 occurring at x = 2

   And, the absolute minimum value of f on [-2, 2] is -8 occurring at x = -2

5. Let I = $\int$ sec^4 x dx, then we have

   I = $\int$ sec^2 x sec^2 x dx = $\int$ (1 + tan^2 x) sec^4 x dx

   Putting tan x = t and sec x dx = dt, we have

   I = $\int$ (1 + t^2) dt = $t + \frac{t^3}{3} + C$ = tan x + $\frac{1}{3}\tan^3 x + C$

OR

Let $I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$ ....(i)

$= \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)} dx$

$= \int_0^\pi \frac{(\pi-x)\sin x}{1+\cos^2 x} dx$ ....(ii)

Adding (i) and (ii), we get

$2I = \int_0^\pi \left[\frac{x \sin x}{1+\cos^2 x} + \frac{(\pi-x)\sin x}{1+\cos^2 x}\right] dx$

$= \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x} dx$

$= \pi\left[-\tan^{-1}(\cos x)\right]_0^\pi$

$= -\pi$ [tan^{-1} (-1) - tan^{-1} (1)]

$= -\pi\left(-\frac{\pi}{4} - \frac{\pi}{4}\right)$

$= \frac{\pi^2}{2}$

Hence, $I = \frac{\pi^2}{4}$

6. Given that, A = [–1, 1]

let $g(x_1) = g(x_2)$

$\Rightarrow |x_1| = |x_2|$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$ and $x_1 = - x_2$ ...{e.g., g(-1) = |-1| = 1 and g(1) = |1| = 1}

$\Rightarrow$ g is not one-one.

We observe that (-1) does not have any pre-image in the domain since g(x) = |x| assumes only non-negative values.

i.e. we cannot find any number in domain which will give (-1) in co-domain.

$\Rightarrow$ g is not onto

Hence, g is neither one one nor onto.

7. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2 \times \frac{1}{2}\right]$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

8. Let the given integral be,

$I = \int \frac{\log(1-x)}{x^2}dx$

$= \int \frac{1}{x^2}\log(1-x)dx$

$= \log(1-x)\int x^{-2}dx - \int \frac{-1}{1-x} \times \left(\frac{x^{-2+1}}{-2+1}\right)dx$

$= \log(1-x)\left[\frac{x^{-2+1}}{-2+1}\right] + \int \frac{-1}{(1-x)x}dx$

$= \log(1-x) \times \left(-\frac{1}{x}\right) + \int \frac{1}{x^2-x}dx$

$= -\frac{\log(1-x)}{x} + \int \frac{1}{x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$= -\frac{\log(1-x)}{x} + \int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$= -\frac{\log(1-x)}{x} + \frac{1}{2\times\frac{1}{2}}\log\left|\frac{x-\frac{1}{2}-\frac{1}{2}}{x-\frac{1}{2}+\frac{1}{2}}\right| + C$

$= -\frac{\log(1-x)}{x} + \log\left|\frac{x-1}{x}\right| + C$

$= -\frac{\log(1-x)}{x}$ + log |(x - 1)|- log x + C

$= -\frac{\log|1-x|}{x}$ + log|1 - x| - log |x| + C

$= \left(1-\frac{1}{x}\right)$ log |1 - x| - log|x| + C

9. Putting x^3 = sin A and y^3 = sin B in the given relation, we get

$\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow$ cos A + cos B = a (sin A - sin B)

$\Rightarrow \quad 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$

$\Rightarrow \quad \cot\left(\frac{A-B}{2}\right) = a$

$\Rightarrow \quad \frac{A-B}{2} = \cot^{-1}(a)$

$\Rightarrow$ A - B = 2 $\cot^{-1}$(a)

$\Rightarrow \sin^{-1}x^3 - \sin^{-1}y^3 = 2\cot^{-1}(a)$.

Differentiating both sides with respect to x, we get

$\frac{1}{\sqrt{1-x^6}} \times \frac{d}{dx}\left(x^3\right) - \frac{1}{\sqrt{1-y^6}} \times \frac{d}{dx}\left(y^3\right) = 0$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^6}} \times 3x^2 - \frac{1}{\sqrt{1-y^6}} \times 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

LHS=RHS

Hence Proved..

