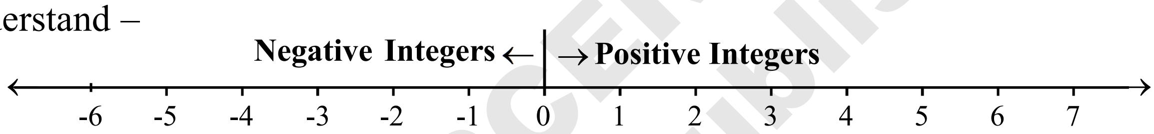
Chapter -1
Integers

D1X9L5

1.1 You have studied preliminary concepts about integers, their representation on the number line in Class VI. We also know that integers are a huge group of numbers where positive and negative integers are arranged in a definite order. In this chapter we will study more about the properties of integers and different operations involved in it.

1.1.1 Let us recall :

We represent positive and negative numbers on the number line. On an infinite line we represent a point as '0' (Zero) and on either side of it, maintaining equal distance between every pair of consecutive numbers we represent positive integers on the right side of zero and negative integers on the left side of it. Observe the line below and try to understand –



Number line

(i) '0' (zero) is neither positive nor negative.

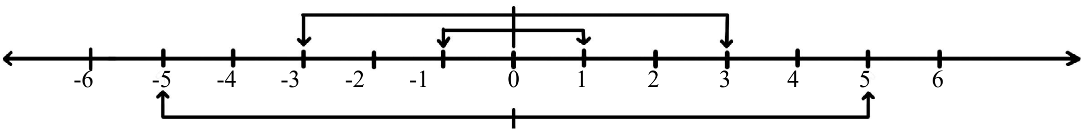
(ii) On right side of '0' are positive numbers and on left side of '0' are negative numbers.

(iii) Natural numbers belong to whole numbers and whole numbers belong to integers ($N \subseteq W \subseteq Z$)

(iv) On the number line numbers on the right side of any number are greater than numbers on the left side of the number.

(v) If a, b are two integers and $a > b$, then $-a < -b$.

For each positive integer there is a corresponding negative integer. For example : corresponding negative numbers of 1, 2, 3, 5 are – 1, –2, –3 and –5. Here the distance between number 1and 0 on the right hand side and the distance between number – 1and 0 on the left hand side are same. Similarly the number 2 and number –2 are equidistant from '0'. In the same way 3 and –3, 5 and –5 etc may be illustrated.



Integers

For addition and subtraction on the number line we proceed in the following way

First number	Second number	Operation	
(i) Positive	Positive	+	Proceed to right hand side of the first number
(ii) Positive	Positive	–	Proceed to left hand side of first number
(iii) Positive	Negative	+	Proceed to left hand side of first number.
(iv) Positive	Negative	–	Proceed to right hand side of first number.

Note : $-(-5)$ means $0-(-5)$, then according to (iv) $-(-5) = 5$.

Let us do :

Fill in the boxes with the sign >, =, <

(i) $2 \boxed{<} 5$

(ii) $4 \;\square\; 7$

(iii) $0 \;\square\; 7$

(iv) $0 \;\square\; (-7)$

(v) $4 \;\square\; (-7)$

(vi) $(-4) \;\square\; 7$

(vii) $(-4) \;\square\; (-7)$

(viii) $(-5) \;\square\; (-13)$

(ix) $(-5) \;\square\; (-2-3)$

(x) $(0-5) \;\square\; (5-0)$

(xi) $(-7)+(-5) \;\square\; (-12)$

(xii) $(-6)+5 \;\square\; 11$

(xiii) $21+3 \;\square\; (-25)+27$

(xiv) $(-3-18) \;\square\; (-25+7)$

(xv) $(-1+4)+(-17+25) \;\square\; (-20)$

Remember :

(i) Sum of two positive integers is a positive integer.

(ii) Sum of two negative integers is a negative integer.

(iii) Sum of a positive integer and negative integer may be a positive integer or a negative integer. In this case if magnitude of positive integer is greater than negative integer than sum is positive and if magnitude of negative integer is greater than positive integer then sum is negative.

