2. MEASURES OF DISPERSION

__EXCERSISE 1.1__

1) Find range of the following data: 575, 609, 335, 280, 729, 544, 852, 427, 967, 250.

SOL: Range = L-S = 967 250 = 717.

2) The following data gives number of typing mistakes done by Radha during a week.

Find the range of the data.

Day	Monday	Tuesday	Wednes-day	Thursday	Friday	Saturday
No. of mistake	15	20	21	12	17	10

Sol. Range = L-S = 21 -10 = 11

3) Find range of the following data.

Classes	62-64	64-66	66-68	68-70	70-72
Frequency	5	3	4	5	3

SOL: Range = L-S = Upper limit of highest class - Lower limit of lowest class

=72-62=210

4) Find the Q.D. for the following data 3,16,8,15,19,11,5,17,9,5,3

Sol: Arranging the data in ascending order, 3, 3, 5, 5, 8, 9, 11, 15, 16, 17, 19.

Here, $n =$11

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ observation

$\left(\frac{11+1}{4}\right)^{th}$ observation

$= 3^{rd}$ observation

$\therefore Q_1 = 5$

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation

$3\left(\frac{11+1}{4}\right)^{th}$ observation

$= 9^{th}$ observation

〖∴Q〗 _(3)= 16

$Q.D. = \frac{Q_3+Q_1}{2}$

$= \frac{16-5}{2}$

$= \frac{11}{2}$

$\therefore \boldsymbol{Q.D. = 5.5}$

5) Given below are the prices of shares of a company for the last 10 days. Find Q.D.

17-2, 164, 188, 214, 190, 237, 200,195, 2'08, 230.

SOL: Arranging the data in ascending order, 164, 172,188, 190, 195, 200, 208, 214, 230, 237.

Here, $n = 10$

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ observation

$= \left(\frac{10+1}{4}\right)^{th}$ observation

= 2.75th observation

= 2nd observation + 0.75 (3rd observation-2nd observation)

= 172 + 0.75 (188-172)

= 172 + 0.75 (16)

= 172 + 12

$\therefore\ Q_1 = 184$

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation

$= 3\left(\frac{10+1}{4}\right)^{th}$ observation

= 3(2.75)th observation
= 8th observation + 0.25 (9th observation-8th observation)

= 214 + 0.25 (230-214)

= 214 + 0.25 (16)

= 214 + 4

$\therefore\ Q_3 = 218$

$Q.D. = \frac{Q_3+Q_1}{2}$

$= \frac{218-184}{2}$

$= \frac{34}{2}$

$\therefore\ \boldsymbol{Q.D. = 17}$

6) Calculate Q. D. for the following data:

X	24	25	26	27	28	29	30
Y	6	5	3	2	4	7	3

SOL:

x	f	$c.f$ (less than type)
24	6	6
25	5	11
26	3	14
27	2	16
28	4	20
29	7	27
30	3	30

	N = 30	

Here, N = 30

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$= \left(\frac{30+1}{4}\right)^{th}$ observation

$= \left(\frac{31}{4}\right)^{th}$

= 7.75th observation

$\therefore \boldsymbol{Q_1} = \mathbf{25}$

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation

$= 3\left(\frac{30+1}{4}\right)^{th}$ observation

= 3(7.75)th observation

= 23.2th observation

$\therefore \boldsymbol{Q_3} = \mathbf{29}$

$Q.D. = \frac{Q_3+Q_1}{2}$

$= \frac{29-25}{2}$

$= \frac{4}{2}$

$\therefore \boldsymbol{Q.D.} = \mathbf{2}$

7) Following data gives the age distribution of 250 employees of a firm. Calculate Q.D. of the distribution.

