CBSE Class 10 Mathematics Basic

Sample Paper - 09 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Without actually performing the long division, Check whether $\frac{129}{2^2 5^7 7^5}$ will have terminating decimal expansion or non-terminating repeating decimal expansion.

   OR

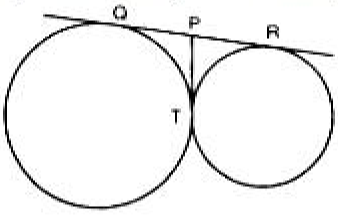
   Using prime factorisation, find the HCF and LCM of 144, 198. verify that HCF $\times$ LCM = product of given numbers.
2. Determine the set of values of k for which the given quadratic equation has real roots: $3x^2 + 2x + k = 0$

3. For what value of k the following pair of linear equation has unique solution?

$kx + 3y = 3$

$12x + ky = 6$

4. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If QP = 3.8 cm, then find length of QR.



5. Write the next term of the A.P $\sqrt{2}, \sqrt{8}, \sqrt{18}$, ...

OR

If the common difference of an A.P. is 3, then what is the value of $a_{20} - a_{15}$?

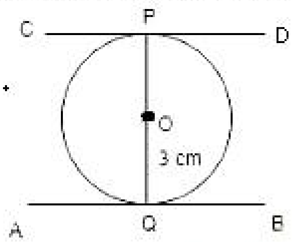
6. Find 10^{th} term of the A.P. 1, 4, 7, 10,...
7. Determine the set of values of k for which the given quadratic equation has real roots:

$kx^2 + 6x + 1 = 0$

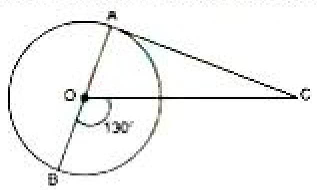
OR

Find the roots of the quadratic equation $x^2 - 3x = 0$.

8. Find the distance between two parallel tangents of a circle of radius 3 cm.



9. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, then find $\angle ACO$



OR

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle.

10. If $\Delta ABC \sim \Delta DEF$ such that AB = 1.2 cm and DE = 1.4 cm. Find the ratio of areas of ΔABC and ΔDEF.

11. Write an A.P. whose first term is 10 and common difference is 3.

12. Prove the trigonometric identity: $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

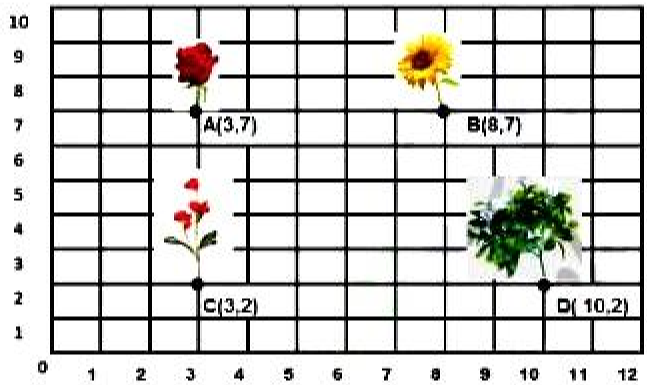
13. Write the value of $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - cosec^2\theta}$?

14. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7} cm^3$. Find the height of the cylinder.

15. How many three-digit numbers are divisible by 9?

16. 17 cards numbered 1 ,2, 3, ...,17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is odd.

17. In the school garden Ajay(A), Brijesh(B), Chinki(C) and Deepak(D) planted their flower plants of Rose, Sunflower, Champa and Jasmine respectively as shown in the following figure. A fifth student Eshan wanted to plant her flower in this area. The teacher instructed Eshan to plant his flower plant at a point E such that CE: EB = 3 : 2.



Answer the following questions:

i. Find the coordinates of point E where Eshan has to plant his flower plant.
   a. (5, 6)
   b. (6, 5)
   c. (5, 5)
   d. (6, 7)

ii. Find the area of $\triangle ECD$.
   a. 9.5 square unit
   b. 11.5 square unit
   c. 10.5 square unit
   d. 12.5 square unit

iii. Find the distance between the plants of Ajay and Deepak.

   a. 8.60 unit
   b. 6.60 unit
   c. 5.60 unit
   d. 7.60 unit

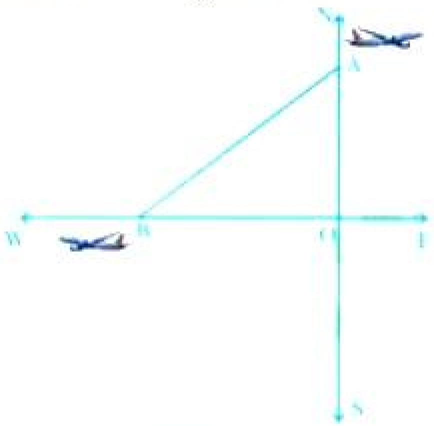
iv. The distance between A and B is:

   a. 5.5 units
   b. 7 units
   c. 6 units
   d. 5 units

v. The distance between C and D is:

   a. 5.5 units
   b. 7 units
   c. 6 units
   d. 5 units

18. An aeroplane leaves an Airport and flies due north at 300 km/h. At the same time, another aeroplane leaves the same Airport and flies due west at 400 km/h.



i. Distance travelled by the first aeroplane in 1.5 hours

   a. 450 km
   b. 300 km
   c. 150 km
   d. 600 km

ii. Distance travelled by the second aeroplane in 1.5 hours

   a. 450 km
   b. 300 km
   c. 150 km

d. 600 km

iii. Which of the following line segment shows the distance between both the aeroplane?

a. OA

b. AB

c. OB

d. WB

iv. Which aeroplane travelled a long distance and by how many km?

a. Second, 150 km

b. Second, 250 km

c. First, 150 km

d. Fist, 250 km

v. How far apart the two aeroplanes would be after 1.5 hours?

a. 600 km

b. 750 km

c. 300 km

d. 150 km

19. 