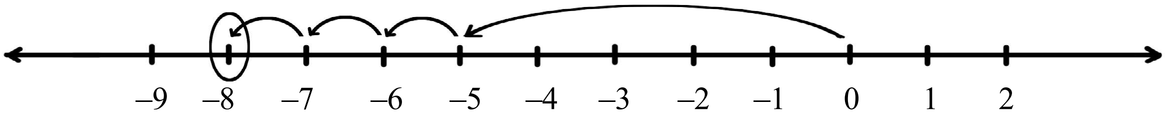
(iv) When we subtract an integer from another integer, we add the first integer with additive inverse of second integer.

Example 1 : Add –5 and –3 on number line.

Solution : Mathematical solution of the statement is

$$(-5)+(-3) = -5-3 = -(5+3) = -8$$

Now according to question to find addition of –5 and –3 on the number line, first we have to go –5 on the left '0' and then again 3 places left of –5 (–6, –7, –8) and thus we get –8. Observe the diagram below



$$(-5) + (-3) = -5-3 = -(5+3) = -8$$

1.2 Properties of addition and subtraction of integers :

1.2.1 Closure property under addition :

We have already seen that – addition of a whole number with another whole number is always a whole number. That is whole numbers obey closure property under addition. Observe the table below :

a	b	$a + b$
2	3	$2 + 3 = 5$
5	–8	$5 + (-8) = -3$
– 4	7	$(-4) + 7 = 3$
– 9	– 11	$(-9) + (-11) = -20$

In all the four cases it is found that sum of two integers is an integer. This property is valid for all the integers for addition. To establish the property you can take any integers and add them.

If a and b are two integers then $a–b$ is always an Integer.
This is called as closure property of integers under subtraction

1.2.2 Closure property under subtraction :

Can we have a whole number when a whole number is subtracted from a whole number ? Observe the following example –

16 – 12 = 4, a whole number, but 10 – 12 = –2, which is not a whole number. That is the whole numbers do not obey closure property under subtraction. Observe the table below:

a	b	$a - b$
– 4	1	$(-4) - 1 = -5$
5	–7	$5 - (-7) = 12$
– 18	– 13	$(-18) - (-13) = -5$

In all the three example it is found that subtraction of two integers is an integer. This property is valid for all the Integers for subtraction.

If a and b are two Integers then $a - b$ is always an Integer.
This is called a closure property of integers under subtraction.

1.3 Integers obey commutative property.

1.3.1 Commutative property under addition :

Whole numbers obey commutative property under addition. Let us see what happens in case of integers.

From the table below it is observed that when we add two integers in any order (changing the position) then there is no change in result (sum) and thus we always get an integer in case of addition.

$a + b$	sum	$b + a$	sum
25 + (–12)	13	(–12) +25	13
(–25)+12	–13	12 + (–25)	–13
(–12) + (–25)	–37	(–25) + (–12)	–37

For any two integers a and b, $a + b = b + a$
This is called as commutative property of integers under addition.

1.3.2 Commutative property under subtraction :

Observe the table below :

$a - b$	difference	$b - a$	difference
(–64) – 24	– 88	24 – (–64)	88
64 – (–24)	88	(–24) – 64	– 88
(–64) – (–24)	– 40	(–24) – (–64)	40

From the above table it is observed that integers do not obey commutative property under subtraction.

Therefore for any two non zero integers *a* and *b*, $a - b \neq b - a$.

1.4 Associative property of integers :

1.4.1 Associative property under addition :

We know that whole numbers obey associative property. Now observe the following examples where any three integers 5, –2, – 6 are taken and added them in following ways. Thus we get–

$$5 + \{(-2) + (-6)\} = 5+(-2-6) = 5 + (-8) = 5 - 8= -3$$

Again $\{5 + (-2)\}+ (-6) = (5-2) - 6 = 3 - 6 = - 3$

So it is found that the same result is coming out while adding them in different groups.

In case of three integers a, b, c we have $a + (b + c) = (a + b) + c$.

You can verify the associative property of integers under addition considering different values for a, b, c.

1.4.2 Associative property under subtraction :

Observe the following subtraction–

Let us consider the numbers – 5, 6, 7

Now $(-5) - 6 - 7 =\{(-5) -6\}-7 = (-11) -7 = (-18)$

Again $(-5) - (6 - 7) = (-5) - (-1) = (-5) +1 = (- 4)$

Thus, $\{(-5)-6\}-7 \neq (-5)-(6-7)$

So it is found that the same result is not coming out while subtracting them in different groups.