Age (in years)	20-25	25-30	30-35	35-40	40-45	45-50
No. of employees	30	40	60	50	46	14

SOL:

$Age\ (in\ years)$	$No.of\ employees\ (f)$	$c.f\ (less\ than\ type)$
20-25	30	30
25-30	40	70
30-35	60	130
35-40	50	180
40-45	46	226
45-50	14	240
	N = 240	

Here, N = 240

$For\ Q_{1,} = \frac{N}{4} = \frac{240}{4} = 60$

$\therefore Q_1$ Lies in the class 25-30

$\therefore$ L = 25; $f = 40;\quad c.f = 30;\quad h = 5$

$Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f\right)$

$= 25 + \frac{5}{40}(60 - 30)$

$= 25 + \frac{1}{8}(30)$

$= 25 + 3.75$

$\boldsymbol{Q_1 = 28.75}$

$For\ Q_{3,} = \frac{3N}{4} = 3\left(\frac{240}{4}\right) = 180$

$\therefore Q_3$ Lies in the class 40-45

$\therefore$ L = 40; $f = 46;\quad c.f = 180;\quad h = 5$

$Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f\right)$

$= 40 + \frac{5}{46}(180 - 180)$

$= 40 + \frac{5}{46}$

$\boldsymbol{Q_3 = 40}$

$Q.D. = \frac{Q_3 + Q_1}{2}$

$= \frac{40 - 28.75}{2}$

$= \frac{11.25}{2}$

$\therefore \boldsymbol{Q.D. = 5.625}$

8) Following data gives the weight of boxes. Calculate Q.D. for the data.

Weight (Kg)	10-12	12-14	14-16	16-18	18-20	20-22
No. of boxes	3	7	16	14	18	2
c.f.	3	10	26	40	58	60

SOL:

Weight (Kg)	**No. of boxes**	$\boldsymbol{c.f\ (less\ than\ type)}$
10-12	3	3
12-14	7	10
14-16	16	26
16-18	14	40
18-20	18	58
20-22	2	60
	N = 60	

Here, N = 60

$For\ Q_1, = \frac{N}{4} = \frac{60}{4} = 15$

$\therefore Q_1$ Lies in the class 14-16

$\therefore$ L = 14; $f = 16; \quad c.f = 10; \quad h = 2$

$Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f\right)$

$= 14 + \frac{2}{16}(15 - 10)$

$= 14 + \frac{2}{16}(5)$

$= 14 + 0.625$

$\boldsymbol{Q_1 = 14.0.625}$

$For\ Q_3, = \frac{3N}{4} = 3\left(\frac{60}{4}\right) = 45$

$\therefore Q_3$ Lies in the class 18-20

$\therefore$ L = 18; $f = 18;\quad c.f = 40;\quad h = 2$

$Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f\right)$

$= 18 + \frac{2}{18}(45 - 40)$

$= 18 + \frac{1}{9}(5)$

$= 18 + 0.556$

$\boldsymbol{Q_3 = 18.556}$

$Q.D. = \frac{Q_3 + Q_1}{2}$

$= \frac{18.556 - 14.625}{2}$

$= \frac{3.931}{2}$

$\therefore \boldsymbol{Q.D. = 1.9655}$

EXERCISE 2.2

Find the various and S.D. for the following sets of numbers.

1) 7,11,2,4,9,6,3,7,11,2,5,8,3,6,8,8,8,2,6

SOL:

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
2	3	6	4	12
3	2	6	9	18
4	1	4	16	16
5	1	5	25	25
6	3	18	36	108
7	2	14	49	98
8	3	24	64	192
9	1	9	81	81
11	2	22	121	242
	N = 18	108		792

Var (x) $=\sigma_x^2 = \frac{\Sigma f_i x_i}{N} - (\bar{x})$ 2

$= \frac{\Sigma f_i x_i}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$

$= \frac{792}{18}$ - $\left(\frac{108}{18}\right)^2$

= 44 – 36
= 8

S.D. $\sigma_x = \sqrt{Var\ (x)} = \sqrt{8} = 2\sqrt{2} = 2 \times (1.414)$

= 2.828

2) 65,77,81,98,100,80,129.

