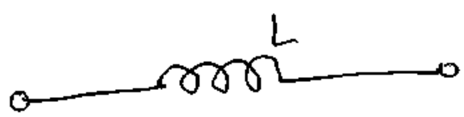
Ch-3 State Space Analysis

-State gives future behaviour of the system based on present input and past history of system.
Past history =(initial condition) of system is described by state variables.

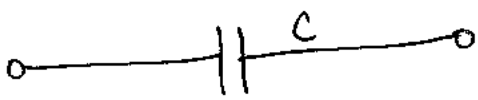
-State of dynamic system is nothing but minimum set of variables such that knowledge of this state variable at $t=t_0$ and with knowledge of their inputs at for $t>t_0$ we can get future behaviour of system for $t>t_0$

-Resistive ckt don't have any state variable because present output doesn't depends on past history of system. So in resistive ckt present output depends only on present input. Resistive ckt can't store any energy.

So i.e. no past history ⇒ no STATE VARIABLES. So pure resistive ckt is called STATIC/MEMORYLESS



$I_L \rightarrow$ state variable



$V_c \rightarrow$ state variable

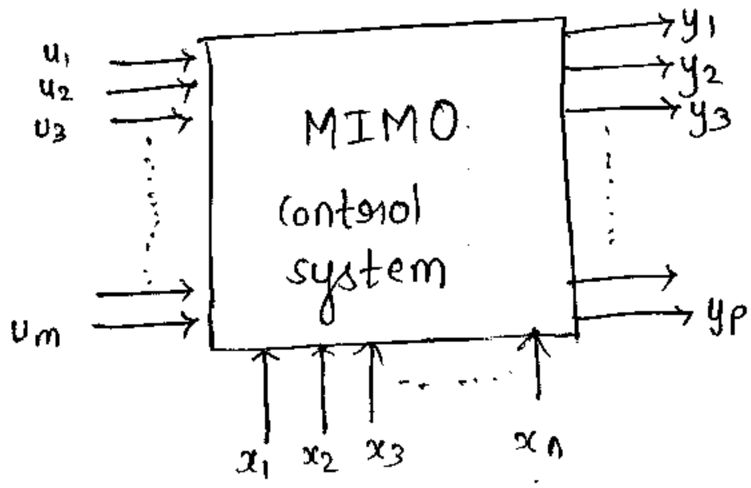
Transfer function approach	State space Analysis
① LTI system only	① Non-linear & linear time invariab and time variant system
② Initial conditions are zero	② Initial conditions are not zero
③ Internal states of the system are not taking into account	③ Internal states of the system are taking into account

Advantages of state space analysis

- State space analysis is applicable to both linear, non-linear and time invariant, Time variant system whereas T.F. approach is applicable to LTI system only
- I.C. are taking into account whereas T.F. approach doesn't.
- The internal states of the system can also be determine by this approach unlike transfer function approach.
- Analysis of multiple input & multiple output is easy with this approach but by using T.F. approach it is very cumbersome.

- Controllability and observability can also be determined by this approach ~~an~~ unless transfer function approach.

*State space representation



m: number of inputs

p: number of output

n: number of state variable

* Standard form

differential state variable

$$\dot{X} = A \cdot X + B \cdot U \quad \text{——— (1)}$$

$$Y = C X + D \cdot U \quad \text{——— (2)}$$

output vector   state variable   input vector

A = state matrix

B = input matrix

C = output matrix

D = transmittance matrix

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$$

$$\dot{X}_{n\times 1} = A_{n\times n} \cdot X_{n\times 1} + B_{n\times m} \cdot U_{m\times 1}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$$

$$Y_{n\times 1} = C_{p\times n} X_{n\times 1} + D_{p\times m} \cdot U_{m\times 1}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix}_{n\times 1}$$

- If RLC circuit is given then no. of state variables is equal to no. of memory storage elements

- If differential eq^n is given the no. of state variables is equal to no. of order of diff. eq^n

* State space representation to differential eq^n:

Q:- Find state model to following system?

$$\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 10U$$

No. of state variables = Order of diff. eq^n

No. of state variable = 3
$$\downarrow$$
x_1, x_2, x_3

→ Differential state variable ⇒ $\dot{x}_1, \dot{x}_2, \dot{x}_3$

Let $\boxed{y = x_1}$

$\dot{y} = \frac{dx_1}{dt} = \boxed{\dot{x}_1 = x_2}$

$\ddot{y} = \boxed{\dot{x}_2 = x_3}$

$\dddot{y} = \dot{x}_3$

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$\dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 10U$

$$\boxed{\dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 10U}$$

$\dot{X}_1 = 0x_1 + x_2 + 0 \cdot x_3 + 0 \cdot U$

$\dot{X}_2 = 0 \cdot x_1 + 0 \cdot x_2 + x_3 + 0 \cdot U$

$\dot{X}_3 = -7x_1 - 5x_2 - 3x_3 + 10U$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix}_{3\times3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}_{3\times1} U$$

$$y_{1\times1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{1\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$$

Short-cut

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \multicolumn{6}{l}{\text{co-efficient from L} \to \text{R with opposite sign (last first)}} \end{bmatrix}_{n \times n}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ K \end{bmatrix} \rightarrow \text{co-efficient of } U$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Q:- $\overset{.....}{y} + 2\overset{....}{y} + 4\dddot{y} + 6\ddot{y} + 8\dot{y} + 10y = 5U$

No. of state variable $= 5$

$\downarrow$

x_1, x_2, x_3, x_4, x_5

let $y = x_1$

$\dot{y} = \boxed{\dot{x}_1 = x_2}$

$\ddot{y} = \boxed{\dot{x}_2 = x_3}$

$\dddot{y} = \boxed{\dot{x}_3 = x_4}$

$\overset{....}{y} = \boxed{\dot{x}_4 = x_5}$

$\overset{.....}{y} = \dot{x}_5$

$$\boxed{\dot{x}_5 = -2x_5 - 4x_4 - 6x_3 - 8x_2 - 10x_1 + 5U}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -10 & -8 & -6 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{1\times 5} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}_{5\times 1} + 0$$

State models are not unique

There are 4 different types of state model

1. Controllable canonical form CCF
2. Observable canonical form OCF
3. Diagonalization OR Normal form
4. Jordan canonical form

Above form is controllable canonical form.

Q:- How to find OCF from CCF

$$A_{OCF} = [A_{CCF}]^T$$

$$B_{OCF} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}_{5\times 1} \begin{matrix} \text{End} \\ \uparrow \\ \text{Start} \end{matrix} \qquad = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{OCF}$$

$$C_{CCF} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{1\times5}$$

end ← start

$$C_{OCF} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{1\times5}$$

* State model to the transfer function

Q:- Write state model to the given transfer function?

$$\frac{Y[s]}{U[s]} = \frac{2s+3}{s^2+5s+6}$$

$$\frac{Y[s]}{X[s]} \frac{X[s]}{U[s]} = \frac{2s+3}{s^2+5s+6}$$

$$\frac{Y[s]}{X[s]} = 2s+3 = N(s) \quad ; \quad \frac{X[s]}{U[s]} = \frac{1}{s^2+5s+6} = \frac{1}{D(s)}$$

$$Y[s] = 2sX[s] + 3X[s]$$

$$s^2 X[s] + 5s X[s] + 6X[s] = U[s]$$

$$\ddot{x} + 5\dot{x} + 6x = u$$

$$y = 2\dot{x} + 3x$$

$$\ddot{x} = \dot{x}_2$$

$$\dot{x} = \boxed{\dot{x}_1 = x_2}$$

$$\boxed{x = x_1}$$

$$y = 2x_2 + 3x_1$$

$$\boxed{\dot{x}_2 = -6x_1 - 5x_2 + u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}_{2\times2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Short-cut