In case of three integers *a, b, c* we may have $a-(b-c) \neq (a-b)-c$.
That is **integers do not obey associative property under subtraction.**

1.4.3 Additive Identity :

We know that when we add a whole numbers with '0' then we get same whole number. For example 25 + 0 = 25 or 0 + 25 = 25. That is '0' is a additive identity for whole numbers. Observe the following example

$(-25)+0=-25$ Or $(-23)+0=(-23)$ Or $0+(-23)=(-23)$

It is seen that when we add an integer with '0' then we get same integer and similarly when we add '0' with an integer then we get same integer $a+0=a=0+a$.

Thus, '0' is the additive identity for integers.

Example 1 : Write four pairs of integers (i) each having sum – 7 (ii) differenc 4

Solution :

(i) $(-3)+(-4)$, $(-10)+3$, $(-5)+(-2)$, $(-22)+15$ etc.

(ii) $(-2)-(-6)$, $(-12)-(-16)$, $(-1)-(-5)$, $(-5)-(-9)$ etc.

Exercise -1.1

1. How many integers are there in between 5 and (–13)?
2. Write greatest and smallest integers in between 13 and (–13)?
3. Plot the following integers on number line

   – 6, 4, –10, 5, –1
4. Write 5 negative integers which are greater than –15.
5. Mention if true or false–

   (i) Positive integers are called as Natural Number.

   (ii) All the integers are whole number.

   (iii) Number line is extended to infinity on the both sides of '0'

   (iv) '0' and negative integers form the collection of whole number.

   (v) If $a+b=0$, then one of them is additive inverse of other and vice versa.

6. Write a pair of integers –
   (i) Whose sum is – 3 (ii) Whose difference is – 5
   (iii) Whose sum is 0 (iv) Whose difference is 2
7. Write a pair of negative integers whose subtraction is 6.
8. Find the integers a and b such that (i) $a + b$ is positive (ii) $a \neq b$ (iii) $a - b = 0$
9. Fill in the boxes
   (i) $(-15) + (-4) = (-4) + \square$
   (ii) $\square + \{(-7) + 8\} = \{5 + (-7)\} + 8$
   (iii) $(-23) + \square = -23 = (-23) + \square$
   (iv) $(-19) + \square = (-27)$
   (v) $x + 12 = 0$ হ'লে $x = \square$
10. A man moved 14 kilometers towards East from the position A. But another man moved 6 Kilometers towards West from the position A. What is the distance between them?
11. A man has a deposited ₹ 35 and another man has a debt of ₹ 40. How much rich is first man compared to second man?
12. On a certain Tuesday temperature of Guwahati at 5 am in the morning was 25^0C. But temperature was increased by 8^0C at 2 pm in the afternoon and at 10 pm the temperature was decreased by 3^0C. On Wednesday at 12 noon again temperature was increased by 5^0C.What was the temperature at 12 noon on Wednesday?
13. Anuradha had deposited ₹ 3200 in the bank and next day she had withdrawn ₹ 2,540. How much money left in the account of Anuradha after withdrawal?
14. Sum of two numbers is –5. If one of the number is 18 then what is the other number?
15. What should be added with –23 to get ‘0’?
16. Sum of two integers is – 48. If one of the number is –20 then what is the other number?
17. Evaluate using number line :
   (i) $(+5) - (+3)$ (iii) $(-6) - (+5)$
   (ii) $(+6) + (-5)$ (iv) $(-8) + (-3)$
18. Find whether of the statements are true or false :
   (i) $(-6) + 23 + (-2) = (-2) + (-6) + 23$
   (ii) $(16-15) + (-7) = 16 - \{15 + (-7)\}$

(iii) Natural numbers are closed under subtraction.

(iv) Of the two numbers 0 and – 670, – 670 is greater.

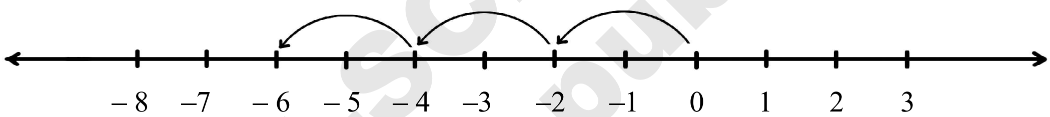
(v) With respect to subtraction of integers commutative property and associative property do not hold.