SOL:

x_i	$u_i = x_i - a$	u_i^2
65	-24	576
77	-12	144
80	-9	81
81	-8	64
98	9	121
100	11	81
129	40	1600
	7	**2667**

a =89 N = Σu_i=7 Σu_i^2=2667

$var(x)\ \sigma_u^2$ = $\frac{\Sigma u_i^2}{N}$ - ($\overline{u}$) 2 = $\frac{2227}{7}$ - $\left(\frac{7}{7}\right)^2$

= 381-1

=380

$\therefore \sigma_x^2 = \sigma_u^2 = 380$

S.D. (x) $= \sigma_x = \sqrt{Var\ (x)}$ = $\sqrt{380}$ = **19.41**

3) Compute variance and standard deviation for the following data:

x	2	4	6	8	10
f	5	4	3	2	1

SOL:

x	f	fx	x^2	fx^2
2	5	10	4	20
4	4	16	16	64
6	3	18	36	108
8	2	16	64	128
10	1	10	100	100
	15	**70**		**420**

Σf = 15; Σfx^2 = 420 Σfx = 70

Mean, $\overline{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{70}{15}$

= 4.67

Variance = $\frac{\Sigma x_i^2}{N} - (\overline{x})$2

$= \frac{420}{15} - (4.67)$2

= 28 – 21.8089

$\therefore$ **Variance = 6.1911**

S.D., $\sigma = \sqrt{\text{Variance}}$

$= \sqrt{6.1911}$

$\therefore \sigma = 2.4882$

4) Compute the variance and S.D.

x	1	3	5	7	9
f	5	10	20	10	5

SOL:

x	f	fx	x^2	fx^2
1	5	5	1	5
3	10	30	9	90
5	20	100	25	500
7	10	70	49	490
9	5	45	81	405
	50	**250**		**1490**

$\Sigma f = 50$; $\Sigma f x^2 = 1490$ $\Sigma f x = 250$

Mean, $\bar{x} = \frac{\Sigma f x}{\Sigma f}$

$= \frac{250}{50}$

$= 5$

Variance $= \frac{\Sigma x_i^2}{N} - (\bar{x})2$

$= \frac{1490}{50} - (5)2$

$= 29.8 - 25$

$\therefore$ **Variance = 4.8**

S.D., $\sigma = \sqrt{\text{Variance}}$

$\sigma = \sqrt{4.8}$

5) Following data gives age of 100 students in a school. Calculate variance and S.D.

Age (in years)	10	11	12	13	14
No. of students	10	20	40	20	10

SOL:

Age (in years)	No. of students	fx	x^2	fx^2
10 11 12 13 14	10 20 40 20 10	100 220 480 260 140	100 121 144 169 196	1000 2420 5760 3380 1960
	100	**1200**		**14520**

$\Sigma f = 100;\quad \Sigma f x^2 = 14520 \quad \Sigma fx = 1200$

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1200}{100}$

$= 12$

Variance $= \frac{\Sigma x_i^2}{N} - (\bar{x})_2$

$= \frac{14520}{100} - (12)_2$

$= 145.2 - 144$

$\therefore$ **Variance = 1.2**

S.D., $\sigma = \sqrt{\text{Variance}}$

$\sigma = \sqrt{1.2}$

6) The mean and variance of 5 observations are 3 and 2 respectively. If three of the five observations are 1, 3 and 5, find the values of other two observations. .

SOL: Here, n = 5, $\overline{x}$=3, σ^2=2.
Let the two unknown values be a and b.
Here,

x_i	x_i^2
1 3 5 a b	1 9 25 a^2 b^2
$9+a+b$	$35+a^2+b^2$

$\sum x_i = 9+a+b$,

$\sum x_i^2\ 35+a^2+b^2$

Mean, $\overline{x} = \frac{\sum x_i}{n}$

$\therefore\quad 3 \quad = \frac{9+a+b}{5}$

$\therefore\quad 9+a+b \quad = 15$

$\therefore\quad a+b \quad = 6 \qquad \ldots. (1)$

Variance = $\frac{\sum x_i^2}{N} - (\overline{x})2$

$\frac{1}{5}(35+a^2+b^2)$ - 32 = 2

$\frac{1}{5}(35+a^2+b^2) - 9 = 2$

$\frac{1}{5}(35+a^2+b^2) = 11$

$\therefore 35+a^2+b^2 \quad = 55$

$\therefore a^2+b^2 \quad = 20 \qquad \ldots. (2)$

$b = 6 - a$ ....from (1)

Using this in (2) we get

$a^2 + (6 - a)^2 = 20$

$\therefore a^2 + 36 - 12a + a^2 - 20 = 0$

$\therefore 2a^2 - 12a + 16 = 0$

$\therefore 2a^2 - 8a - 4a + 16 = 0$

$\therefore (a - 4)(2a - 4) = 0$

$\therefore a - 4 = 0$ or $2a - 4 = 0$

$a = 4$ or $a = 2$

When $a = 4$; When $a = 2$

$b = 6 - 4$ $b = 6 - 2$

$b = 2$ $b = 4$

i.e. the two unknown values are 2 and 4.