A medical camp was held in a school to impart health education and the importance of exercise to children. During this camp, a medical check of 35 students was done and their weights were recorded as follows:

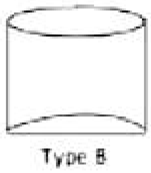
Weight (in Kg)	Number of Students
below 40	3
below 42	5
below 44	9
below 46	14

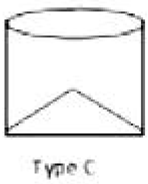
below 48	28
below 50	31
below 42	35

i. Modal class of given data:
   a. 46-48
   b. 48-50
   c. 44-46
   d. 50-52
ii. Compute the modal weight.
   a. 47.9 kg
   b. 46.9 kg
   c. 57.9 Kg
   d. 77.9 Kg
iii. Which of the following can not be determined graphically?
   a. Mean
   b. Median
   c. Mode
   d. None of these
iv. The mean of the following data:
   a. 55.58
   b. 50.85
   c. 45.85
   d. 35.56
v. Mode and the mean of data are 12k and 15K. The median of the data is:
   a. 12k
   b. 14k
   c. 15k
   d. 16k

20. Ganesh a juice seller has his juice shop near Kutub Minar in Delhi. He has three types of glasses, Type A - A glass with a plane bottom, Type B - A glass with a hemispherical raised bottom, and Type C - A glass with the conical raised bottom of height 1.5 cm. The inner diameter of all types of glass is the same as 5cm to serve the customer. The height of the

glasses is 10cm (use pi = 3.14)





i. The volume of the glass of type A:

a. 196.25 cm^3

b. 169.52 cm^3

c. 187.25 cm^3

d. 172.55 cm^3

ii. The volume of the hemisphere in the glass of type B:

a. 37.71cm^3

b. 32.71 cm^3

c. 33.71 cm^3

d. 43.34 cm^3

iii. The volume of a glass of type B:

a. 136.54 cm^3

b. 166.45 cm^3

c. 163.54 cm^3

d. 176.54 cm^3

iv. The volume of the cone in the glass of type C:

a. $8.33\ cm^3$

b. $9.81\ cm^3$

c. $10.81\ cm^3$

d. $11.88\ cm^3$

v. The volume of a glass of type C:

a. $188.88\ cm^3$

b. $189.99\ cm^3$

c. $196.89\ cm^3$

d. $186.44\ cm^3$

Part-B

21. Given that $\sqrt{3}$ is an irrational number, prove that $(2 + \sqrt{3})$ is an irrational number.

22. Find the lengths of the medians AD and BE of ΔABC whose vertices are A(7, -3), B(5, 3) and C(3, -1).

OR

Prove that in a right-angled triangle, the mid-point of the hypotenuse is equidistant from the vertices.

23. Find a cubic polynomial whose zeros are 2, -3 and 4.

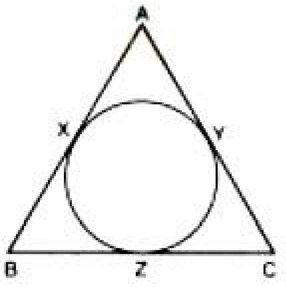
24. Draw a circle of radius 5 cm. From a point P, 13 cm away from its centre, draw two tangents to the circle.

25. If $\cos\theta = \frac{3}{4}$, then find the value of $9\tan^2\theta + 9$.

OR

If $\sin A = \frac{3}{4}$, calculate cos A and tan A.

26. ABC is an isosceles triangle in which AB = AC which is circumscribed about a circle as shown in the figure. Show that BC is bisected at the point of contact.



27. Show that there are infinitely many positive primes.

28. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two fifth of Varun's age. Find present ages of Varun and Swati.

OR

Solve: $4^{(x+1)} + 4^{(1-x)} = 10$

29. Find the zeros of polynomial p(y) = $y^2 + \frac{3\sqrt{5}}{2}y - 5$ and verify the relationship between the zeros and its coefficients.

30. ABCD is a trapezium in which AB || DC and AB = 2DC. If the diagonals of the trapezium intersect each other at point O, find the ratio of the areas of $\triangle AOB$ and $\triangle COD$.

OR

In ΔABC, D is the mid-point of BC and ED is the bisector of the $\angle$ADB and EF is drawn parallel to BC cutting AC in F. Prove that $\angle$EDF is a right angle.

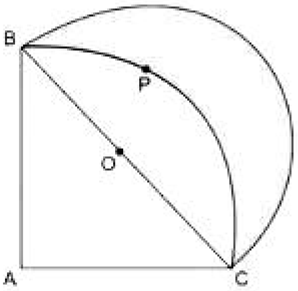
31. In a bag there are 44 identical cards with figure of circle or square on them. There are 24 circles, of which 9 are blue and rest are green and 20 squares of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of
   i. square
   ii. green colour,
   iii. blue circle and
   iv. green square.

32. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30° and 60°. Find the height of the poles and distance of point from poles.

33. The number of students in a hostel speaking different languages is given below. Present the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

34. In given figure ABPC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



35. Show graphically that the system of equations

    $2x + 4y = 10$

    $3x + 6y = 12$

    has no solution.

36. Two pillars of equal heights stand on either side of a road which is 100m wide. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are 60° and 30°. Find the height of each pillar and position of the point on the road. [Take $\sqrt{3}$ = 1.732.]

CBSE Class 10 Mathematics Basic

Sample Paper - 09 (2020-21)

Solution

Part-A

1. The given number is $\frac{129}{2^2 5^7 7^5}$

   Since, the denominator is not in the form of $2^m \times 5^n$, as it has 7 in denominator.

   So, the decimal expansion of $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

OR

   Given numbers are: 144, 198

   Prime factorization:

   $144 = 2^4 \times 3^2$

   $198 = 2 \times 3^2 \times 11$

   HCF = product of smallest power of each common prime factor in the numbers

   $= 2 \times 3^2 = 18$

   LCM = product of greatest power of each prime factor involved in the numbers

   $= 2^4 \times 3^2 \times 11 = 1584$

   HCF $\times$ LCM = 18 $\times$ 1584 = 28512 .... (i)

   Product of given numbers = 144 $\times$ 198 = 28512 .... (ii)

   From (i) and (ii)

   HCF $\times$ LCM = product of given numbers

2. $3x^2 + 2x + k = 0$

   The given equation has real roots,

   So, D $\geq$ 0

   $b^2 - 4ac \geq 0$

   $2^2 - 4 \times 3 \times k \geq 0$

   $4 - 12k \geq 0$

   $k \leq \frac{4}{12}$

   $k \leq \frac{1}{3}$

   So, the values of k will be less than or equal to $\frac{1}{3}$.

3. Given pair of equations

   kx + 3y = 3, 12x + ky = 6

   For unique solutions $\frac{k}{12} \neq \frac{3}{k}$

   $\Rightarrow k^2 \neq 36$

   $\Rightarrow k \neq \pm 6$.
4. QP = 3.8

   QP = PT (Length of tangents from the same external point are equal)

   Therefore, PT = 3.8 cm

   Also, PR = PT = 3.8 cm

   Now, QR = QP + PR

   QR = 3.8 + 3.8 = 7.6 cm.
5. Here the given series is:

   $\sqrt{2}, \sqrt{8}, \sqrt{18}$,....

   This series can be written as:

   $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$,,.....

   Therefore an A.P is formed.

   Here a = $\sqrt{2}$ and d = $\sqrt{2}$

   $\therefore$ Next term, T_4 = a + 3d = $\sqrt{2}$ + $3\sqrt{2}$ = $4\sqrt{2}$ = $\sqrt{32}$

   OR

   Let the first term of the AP be a.

   Given, common difference (d) = 3

   a_n = a + (n – 1)d

   Now,

   $a_{20} - a_{15}$ = [a + (20 – 1) d] – [a + (15 – 1)d]

   = 19d – 14d

   = 5d

   = 5 × 3

   = 15.
6. First term(a) $= 1$

   Common difference(d) $= 4 - 1 = 3$

   We have,

n^{th} term$(a_n) = a + (n-1)d$

Then, 10^{th} term$(a_{10}) = 1 + (10-1)3$

= 1 + 9 $\times$ 3

= 1 + 27

= 28

7. Given equation is;

   $kx^2 + 6x + 1 = 0$

   The given equation has real roots if Discriminant, D $\geq$ 0

   $6^2 - 4 \times k \times 1 \geq 0$

   $36 - 4k \geq 0$

   $4k \leq 36$

   $k \leq 9$

   Thus, the values of k will be less than or equal to 9.

OR

$x^2 - 3x = 0$

$\Rightarrow x(x-3) = 0$

$\Rightarrow$ x = 0 or x = 3.

8. Distance between two parallel tangents = diameter = PQ

   PQ = OP + OQ = 3 + 3 = 6cm

   The total distance between two parallel tangents lines is 6 cm.

9. In Fig , OA is the radius and AC is the tangent from the external point C.

   Therefore, $\angle OAC = 90^\circ$ (Theorem:Radius and tangent are always perpendicular to each other at the point of contact)

   $\angle BOC = \angle OAC + \angle ACO$ (Exterior angle property)

   $\Rightarrow 130^\circ = 90^\circ + \angle ACO \Rightarrow \angle ACO = 130^\circ - 90^\circ = 40^\circ$

OR

AP = $5^2 - 3^2$

= 4cm

$\Rightarrow AB = 2 \times 4$

= 8cm

10. Given: $\Delta ABC \sim \Delta DEF$

According to theorem, the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$\therefore \quad \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{BC}{EF}\right)^2$

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$

$= \left(\frac{1.2}{1.4}\right)^2$

$= \left(\frac{6}{7}\right)^2$

$= \frac{36}{49}$

Hence, $\frac{area(\Delta ABC)}{area(\Delta DEF)} = \frac{36}{49}$

11. We know that if a is the first term and d is the common difference, then the arithmetic progression is

a,a + d, a + 2d, a + 3d,...

