# Sample Question Paper - 32
# Mathematics-Standard (041)
# Class- X, Session: 2021-22
# TERM II

***Time Allowed : 2 hours*** | ***Maximum Marks : 40***

## *General Instructions :*

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

## SECTION - A

**1.** Find the value of $\sqrt{30+\sqrt{30+\sqrt{30+...}}}$ by factorisation.

**2.** In a flower bed, there are 43 rose plants in the first row, 41 in the second, 39 in the third and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed?

**3.** A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal, then find the ratio of its radius and the slant height of the conical part.

**OR**

A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice cream cones of radius 2 cm and height 7 cm upto its brim. How many children will get ice cream cones?

**4.** If $p$, $q$, $r$ are real and $p \neq q$, then show that the roots of the equation $(p - q)x^2 + 5(p + q)x - 2(p - q) = 0$ are real and unequal.

**5.** Find the mean of the following data:

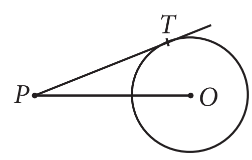
| **Class** | 1-3 | 3-5 | 5-7 | 7-9 |
|---|---|---|---|---|
| **Frequency** | 12 | 22 | 27 | 19 |

**OR**

Find the sum of upper limit and lower limit of the class interval in which the 20th observation of the following data lies :

| **Class interval** | 0- 100 | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 | 600-700 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 5 | 7 | 6 | 3 | 20 | 4 | 8 |

**6.** In the given figure, point $P$ is 26 cm away from the centre $O$ of a circle and the length $PT$ of the tangent drawn from $P$ to the circle is 24 cm. Find the radius of the circle.



## SECTION - B

**7.** Find the median of the following data :

| **Class Interval** | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | Total |
|---|---|---|---|---|---|---|
| **Frequency** | 8 | 16 | 36 | 34 | 6 | 100 |

**8.** Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

**9.** If two towers of height $h_1$ units and $h_2$ units subtends angles of 60° and 30° respectively at the mid point of line joining their feet, find $h_1 : h_2$.

**OR**

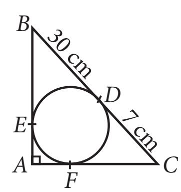
The angle of elevation of the top of a tower at a point on the ground is 30°. If the height of the tower is tripled, find the angle of elevation of the top at the same point.

**10.** Find the values of frequencies $x$ and $y$ in the following frequency distribution table, if $N = 100$ and median is 32.

| **Marks** | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | Total |
|---|---|---|---|---|---|---|---|
| **No. of students** | 10 | $x$ | 25 | 30 | $y$ | 10 | 100 |

## SECTION - C

**11.** In the given figure, $BDC$ is a tangent to the given circle at point $D$ such that $BD = 30$ cm and $CD = 7$ cm. The other tangents $BE$ and $CF$ are drawn respectively from $B$ and $C$ to the circle and meet when produced at $A$ making $BAC$ a right angled triangle. Calculate (i) radius of the circle (ii) $AF$ (iii) $AB$ (iv) $AC$.



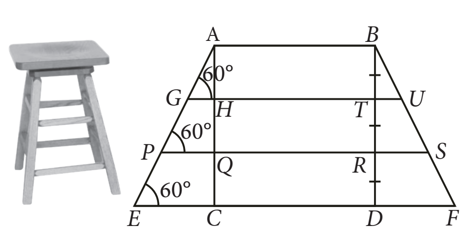

**12.** Two trains leave a railway station at the same time, the first travels towards west and the second to south. The speed of the first train is 30 km/hr more than that of second train. If after 30 minutes, they are 75 km apart, then find the average speed of both trains.

**OR**

Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

## Case Study - 1

**13.** Umesh purchase a wooden bar stool for her living room with square top of side 2 m and having height of 6 m above the ground. Also each leg is inclined at an angle of 60° to the ground as shown in the figure (not drawn to scale).



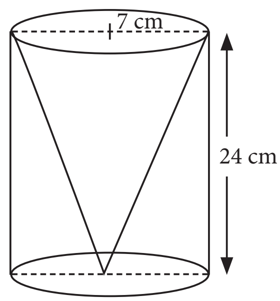
Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.73$)

(i) Find the length of the each leg.

(ii) Find the length of second step.

## Case Study - 2

**14.** One day Rinku was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Rinku's mind. Help Rinku to find the answer of the following questions.




(i) Find the curved surface area of the conical cavity so formed.