OR

$\because x = e^{\cos 2t}$ and $y = e^{\sin 2t}$

$\therefore \frac{dx}{dt} = \frac{d}{dt}e^{\cos 2t} = e^{\cos 2t}.\frac{d}{dt}\cos 2t$

$= e^{\cos 2t}.(-\sin 2t).\frac{d}{dt}(2t)$

$\frac{dx}{dt} = -2e^{\cos 2t}.\sin 2t$ ....(i)

and $\frac{dy}{dt} = \frac{d}{dt}e^{\sin 2t} = e^{\sin 2t}.\frac{d}{dt}\sin 2t$

$= e^{\sin 2t}\cos 2t.\frac{d}{dt}2t$

$= 2e^{\sin 2t}.\cos 2t$ ..(ii)

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^{\sin 2t}.\cos 2t}{-2e^{\cos 2t}.\sin 2t}$

$= \frac{e^{\sin 2t}.\cos 2t}{e^{\cos 2t}.\sin 2t}$...(iii)

Now, $x = e^{cos 2t}$

Taking log on both sides,we get,

$\log x = \cos 2t.\log e = \cos 2t$ ...(iv)

Also, $y = e^{sin 2t}$

Taking log on both sides, we get

$\log y = \sin 2t.\log e = \sin 2t$ ...(v)

$\therefore \frac{dy}{dx} = \frac{-y\log x}{x\log y}$

[Using Eqs. (iv) and (v) in Eq. (iii) and $x = e^{\cos 2t}$, $y = e^{\sin 2t}$]

Hence proved.

10. **Read the text carefully and answer the questions:**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(ii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

= $P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

(iii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 2 subjects)

$\Rightarrow$ P(grade A in M and P not in C) + P(grade A in P & C not in M) + P(grade A in M & C not in P)

$\Rightarrow$ P$\left(M \cap P \cap \overline{C}\right)$ + P$\left(P \cap C \cap \overline{M}\right)$ + P$(M \cap C \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.3 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.2 $\times 0.5 \times$ 0.7 = 0.03 + 0.12 + 0.07

P(getting grade A in 2 subjects) = 0.22

(iv)P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 1 subjects)

$\Rightarrow$ P(grade A in M not in P and C) + P(grade A in P not in M and C) + P(grade A in C not in P and M)

$\Rightarrow$ P$\left(M \cap \overline{P} \cap \overline{C}\right)$ + P$\left(P \cap \overline{C} \cap \overline{M}\right)$ + P$(C \cap \overline{M} \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.7 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.5 $\times 0.8 \times$ 0.7 = 0.07 + 0.12 + 0.028

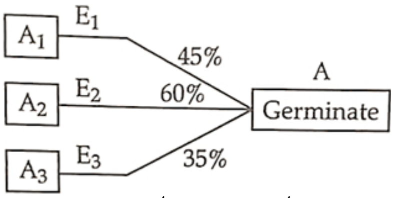
P(getting grade A in 1 subjects) = 0.47

OR

Read the text carefully and answer the questions:

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i)


Here, P(E_1) = $\frac{4}{10}$, P(E_2) = $\frac{4}{10}$, P(E_3) = $\frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore$ P(A) = P(E_1) $\cdot$ P$\left(\frac{A}{E_1}\right)$ + P(E_2) $\cdot$ $P\left(\frac{A}{E_2}\right)$ + P(E_3) $\cdot$ $P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

$= \frac{490}{1000}$ = 4.9

(ii) Required probability = $P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

(iii)Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

= $\frac{1}{2}$

and P(E_2) = $\frac{13}{52} = \frac{1}{4}$

Then, P($E_1 \cap E_2$) = P(E_1) · P(E_2)

= $\frac{1}{2}, \frac{1}{4} = \frac{1}{8}$

(iv) P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A $\cap$ B) = P(A)

$\Rightarrow$ P(B) - P(A $\cap$ B) = 0

$\Rightarrow$ P(A $\cap$ B) = P(B)

$\therefore$ P(A|B) = $\frac{P(A \cap B)}{P(B)}$

= $\frac{P(B)}{P(B)}$

= 1

11. **Read the text carefully and answer the questions:**

In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories type I, type II and type III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(i) In factory A, number of units of type I, II and III for boys are 80, 70, 65 respectively and for girls number of units of type I, II and III are 80, 75, 90 respectively.

$$\therefore P = \begin{matrix} I \\ II \\ III \end{matrix} \overset{\begin{matrix} Boys & Girls \end{matrix}}{\begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix}}$$

(ii) In factory B, number of units of type I, II and III for boys are 85, 65, 72 respectively and for girls number of units of type I, II and III are 50, 55, 80 respectively.

$$\therefore Q = \begin{matrix} I \\ II \\ III \end{matrix} \overset{\begin{matrix} Boys & Girls \end{matrix}}{\begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix}}$$