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \multicolumn{6}{l}{[\text{co-efficient of } D(s) \text{ from Last} \to \text{First with opp. sign}]} \end{bmatrix}_{n \times n}$$

$$C = \left[\text{co-efficient of } N(s) \text{ from } L \to F \text{ with same sign}\right]_{1 \times p}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ K \end{bmatrix}.$$

NOTE:-

$$\boxed{s^n = \dot{X}_n}$$

Q:- $\dfrac{Y[s]}{U[s]} = \dfrac{2s^3+4s^2+6}{s^5+3s^4+5s^3+7s^2+9s+10}$

$$\frac{Y[s]}{X[s]} \cdot \frac{X[s]}{U[s]} = \frac{2s^3+4s^2+6}{s^5+3s^4+5s^3+7s^2+9s+10}$$

$$\frac{Y[s]}{X[s]} = 2s^3+4s^2+6$$

$$Y[s] = 2s^3X[s] + 4s^2X[s] + 6X[s]$$

$$y = 2\dddot{x} + 4\ddot{x} + 6x$$

$$\begin{array}{cccccc} s^5 & +3s^4 & +5s^3 & +7s^2 & +9s & +10 \\ \dot{x}_5 & \dot{x}_4 & \dot{x}_3 & \dot{x}_2 & \dot{x}_1 & x_1 \\ & \| & \| & \| & \| & \\ & x_5 & x_4 & x_3 & x_2 & \end{array}$$

~~Let $y = x_1$~~

$\ddot{x} = \dot{x}_2 = x_3$, $\dddot{x} = \dot{x}_3 = x_4$

$\dot{x} = \dot{x}_1 = x_2$

$x = x_1$

$\therefore\ y = 2x_4 + 4x_3 + 6x_1$

$$\frac{X[s]}{U[s]} = \frac{1}{s^5+3s^4+5s^3+7s^2+9s+10}$$

$$X[s]s^5 + 3s^4X[s] + 5s^3X[s] + 7s^2X[s] + 9sX[s] + 10X[s] = U[s]$$

$$\overset{\cdot\cdot\cdot\cdot\cdot}{x} + 3\overset{\cdot\cdot\cdot\cdot}{x} + 5\dddot{x} + 7\ddot{x} + 9\dot{x} + 10x = U$$

$$\dot{x}_5 + 3x_5 + 5x_4 + 7x_3 + 9x_2 + 10x_1 = U$$

$$x_5 = U - 3x_5 - 5x_4 - 7x_3 - 9x_2 - 10x_1$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -10 & -3 & -7 & -5 & -3 \end{bmatrix}_{5\times5}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}_{5\times1} \qquad C = \begin{bmatrix} 3 & 0 & 2 & 1 & 0 \end{bmatrix}_{1\times5}$$

Q:- $$\frac{Y[s]}{U[s]} = \frac{1}{(s+1)(s+2)(s+3)}$$

$$\frac{Y[s]}{U[s]} = \frac{k_1}{s+1} + \frac{k_2}{s+2} + \frac{k_3}{s+3}$$

$$\frac{Y[s]}{U[s]} = \frac{1/2}{s+1} + \frac{-1}{s+2} + \frac{1/2}{s+3}$$

$$Y[s] = \frac{U[s]}{2(s+1)} + \frac{(-1)\,U[s]}{s+2} + \frac{U[s]}{2(s+3)}$$

$$Y[s] = \underbrace{\frac{1/2\, U[s]}{s+1}}_{X_1} + \underbrace{\frac{(-1)\, U[s]}{s+2}}_{X_2} + \underbrace{\frac{1/2\, U[s]}{s+3}}_{X_3}$$

$$\therefore \; y = X_1 + X_2 + X_3$$

$$X_1 = \frac{\frac{1}{2}u[s]}{s+1}$$

$$sX_1 + X_1 = \frac{1}{2}u[s]$$

$$\dot{x}_1 + x_1 = \frac{1}{2}u$$

$$\boxed{\dot{x}_1 = -x_1 + \frac{1}{2}u}$$

$$x_2 = \frac{(-1)\ u[s]}{s+2}$$

$$sx_2 + 2x_2 = -u(s)$$

$$\dot{x}_2 + 2x_2 = -u$$

$$\boxed{\dot{x}_2 = -u - 2x_2}$$

$$x_3 = \frac{\frac{1}{2}u[s]}{s+3}$$

$$\boxed{\dot{x}_3 = -3x_3 + \frac{1}{2}u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}}_{\text{no. of poles location}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overset{B}{\underbrace{\begin{bmatrix} 1/2 \\ -1 \\ 1/2 \end{bmatrix}}_{\text{Partial fraction co-efficient}}} U$$

$$Y = \underset{C}{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Matrix B & C can be interchanged in diagonlization form.

Q:- $\frac{Y[s]}{U[s]} = \frac{2(s+1)}{(s+3)(s+4)(s+6)}$

$$= \frac{k_1}{s+3} + \frac{k_2}{s+4} + \frac{k_3}{s+6}$$

$\frac{2(-2)}{(1)(3)}$

$\frac{-3 \cdot 2}{-1(2)}$

$\frac{2(-5)}{(-2)(-3)}$

$$= \frac{-4/3}{s+3} + \frac{3}{s+4} + \frac{-5/3}{s+6}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -6 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} -4/3 \\ 3 \\ -5/3 \end{bmatrix}}_{B} U$$

$$Y = \underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

* Jordan canonical form

Q:- $\frac{Y[s]}{U[s]} = \frac{1}{(s+2)^2 (s+3)}$

$$\frac{Y[s]}{U[s]} = \frac{K'}{(s+2)^2} + \frac{K''}{s+3}$$

$$\frac{1}{(s+2)^2(s+3)} = \frac{k_1}{s+2} + \frac{k_2}{(s+2)^2} + \frac{k_3}{s+3}$$

$$1 = k_1(s+2)(s+3) + k_2(s+3) + k_3(s+2)^2$$

$1 = k_2(1)$ $\quad \boxed{1 = k_3} \quad$ $1 = 6k_1 + 3k_2 + 4k_3$

$\boxed{k_2 = 1}$ $\quad \boxed{k_1 = -1}$

$$\frac{Y[s]}{U[s]} = \frac{-1}{s+2} + \frac{1}{(s+2)^2} + \frac{1}{(s+3)}$$

$$\frac{Y[s]}{U[s]} = 1 \cdot \underbrace{\frac{U[s]}{(s+2)^2}}_{x_1} - 1 \underbrace{\frac{U(s)}{s+1}}_{x_2} + 1 \cdot \underbrace{\frac{U(s)}{s+3}}_{x_3}$$

$$y = k_1 x_1 + k_2 x_2 + k_3 x_3$$

$$x_2 = \frac{U(s)}{s+2}$$

$$\boxed{\dot{x}_2 = -2x_2 + U}$$

$$x_1 = \frac{U(s)}{(s+2)^2} = \underbrace{\frac{U(s)}{(s+2)}}_{x_2} \cdot \frac{1}{(s+2)}$$

$$x_1 = x_2 \cdot \frac{1}{(s+2)}$$

$$s x_1 + 2x_1 = x_2$$

$$\boxed{\dot{x}_1 = -2x_1 + x_2}$$

$$\boxed{\dot{x}_3 = -3x_3 + U}$$

Partial fraction coefficient

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{B} U$$

$$Y = \underbrace{\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q: $\frac{Y[s]}{U[s]} = \frac{1}{(s+5)^3(s+3)}$