(ii) Find the volume of the conical cavity.

**Solution**

**MATHEMATICS STANDARD 041**

**Class 10 - Mathematics**

**1.** Let $x = \sqrt{30 + \sqrt{30 + \sqrt{30 + \ldots}}}$

$\Rightarrow x = \sqrt{30 + x}$ ...(i)

On squaring both sides of (i), we get

$x^2 = 30 + x \Rightarrow x^2 - x - 30 = 0$

$\Rightarrow x^2 - 6x + 5x - 30 = 0 \Rightarrow x(x - 6) + 5(x - 6) = 0$

$\Rightarrow (x - 6)(x + 5) = 0 \Rightarrow x = 6$ or $x = -5$

But $x$ is a positive quantity.

So, $x = 6$.

**2.** The number of rose plants in the 1st, 2nd, 3rd, ... rows are : 43, 41, 39, ...., 11, respectively which is an A.P.

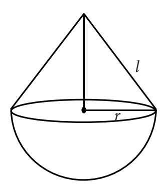
Let there be $n$ terms in the A.P.

Here, $a = 43$, $d = 41 - 43 = -2$, $a_n = 11$

$\because \; a_n = a + (n - 1)d \Rightarrow 11 = 43 + (n - 1)(-2)$

$\Rightarrow -32 = (n - 1)(-2) \Rightarrow 16 = n - 1 \Rightarrow n = 17$

**3.** Let $r$ be the radius of hemisphere and conical part. Also, let $l$ be the slant height of conical part.




Given that, surface area of hemisphere = surface area of conical part

$\Rightarrow 2\pi r^2 = \pi rl \Rightarrow 2r = l$

$\Rightarrow \frac{r}{l} = \frac{1}{2}$ i.e., $r : l = 1 : 2$

**OR**

Volume of brick = $(22)^3$ cm$^3$.

Volume of 1 cone $= \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 7 = \frac{22 \times 4}{3}$ cm$^3$

Let number of cones = $n$

$\Rightarrow n \times 22 \times \frac{4}{3} = 22 \times 22 \times 22 \Rightarrow n = \frac{22 \times 22 \times 3}{4}$

$\Rightarrow n = 121 \times 3 = 363$

Hence, the number of ice cream cones is 363.

**4.** The given equation is

$(p - q)x^2 + 5(p + q)x - 2(p - q) = 0$.

Here, $a = p - q$, $b = 5(p + q)$ and $c = -2(p - q)$.

$\therefore \; D = b^2 - 4ac = 25(p + q)^2 - 4(p - q)(-2(p - q))$

$= 25(p + q)^2 + 8(p - q)^2$

We find that, $25(p + q)^2 > 0$ and $8(p - q)^2 > 0$ $[\because p \neq q]$

$\therefore \; D = 25(p + q)^2 + 8(p - q)^2 > 0$

Hence, roots of the given equation are real and unequal.

**5.** The frequency distribution table from the given data can be drawn as :

| Class | Class marks ($x_i$) | Frequency ($f_i$) | $f_i x_i$ |
|---|---|---|---|
| 1-3 | 2 | 12 | 24 |
| 3-5 | 4 | 22 | 88 |
| 5-7 | 6 | 27 | 162 |
| 7-9 | 8 | 19 | 152 |
| | | $\Sigma f_i = 80$ | $\Sigma f_i x_i = 426$ |

$\therefore$ Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{426}{80} = 5.325$

**OR**

| Class | Frequency | Cumulative frequency |
|---|---|---|
| 0-100 | 5 | 5 |
| 100-200 | 7 | 12 |
| 200-300 | 6 | 18 |
| 300-400 | 3 | 21 |
| 400-500 | 20 | 41 |
| 500-600 | 4 | 45 |
| 600-700 | 8 | 53 |

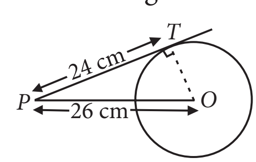
Clearly 20th observation lies near to 21 *i.e.*, between the class 300-400 so by adding upper limit and lower limit we get 300 + 400 = 700

**6.** Let us join $OT$

We have, $OP = 26$ cm, $PT = 24$ cm

Since, radius is perpendicular to the tangent at the point of contact.