(iii) Let matrix X represent the number of units of each type produced by factory A for boys and matrix Y represents the number of units of each type produced by factory B for boys.

$$\therefore X = \overset{\begin{matrix} I & II & III \end{matrix}}{\begin{bmatrix} 80 & 70 & 65 \end{bmatrix}}$$

$$Y = \overset{\begin{matrix} I & II & III \end{matrix}}{\begin{bmatrix} 85 & 65 & 72 \end{bmatrix}}$$

Now, total production of sports clothes of each type for boys = X + Y

= [80 70 65] + [85 65 72]

= [165 135 137]

12. We have (1 + x^2)$\frac{dy}{dx}$ = $\left(e^{m \tan^{-1} x} - y\right)$

$\Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{1}{1+x^2}e^{m \tan^{-1} x}$

It is linear difference equation of the form $\frac{dy}{dx}$ + P(x)y = Q(x) where P(x) = $\frac{1}{1+x^2}$ and Q(x) = $\frac{1}{1+x^2}e^{m \tan^{-1} x}$ }

Now I.F. = $\underset{e}{\int} \frac{1}{1+x^2}dx = e^{\tan^{-1} x}$

Required solution is given by ==> y(I.F.) = $\int$ {Q(x) (I.F.)} dx

$\Rightarrow ye^{\tan^{-1} x} = \int e^{\tan^{-1} x} \frac{1}{1+x^2} e^{m \tan^{-1} x}dx$ + C $\Rightarrow ye^{\tan^{-1} x} = \int \frac{1}{1+x^2}e^{(m+1) \tan^{-1} x}dx$ + C

Put $\tan^{-1}$ x = t $\Rightarrow$ $\frac{dx}{1+x^2}$ = dt, we get $ye^{\tan^{-1} x} = \int e^{(m+1)t}dt$ + C

$\Rightarrow ye^{\tan^{-1}x} = \frac{1}{(m+1)}e^{(m+1)t} + C$

$\Rightarrow ye^{\tan^{-1}x} = \frac{1}{(m+1)}e^{(m+1)\tan^{-1}x} + C$

Since y = 1 when x = 0 so, $1.\mathrm{e}^{\tan^{-1}0} = \frac{1}{(\mathrm{m}+1)}\mathrm{e}^{(\mathrm{m}+1)\log^{-1}0} + \mathrm{C} \Rightarrow \mathrm{C} = \frac{m}{m+1}$

Hence the solution is $ye^{\tan^{-1}x} = \frac{1}{(m+1)}e^{(m+1)\tan^{-1}x} + \frac{m}{m+1}$

OR

$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$

$\Rightarrow \frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{y}{x}\left\{\cos\frac{y}{x}+\frac{y}{x}.\sin\frac{y}{x}\right\}}{\frac{y}{x}.\sin\left(\frac{y}{x}\right)-\cos\frac{y}{x}}$ (1)

Let y = vx, then, $\frac{dy}{dx} = v + x\frac{dy}{dx}$, put $\frac{dy}{dx}$ in eq (1), we get,

$v + x\frac{dv}{dx} = v\frac{(\cos v+v\sin v)}{v\sin v-\cos v}$

$\Rightarrow x\frac{dv}{dx} = v\frac{(\cos v+v\sin v)}{v\sin v-\cos v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v-\cos v}$

$\Rightarrow \int\left(\frac{v\sin v-\cos v}{v\cos v}\right)dv = \int\frac{2}{x}dx$

$\Rightarrow \int\left(\tan v - \frac{1}{v}\right)dv = \int\frac{2}{x}dx$

$\Rightarrow -\log(\cos v) - \log v = 2\log x + c$

$\Rightarrow -\log v\cos v = 2\log x + c$

$\Rightarrow \log\left((v\cos v).x^2\right) = -c$

$\Rightarrow (v\cos v).x^2 = e^{-c}$

$\Rightarrow x^2.\frac{y}{x}.\cos\frac{y}{x} = A[putting, A = e^{-c}]$

$\Rightarrow xy\cos\frac{y}{x} = A$

13. We have, $f(x) = \sin x - \cos x, 0 < x < 2\pi$

On differentiating both sides w.r.t. x ,we get,

f'(x) = cosx + sinx..........(i)

For local maximum and local minimum,

Put f'(x) = 0,

i.e. $\cos x + \sin x = 0 \Rightarrow \cos x = -\sin x$

$\Rightarrow \quad \tan x = -1 \Rightarrow x = \pi - \frac{\pi}{4}$ or $2\pi - \frac{\pi}{4}$