$$A = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ \hline 0 & 0 & 0 & -3 \end{bmatrix}_{4\times4}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \hline 1 \end{bmatrix}$$

$$C = [k_1 \;\; k_2 \;\; k_3 \;\; k_4] = [-1/2 \;\; -1/4 \;\; -1/8 \;\; 1/8]$$

$$\frac{1}{(s+5)^3(s+3)} = \frac{k_1}{(s+5)^3} + \frac{k_2}{(s+5)^2} + \frac{k_3}{(s+5)} + \frac{k_4}{s+3}$$

$$1 = k_1(s+3) + k_2(s+5)(s+3) + k_3(s+5)^2(s+3) + k_4(s+5)^3$$

At $s=-3$

$1 = k_4(2)^3$

$\boxed{k_4 = \frac{1}{8}}$

At $s=-5$

$1 = k_1(-2)$

$\boxed{k_1 = \frac{-1}{2}}$

Co-efficient of s^3

$0 = k_3 + k_4$

$\boxed{k_3 = -1/8}$

At $s=0$

$$1 = 3k_1 + 15k_2 + 75k_3 + 125k_4$$

$$1 = 3\left(\frac{-1}{2}\right) + 15k_2 + 75\left(\frac{-1}{8}\right) + 125 \cdot \frac{1}{8}$$

$$1 = \frac{-3}{2} + 15k_2 + \frac{\cancel{50}\;25}{\cancel{8}\;4}$$

$$1 = \frac{-6+25}{4} + 15k_2 \qquad 1 = \frac{19}{4} + 15k_2$$

$\boxed{k_2 = -1/4}$

* State Model to SFG (signal flow graph)

$\frac{1}{s} \rightarrow$ term $\rightarrow \boxed{\int}$

$\boxed{\int} \rightarrow 1/s$

$\boxed{d/dt} \rightarrow s$

$\Rightarrow \quad \dot{x}_1 \xrightarrow{1/s} x_1$

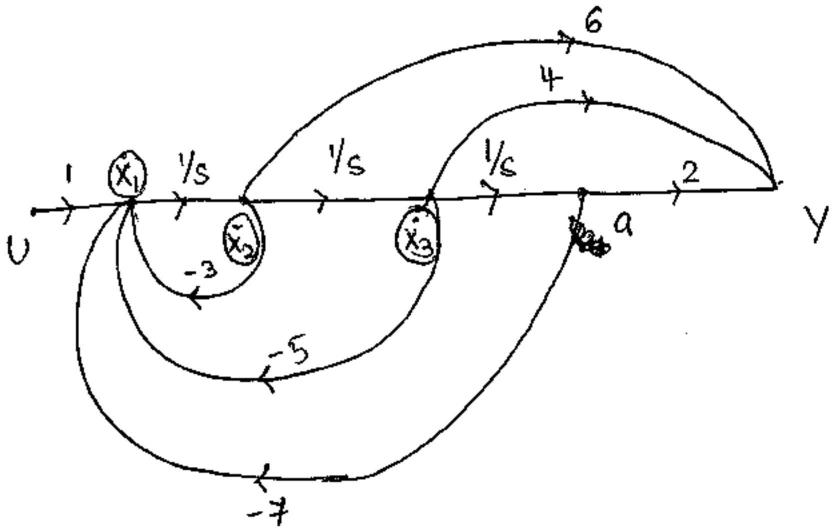
$$\dot{x} = \frac{d}{dt} x$$

$$\int \frac{d}{dt} x \, dt = x$$

NOTE:-

$\Rightarrow$ To select a node as a state variable the incoming branch ~~has~~ to particular node must be integrator

Q:-



For state variable, no. of 1/s terms = no. of state variable

$$\dot{X}_1 = -3\dot{X}_2 - 5\dot{X}_3 - 7a + U$$

$$\dot{X}_2 = \frac{1}{s}\dot{X}_1 = X_1 \qquad \boxed{\dot{X}_2 = X_1}$$

$$\dot{X}_3 = \frac{1}{S}\dot{X}_2 = X_2$$

$$\boxed{\dot{X}_3 = X_2}$$

$$a = \dot{X}_3 \cdot \frac{1}{S} = X_3$$

$$\boxed{\dot{X}_1 = -3X_1 - 5X_2 - 7X_3 + U}$$

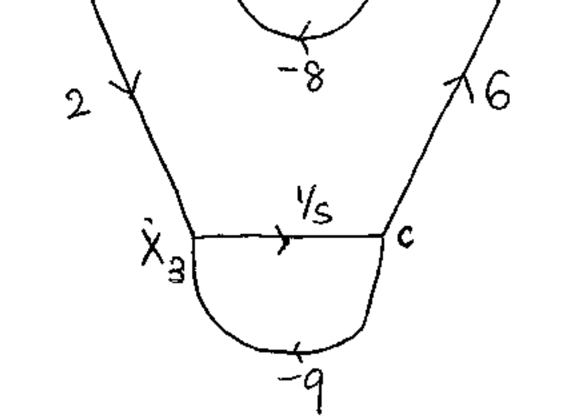
$$Y = 2a + 4\dot{X}_3 + 6\dot{X}_2$$

$$\boxed{Y = 6X_1 + 4X_2 + 2X_3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & -5 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B} U$$

$$Y = \underbrace{\begin{bmatrix} 6 & 4 & 2 \end{bmatrix}}_{C} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Q:-



$$\dot{X}_1 = -7X_1 + 3b$$

$$\boxed{\dot{X}_1 = -7X_1 + 3X_2}$$

$$\boxed{a = X_1}$$

$$\boxed{b = X_2}$$

$$\boxed{c = X_3}$$

$$\boxed{\dot{x}_2 = u - 8x_2}$$

$$y = 5b + 4a + 6c$$

$$\boxed{y = 5x_2 + 4x_1 + 6x_3}$$

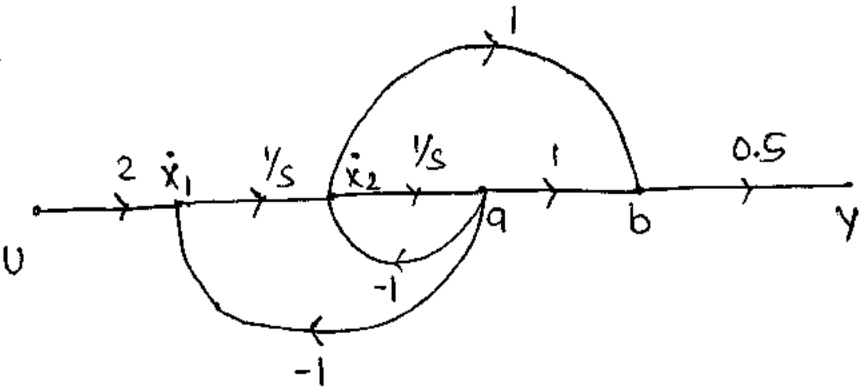
$$\dot{x}_3 = 2u - 9\dot{c}$$

$$\boxed{\dot{x}_3 = 2u - 9x_3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -7 & 3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -9 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}_{c} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q:-