1.5 Multiplication of Integers :

We have already discussed about addition and subtraction of integers. Now we will discuss how to multiply integers -

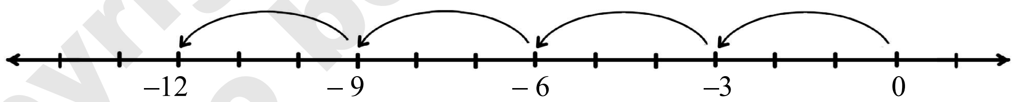
1.5.1 Multiplying a positive integer by a negative integer :

We have already learnt how to multiply whole numbers. Multiplication of whole number is repeated addition of the same whole number. For example, $3 + 3 + 3 + 3 = 4\times3 = 12$. When a positive number is multiplied by another positive number the product is always a positive number. That is **positive number × positive number = positive number.** Now let us observe what happens in case of multiplication of integers. Let us evaluate $3\times(-2)$. $3\times(-2)$ means (-2) is to be added three times. From the number line below we get, $3\times(-2) = (-2) + (-2) + (-2) = -6$.



On the above number line while plotting $3\times(-2)$, the number -2 will be shifted thrice to get -6 towards left side of 0.

Similarly $(-3) + (-3) + (-3) + (-3) = 4 \times(-3) = -12$ ।



Now we will find multiplication of positive integers with negative integers without using number line

Let us consider the above example $4\times(-3)$. First we find the multiplication of 3×4 without considering '–'. We will get 12. Now placing the '–' in front of 12, we will get the required product i.c. (–12). That is $4\times(-3) = -12$.

We can write in this way also $(-4)\times3 = -12$

Therefore $4\times(-3) = (-4)\times3 = -12 = -(4\times3)$

For any two positive integers 'a' and 'b', the integers $-a$ and $-b$ are negative and then

$$a\times(-b) = (-a)\times b = -(a\times b)$$

That is, **Product of a positive integer and a negative integer is a negative integer.**

1.5.2 Product of two negative integers :

Let (– 4) and (– 3) be two integers whose product is to be obtained

We know that

$$(-4) \times 2 = (-8)$$
$$(-4) \times 1 = (-4)$$
$$(-4) \times 0 = 0$$
$$(-4) \times (-1) = 4$$
$$(-4) \times (-2) = 8$$
$$(-4) \times (-3) = 12$$

From the above pattern the numbers multiplying (– 4) are gradually decreased by 1; (2, 1, 0, – 1, – 2, –3) and products are increasing by 4 (– 8, – 4, 0, 4, 8, 12) i.e. from negative to positive. Thus, the product becomes positive when, two negative integers are multiplied.

From above example it follows that the product of two negative integers is always a positive integer.

For any two negative integers $(-a)$ and $(-b)$, $(-a) \times (-b) = a \times b$.

Remember :

(a) Product of integers having same sign (positive and positive or negative and negative) is always a positive integer. $[(+)\times(+) = (+)$ and $(-)\times(-) = (+)]$.

(b) Product of integers having opposite sign (positive and negative or negative and positive) is always a negative integer $[(-)\times(+) = (-)$ and $(+)\times(-) = (-)]$.

Class work : Complete the table –

×	+2	–28	–54	13	0	–1	11
–9							
–12							
30							
–25							
50							
–40							
–115							

1.6 Properties of multiplication of integers :

1.6.1 Closure property under multiplication :

Observe the following multiplication –

$25 \times 25 = 625$ (Product is an integer)

$(-25) \times 25 = -625$ (Product is an integer)

$25 \times (-25) = -625$ (Product is an integer)

$(-25) \times (-25) = 625$ (Product is an integer)

It is observed that, the product of two integers is an integer.

Conclusion : If *a* and *b* are two integers then ($a \times b$) is also an integer.

1.6.2 Commutative property under multiplication :

Observe the following multiplication –

$4 \times 16 = 64 = 16 \times 4$
$(-4) \times 16 = -64 = 16 \times (-4)$
$(-4) \times (-16) = 64 = (-16) \times (-4)$
$4 \times (-16) = -64 = (-16) \times 4$

It is observed that, the product of two integers does not change if we alter the position of integers. If a and b are two integers then $a \times b = b \times a$

That is, integers obey commutativity under multiplication.

Note : Teachers will show the commutative property under multiplication taking different integers.

1.6.3 Property of multiplication of integers by ‘0’ (zreo).

When we multiply a whole number by ‘0’ (zero) the result is ‘0’. In the same way when we multiply an integer by ‘0’ (zero) the result is ‘0’.