7) Obtain standard deviation for the following date:

Height(in inches)	60-62	62-64	64-66	66-68	68-70
No. of students	4	30	45	15	6

SOL:

Class	f_i	$Mid\_pt\ x_i$	$u_i = \frac{x_i - 65}{2}$	$f_i u_i$	$f_i {u_i}^2$
60-62	4	61	-2	-8	16
62-64	30	63	-1	-30	30
64-66	45	65	0	0	0
66-68	15	67	1	15	15
68-70	6	69	2	12	24

Total	100			-11	85

Since x_i Values are large and also have common difference of 2, we use step-deviation method for easy calculation.

We take $u_i = \frac{x_i - a}{h}$, where $a = 65, h = 2,$ $\Sigma f_i = 100,$ $\Sigma f_i u_i 2 = 85 \Sigma f_i u_i = -11,$

$\therefore \overline{u} = \frac{\Sigma f_i u_i}{\Sigma f_i}$

$= \frac{1}{100} \times (-11) = -0.11$

$\therefore \overline{u} \quad = -0.11$

$\sigma_u^2 = \frac{\Sigma f_i u_i 2}{\Sigma f_i} - (\overline{u})2$

$= \frac{1}{100} \times 85 - (-0.11)2$

$= 0.85 - 0.0121$

$\sigma_u^2 = 0.8379$

$\sigma_u^2 = \sqrt{0.8379}$

$\sigma_u = 0.9154$

$\sigma_x = h.\ \sigma_u = 2 \times 0.9154 = 1.8307$

$\therefore\ \boldsymbol{Standard\ deviation = 1.8307}$

8) The following distribution was obtained change of origin and scale of variable X.

d_i	-4	-3	-2	-1	0	1	2	3	4
f_i	4	8	14	18	20	14	10	6	6

If it is given that mean and variance are 59.5 and 413 respectively, determine actual class intervals.

SOL: We are given means and variance of variable X
i.e.$\overline{x} = 59.5$ and $\sigma_x^2 = 413$

From the given data, we can find means and variance of d_i
i.e. we will find $\bar{d}$ and σ_d^2 as follows:

d_i	f_i	$f_i d_i$	$f_i d_i 2$
-4	4	-16	64
-3	8	-24	72
-2	14	-28	56
-1	18	-18	18
0	20	0	0
1	14	14	14
2	10	20	40
3	6	18	54
4	6	24	96
Total	**100**	**-10**	**414**

$\Sigma f_i = 100$, $\Sigma f_i d_i = -10$, $\Sigma f_i d_i 2 = 414$

$\therefore \bar{d} = \frac{\Sigma f_i u_i}{\Sigma f_i}$

$= \frac{1}{100} \times (-10) = -0.1$

$\therefore \bar{d} = -0.1$

We have $d = \frac{x-a}{h}$ where x is mid-point of class intervals

a = assumed mean

h = class width

$\therefore \quad \bar{x} = a + h\bar{d}$

$\therefore \quad 59.5 = a + h(-0.1)$

$\therefore \quad 59.5 = a - 0.1h$ ... (1)

Next $\sigma_d^2 = \frac{1}{N} \Sigma f_i d_i 2 - \bar{d}$

$\sigma_d^2 = \frac{1}{100}(414) - (-0.01)2$
$\sigma_d^2 = 4.14 - 0.01$

$\therefore \quad \sigma_d^2 = 4.13$

Now, $\quad \sigma_d^2 = h2\, \sigma_d^2$

$\therefore \quad 413 = h2\ (4.13)$

$\therefore \quad h2 = \frac{413}{4.13}$

$\therefore \quad h2 = 100$

$\therefore \quad h = 2$

Substituting $h = 10$ in equation (1)