Here, a = 10 and d = 3.

a = 10

a + d = 10 + 3 = 13

a + 2d = 10 + 2(3) = 10 + 6 = 16

a + 3d = 10 + 3(3) = 10 + 9 = 19

....

So, the arithmetic progression is 10,13,16,19,22,...

12. To prove : $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

We have,

LHS = $\sin^4 A + \cos^4 A$

$= (\sin^2 A)^2 + (\cos^2 A)^2 + 2\sin^2 A \cos^2 A - 2\sin^2 A \cos^2 A$ [By Adding and subtracting $2\sin^2 A\cos^2 A$]

$= (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A$

$= 1 - 2\sin^2 A\cos^2 A$ = RHS

13. $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - cosec^2\theta}$

$= \frac{-1}{-1}$ [Since, $1 + \tan^2\theta = \sec^2\theta$ & $1 + \cot^2\theta = \text{cosec}^2\theta$]

= 1.

14. Given that the height of the cylinder = radius of the cylinder = r (say)

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow 25\frac{1}{7} = \frac{22}{7} \times r^2(r)$

$\Rightarrow \frac{176}{7} = \frac{22}{7} \times r^2(r)$

$\Rightarrow r^3 = \frac{176}{22}$

$\Rightarrow$ r^3 = 8

$\Rightarrow$ r = 2 cm

15. The three-digit numbers divisible by 9 start from 108,117,126,135,...,999

Here,

$a = 108$

$d = 9$

$a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1)(9)$

$\Rightarrow 999 = 108 + 9n - 9$

$\Rightarrow 900 = 9n$

$\Rightarrow$ n=100

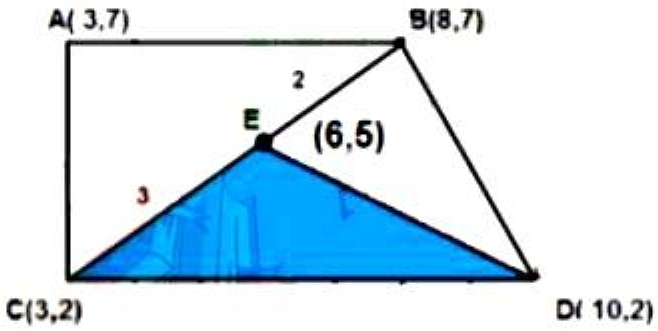
Thus, 100 three-digit numbers are divisible by 9.

16. There 9 odd numbered cards, namely, 1,3,5,7,9 ,1 1 ,1 3 ,1 5 ,1 7 . Out of these 9 cards one card can be drawn in 9 ways.

∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{17}$

17. i. (b) Let us redraw the figure as follows:



Now, Coordinates of C and B are C(3, 2) and B(8, 7).

E divides CB internally in the ratio 3 : 2.

Therefore, the coordinates of E, by applying the section formula, are

$\left(\frac{3\times8+2\times3}{3+2}, \frac{3\times7+2\times2}{3+2}\right)$ = (6, 5)

ii. (c) Area of the $\triangle$ECD $= \frac{1}{2}[6(2-2) + 3(2-5) + 10(5-2)]$

$= \frac{1}{2}[3 \times -3 + 10 \times 3]$ = 10.5 square unit

iii. (a) The coordinates of the flower plants of Ajay and Deepak are (3, 7) and (10, 2) respectively.

Therefore, the distance between plants of Ajay and Deepak

$= \sqrt{(10-3)^2 + (7-2)^2}$

$= \sqrt{74}$ = 8.60 unit

iv. (d) 5 units

v. (b) 7 units

18. i. (a) 450 km

ii. (d) 600 km

iii. (b) AB

iv. (a) Second, 150 km

v. (b) 750 km

19. i. (a) 46-48

ii. (b) 46.9 kg

iii. (a) Mean

iv. (c) 45.85

v. (b) 14K

20. i. (a) 196.25 cm^3

ii. (b) 32.71 cm^3

iii. (c) 163.54 cm^3

iv. (b) 9.81 cm^3

v. (d) 186.44 cm^3

Part-B

21. To Prove: 2+$\sqrt{3}$ is an irratinal number.

Given: $\sqrt{3}$ is irrational number.

Proof: Let 2 + $\sqrt{3}$ be a rational number.

$\Rightarrow 2 + \sqrt{3} = \frac{p}{q}$, p, q $\in$ I, q $\neq 0$

$\Rightarrow \sqrt{3} = \frac{p}{q} - 2$

$= \frac{p-2q}{q}$

$= \frac{integer}{integer}$

$\Rightarrow \sqrt{3}$ is rational number

which is a contradiction to the fact that $\sqrt{3}$ is a rational hence 2 + $\sqrt{3}$ is irrational number.

22. According to the question, A(7, -3), B(5, 3) and C(3, -1).

AD and BE are medians of ΔABC.