For example,

$2 \times 0 = 0$ $\quad -4 \times 0 = 0$ $\quad -26 \times 0 = 0$ $\quad$ বা $0 \times (-26) = 0$

Thus when any integer is multiplied with ‘0’ (zero) or when ‘0’ (zero) is multiplied with any integer then we have $a \times 0 = 0 = 0 \times a$

On the other hand if, a and b are any two integers and $a \times b = 0$ then any one of a and b must be ‘0’ (zero).

1.6.4 Multiplicative Identity of Integers :

1 is the multiplicative identity of all the whole numbers (positive integers) because when we multiply a whole number by ‘1’ the result is the whole number.

Itself observe now, the following multiplication–

$$(-7) \times 1 = -7 = 1 \times (-7)$$
$$(-15) \times 1 = -15 = 1 \times (-15)$$
$$(-101) \times 1 = -101 = 1 \times (-101)$$

It is observed that when we multiply an integer by '1' the result is the same integer. That is '1' is the multiplicative identity of integers.

If a is any integer then $a \times 1 = a = 1 \times a$

Remember : '0' is the additive identity integer and '1' is the multiplicative identity of integers. When we multiply any integers by (–1), then we get additive inverse of the integers.
That is, $a \times (-1) = (-1) \times a = -a$

1.6.5 Associative property of integers under multiplication :

We can apply associative property of integers under multiplication the way in which we have applied for associative property of whole number under multiplication. Let us consider three integers – 3, 2, –5. Observe how these three integers are multiplied using brackets :

$(-3) \times \{2 \times (-5)\} = (-3) \times (-10) = 30$ — first we have multiplied $2 \times (-5)$

Again $\{(-3) \times 2\} \times (-5) = (-6) \times (-5) = 30$ — similarly we have multiplied by $(-3) \times 2$

It is observed that if three integers are multiplied using brackets in different ways the product remains same.

If a, b, c are any three integers.

Then $(a \times b) \times c = a \times (b \times c) = (a \times c) \times b = a \times (c \times b)$.

That is the, integers obey associative property under multiplication.

Activity : Observe whether this property can be applied for multiplication of four integers.

1.6.6 Distributive property of integers :

We have come across that in case of whole number that, $6 \times (8+5) = 6 \times 8 + 6 \times 5$

That is in case of whole numbers $a \times (b + c) = a \times b + a \times c$

Now let us see whether it can be applied in case of integers or not.

Let $a = -2$, $b = -3$, $c = -6$ Then

$$a \times (b + c) = (-2) \times \{-3 + (-6)\}$$
$$= (-2) \times (-9) = 18$$

Again, $a \times b + a \times c$

$= (-2)\times(-3) + (-2)\times(-6)$

$= 6 + 12 = 18$

Therefore $(-2) \times \{(-3)+(-6)\} = \{(-2) \times (-3)\}+\{(-2) \times (-6)\}$

That is, if a, b, c are any three integers then $a\times(b+c) = (a\times b)+(a\times c)$

Integers obey distributive property over addition.

Similarly, $a\times(b-c) = (a\times b)-(a\times c)$

Integers obey distributive property over subtraction also.

1.6.7 Easy technique of finding product :

You have already learnt about different properties under multiplication. Now we will discuss some techniques to find larger product more easily (to some extent orally) by using these properties –

Example 3 : Find $(-25) \times 29 \times (-4)$.

Solution : $\{(-25) \times 29\} \times (-4) = (-725) \times (-4) = 2900$

Easy method : $(-25) \times 29 \times (-4) = \{(-25) \times (-4)\} \times 29$ (Commutative property)

$= 100 \times 29 = 2900$

Example 4 : $75 \times (-6) + (-75) \times 4$

Easy method : $75\times(-6) + (-75) \times 4$

$= 75\times(-6) + 75\times(-4)$

$= 75\times\{(-6) + (-4)\}$ (Distributive property)

$= 75 \times (-10) = -750$

Example 5 : -32×53

Easy method : $(-32)\times(50+3) = \{(-32)\times 50\} + \{(-32)\times 3\}$ (Distributive property)

$= -1600 - 96 = -1696$

Example 6 : $16 \times (-18)$

$= 16\times\{-(20-2)\}$

$= 16\times(-20+2)$

$= 16 \times (-20) + 16\times 2$ (Distributive property)