$\Rightarrow \quad x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

Again, on differentiating both sides of Eq. (i)

w.r.t. x, we get,

f'' (x)=-sin x + cos x

When $x = \frac{3\pi}{4}$, then

$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}$

$= -\sin\left(\pi - \frac{\pi}{4}\right) + \cos\left(\pi - \frac{\pi}{4}\right)$

$= -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} < 0$

When $x = \frac{7\pi}{4}$, then $f''\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4}$

$= -\sin\left(2\pi - \frac{\pi}{4}\right) + \cos\left(2\pi - \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} > 0$

Thus, $x = \frac{3\pi}{4}$ is a point of local maxima and $x = \frac{7\pi}{4}$ is a point of local minima.

Now, the local maximum value,

$f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}$

$= \sin\left(\pi - \frac{\pi}{4}\right) - \cos\left(\pi - \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

and the local minimum value,

$f\left(\frac{7\pi}{4}\right) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4}$

$= \sin\left(2\pi - \frac{\pi}{4}\right) - \cos\left(2\pi - \frac{\pi}{4}\right)$

$= -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}} = -\sqrt{2}$

OR

Let P(x,y) be a point on the curve,

$x^2 = 8y$ ... equation(i)

Let A = (2, 4) be a point and

let S = square of the distance between P and A.

$\because s = (x - 2)^2 + (y - 4)^4$ ...equation(ii)

Using equation(i), we get

$S = (x - 2)^2 + \left(\frac{x^2}{8} - 4\right)^2$

$\because \frac{dS}{dy} = 2(x - 2) + 2\left(\frac{x^2}{8} - 4\right) \times \frac{2x}{8}$

$= 2(x - 2) + \frac{(x^2-32)x}{16}$

Also, $\frac{d^2s}{dx^2} = 2 + \frac{1}{16}[3x^2 - 32]$

$= 2 + \frac{1}{16}[3x^2 - 32]$

For maxima and minima,

$\frac{dS}{dy} = 0$

$\Rightarrow 2(x - 2) + \frac{x(x^2-32)}{16} = 0$

$\Rightarrow 32x - 64 + x^3 - 32x = 0$

$\Rightarrow x^3 - 64 = 0$

$\Rightarrow x = 4$

Now,

At x = 4, $\frac{d^2s}{dx^2} = 2 + \frac{1}{16}[16 \times 3 - 32] = 2 + 1 = 3 > 0$

$\because$ x = 4 is point of local minima

From equation(i)

$y = \frac{x^2}{8} = 2$

Thus, P (4, 2) is the nearest point.

14. **Read the text carefully and answer the questions:**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(i) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that daughter is at one end n(A) = 12 and B denotes the event that father, and mother are in the middle n(B) = 4

Also, $n(A \cap B) = 4$

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{4}{24}} = 1$$

(ii) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that mother is at right end. n(A) = 6 and B denotes the event that son and daughter are together. n(B) = 12

Also, n (A ∩ B) = 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{12}{24}} = \frac{1}{3}$$

(iii) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that father, and mother are in the middle. n(A) = 4 and B denote the event that son is at right end. n(B) = 6

Also, n (A ∩ B) = 2

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{24}}{\frac{6}{24}} = \frac{1}{3}$$

(iv) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM},

where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that father and son are standing together. n(A) = 12 and B denote the event that mother and daughter are standing together. n(B) = 12

Also, n (A ∩ B) = 8

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{8}{24}}{\frac{12}{24}} = \frac{2}{3}$$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(d)** 3

Explanation: {

3

(ii) Let θ be the angle between the given lines.

we have,

$a_1 = 4$, $b_1 = -3$, $c_1 = 5$ and $a_2 = 3$, $b_2 = 4$, $c_2 = 5$

$\therefore \cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{12-12+25}{\sqrt{16+9+25}\sqrt{9+16+25}} = \frac{25}{50} = \frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

Thus, the measure of the angle between the given lines is 60°.

(iii) Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} + 2\hat{k}$

Since $\vec{c} = \vec{a} + \vec{b}$, three vectors form a triangle.

Also, $\vec{a} \cdot \vec{b}$ = 0.

So, triangle is a right angled triangle.