D is the mid-point of BC and

E is the mid-point of AC

$\therefore$ Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right) = (4, 1)$

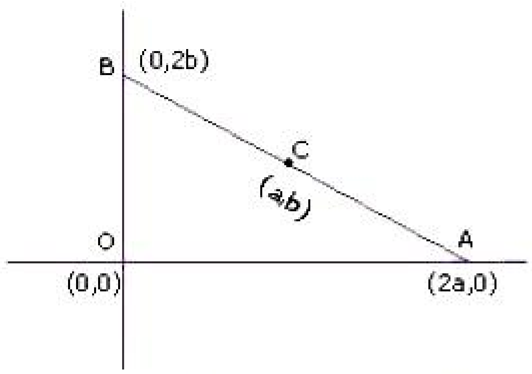
Coordinates of $E = \left(\frac{7+3}{2}, \frac{-3-1}{2}\right) = \left(\frac{10}{2}, \frac{-4}{2}\right) = (5, -2)$

Now, median $AD = \sqrt{(4-7)^2 + (1+3)^2} = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5\ units$

And, median $BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0 + (-5)^2} = \sqrt{25} = 5\ units$

OR



Let A(2a, 0), B(0, 2b) and O(0, 0) are the vertices of right-angled triangle

Coordinate of $C\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

i.e. (a, b)

$OC = \sqrt{a^2 + b^2}$

$AC = \sqrt{a^2 + b^2}$

$BC = \sqrt{a^2 + b^2}$

Hence, C is Equidistant from the vertices.

23. Suppose $\alpha,\ \beta\ and \gamma$ are the zeros of the said polynomial p(x)

Then, we have α = 2, β = -3 and γ = 4

Now,

$\alpha + \beta + \gamma$ = 2 - 3 + 4 = 3......(1)

$\alpha\beta + \beta\gamma + \gamma\alpha$ = 2(-3) + (-3)(4) + (4)(2) = -6 - 12 + 8 = -10 .... (2)

$\alpha\beta\gamma$ = 2(-3)(4) = -24 ..... (3)

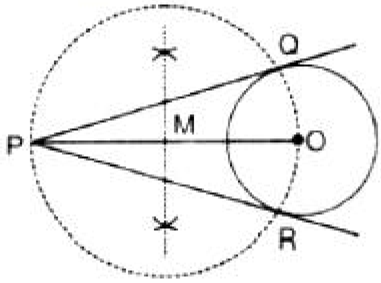
Now, a cubic polynomial whose zeros are $\alpha,\ \beta\ and\ \gamma$ is given by

p(x) = x^3 - $(\alpha+\beta+\gamma)x^2$ + $(\alpha\beta+\beta\gamma+\gamma\alpha)x$ - $\alpha\beta\gamma$

Now putting the values from (1),(2) and (3) we get

p(x) = x^3 -(3)x^2 + (-10)x - (-24)

= x^3 -3x^2 -10x +24

24. 

i. Draw a line segment PO = 13 cm.
ii. From the point O, draw a circle of radius = 5 cm.
iii. Draw a perpendicular bisector of PO. Let M be the mid-point of PO.
iv. Taking M as centre and OM as radius draw a circle.
v. Let this circle intersects the given circle at the point Q and R.
vi. Join PQ and PR which are two tangents to the circle.

25. Given:

$\cos\theta=\frac{3}{4}$
$\Rightarrow\frac{1}{\cos\theta}=\frac{4}{3}$
$\Rightarrow\sec\theta=\frac{4}{3}$

We know that

$\sec^2\theta-\tan^2\theta=1$
$\Rightarrow\left(\frac{4}{3}\right)^2-\tan^2\theta=1$
$\Rightarrow\tan^2\theta=\frac{16}{9}-1$
$\Rightarrow\tan^2\theta=\frac{7}{9}$

Therefore,

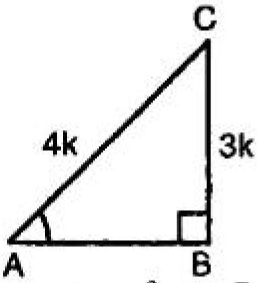
$9\tan^2\theta+9=9\times\frac{7}{9}+9$

= 7 + 9

= 16

OR

Given: A triangle ABC in which $\angle B=90^\circ$



$SinA = \frac{3}{4} = \frac{P}{H}$

Let BC = 3k and AC = 4k where k is a positive integer.

Using Pythagoras theorem,

$AB^2 = AC^2 - BC^2$

$AB = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = \sqrt{7k^2} = k\sqrt{7}$

Therefore, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

$\tan \mathrm{A} = \frac{\mathrm{P}}{\mathrm{B}} = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

26. $AX = AY \quad \rightarrow (1)$

$BX = BZ \quad \rightarrow (2)$

$CZ = CY \quad \rightarrow (3)$

(Tangents from an external

point to a circle are equal)

AB = AC, (Given)

or, AX + XB = AY + YC

or, XB = YC

or, BZ = CZ

∴ Z is the mid-point of BC and Z is the point of contact.

Hence, BC is bisected at the point of contact.

27. For any finite set of primes $\{p_1, p_2, p_3, \ldots\ldots, p_n\}$, Euclid considered the number

$n = 1 + p_1 \times p_2 \times p_3 \times \ldots\ldots \times p_n$

n has a prime divisor p (every integer has at least one prime divisor). But p is not equal to any of the p_i .(If p were equal to any of the p_i , then p would have to divide 1, which is impossible).

So for any finite set of prime numbers, it is possible to find another prime that is not in that set.

In other words, a finite set of primes cannot be the collection of all prime numbers.

Hence, there are infinitely many positive primes.

28. Let seven years ago Swati's age be x years.

As per given condition

Seven years ago Varun's age was five times the square of Swati's age.

Then, seven years ago Varun's age was $5x^2$ years.