$$\dot{x}_1 = 2u - a$$

$$a = \dot{x}_2 \cdot \frac{1}{s} = x_2$$

$$\boxed{\dot{x}_1 = 2u - x_2}$$

$$\boxed{a = x_2}$$

$$b = a + \dot{x}_2$$

$$\dot{x}_2 = x_1 - a$$

$$b = x_1 - \dot{x}_2 + \dot{x}_2$$

$$\boxed{\dot{x}_2 = x_1 - x_2}$$

$$\boxed{b = x_1}$$

$$y = 0.5b$$

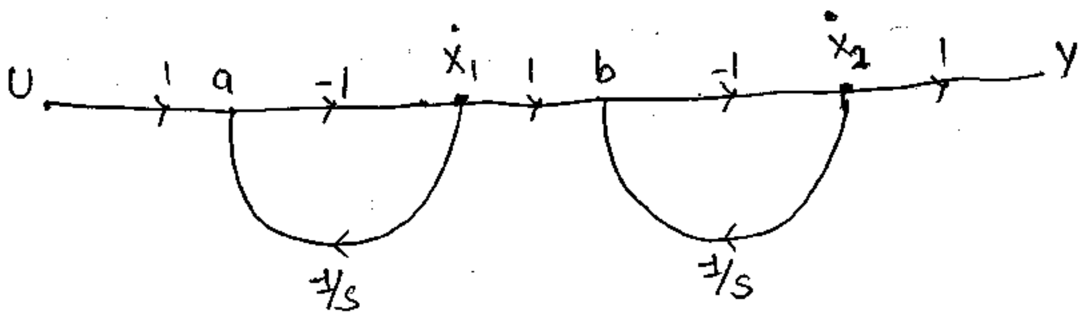
$$y = 0.5x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

GATE-13

Q.-



$a = U + x_1$

$b = \dot{x}_1 + x_2$

Now,

$\dot{x}_1 = -a = -U - x_1$

$\boxed{\dot{x}_1 = -U - x_1}$

$\boxed{y = \dot{x}_2 = +x_1 - x_2 + U}$

$\dot{x}_2 = -b$

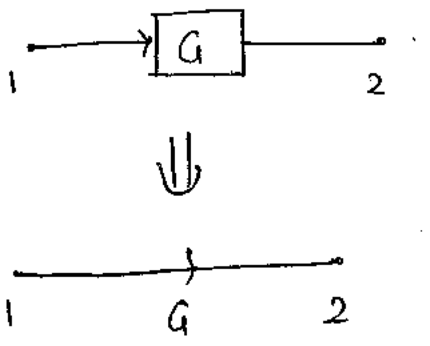
$\therefore\ b = -U - x_1 + x_2$

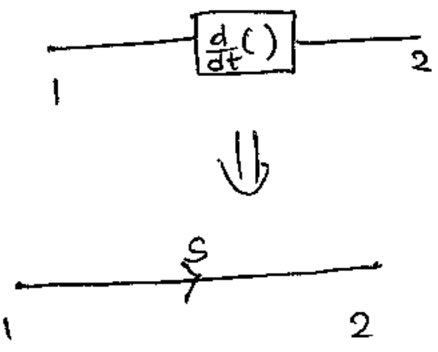
$\boxed{\dot{x}_2 = +x_1 - x_2 + U}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underset{A}{\begin{bmatrix} -1 & 0 \\ +1 & -1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{B}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}} U$$

$$y = \underset{\uparrow C}{\begin{bmatrix} +1 & -1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{\uparrow D}{\begin{bmatrix} 1 \end{bmatrix}} U$$

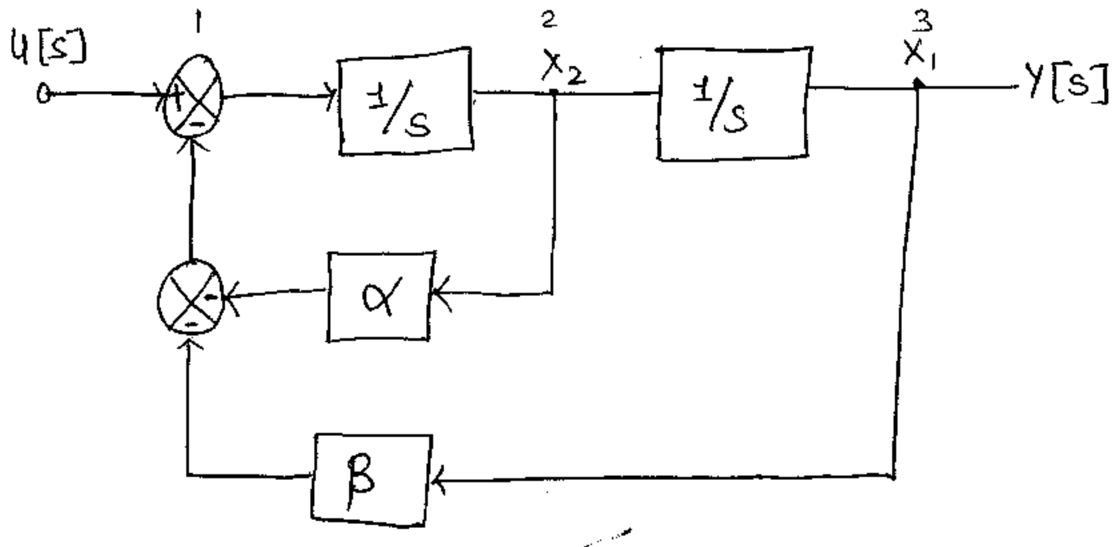
* State model from the block diagram

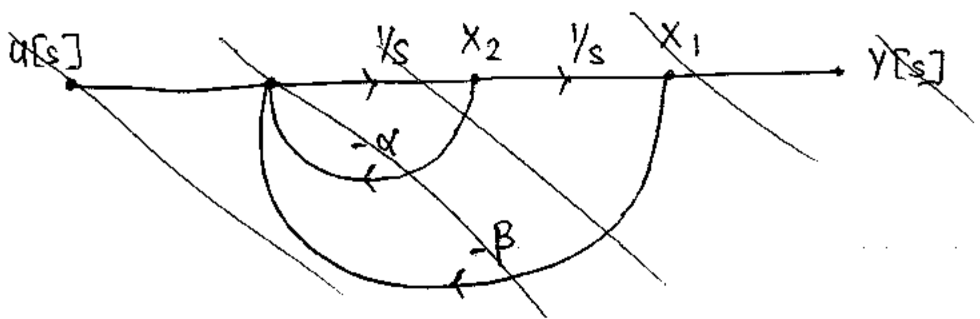


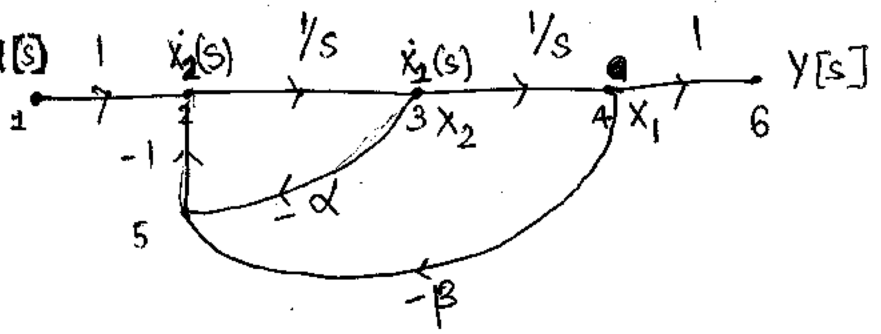


$$\boxed{\text{Block diagram} \longrightarrow \text{SFG} \longrightarrow \text{STATE MODEL}}$$

Q:- Consider the closed loop system shown in figure. The state model of the system is ____.