(iv) **(c)** $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Explanation: {

Rewrite the given line as

$r\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$

or $\frac{x-\frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$ or $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

(v) We need to find the direction cosines of the line. Here, we are given equation of a line in the following form.Now, we have,

5x - 3 = 15y + 7 = 3 - 10z ....(i)

Let us first convert the equation in standard form

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ .... (ii)

Let us divide Eq. (i) by LCM (coefficients of x, y and z). i.e. LCM (5, 15, -10) = 30

Now, the Eq. (i) becomes

$\frac{5x-3}{30}=\frac{15y+7}{30}=\frac{3-10z}{30}$

$\Rightarrow \quad \frac{x-\frac{3}{5}}{6}=\frac{y+\frac{7}{15}}{2}=\frac{z-\frac{3}{10}}{-3}$

On comparing the above equation with Eq.(ii), we get 6, 2, -3 are the direction ratios of the given line.

Now, the direction cosines of given line are

$\frac{6}{\sqrt{6^2+2^2+(-3)^2}}\frac{2}{\sqrt{6^2+2^2+(-3)^2}}$ and $\frac{-3}{\sqrt{6^2+2^2+(-3)^2}}$ i.e, $\left(\frac{6}{7},\frac{2}{7},\frac{-3}{7}\right)$

16. Let V be the volume of the parallelopiped whose adjacent sides are determined by the vectors $\vec{a},\vec{b}$ and $\vec{c}$.

Then,

V = $[\vec{a}\vec{b}\vec{c}]$ [scalar triple product of adjacent sides]

$\Rightarrow \quad V=\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -1\\ 1 & 1 & 3\end{vmatrix}$ = (12 + 1) -(6 +1) + (2 -4) = 4 cubic units.

Let A be the area of the base of the parallelopiped. Then, A = $|\vec{a}\times\vec{b}|$

Now,

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 4 & -1\end{vmatrix}=-5\hat{i}+3\hat{j}+2\hat{k}$

$\therefore$ A = $|\vec{a}\times\vec{b}|=\sqrt{25+9+4}=\sqrt{38}$

We know that:

Volume of the parallelopiped = Area of the base $\times$ Altitude

$\therefore$ Altitude = $\frac{V}{A}=\frac{4}{\sqrt{38}}$ units

OR

$\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right)=0$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{a}.\vec{c}=-|\vec{a}|^2$

$\Rightarrow \vec{a}.\vec{b}+\vec{a}.\vec{c}=-1$ ...(i)

Similarly, $\vec{b}.\vec{a}+\vec{b}.\vec{c}=-|\vec{b}|^2=-(4^2)=-16$ ...(ii)

Similarly, $\vec{a}.\vec{c}+\vec{b}.\vec{c}=-|\vec{c}|^2=-(2)^2=-4$ ...(iii)

Adding (i) (ii) and (iii), we get,

$2(\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{a}.\vec{c})=-21$

$\Rightarrow \mu=\frac{-21}{2}$

17. Clearly, we have to find the direction cosines of the normal to the given plane.

The given equation may be written as

$\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})=-3\Rightarrow\vec{r}\cdot(-6\hat{i}+3\hat{j}+2\hat{k})=3$

$\Rightarrow \vec{r}\cdot\vec{n}=3$, where $\vec{n}=(-6\hat{i}+3\hat{j}+2\hat{k})$

$\Rightarrow \vec{r}\cdot\frac{\vec{n}}{|\vec{n}|}=\frac{3}{|\vec{n}|}$ where $|\vec{n}|=\sqrt{(-6)^2+3^2+2^2}=7$

$\Rightarrow \vec{r}\cdot\frac{(-6\hat{i}+3\hat{j}+2\hat{k})}{7}=\frac{3}{7}\Rightarrow\vec{r}\cdot\left(-\frac{6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{3}{7}$

Hence, the direction cosines of the normal to the given plane are

$\left(-\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$.

OR

We have, $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

And $\vec{r}=\left(2\hat{i}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right)$

Where, $\vec{a}_1=3\hat{i}-2\hat{j}+6\hat{k},\vec{b}_1=2\hat{i}+\hat{j}+2\hat{k}$

And $\vec{a}_2=2\vec{j}-5\vec{k},\vec{b}_2=6\hat{i}+3\hat{j}+2\hat{k}$

If θ is angle between the lines, then

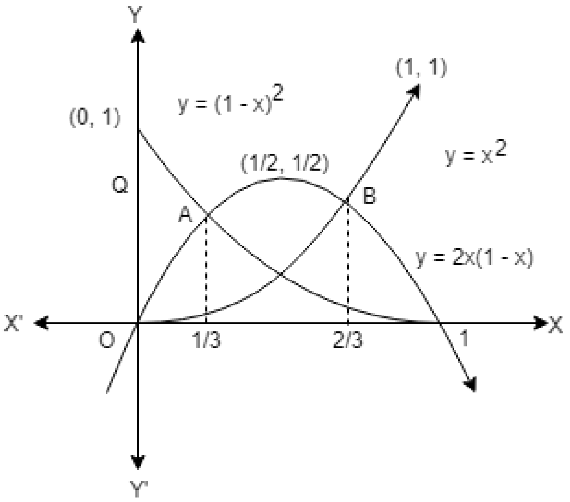
$\cos\theta=\frac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|.\left|\vec{b}_1\right|}$