Therefore, Swati's present age = (x + 7) years

And Varun's present age = $(5x^2 + 7)$ years

Three years hence Swati's age = (x + 7 + 3) years = (x + 10) years

And three years hence Varun's age = $(5x^2 + 7 + 3)$ years = $(5x^2 + 10)$ years

As per given condition

Three years hence Swati's age will be $\frac{2}{5}$ of Varun's age.

Therefore, $x + 10 = \frac{2}{5}(5x^2 + 10)$

$\Rightarrow x + 10 = 2(x^2 + 2)$

$\Rightarrow x + 10 = 2x^2 + 4$

$\Rightarrow 2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (2x + 3)(x - 2) = 0$

$\Rightarrow x - 2 = 0$ [$\because 2x + 3 \neq 0$ as $x > 0$]

$\Rightarrow x = 2$

Hence, Swati's present age = (2 + 7) years = 9 years

Varun's present age = $(5 \times 2^2 + 7)$ years = 27 years

OR

The given equation is:

$4^{(x+1)} + 4^{(1-x)} = 10$

$4^x.4^1 + 4^1.4^{-x} = 10$ ($\because a^m.a^n = a^{m+n}$)

$4y + \frac{4}{y} = 10$, where we put $4^x = y$

$4y^2 - 10y + 4 = 0$

By middle term splitting,

$4y^2 - 8y - 2y + 4 = 0$

$\Rightarrow 4y(y - 2) - 2(y - 2) = 0$

$\Rightarrow (y - 2)(4y - 2) = 0$

$\Rightarrow y - 2 = 0$ or $4y - 2 = 0$

Therfore, either y = 2 or $y = \frac{2}{4} = \frac{1}{2}$.

Now we have $4^x = y$.

Substituting the value of y we will get the value of x,

Case I: $4^x = 2 \Rightarrow (2)^{2x} = (2)^1 \Rightarrow 2x = 1$

$x = \frac{1}{2}$

Case II: $4^x = 2^{-1}$

$(2)^{2x} = (2)^{-1} \Rightarrow 2x = -1$

$x = -\frac{1}{2}$

Hence the roots of the given equation are $\frac{1}{2}, -\frac{1}{2}$.

29. The given polynomial is:

$p(y) = y^2 + \frac{3}{2}\sqrt{5}y - 5$

For zeroes of f(y), put f(y) = 0

$\Rightarrow y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$

$\Rightarrow 2y^2 + 3 \cdot \sqrt{5}y - 10 = 0$

$\Rightarrow 2y^2 + 4\sqrt{5}y - 1\sqrt{5}y - 10 = 0$

$\Rightarrow 2y(y + 2\sqrt{5}) - \sqrt{5}[y + 2\sqrt{5}] = 0$

$\Rightarrow (y + 2\sqrt{5})(2y - \sqrt{5}) = 0$

Therefore, either $y + 2\sqrt{5} = 0$ or $2y - \sqrt{5} = 0$

$\Rightarrow y = -2\sqrt{5}$ or $y = \frac{\sqrt{5}}{2}$

Now Verification of the relations between α, β and a, b, c

We have, $\alpha = -2\sqrt{5},\ \beta = \frac{\sqrt{5}}{2}$, a = 1, $b = \frac{3}{2}\sqrt{5}$ and c = -5

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow -2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{\frac{-3}{2}\sqrt{5}}{1}$

$\Rightarrow \frac{-4\sqrt{5}+\sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$

$\Rightarrow \frac{-3\sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$

$\Rightarrow$ LHS = RHS

Hence, verified.

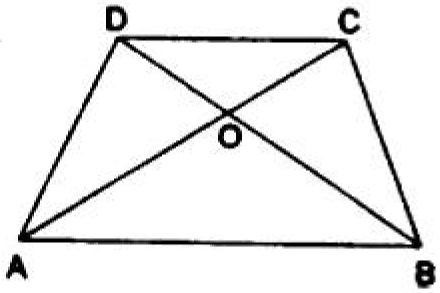
Also we know that $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow (-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right) = \frac{-5}{1}$

$\Rightarrow$ -5 = -5

$\Rightarrow$ LHS = RHS

Hence, verified.

30. 

Given:- ABCD is a trapezium where it is given that AB || DC and AB = 2 × DC. Its diagonals, AC and BD, intersect each other at the point O.

To Find :- $\frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)}$

solution :- In $\triangle AOB$ and $\triangle COD$, we have

$\angle AOB = \angle COD$ [vertical opposite angle]

$\angle OAB = \angle OCD$ [alternate interior angles]

$\angle OBA = \angle ODC$ [Alternate interior angles]

$\therefore \quad \triangle AOB \sim \Delta COD$ [by AAA-similarity criteria].

By using Area Theorem,

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.i.e.,

$\therefore \quad \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{AB^2}{DC^2} = \frac{(2\times DC)^2}{DC^2}$ [∵ AB = 2 × DC, Given]

$= \frac{4\times DC^2}{DC^2} = \frac{4}{1}$

$\Rightarrow \quad \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{4}{1}$

Hence, $\text{ar}(\triangle AOB) : \text{ar}(\triangle COD)$

= 4 : 1.

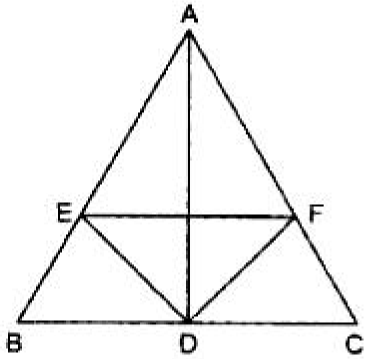
OR

GIVEN : ΔABC in which D is the mid-point of side BC and ED is the bisector of $\angle$ADB, meeting AB in E. EF is drawn parallel to BC meeting AC in F.