$59.5 = a - 0.1(10)$

$\therefore \quad 59.5 = a - 1$

$\therefore \quad a = 60.5$

With value of a = 60.5 and h = 10, we work out the actual class – intervals in the following tale;

d_i	Mid-point	Class-intervals
-4	20.5	15.5-25.5
-3	30.5	25.5-35.5
-2	40.5	35.5-45.5
-1	50.5	45.5-55.5
0	60.5	55.5-65.5
1	70.5	65.5-75.5
2	80.5	75.5-85.5
3	90.5	85.5-95.5
4	100.5	95.5-105.5

Note that the value of a = 60.5 corresponds to the value of d = 0. Also since width h = 10, we have to add 5 and subtract 5 for each midpoint to get the class-intervals.

EXERCISE 2.3

1) Mean and standard deviation of two distributions of 100 and 150 items are 50, 5 and 40, 6 respectively. Find the mean and standard deviations of all the 250 items taken together.

SOL: $n_1 = 100$; $n_2 = 150$

$\bar{x}_1 = 50$; $\bar{x}_2 = 40$

$\sigma_1 = 5$; $\sigma_2 = 6$

$\bar{x} = ?$; $\sigma = ?$

Combined Mean,

$$\bar{x} = \frac{n_1 x_2 + n_2 x_2}{n_1 + n_2}$$

$$= \frac{(100)(50)+(150)(40)}{100+150}$$

$$= \frac{5000+6000}{250}$$

$$= \frac{11000}{250}$$

$\therefore \quad \bar{x} = 44$

Now, $d_1 = \bar{x}_1 - \bar{x}$; $d_2 = \bar{x}_2 - \bar{x}$

$= 50 - 44 \qquad = 40 - 44$

$\therefore \quad d_1 = 6 \qquad \therefore d_2 = -4$

Combined Standard Deviation,

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

$$= \sqrt{\frac{100(5^2+6^2)+150(6^2+(-4)^2)}{100+150}}$$

$$= \sqrt{\frac{100(25+36)+150(36+16)}{250}}$$

$$= \sqrt{\frac{100(61)+150(52)}{250}}$$

$$= \sqrt{\frac{6100+7800}{250}}$$

$$= \sqrt{\frac{13900}{250}}$$

$$= \sqrt{55.6}$$

$\therefore \quad \sigma = 7.46$

2) For a certain bivariate data, following information is available.

	X	Y
Mean	13	17
S.D.	3	2
Size3.	10	10

Obtain the combined standard deviation.

SOL: $\bar{x} = 13; \quad \bar{y} = 17$

$\sigma_x = 3; \quad \sigma_y = 2$

$n_x = 20; \quad n_y = 30$

Combined Mean,

$$\overline{xy} = \frac{n_x \bar{x} + n_y \bar{y}}{n_x + n_y}$$

$$= \frac{20(13)+(30)(17)}{20+30}$$

$$= \frac{360+510}{50}$$

$$= \frac{770}{50}$$

$\therefore \quad \overline{xy} = 15.4$

Now, $d_x = \overline{x} - \overline{xy}$; $d_y = \overline{x} - \overline{xy}$

$= 13 - 15.4$ $= 17 - 15.4$

$\therefore$ $d_1 = 2.4$ $\therefore d_2 = 1.6$

Combined variance $\sigma_{xy}^2 = \frac{n_x\left(\sigma_x^2 + d_x^2\right) + n_y\left(\sigma_x^2 + d_y^2\right)}{n_x + n_y}$

$= \frac{20(9+5.76)+30\,(4+2.56)}{20+30}$

$= \frac{20(14.76)+30\,(2.56)}{50}$

$= \frac{295.2+196.8}{50}$

$= \frac{492}{50}$

$\sigma_{xy}^2 = 9.84$

Combined S.D. $\sigma_{xy}^2 = \sqrt{9.84} = 3.1369$

3) Calculate coefficient of variation of marks secured by a student in the exam, where the marks are : 2,4,6,8,10
(Given: $\sqrt{9.6} = 1.8976$)

Sol:

x	x^2
2 4 6 8 10	4 16 36 64 100
30	220

$\therefore \sum x = 30$; $\sum x^2 = 220$

$n = 5$

Mean, $\overline{x} = \frac{\sum x}{n}$

$= \frac{30}{5}$

$= 6$

Standard deviation,

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{220}{5} - (6)^2}$

$= \sqrt{44 - 36}$

$= \sqrt{8}$

$= 2.83$

Co-efficient of variance;

$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{2.83}{6} \times 100$

C.V. = 47.17%

4) Find the coefficient of Variation of a sample which has mean equal to 25 and standard deviation of 5.