$=\frac{\left|\left(2\hat{i}+\hat{j}+2\hat{k}\right).\left(6\hat{i}+3\hat{j}+2\hat{k}\right)\right|}{\left|2\hat{i}+\hat{j}+2\hat{k}\right|\left|6\hat{i}+3\hat{j}+2\hat{k}\right|}$

$= \frac{|12+3+4|}{\sqrt{9}\sqrt{49}} = \frac{19}{21}$

$\theta = \cos^{-1}\frac{19}{21}$

18. Given , y = x^2, y = (1 - x^2) and y = 2x(1 - x)



To get the point of intersection of y = x^2 and y = 2x(1 - x),

$x^2 = 2x(1-x) \Rightarrow 3x^2 = 2x$

$\Rightarrow x(3x-2) = 0 \Rightarrow x = 0, \frac{2}{3}$

the point of intersection of y = x^2 and y = 2x(1 - x) are 0,$\frac{2}{3}$

To get the point of intersection of y = (1 - x^2) and y = 2x(1 - x) ,

$1 - x^2 = 2x(1-x) \Rightarrow x^2 - 2x + 1 = 0$

$(x-1)^2 = 0 \Rightarrow x = 1$

The points of intersection of y = (1 - x^2) and y = 2x(1 - x) is x = 1

if x=1 then ,

y = (1 - x^2)

y=1-1

y =0.

point of intersection is (1,0)

From the figure, it is clear that,

(1 - x^2), if $0 \leq x \leq \frac{1}{3}$

f(x) = 2x(1-x), if $\frac{1}{3} \leq x \leq \frac{2}{3}$

x^2, if $\frac{2}{3} \leq x \leq 1$

The required area = The area of the region bounded by the curves

y= $Max\{x^2, (1-x)^2, 2x(1-x)\}$, X-axis, (x=0) and (x=1)

A=$\int_0^1 f(x)dx$

=$\int_0^{\frac{1}{3}} (1-x)^2 dx + \int_{\frac{1}{3}}^{\frac{2}{3}} 2x(1-x)dx + \int_{\frac{2}{3}}^{1} x^2 dx$

=$\left[-\frac{1}{3}(1-x)^3\right]_0^{\frac{1}{3}} + \left[x^2 - \frac{2x^3}{3}\right]_{\frac{1}{3}}^{\frac{2}{3}} + \left[\frac{1}{3}x^3\right]_{\frac{2}{3}}^{1}$

$= \left[-\frac{1}{3}(\frac{2}{3})^3 + \frac{1}{3}\right] + \left[(\frac{2}{3})^2 - \frac{2}{3}(\frac{2}{3})^3 - (\frac{1}{3})^2 + \frac{2}{3}(\frac{1}{3})^3\right] + \left[\frac{1}{3}(1) - \frac{1}{3}(\frac{2}{3})^3\right]$

$= \frac{19}{81} + \frac{13}{81} + \frac{19}{81} = \frac{17}{27}$ sq.units

The area of the region bounded by the curves y= $Max\{x^2, (1-x)^2, 2x(1-x)\}$, X-axis, (x=0) and (x=1) = $\frac{17}{27}$ sq. units

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(c)** $2x + 1 - \frac{4}{x}$

Explanation: {

AC = $\frac{C}{x} = \frac{2x^2+x-4}{x}$

$\Rightarrow AC = 2x + 1 - \frac{4}{x}$

(ii) **(c)** first quadrant

Explanation: {

All the positive value of x and y always lie in first quadrant.