TO PROVE :$\angle$EDF is a right angle.

PROOF: In ΔADB, DE is the bisector of $\angle$ADB.

Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.



$\therefore \quad \frac{AD}{DB} = \frac{AE}{EB}$

$\Rightarrow \quad \frac{AD}{DC} = \frac{AE}{EB}$ [$\because$ D is the mid-point of BC $\therefore$ DB = DC] .....(i)

In ΔABC, we have,

$EF \| BC$

Therefore, by basic proportionality theorem,we have,

$\frac{AE}{EB} = \frac{AF}{FC}$ ..... (ii)

From (i) and (ii), we get

$\frac{AD}{DC} = \frac{AF}{FC}$

$\Rightarrow$ In ΔADC, DF divides AC in the ratio AD: DC

$\Rightarrow$ DF is the bisector of $\angle$ADC

Thus, DE and DF are the bisectors of adjacent supplementary angles $\angle$ADB and $\angle$ADC respectively.

Hence, $\angle$EDF is a right angle.

31. Number of identical cards = 44

Out of 44 cards, one card can be drawn in 44 ways.

$\therefore$ Total number of elementary events = 44

Number of circles = 24

Number of blue circles = 9

$\therefore$ Number of green circles = 24 - 9 = 15

Number of squares = 20

Number of blue squares = 11

$\therefore$ Number of green squares = 20 - 11 = 9

i. Number of square = 20

∴ Favourable number of elementary events = 20

Hence, required probability = $\frac{20}{44} = \frac{5}{11}$

ii. Number of green figures = Number of green circles + Number of green square

= 15 + 9 = 24

∴ Favourable number of elementary events = 24

Hence, required probability = $\frac{24}{44} = \frac{6}{11}$

iii. Number of blue circles = 9

∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$

iv. Number of green squares = 9

∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$.

32. Let the distance between pole AB and man be x

∴ Distance between pole CD and man = 80 - x

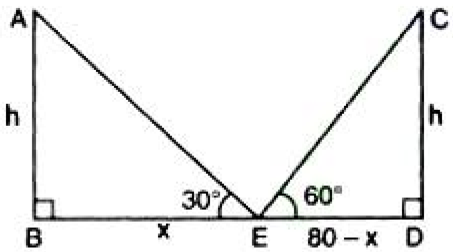
In $\triangle$ ABE, $\angle AEB = 30^\circ$

$\tan 30^\circ = \frac{h}{x}$ (using Pythagoras theorem)

$\sqrt{3} = \frac{h}{x}$

$h = \frac{x}{\sqrt{3}}$ ...... (I)

In $\triangle$ CDE, $\angle CED = 60^\circ$



$\tan 60^\circ = \frac{h}{80-x}$

$\Rightarrow \sqrt{3} = \frac{h}{80-x}$

$\Rightarrow h = 80\sqrt{3} - x\sqrt{3}$ ....(ii)

Comparing (i) and (ii) we get

$\Rightarrow \frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$

$\Rightarrow x = 80 \times \sqrt{3} \times \sqrt{3} - x\sqrt{3} \times \sqrt{3}$

$\Rightarrow$ 4x = 240

$= \frac{240}{4} = 60\text{m}$

Substituting this value of x in (i)

$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

Hence, height of the pole = 60m

Distance between pole CD and man = 80 - x

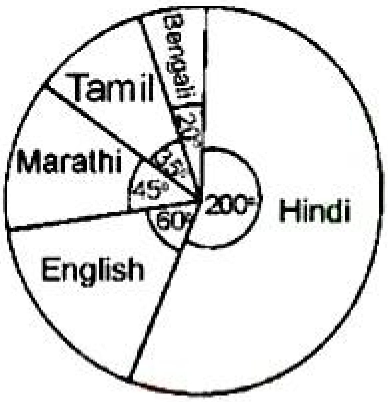
= 80 - 60 = 20m

33.

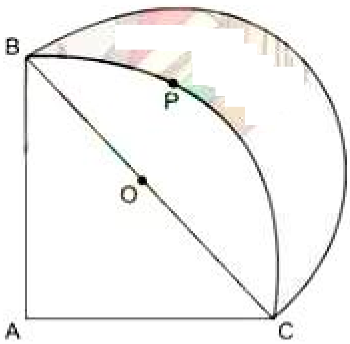
Language	Number of students	Measure of central Angle
Hindi	40	$\frac{72}{\times}360^\circ = 200^\circ$
English	12	$\frac{72}{\times}360^\circ = 60^\circ$
Marathi	9	$\frac{72}{\times}360^\circ = 45^\circ$
Tamil	7	$\frac{72}{\times}360^\circ = 35^\circ$
Bengali	4	$\frac{72}{\times}360^\circ = 20^\circ$
Total	72	360°

A circle of a convenient radius is now drawn and the above angles are marked at the centre of the circle, 5 radii will then divide the whole circle into five required sectors. The different sectors may be shaded with different designs.

The figure gives the required pie chart.



34.