$$\dot{x}_2 = \alpha x_2 + \beta x_1 + U$$

$$\dot{x}_1 = x_2$$

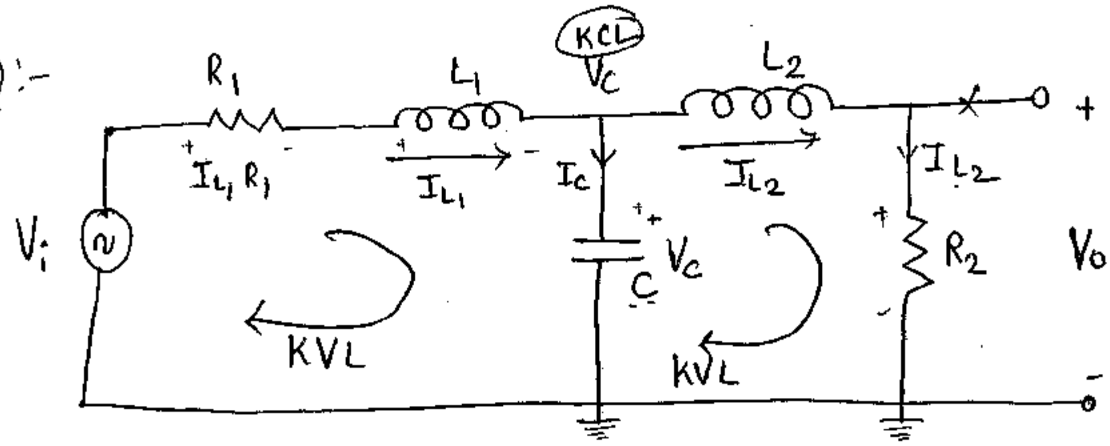
$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

*STATE MODEL to ELECTRICAL NETWORK

Q:-



No. of state variable = No. of memory storage elements

No. of " " = 3

1. Select the state variable as current flowing through inductor and voltage across capacitor.

2. No. of state variables = no. of memory storage element OR Sum of inductor & capacitor

3. Write independent KCL, KVL eq^n

4. At capacitance junction apply KCL and applie KVL through inductor

5. The resultant eq^n should consists diff. state variable, state variable, output variable and input variable

⇒ No. of state variables = 3

$$\begin{bmatrix} I_{L_1} \\ I_{L_2} \\ V_c \end{bmatrix}$$

Differential state variables

$$\begin{bmatrix} \dot{I}_{L_1} \\ \dot{I}_{L_2} \\ \dot{V}_c \end{bmatrix}$$

* KVL$_1$

$$V_i - I_{L_1}R_1 - L_1\frac{dI_1}{dt} - V_c = 0$$

$$L_1 \dot{I}_{L_1} = -I_{L_1}R_1 - V_c + V_i$$

$$\boxed{\dot{I}_{L_1} = \frac{-R_1}{L_1} I_{L_1} - \frac{1}{L_1} V_c + \frac{1}{L} V_i} \quad \text{— (1)}$$

KVL$_2$

$$V_c - L_2 \frac{dI_{L_2}}{dt} - I_{L_2} R_2 = 0$$

$$L_2 \frac{dI_{L_2}}{dt} = -R_2 I_{L_2} + V_c$$

$$L_2 \cdot \dot{I}_{L_2} = -R_2 I_{L_2} + V_c$$

$$\boxed{\dot{I}_{L_2} = \frac{-R_2}{L_2} I_{L_2} + \frac{V_c}{L_2}} \quad \text{—— (2)}$$

KCL

$$I_{L_1} = I_c + I_{L_2}$$

$$I_{L_1} = C \frac{dV_c}{dt} + I_{L_2}$$

$$C \frac{dV_c}{dt} = I_{L_1} - I_{L_2}$$

$$C \dot{V}_c = I_{L_1} - I_{L_2}$$

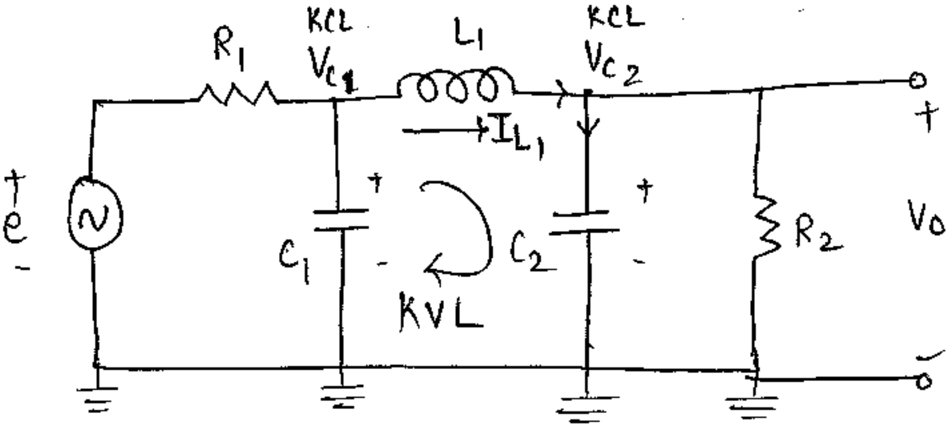
$$\boxed{\dot{V}_c = \frac{I_{L_1}}{C} - \frac{I_{L_2}}{C}}$$

$$\boxed{V_0 = R_2 I_{L_2}}$$

$$\begin{bmatrix} \dot{I}_{L_1} \\ \dot{I}_{L_2} \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & -1/L_1 \\ 0 & \frac{-R_2}{L_2} & 1/L_2 \\ 1/c & -1/c & 0 \end{bmatrix} \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ V_c \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix} U$$

$$V_0 = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ V_c \end{bmatrix}$$

Q:-



* KCL₁

$$I_{R_1} = I_C + I_{L_1}$$

$$\frac{e}{R_1} = C_1\frac{dV_{c_1}}{dt} + I_{L_1}$$

$$\frac{e - V_{c_1}}{R_1} = C_1 \dot{V}_{c_1} + I_{L_1} \quad \therefore \quad \frac{e}{R_1} - \frac{V_{c_1}}{R_1} = C_1\dot{V}_{c_1} + I_{L_1}$$

$$\dot{V}_{c_1} = \frac{e}{R_1 C_1} - \frac{I_{L_1}}{C_1} \quad \therefore \quad \dot{V}_{c_1} = \frac{e}{R_1 C_1} - \frac{V_{c_1}}{R_1 C_1} - \frac{I_{L_1}}{C_1}$$