Sol: $\bar{x} = 25$ S.D. $(\sigma_x)_x = 5$

C.V. $\frac{\sigma}{\bar{x}} \times 100 = \frac{5}{25} \times 100$

$\therefore$ C.V. $= 20$

5) A group of 65 students of class XI have their average height is 150.4 cm with coefficient of variation 2.5%. What is the standard deviation of their height?

Sol: No. of students $n = 65$

Average height of students $= \bar{x} = 150.4$ cm.

C.V. $(\bar{x}) = 2.5$

S.D. $\sigma_x = ?$

C.V. $= \frac{\sigma_x}{\bar{x}} \times 100$

$\therefore \quad 2.5 = \frac{\sigma_x}{150.4} \times 100$

$\therefore \quad \frac{2.5 \times 150.4}{100} = \sigma_x$

$\therefore \quad \sigma_x = \frac{376}{100}$

$\therefore \quad \sigma_x = 3.76$

6) Two workers on the same job show following results:

Sol:

	Worker P	Worker Q
Mean time for completing the job (hours)	33	21
Standard Deviation (hours)	9	7

(I) Regarding the time r5equired to complete the job, which worker is more consistent?

(II) Which worker seems to be faster in completing the job?

Sol:

	Worker P	Worker Q
Mean time for completing the job (hours)	$\overline{x}_p = 33$	$\overline{x}_p = 21$
Standard Deviation (hours)	$\sigma_P = 9$	$\sigma_Q = 7$

$C.V_P = \frac{\sigma_P}{\overline{x}_P} \times 100 \qquad C.V_Q = \frac{\sigma_Q}{\overline{x}_Q} \times 100$

$= \frac{9}{33} \times 100 \qquad = \frac{9}{33} \times 100$

$C.V_P = 27.2727 \qquad C.V_Q = 27.2727$

(I) $C.V_P < C.V_Q$

∴ Worker P is more consistent

(II) Average time in hours taken by worker Q is less than worker P.

∴ Worker Q is faster than worker P.

7) A company has two departments with 42 and 60 employees respectively. Their average weekly wages are Rs. 750 and Rs. 400. The standard deviations are 8 and 10 respectively.

(I) Which department has a larger bill?
(II) Which department has larger variability in wages?

Sol: Given information can be consolidated as follows:

	Department A	**Department B**
No. of employees	$n_A = 42$	$n_B = 60$
Average weekly wages in Rs.	$\bar{x}_A = 750$	$\bar{x}_B = 400$
S.D.	$\sigma_A = 8$	$\sigma_B = 10$

(I) Average monthly bill

$\bar{x} = \frac{Total\ amount\ of\ bill}{no.\ of\ employees} = \frac{\sum x_i}{n}$

∴ Total amount of bill $= \sum x_i = n\bar{x}$

Here, Total bill of dept A is $\sum x_A$ and Total bill of dept is B is $\sum x_B$

$\bar{x}_A = \frac{\sum x_A}{n}$ $\qquad$ $\bar{x}_B = \frac{\sum x_B}{n}$

∴ $750 = \frac{\sum x_A}{42}$ $\qquad$ ∴ $400 = \frac{\sum x_B}{60}$

∴ $\sum x_A = 750 \times 42$ $\qquad$ ∴ $\sum x_B = 400 \times 60$

$= 31500$ $\qquad$ $= 24000$

$31500 > 24000$

∴ Department A has larger total bill.

(II) C.VA $= \frac{\sigma_A}{\bar{x}_A} \times 100 = \frac{8}{750} \times 100 = 1.0667$

C.VB $= \frac{\sigma_B}{\bar{x}_B} \times 100 = \frac{10}{400} \times 100 = 2.5$

C.VB > C.VA

∴ Department A has larger variability is wages.