(iii) Given, Σx = 15, Σy = 25, Σx^2 = 55, Σy^2 = 135, Σxy = 83 and n = 5

$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\Sigma y)^2}{n}}$

$= \frac{83 - \frac{15 \times 25}{5}}{135 - \frac{(25)^2}{5}}$

$= \frac{83-75}{135-125}$

$= \frac{8}{10}$

= 0.8

and $b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$

$= \frac{83 - \frac{15 \times 25}{5}}{55 - \frac{(15)^2}{5}}$

$= \frac{83-75}{55-45}$

$= \frac{8}{10}$ = 0.8

(iv) Given R = 2q and C = 500 + $\frac{1}{2}\left(\frac{q}{20}\right)^2$

So, profit function P = R - C

$\Rightarrow$ P = 2q - 500 - $\left(\frac{q^2}{800}\right)$

$\therefore \frac{dp}{dq}$ = 2 - $\frac{2q}{800} \Rightarrow \frac{dp}{dq}$ = 2 - $\frac{q}{400}$

$\left[\frac{dP}{dq}\right]_{q=500}$ = 2- $\frac{500}{400}$ = 2 - 1.25 = 0.75

(v) Given TFC = ₹35000, TVC = ₹500x

$\Rightarrow$ C(x) = 35000 + 500x

Also, p = 5000 - 100x $\Rightarrow$ R(x) = 5000x - 100x^2

At breakeven point: R(x) = C(x)

$\Rightarrow$ 5000x - 100x^2 = 35000 + 500z

$\Rightarrow$ x^2 - 45x + 350 = 0

$\Rightarrow$ (x - 10) (x - 35) = 0 $\Rightarrow$ x = 10, 35

20. i. Total fixed cost = ₹6000,

variable cost of producing one unit = ₹20

$\therefore$ Total cost, TC = ₹(6000 + 20x), where x is the number of units produced.

ii. Total cost of producing 15 units = $\mathrm{TC}|_x$ = 15 = ₹(6000 + 20 $\times$ 15) = ₹6300

$\therefore$ Average cost of producing 15 units = ₹ $\frac{6300}{15}$ = ₹420

iii. Total cost of producing 100 units = ₹(6000 + 20 $\times$ 100) = ₹8000

$\therefore$ Average cost of producing 100 units = ₹8000 = ₹80

OR

We have, x = 100 - 4p

$\Rightarrow$ p = $\frac{100-x}{4}$

i. Let R be the total revenue. Then,

R = px $\Rightarrow$ R = $\left(\frac{100-x}{4}\right)x \Rightarrow$ R = 25x - $\frac{x^2}{4}$

ii. AR = $\frac{R}{x} \Rightarrow$ AR = $\frac{1}{x}\left(25x - \frac{x^2}{4}\right)$ = 25 - $\frac{x}{4}$

iii. R = 25x - $\frac{x^2}{4} \Rightarrow \frac{dR}{dx}$ = 25 - $\frac{x}{2}$ i.e. MR = 25 - $\frac{x}{2}$

iv. MR = 0 $\Rightarrow$ 25 - $\frac{x}{2}$ = 0 $\Rightarrow$ x = 50

Putting x = 50 in p = $\frac{100-x}{4}$ we get, p = $\frac{50}{4} = \frac{25}{2}$ = 12.5

21. Given, $\bar{x}$ = 25, $\bar{y}$ = 30, b_{yx} = 1.6 and b_{xy} = 0.4

i. Regression equation y on x:

y - $\bar{y}$ = b_{yx}(x - $\bar{x}$)

$\Rightarrow$ y - 30 = 1.6(x - 25)

$\Rightarrow$ y - 30 = 1.6x - 40

$\Rightarrow$ y = 1.6x - 10 ...(i)

ii. From eq. (i)

When x = 60, y = (1.6) $\times$ (60) - 10

= 96 - 10 = 86

iii. Coefficient of correlation:

r = $\pm\sqrt{b_{yx}b_{xy}}$

= $\pm\sqrt{(1.6)(0.4)}$

= $\pm\sqrt{0.64}$

($\because$ both regression coefficients are +ve)

22. First, we will convert the given inequations into equations, we obtain the following equations:

$x_1 - x_2 = -1$, $-x_1 + x_2 = 0$, $x_1 = 0$ and $x_2 = 0$

Region represented by $x_1 - x_2 \leq -1$:

The line $x_1 - x_2 = -1$ meets the coordinate axes at A(- 1,0) and B(0,1) respectively. By joining these points we obtain the line $x_1 - x_2 = -1$.