$= -320 + 32 = -288$

Example 7 : $(-95) \times (-98)$

Easy method : $= -95 \times (-100 + 2)$

$= -95 \times (-100) + (-95) \times 2$ (Distributive property)

$= 9500 - 190 = 9310$

Exercise - 1.2

1. Find the product

   (i) $5 \times (-2)$ (ii) $(-3) \times 7$ (iii) $(-4) \times (-3)$

   (iv) $(-129) \times (-1)$ (v) $(-12) \times 0 \times (-17)$ (vi) $(-22) \times (-11) \times 10$

   (vii) $13 \times (-5) \times (-3)$ (viii) $(-27) \times (-31) \times (-2)$ (ix) $(-3) \times (-1) \times (-2) \times 5$

2. Verify whether true or false

   (i) $27 \times \{(-5)+10\} = 27 \times (-5) + 27 \times 10$

   (ii) $(-25) \times \{(-16)+(-24)\} = (-25) \times (-16) \times (-24)$

   (iii) $a - (-b) = a + b$, where $a = (-75)$, $b = (-20)$

3. (i) Product of any two integers is -33. If one of them is 11, what is the other number?

   (ii) Product of any two integers is 51. If one of them is -1, what is the other number?

   (iii) What is the value of $(-1 \times a)$ for any integer 'a'?

4. Find the product applying appropriate property–

   (i) $125 \times (-54) \times 8$ (ii) $(-25) \times (-97) \times 4$

   (iii) $(-27) \times (-33)$ (iv) $25 \times (-58) + (-58) \times (-35)$

   (v) $15 \times (-25) \times (-4) \times (-10)$ (vi) $(-57) \times (-19) \times 57$

5. Evaluate with the help of distributive and associative property :

   (i) $125 \times (54) \times 8$ (ii) $(-25) \times 75 \times 8 \times (-4)$

   (iii) $225 \times 67 \times 3$

6. Evaluate using distributive property :

   (i) $172 \times 25 + 172 \times 35$ (ii) $159 \times 82 + 159 \times 16 + 159 \times 2$

   (iii) $67 \times 78 + 67 \times (-43) + 67 \times (-25)$ (iv) $999 \times 99 + 99$

   (v) $58 \times 47 + 94$

7. Justify whether right or wrong :

(i) $(-7)\times15\times(-4) = (-7)\times15 + (-7)\times(-4)$ (ii) $(-6)\times23\times(-2) = (-2)\times(-6)\times23$

(iii) $(-5)\times\{(-3)\times2\} = \{(-5)\times(-3)\}\times2$ (iv) $(-175)\times(-1) = -175$

(v) $(-25)\times(-4)\times0 = 100$

1.7 Division of integers :

Division is the opposite of multiplication. You have learnt about multiplication and division of whole numbers. Observe the example below :

$3 \times 4 = 12$

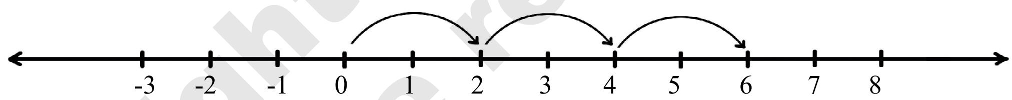
Thus, $12 \div 3 = 4$ and $12 \div 4 = 3$

In the same way from $3 \times 5 = 15$, we get the statements $15 \div 3 = 5$ and $15 \div 5 = 3$. Therefore with any multiplication of whole numbers there are involved two statements. Now let us discuss about division of inetgers –

1.7.1 Division of a positive integer by a positive integer :

We know that, $6 \div 2 = 3$. This means when 6 is divided by 2 we get 3. Let us represent this fact on number line –

To get 6, we have to move three steps towards positive direction on number line each step comprising of two units. Its mathematical representation is $6 \div 2 = 3$. Thus $64 \div 16 = 4$, $500 \div 25 = 20$ etc.