* KVL₁

$$L_1\frac{dI_{L_1}}{dt} + V_{c_2} - V_{c_1} = 0$$

$$L_1 \cdot \dot{I}_{L_1} + V_{c_2} - V_{c_1} = 0$$

$$\dot{I}_{L_1} = \frac{V_{c_1}}{L_1} - \frac{V_{c_2}}{L_1}$$

* KCL₂

$$I_{L_1} = C_2\frac{dV_{c_2}}{dt} + \frac{V_{c_2} - 0}{R_2}$$

$$I_{L_1} = C_2 \cdot \dot{V}_{c_2} + \frac{V_{c_2}}{R_2}$$

$$\dot{V}_{c_2} = \frac{I_{L_1}}{C_2} - \frac{V_{c_2}}{C_2 R_2}$$

$$\begin{bmatrix} \dot{V}_{C_1} \\ \dot{V}_{C_2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_1C_1} & 0 & -1/C_1 \\ 0 & -1/C_2R_2 & 1/C_2 \\ 1/L_1 & -1/L_1 & 0 \end{bmatrix} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ I_L \end{bmatrix} + \begin{bmatrix} 1/R_1C_1 \\ 0 \\ 0 \end{bmatrix} U$$

$$V_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ I_L \end{bmatrix}$$

* Transfer Function from State model

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$T.F. = \left. \frac{L[o/p]}{L[i/p]} \right|_{I.C.=0}$$

$$= \left. \frac{Y[s]}{U[s]} \right|_{I.C.=0}$$

L

$$sX[s] = A_{n\times n}X[s] + BU[s]$$

$$[sI_{n\times n} - A]X[s] = BU[s]$$

L

$$\Rightarrow \underline{X[s]} = [sI - A]^{-1}_{n\times n} B_{n\times m} U[s]$$

$$Y[s] = CX[s] + DU[s]$$

$$Y[s] = C_{p\times n} [sI - A]^{-1}_{n\times n} B_{n\times m} U[s] + D \cdot U[s]$$

$$T.F. = \frac{Y[s]}{U[s]} = C_{p\times n}[sI - A]^{-1}_{n\times n} B_{n\times m} + D_{p\times m}$$

$$\boxed{T.F. = C\,[SI-A]^{-1}B + D}$$

Q:-

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad C = [1 \quad -1], \quad D = 1$$

$$T.F. = C\,[SI-A]^{-1}B + D$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} S+1 & 0 \\ -1 & S+1 \end{bmatrix}$$

$$\text{adj}\,(SI-A) = \begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix}$$

$$|SI-A| = (S+1)^2 = s^2+2s+1$$

$$T.F. = [1 \quad -1]\begin{bmatrix} \frac{s+1}{s^2+2s+1} & 0 \\ \frac{1}{s^2+2s+1} & \frac{s+1}{(s+1)^2} \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + [1]$$

$$= [1 \quad -1]_{1\times 2}\begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{1}{(s+1)^2} & \frac{1}{s+1} \end{bmatrix}_{2\times 2}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + I$$

$$= \begin{bmatrix} \frac{1}{s+1} - \frac{1}{(s+1)^2} & \frac{-1}{s+1} \end{bmatrix}_{1\times 2}\begin{bmatrix} -1 \\ 1 \end{bmatrix}_{2\times 1} + I$$

$$= \frac{-1}{s+1} + \frac{1}{(s+1)^2} - \frac{1}{s+1} = \frac{-(s+1)+1-(s+1)}{(s+1)^2} =$$

$$T.F. = \frac{C\,adj[sI-A]B}{|sI-A|} + D$$

$$T.F. = \frac{C\,adj[sI-A]B + D|sI-A|}{|sI-A|}$$

Characteristics $eq^n = |sI-A| = 0 \Rightarrow$ closed loop poles — stable

$$T.F. = \frac{\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 1|sI-A|}{(s+1)^2}$$

$$= \frac{\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} -(s+1) \\ s \end{bmatrix} + (s+1)^2}{(s+1)^2}$$

$$= \frac{-(s+1) - s + (s+1)^2}{(s+1)^2}$$

$$= \frac{-\cancel{2s} - \cancel{1} + s^2 + \cancel{2s} + \cancel{1}}{(s+1)^2}$$

$$\boxed{T.F. = \frac{s^2}{(s+1)^2}}$$

Using C.E.

$$T.F. = \frac{1}{1 + \frac{1}{s} + \frac{1}{s} + \frac{1}{s^2}}$$

$$T.F. = \frac{s^2}{s^2 + s + s + 1}$$

$$= \frac{s^2}{(s+1)^2}$$

Q:- $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underset{A}{\begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{B}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} U$

$[Y] = \underset{C}{[1 \quad 1]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Find T.F. of given state model.

$T.F. = C[sI - A]^{-1} B$

$= [1 \quad 1] \begin{bmatrix} s+2 & -3 \\ 4 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$= [1 \quad 1] \dfrac{\begin{bmatrix} s-2 & 3 \\ -4 & s+2 \end{bmatrix}}{(s+2)(s-2)+12} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$= [1 \quad 1] \dfrac{\begin{bmatrix} 3s-6+15 \\ -12+5s+10 \end{bmatrix}}{s^2-4+12} = [1 \quad 1] \dfrac{\begin{bmatrix} 3s+9 \\ 5s-2 \end{bmatrix}}{s^2+8}$

$= \dfrac{3s+9+5s-2}{s^2+8}$

$T.F. = \dfrac{8s+7}{s^2+8}$

Q:- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Sol^n:- $$T.F. = C\,[sI - A]^{-1} B$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

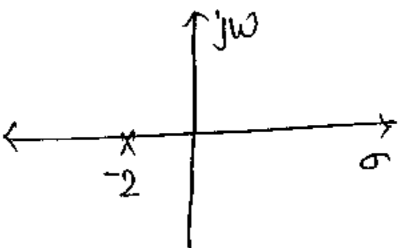
$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s & -3 \\ 2 & s+5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix}}{s(s+5)+6} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+8 \\ -2+s \end{bmatrix}}{s(s+5)+6}$$

$$T.F. = \frac{s+8-2+s}{s(s+5)+6} = \frac{2s+6}{s(s+5)+6} = \frac{2s+6}{s^2+5s+6} = \frac{2\cancel{(s+3)}}{\cancel{(s+3)}(s+2)}$$

$$= \frac{2}{s+2}$$



Stable

$$\frac{Y[s]}{U[s]} = \frac{2}{s+2}$$

If we want to find I.R.

$\Rightarrow U[s] = 1$

$$Y[s] = \frac{2}{s+2}$$

$$y(t) = 2e^{-2t} \cdot u(t)$$

If we want to find USR

$U[s] = 1/s$

$$Y[s] = \frac{2}{s(s+2)}$$

$$= 2\left[\frac{1}{s} - \frac{1}{s+2}\right] \cdot \frac{1}{2}$$

$$y(t) = (1 - e^{-2t})u(t)$$

Q:- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$[Y] = [6 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T.F. = C(sI - A)^{-1} B$$

$$= [6 \quad 1] \left\{ \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

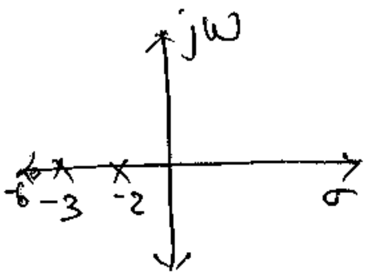
$$= [6 \quad 1] \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T.F. = \frac{\begin{bmatrix} 6 & 1 \end{bmatrix} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s(s+5)+6}$$

$$= \frac{\begin{bmatrix} 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}}{s(s+5)+6}$$

$$T.F. = \frac{6+s}{s(s+5)+6}$$

$$T.F. = \frac{6+s}{s^2+5s+6} = \frac{s+6}{(s+2)(s+3)}$$



stable

To find system stablility,

$$T.F. = \frac{N(s)}{D(s)}$$

$$D(s) = |sI - A| = 0$$

$$\underline{s = -2, -3}$$

(1) I.R.

$$U[s] = 1$$

$$Y[s] = \frac{s+6}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3} = \frac{4}{s+2} + \frac{-3}{s+3}$$

$$y(t) = \left(4e^{-2t} - 3e^{-3t}\right)u(t)$$

(2) $\overset{U}{S}$.R.