Clearly (0,0) does not satisfies the inequation $x_1 - x_2 \leq -1$. So, the region in the plane which does not contain the origin represents the solution set of the
inequation $x_1 - x_2 \leq -1$

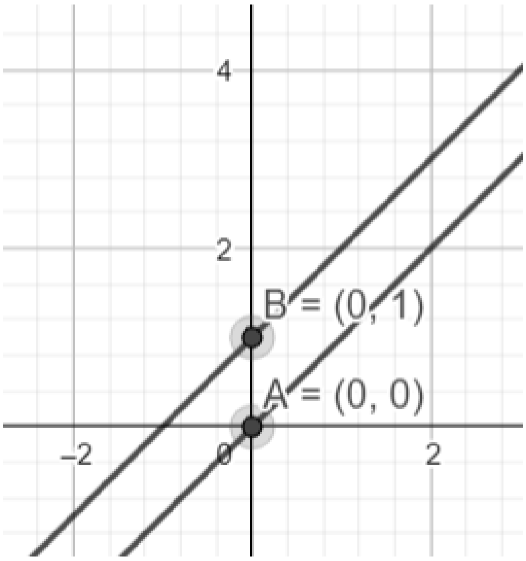
Region represented by $-x_1 + x_2 \leq 0$ or $x_1 \geq x_2$

The line $-x_1 + x_2 = 0$ or $x_1 = x_2$ is the line passing through (0, 0).The region to the right of the line $x_1 = x_2$ will satisfy the given inequation $-x_1 + x_2 \leq 0$ If we take a point (1, 3) to the left of the line $x_1 = x_2$. Here, $1 \leq 3$ which is not satifying the inequation $x_1 \geq x_2$. Therefore, region to the right of the line $x_1 = x_2$ will satisfy the given inequation $-x_1 + x_2 \leq 0$

Region represented by $x_1 \geq 0$ and $x_2 \geq 0$

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq 0$ and $x_2 \geq 0$

The feasible region determined by subject to the constraints are $x_1 - x_2 \leq -1$, $-x_1 + x_2 \leq 0$, and the non-negative restrictions; $x_1 \geq 0$, and $x_2 \geq 0$, are as follows.



We observe that the feasible region of the given LPP does not exist because the following equations have no common region.

OR

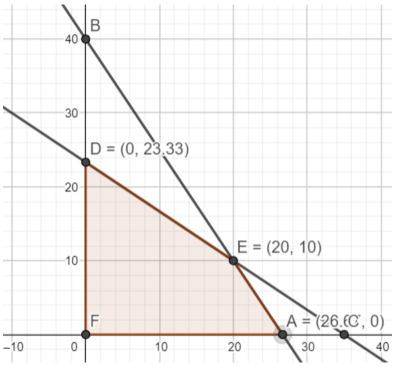
First, we will convert the given inequations into equations, we obtain the following equations:

3x + 2y = 80, 2x + 3y = 70, x = 0 and y = 0

Region represented by $3x + 2y \leq 80$:

The line 3x + 2y = 80 meets the coordinate axes at $A\left(\frac{80}{3}, 0\right)$ and B(0, 40) respectively. By joining these points we obtain the line 3x + 2y = 80. Clearly (0,0) satisfies the inequation $3x + 2y \leq 80$. So, the region containing the origin represents the solution set of the inequation $3x + 2y \leq 80$. Region represented by $2x + 3y \leq 70$:

The line 2x + 3y = 70 meets the coordinate axes at C(35,0) and $D\left(0, \frac{70}{3}\right)$ respectively. By joining these points we obtain the line 2x + 3y $\leq$ 70. Clearly (0,0) satisfies the inequation 2x + 3y $\leq$ 70. So, the region containing the origin represents the solution set of the inequation 2x + 3y $\leq$ 70. The region represented by x $\geq$ 0 and y $\geq$ 0 since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 andy $\geq$ 0 The feasible region determined by subject to the constraints are, 3x + 2y $\leq$ 80, 2x + 3y $\leq$ 70,and the non-negative restrictions, x $\geq$ 0, and y $\geq$ 0 are as follows:



The corner points of the feasible region are o (0, 0) $A\left(\frac{80}{3}, 0\right)$ E(20, 10) and $D\left(0, \frac{70}{3}\right)$

The values of objective function z at these corner points are as follows.

Corner point	Z = 15x + 10y
O(0, 0)	$15 \times 0 + 10 \times 0 = 0$
$A\left(\frac{80}{3}, 0\right)$	$15 \times \frac{80}{3} + 10 \times 0 = 400$
E(20, 10)	$15 \times 20 + 10 \times 10 = 400$
$D\left(0, \frac{70}{3}\right)$	$15 \times 0 + 10 \times \frac{70}{3} = \frac{700}{3}$

We see that the maximum value of the objective function Z is 400 which is at $A\left(\frac{80}{3}, 0\right)$ and E(20, 10) Thus, the optimal value of objective function Z is 400.