Thus, **the division of a position integer by another positive integer results in a positive integer.**

1.7.2 Division of a positive integer by a negative integer and that of a negative integer by a positive integer :

Observe the table below and fill up the blanks :

Result of multiplicatin	corresponding results of division	
$2\times(-5) = -10$	$(-10) \div 2 = -5$	$(-10) \div (-5) = 2$
$(-3)\times4 = -12$	$(-12) \div (-3) = 4$	$(-12) \div 4 =$ ______
$(-6)\times(-7) = 42$	$42 \div$ ____ $=$ ____	$42 \div$ ____ $=$ ____
$4\times(-8) =$ ______	______	______
$(-11)\times(-15) =$ ____	______	______

It is observed from the examples –

$42 \div (-6) = (-7)$ $\quad$ $(-10) \div 2 = (-5)$

$42 \div (-7) = (-6)$ $\quad$ $(-12) \div 4 = (-3)$

$165 \div (-11) = (-15)$ $\quad$ $(-32) \div 4 = (-8)$

$165 \div (-15) = (-11)$ $\quad$ $(-54) \div 6 = (-9)$

Thus, when a positive integer is divided by a negative integer or when a negative integer is divided by positive number, first we have to proceed like division of a whole numbers and then we have to put '–' sign in front of the result. In this case we get a negative integer in the process of division.

Conclusion : For two integers a, b we have $a \div (-b) = (-a) \div b$, where $b \neq 0$.

1.7.3 Division of a negative integer by a negative integer :

From above examples we also get statements like

$(-10) \div (-5) = 2$, $\quad$ $(-12) \div (-3) = 4$, $\quad$ $(-32) \div (-8) = 4$ etc.

Thus, when a negative integer is divided by a negative integer the result is always a positive integer.

For any two positive a, b we have $(-a) \div (-b) = a \div b$, where $b \neq 0$.

1.8 Properties of divison under integers :

1.8.1 Closure property under division :

We have seen that whole numbers are not closed under division. Now let us observe few examples in case of integers –

$(-49) \div 7 = -7$ (integers); on the other hand $7 \div (-14) = -\frac{1}{2}$ (not an integers)

Similarly $(-16) \div (-2) = 8$ (integers); on the other hand $(-16) \div (-64) = \frac{1}{4}$ (not an integers)

It is observed that, when an integer is divided by another integer the result is not an integer always. That is, **integers are not closed under division.**

Conclusion : If a, b are two integers then $a \div b$ (where $b \neq 0$) may not be an integer.

1.8.2 Commutative property of integers under division :

We have already come across that whole numbers do not obey commutative property under division. Let us verify for integers –

$(-64) \div 16 = -4$ (Integer), on the other hand $16 \div (-64) = -\frac{1}{4}$ (not an Integer)

Similarly, $(-25) \div 5 = -5$ (Integer). On the other hand $5 \div (-25) = -\frac{1}{5}$ (not an Integer)

It is observed that when an integer is divided by another integer the result may not be an integer and also when their positions are interchanged their values are changes.

That is, integers do not obey commutative property under division
If a and b are any two integers then $a \div b \neq b \div a$.

1.8.3 Associative property of integers under division :

We have already come across that whole numbers do not obey associative property under division.

Observe the following statements

$(18 \div 6) \div 3 = 3 \div 3 = 1$; on the other hand $18 \div (6 \div 3) = 18 \div 2 = 9$

That is, $(18 \div 6) \div 3 \neq \quad 18 \div (6 \div 3)$

Similarly $\{(-72) \div 6\} \div 2 = -12 \div 2 = -6$; That is $(-72) \div (6 \div 2) = (-72) \div 3 = -24$

That is, $\{(-72) \div 6\} \div 2 \neq (-72) \div 6 \div 2$

From above examples we observe that, when integers are associated differently their values also change.

That is, **the integers do not obey associative property under division.**
If a, b, c are any three integers then $(a \div b) \div c \neq a \div (b \div c)$.

1.8.4 Division of integers by '0' (zero)

According to property of division when we evaluate $2 \div 0$, we are supposed to find a number such that, the number when multiplied by '0'. The product will be 2. But we know that when we multiply a number by '0', the product is '0'. There is no such number which when multiplied by '0'would result in product 2. Therefore, $2 \div 0$ is meaningless.

'0' is an integer and integer and like the whole numbers, it is meaningless to divide any integer by 0. However, when 0 is divide by any integer other than 0, we get 0. In the same way $0 \div 0$ is also meaningless.

For any integer a, $a \div 0$ can not be determined (not defined). But $0 \div a = 0$, where $a \neq 0$.