$$\frac{Y[s]}{U[s]} = \frac{s+6}{(s+2)(s+3)}$$

$U[s] = 1/s$

$$Y[s] = \frac{s+6}{s(s+2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$= \frac{1}{s} + \frac{-2}{s+2} + \frac{+1}{s+3}$$

$$y(t) = (1 - 2e^{-2t} + e^{-3t})\,u(t)$$

* Solution to the state eq^n's:-

$$\dot{X} = \frac{dx}{dt} = AX + BU \quad \text{———} \quad (1)$$

$$Y = CX + DU \quad \text{———} \quad (2)$$

Method: 1 - Laplace Transform Method

L

L

$$Y[s] = C\,X[s] + D\,U[s]$$

$$sX(s) - X(0) = AX[s] + B\,U[s]$$

$$X[s] = [sI-A]^{-1}X[0] + [sI-A]^{-1}B\,U[s]$$

$\downarrow L^{-1}$

$$\boxed{x(t) = \underbrace{L^{-1}\{[sI-A]^{-1}X[0]\}}_{\text{ZIR due to input}} + \underbrace{L^{-1}\{[sI-A]^{-1}B\cdot U[s]\}}_{\text{ZSR due to input}}} \quad \text{———} \quad (1)$$

Method : 2 Classical Method

$$x(t) = \underbrace{e^{At} x[0]}_{ZIR} + \underbrace{\int_0^t e^{A(t-\tau)} B\, u(\tau) \cdot d\tau}_{ZSR} \quad \text{——} (2)$$

(1) Compare ZIR term of (1) & (2)

$$\underbrace{\phi(t) = e^{At} = L^{-1}\{[sI-A]^{-1}\}}_{\text{State transmission matrix (STM)}}$$

$$\phi(s) = [sI-A]^{-1}$$

$$\boxed{x(t) = e^{At} x(0) + L^{-1}\{\phi(s) B \cdot U[s]\}}$$

$$y(t) = C\left[e^{At} x(0) + L^{-1}\{\phi(s) \cdot B \cdot U[s]\}\right] + DU$$

Steps :-

1. A, B, C, D, U are given
2. $SI - A$
3. $\phi[s] = [sI-A]^{-1}$
4. $\phi(t)$
5. $\phi(t) \cdot x(0)$

* Properties of STM :-

$$\phi(t) = e^{At}$$

(1) $\phi(0) = e^{0} = I$

$$\boxed{\phi(0) = I}$$

(2) $\phi^{k}(t) = e^{Akt} = \phi(kt)$

$$\boxed{\phi^{k}(t) = \phi(kt)}$$

(3) $\boxed{\phi^{-1}(t) = \phi(-t)}$

(4) $\phi(t_1+t_2) = e^{A(t_1+t_2)} = e^{At_1} \cdot e^{At_2}$

$$\phi(t_1+t_2) = \phi(t_1)\,\phi(t_2)$$

$$\boxed{\phi(t_1+t_2) = \phi(t_1)\,\phi(t_2)}$$

(5) $\boxed{\phi(t_1-t_0)\,\phi(t_0-t_2) = \phi(t_1-t_2) = \dfrac{\phi(t_1)}{\phi(t_2)}}$

$\phi(t_1-t_0)\ \phi(t_0-t_2)$

$= e^{A(t_1-t_0)} \cdot e^{A(t_0-t_2)}$

$= e^{A(t_1-t_2)} = \dfrac{e^{At_1}}{e^{At_2}} = \dfrac{\phi(t_1)}{\phi(t_2)}$

$= \phi(t_1-t_2)$

$= \dfrac{\phi(t_1)}{\phi(t_2)}$

Q:- Obtain complete system response of given system below:-

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$X[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad Y = [1 \ -1] X$$

<u>Sol:</u> We can't use T.F. method bez. I.C. are given and are not zero.

$$y = CX + DU$$

But $D = 0$

$$\therefore \; y(t) = c\,x(t)$$

$$\Rightarrow x(t) = e^{At} X[0] + L^{-1}\{\phi(s) B\, U(s)\}$$

But here $u(t) = 0$

$$x(t) = e^{At} \cdot X[0]$$

$$\phi(t) = e^{At} = [sI - A]^{-1}$$

$$SI - A = \begin{bmatrix} s & -1 \\ +2 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{adj}\,[sI - A]}{|sI - A|}$$

$$\phi[s] = \frac{\begin{bmatrix} s & +1 \\ -2 & s \end{bmatrix}}{s^2 + 2} = \begin{bmatrix} \frac{s}{s^2+2} & \frac{+1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}} \sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$x(t) = \phi(t)\, X[0]$$

$$x(t) = \begin{bmatrix} \cos\sqrt{2}t & 1/\sqrt{2}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$y(t) = c\, x(t)$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$y(t) = \cancel{\cos\sqrt{2}t} + \frac{1}{\sqrt{2}}\sin\sqrt{2}t - \sqrt{2}\sin\sqrt{2}t - \cancel{\cos\sqrt{2}t}$$

$$y(t) = \left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)\sin\sqrt{2}t$$

Q: $\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} U$ (IMP)

$X[0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $Y = [0 \;\; 1] X$. Find ZSR, ZIR and total response.

Sol:. $Y = CX + DU$

But $D = [0]$

$y(t) = c \cdot x(t)$

$$\Rightarrow x(t) = \underbrace{e^{At} x(0)}_{\text{ZIR}} + L^{-1}\{\underbrace{\phi(s) \cdot B \cdot U(s)}_{\text{ZSR}}\}$$

$U[s] = 1/s$

$$[sI - A] = \begin{bmatrix} s & -1 \\ 2 & s-3 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\begin{bmatrix} s-3 & 1 \\ -2 & s \end{bmatrix}}{s(s-3)+2}$$

$$\phi[s] = \frac{\begin{bmatrix} s-3 & 1 \\ -2 & s \end{bmatrix}}{s^2-3s+2}$$

$$\phi[s] = \begin{bmatrix} \frac{s-3}{(s-1)(s-2)} & \frac{1}{(s-1)(s-2)} \\ \frac{-2}{(s-1)(s-2)} & \frac{s}{(s-1)(s-2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{s-1} + \frac{-1}{s-2} & \frac{-1}{s-1} + \frac{1}{s-2} \\ \frac{2}{s-1} + \frac{-2}{s-2} & \frac{-1}{s-1} + \frac{2}{s-2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} 2e^{+t} - e^{+2t} & -e^{+t} + e^{+2t} \\ 2e^{t} - 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$$

$$\Rightarrow ZIR \Rightarrow \phi(t) \cdot x[0] = \begin{bmatrix} 2e^{t} - e^{2t} & -e^{t} + e^{2t} \\ 2e^{t} - 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{t} - e^{2t} \\ 2e^{t} - 2e^{2t} \end{bmatrix}$$

$$y_{ZIR}(t) = C\,[ZIR]$$

$$= [0 \;\; 1] \begin{bmatrix} 2e^{t} - e^{2t} \\ 2e^{t} - 2e^{2t} \end{bmatrix}$$

$$y_{ZIR}(t) = 2e^{t} - 2e^{2t}$$

$$\Rightarrow ZSR \Rightarrow \phi[s]\cdot B\cdot U[s] = \begin{bmatrix} \frac{s-3}{(s-1)(s-2)} & \frac{1}{(s-1)(s-2)} \\ \frac{-2}{(s-1)(s-2)} & \frac{s}{(s-1)(s-2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \cdot \frac{1}{s}$$

$$= \begin{bmatrix} \frac{5}{s(s-1)(s-2)} \\ \frac{5\cancel{s}}{\cancel{s}(s-1)(s-2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{2s} + \frac{-5}{s-1} + \frac{5}{2(s-2)} \\ \frac{-5}{s-1} + \frac{5}{s-2} \end{bmatrix}$$

$$ZSR \; L^{-1}\{\phi(s)\cdot B\cdot U[s]\} = \begin{bmatrix} \frac{5}{2} - 5e^{t} + \frac{5}{2}e^{2t} \\ -5e^{t} + 5e^{2t} \end{bmatrix}$$

$$\cancel{ZSR} \; y_{ZSR}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 5/2 - 5e^{t} + 5/2\, e^{2t} \\ -5e^{t} + 5e^{2t} \end{bmatrix}$$

$$y_{ZSR}(t) = -5e^{t} + 5e^{2t}$$

$$\therefore \; y(t) = y_{ZIR}(t) + y_{ZSR}(t)$$

$$= 2e^{t} - 2e^{2t} - 5e^{t} + 5e^{2t}$$

$$= (-3e^{t} + 3e^{2t})u(t)$$

$$\boxed{y(t) = 3\left[e^{2t} - e^{t}\right]u(t)} \longrightarrow US$$

Q The state space representation of system is given by & when T.F. of system is ______ .