Given, radius of the quadrant AB = AC = 14 cm

$BC^2 = AB^2 + AC^2$

$BC^2 = 14^2 + 14^2$

$\therefore \quad BC = \sqrt{14^2 + 14^2} = 14\sqrt{2}\text{cm}$

$\therefore$ radius of semicircle = $7\sqrt{2}\text{cm}$

Area of semicircle = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}$

$= 11 \times \sqrt{2} \times 7\sqrt{2}$

$= 11 \times 7 \times 2$

= 154 cm^2

Area of segment BPCO $= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2}\sin\theta r^2$

$= r^2\left(\frac{\pi\theta}{360} - \frac{1}{2}\sin\theta\right)$

$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\sin 90^\circ\right)$

$= 14 \times 14 \times \frac{2}{7}$

$= 56 \text{ cm}^2$

Hence, area of shaded region = 56 cm^2

35. **Graph of $2x + 4y = 10$:**

We have,

$2x + 4y = 10$

$\Rightarrow 4y = 10 - 2x$

$\Rightarrow y = \frac{5-x}{2}$

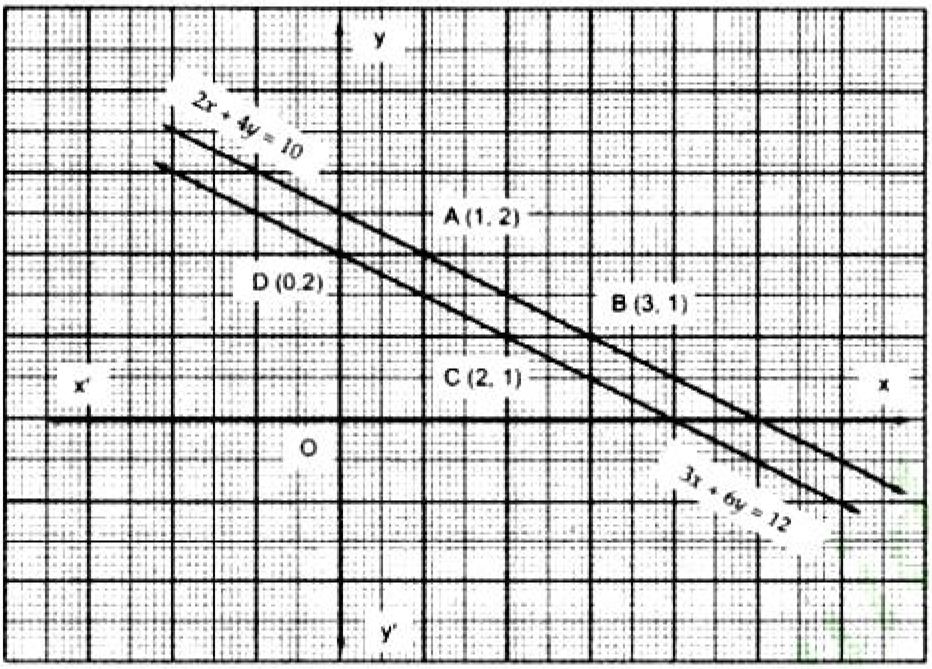
When x=1, we have

$y = \frac{5-1}{2} = 2$

When x=3, we have $y = \frac{5-3}{2} = 1$

x	1	3
y	2	1

Plot the points A (1,2) and B (3,1) on a graph paper. Join A and B and extend it on both sides to obtain the graph of 2x + 4y =10 as shown in Fig.



Graph of 3x + 6y = 12 :

We have, 3x + 6y = 12

$\Rightarrow 6y = 12 - 3x$

$\Rightarrow y = \frac{4-x}{2}$

When x=2, we have

$y = \frac{4-2}{2} = 1$

when x=0, we have

$y = \frac{4-0}{2} = 2$

Thus, we have the following table:

x	2	0
y	1	2

Plot the points C(2,1) and D (0, 2) on the same graph paper. Join C and D and extend it on both sides to obtain the graph of $3x + 6y = 12$ as shown in Fig.

We find the lines represented by equations $2x + 4y = 10\ and\ 3x + 6y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

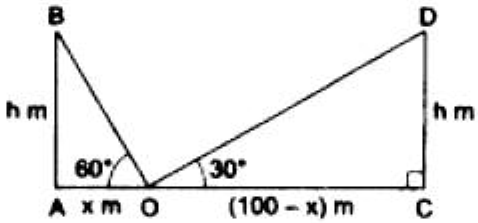
36. Let the height of the equal pillars be AB = CD = h

Given width of the road as AC = 100 m

Let O be the point of observation on AC

Let OA= x metres then

OC= AC - AO = (100 - x)m.



Also according to question $\angle AOB = 60^\circ$ and $\angle COD = 30^\circ$.

$AB \perp AC$ and $CD \perp AC$.

From right ΔOAB, using Pythagoras theorem

$\frac{AB}{OA} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x$ ....(i)

From right ΔOCD, by Pythagoras theorem

$\frac{CD}{OC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h}{(100-x)} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{(100-x)}{\sqrt{3}}$ ....(ii)

Equating the values of h from (i) and (ii), we get

$\sqrt{3}x = \frac{(100-x)}{\sqrt{3}} \Rightarrow 3x = (100 - x) \Rightarrow 4x = 100 \Rightarrow x = 25.$

Putting x = 25m in (i), we get

h = $(25 \times \sqrt{3})$ = (25×1.732) = 43.3.

Hence, the height of each pillar is 43.3 m and the point of observation is 25m away from the first pillar.