1.8.5 Division of integers by 1 and –1 :

We know that, when we divide a positive integer (whole number) by 1 then there is no change in the value of the integer. Now, let us observe what happens in case of negative integers –

$(-11) \div 1 = (-11)$, $(-23) \div 1 = (-23)$, $(-6) \div 1 = (-6)$

We have observed that, when any negative integer is divided by 1, the result is same negative integer. For any integer a, $a \div 1 = a$.

Now, let us see what happens when any integer is divided by –1

$(-6) \div (-1) = 6$, $(-11) \div (-1) = 11$, $(-23) \div (-1) = 23$

From above examples we observed that, when any integer is divided by (–1), the result is the integer opposite in sign to the given integer.

Example 7 : In a quiz competition of a school, for a correct answer 10 marks is awarded and for a wrong answer –5 marks is awarded. Statistics of two competitors are as follows –

(i) First competitor answered all the questions but only 9 answers were correct and secured 35 marks.

(ii) Second competitor answered all the questions but his 6 answer were correct and secured 10 marks. How many questions were answered incorrectly by each of them?

Solution :

(i) For a correct answer marks awarded = 10

Then first competitor will get (for 9 correct answers) = $9 \times 10 = 90$

But his total marks = 35

$\therefore$ For his incorrect answer he was awarded = $35 - 90 = -55$

Again for each incorrect answer marks awarded = –5

$\therefore$ No of questions answered incorrectly = $(-55) \div (-5) = 11$

(ii) For a correct answer marks awarded = 10

Then second competitor will get (for 6 correct answers) = $6 \times 10 = 60$

But his total marks = –10

$\therefore$ For his incorrect answer he was awarded = $(-10) - 60 = -70$

Again for each incorrect answer marks awarded = –5

$\therefore$ No of question answered incorrectly = $(-70) \div (-5) = 14$.

Answer : No. of wrong answers by the first compitition is 11 and by the second competitor is 14.

Exercise -1.3

1. Find the quotient

   (i) $14 \div (-5)$ (ii) $(-60) \div 10$ (iii) $(-54) \div (-6)$

   (iv) $0 \div (-15)$ (v) $(-61) \div \{(-60)+(-1)\}$ (vi) $\{(-72) \div (-6)\} \div (-3)$

2. Fill in the blanks

   (i) $(-600) \div 25 =$ ——— (ii) $\{(-4)\times 18\} \div$ ——— $=12$

   (iii) ——— $\div (5-6) = -20$ (iv) $(-123) \div (-1) =$ ———

3. (i) If $a \div (-7) = 8$, then find the value of integer 'a'.

   (ii) If $125 \div b = -5$, then find the value of integer 'b'.

4. Write three pairs of integers a, b such that $a \div b = -5$

5. In a class test of a school 20 questions were given. For each correct answer 5 marks is awarded and for each wrong answer (–2) is awarded.

   (i) A student answered all the questions. But her 10 answers were correct. How much marks did she score ?

   (ii) Other student could answer only 5 answers correctly. What was the score by the student ?

6. In an examination for each correct answer 5 marks is awareded and for each wrong answer (–2) is awarded.

   (i) Sumon answered all the questions. But her 16 answers were correct and obtained 64.

   (ii) Jaya answered all the questions. But she could answer only 5 questions correctly and obtained (– 6). How many questions were answered incorrectly by them.

7. A rubber company makes a profit of ₹ 15 per bag of rubber. Loss incured on every waste bag of rubber is of ₹ 8.

   (i) The Company sold 1500 bags of good rubber and 500 bags of waste rubber. How much profit or loss incurred by the company?

   (ii) If a the company sold 750 bags of waste rubber then how many bags of good rubber are to be sold so that company incured neither profit nor loss?

What we have learnt

1. Integers comprise of whole numbers and negative of whole numbers.
2. On the number line the right side of '0' (zero) contains positive integers and left side the negative integers.
3. Integers are closed under addition, subtraction and multiplication. But these are not closed under division.
4. Integers obey commutative and associative property under addition and multiplication but do not obey commutative and associative property under subtraction and division.
5. '0' (zero) is the additive identity of integers.
6. Product of integers having same sign is always positive and product of integers having opposite sign is always negative.
7. Integers obey distributive property of multiplication over addition and subtraction.
8. For any integer a, $a \div 0$ can not be determined (not defined). But $0 \div a = 0$, where $a \neq 0$.
9. For any integer a, $a \div 1 = a$ and $a \div (-1) = -a$.