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T X$$

As we know,

$\Rightarrow$ Here $[D] = 0$ $\quad$ $Y = CX + DU$

$$\therefore \; y(t) = Cx(t)$$

$$x(t) = e^{At}x[0] + L^{-1}\{\phi(s) B \cdot U[s]\}$$

$$T.F. = C[sI - A]^{-1} B + D$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}}{(s+1)(s+2)} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\begin{bmatrix} s+2 \end{bmatrix}}{(s+1)(s+2)}$$

$$T.F. = \frac{\cancel{s+2}}{(s+1)\cancel{(s+2)}} = \frac{1}{s+1}$$

Q:- Find T.F. to the given state variable

$$\dot{x}_1 = 2x_1 - x_2 + 3u$$

$$\dot{x}_2 = -4x_2 - u$$

$$y = 3x_1 - 2x_2$$

Ans: $T.F. = C[sI-A]^{-1}B + D$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

$$A = \begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad C\begin{bmatrix} 3 & -2 \end{bmatrix}, \quad D = 0$$

$$T.F. = \begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} s-2 & +1 \\ 0 & +4+s \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 0$$

$$= \frac{\begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} +4+s & -1 \\ 0 & s-2 \end{bmatrix}_{2\times 2}\begin{bmatrix} 3 \\ -1 \end{bmatrix}_{2\times 1}}{(s-2)(-4-s)}$$

$$= \frac{\begin{bmatrix} 3 & -2 \end{bmatrix}_{1\times 2}\begin{bmatrix} +12+3s+1 \\ -s+2 \end{bmatrix}_{2\times 1}}{(s-2)(+4+s)} = \frac{+39+9s+2s-4}{(s-2)(s+4)}$$

$$= \frac{35+11s}{(s-2)(s+4)}$$

Q:- $\dot{X} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} X$, $X[0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $Y = [0 \;\; 1] X$

Find $y(t)$.

(A) $\sin t$

(B) $1 - e^{-t}$

(C) $1 - \cos t$

(D) 0 ✓

$$x(t) = e^{At} x[0] + L^{-1}\{ \phi(s) B \cdot U[s] \}$$

$$U[s] = 0$$

$$x(t) = e^{At} x[0]$$

$$\phi(t) = e^{At} = [sI - A]^{-1}$$

$$sI - A = \begin{bmatrix} s & -1 \\ 0 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+1 & 1 \\ 0 & s \end{bmatrix}}{s(s+1)}$$

$$y(t) = \phi(t) \cdot x[\cancel{t}]$$

$$= [0 \;\; 1] \, \phi(t) \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1}$$

$$y(t) = 0 \cdot \phi(t) = 0$$

Q:- $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find $\phi(t)$ matrix & comment on stability

$$\phi(t) = [sI - A]^{-1}$$

$$= \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}^{-1}$$

$$= \frac{\begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}}{(s-1)^2} \longrightarrow \text{Poles location } 1, 1$$

$$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{bmatrix}$$

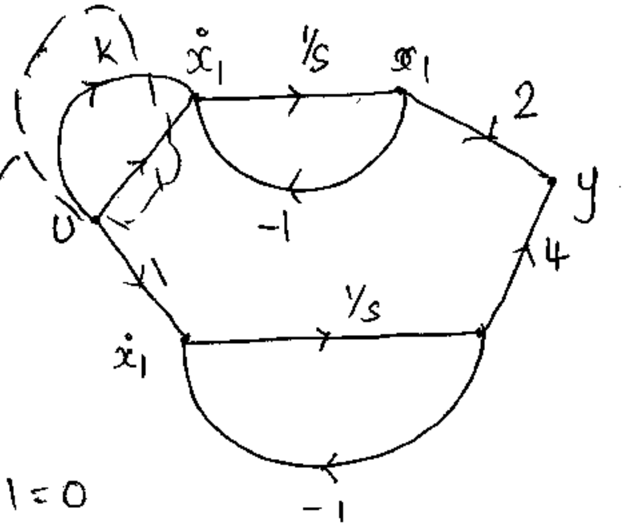
$$\phi(t) = \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix} \longrightarrow \text{US}$$

* Controllability and Observability

- A system is said to be controllable if it is possible to transfer initial state of system to the desired state of system in a finite time interval. by the controlled input.

- If the SFG is given then to check the controllability observe the continuous path from input to each and every state variable if the path exists then it is controllable.

Q:- Find the k value to become system uncontrollable



Gain $k+1=0$

$\boxed{k=-1}$ → system becomes uncontrollable bcz. path doesn't exists. to input to state variable.

* Kalaman's Test for controllability

Find A, B, C, D matrix

$$Q_c = [B \quad AB \quad A^2B \quad A^3B \; \ldots\ldots \; A^{n-1}B]$$

$\boxed{|Q_c| \neq 0 \rightarrow \text{controllable}}$

$\boxed{\text{Rank of } Q_c = \text{Rank of } A}$

Q:- Check the controllability of given system:-

$$\frac{Y[s]}{U[s]} = \frac{1}{s^3+2s^2+3s+4}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}_{3\times3}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1}$$

$$AB = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -2 \\ -3+4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -4 & -3 & -2 \\ 8 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{matrix} -4+6 \\ -3+4 \end{matrix}$$

$$A^2B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$Q_c = \begin{array}{c} \begin{matrix} B & AB & A^2B \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix} \end{array}$$

$|Q_c| = -1 \neq 0 \rightarrow$ controllable

* Cont Observability

A system is said to be observable if it is possible to determine the initial state of system by observing the output in finite time interval

* Kalaman's Test for Observability

Find A, B, C, D

$$Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$\boxed{\text{If } |Q_0| \neq 0 \Rightarrow \text{observable}}$$

Rank of Q_0 = Rank of A

Q:- Check controllability & observability of system.

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \qquad \cancel{A^2B = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}}$$

$$Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$|Q_c| = -2+2 = 0$$

$$|Q_c| = 0 \Rightarrow \text{not controllable}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times 2} \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}_{2\times 2}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0$$

$$|Q_o| = 0 \Rightarrow \text{unobservable}$$

Q:- $\dot{x}_1 = -2x_1 + x_2 + u$

$\dot{x}_2 = -x_2 + u$

$y = x_1 + x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad AB = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = -1+1 = 0 \Rightarrow \text{uncontrollable}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times 2} \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} -2 & 0 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} = 0 + 2 = 2 \neq 0 \Rightarrow \text{Observable}$$