8) The following table gives weight of the students of class A. Calculate the Coefficient of variation.

(Given: $\sqrt{0.8} = 0.8944$)

Weight (in kg)	Class A
25-35	8
35-45	4
45-55	8

Sol:

Weight (in kg)	Class A (f)	x	fx	x^2	fx^2
25-35	8	30	240	900	7200
35-45	4	40	160	1600	6400
45-55	8	50	400	2500	20000
	20		**800**		**33600**

∴ $\Sigma f = 20$; $\Sigma f x^2 = 33600$ $\Sigma fx = 800$

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{800}{20}$

∴ $\bar{x} = 40$

Standard deviation,

$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$= \sqrt{\frac{33600}{20} -}$ $(40)2$

$= \sqrt{1680 - 1600}$

$= \sqrt{80}$

$\therefore \quad \sigma \quad = 8.944$

Co-efficient of variance;

$C.V. = \frac{\sigma}{x} \times 100$

$= \frac{8.944}{40} \times 100$

$\therefore$ **C.V. = 22.36%**

9) Compute coefficient of variation for team A and team B.

(Given: $\sqrt{26} = 5.099, \sqrt{22} = 4.6904$)

No. of goals	0	1	2	3	4
No. of matches played by team A	18	7	5	16	14
No. of matches played by team B	14	16	5	16	17

Which team is more consistent?

Sol: <u>→ For team A</u>

No. of goals (x)	No. of matches (f)	fx	$x2$	$fx2$
0	18	0	0	0
1	7	7	1	7
2	5	10	4	20
3	16	48	9	144
4	14	56	16	224
	60	**121**		**395**

$\therefore \Sigma f = 60; \quad \Sigma f x^2 = 395 \quad \Sigma fx = 121$

$$\text{Mean, } \bar{x}_A = \frac{\Sigma fx}{\Sigma f} = \frac{121}{60}$$

$\therefore \quad \bar{x}_A = 2.017$

Standard deviation,

$$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$$

$$= \sqrt{\frac{395}{60} - (2.017)^2}$$

$$= \sqrt{6.583 - 4.068}$$

$$= \sqrt{2.515}$$

$\therefore \quad \sigma A = 1.59$

Co-efficient of variance;

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.59}{2.027} \times 100$$

$\therefore$ **C.V.A = 78.83%**

→ For team B

No. of goals (x)	**No. of matches (f)**	fx	x^2	fx^2
0	14	0	0	0
1	16	16	1	16
2	5	10	4	20
3	18	54	9	162
4	17	68	16	272
	70	**148**		**470**

$\therefore \Sigma f = 70; \quad \Sigma f x^2 = 470 \quad \Sigma fx = 148$

Mean, $\bar{x}_B = \frac{\Sigma fx}{\Sigma f}$

$= \frac{148}{70}$

$\therefore \quad \bar{x}_B = 2.114$

Standard deviation,

$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$

$= \sqrt{\frac{470}{70} - (2.114)^2}$

$= \sqrt{6.714 - 4.469}$

$= \sqrt{2.245}$

$\therefore \quad \sigma B = 1.5$

Co-efficient of variance;

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.5}{2.114} \times 100$$

$\therefore$ **C.V.B = 70.96%**

$\therefore$ **C.V.B > C.V.A**

$\therefore$ Team B is more consistent

10) Given below is the information about marks obtained in Mathematics and Statistics by 100 students in a class. Which subject shows A the highest variability in marks?

	Mathematics	Statistics
Mean	20	25
S.D.	2	3

Sol: From given information

	Mathematics	Statistics
Mean	$\overline{x}_M = 20$	$\overline{x}_S = 25$
S.D.	$\sigma M = 2$	$\sigma M = 3$

$$\text{C.V. } (M) = \frac{\sigma M}{\overline{x}_M} \times 100 = \frac{2}{20} \times 100 = 10\%$$

$$\text{C.V. } (S) = \frac{\sigma S}{\overline{x}_S} \times 100 = \frac{3}{25} \times 100 = 12\%$$

$$\text{C.V. } (S) > \text{C.V. } (M)$$

∴ Statistics shows more variability than mathematics.