$\therefore \; \angle PTO = 90°$




In right angled $\Delta PTO$,

$OP^2 = PT^2 + OT^2$

$\Rightarrow (26)^2 = (24)^2 + OT^2$

$\Rightarrow OT^2 = 676 - 576 = 100$

$\Rightarrow OT = 10$ cm

Hence, radius of the circle is 10 cm

**7.** The frequency distribution table from the given data can be drawn as :

| Class Interval | Frequency ($f_i$) | Cumulative frequency ($c.f.$) |
|---|---|---|
| 0-10 | 8 | 8 |
| 10-20 | 16 | 24 |
| 20-30 | 36 | 60 |
| 30-40 | 34 | 94 |
| 40-50 | 6 | 100 |
| Total | 100 | |

Here $N = 100$, $\frac{N}{2} = 50$, which lies in the class interval 20-30

So, median class is 20-30

$$\therefore \quad \text{Median} = l + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 20 + \left[\frac{50 - 24}{36}\right] \times 10 = 20 + 7.22 = 27.22$$

**8. Steps of construction :**

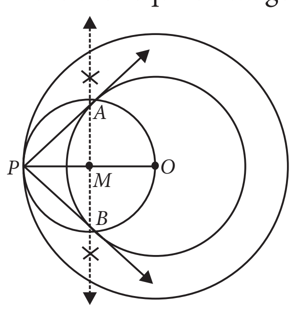
**Step-I :** Draw two concentric circles with centre $O$ and radii 4 cm and 6 cm. Take a point $P$ on the outer circle and then join $OP$.

**Step-II :** Draw the perpendicular bisector of $OP$. Let the bisector intersects $OP$ at $M$.

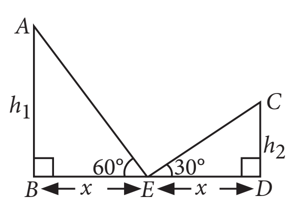
**Step-III :** With $M$ as the centre and $OM = MP$ as the radius, draw a circle, which intersect the inner circle at $A$ and $B$.

**Step-IV :** Join $PA$ and $PB$.

Thus, $PA$ and $PB$ are the required tangents.




**9.** In the figure, let $AB$ and $CD$ are towers of height $h_1$ units and $h_2$ units respectively. Let $E$ is the midpoint of $BD$ such that $BE = DE = x$ units




In right $\Delta ABE$,

$$\frac{h_1}{x} = \tan 60° = \sqrt{3} \Rightarrow h_1 = \sqrt{3}x \text{ units}$$

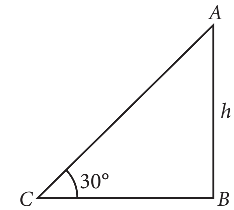
In right $\Delta CDE$,

$$\frac{h_2}{x} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow h_2 = \frac{x}{\sqrt{3}} \text{ units}$$

$$\text{Now, } \frac{h_1}{h_2} = \frac{\sqrt{3}x}{\frac{x}{\sqrt{3}}} = \frac{3}{1} \Rightarrow h_1 : h_2 = 3 : 1$$

**OR**

Let $AB = h$ m be the height of tower.

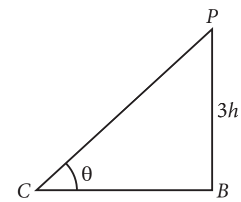



In $\Delta ABC$, $\tan 30° = \frac{h}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC}$$

$$\Rightarrow BC = h\sqrt{3} \qquad \text{...(i)}$$

Again, if $h$ is tripled *i.e.*, equal to $3h$, then




In $\Delta PCB$, $\tan \theta = \frac{3h}{BC}$

$$\Rightarrow \tan \theta = \frac{3h}{h\sqrt{3}} \qquad \text{(Using (i))}$$

$$= \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

**10.** The frequency distribution table for the given data is as follows :

| **Class** | **Frequency ($f_i$)** | **Cumulative frequency (*c.f.*)** |
|---|---|---|
| 0-10 | 10 | 10 |
| 10-20 | $x$ | $10 + x$ |
| 20-30 | 25 | $35 + x$ |
| 30-40 | 30 | $65 + x$ |
| 40-50 | $y$ | $65 + x + y$ |
| 50-60 | 10 | $75 + x + y$ |
| Total | 100 | |

Here, $N = 100$, median $= 32$, it lies in the interval 30-40.

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

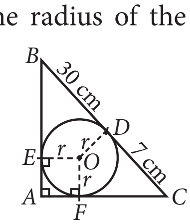
$$\Rightarrow \quad 32 = 30 + \left(\frac{50 - (35 + x)}{30}\right) \times 10$$

$$\Rightarrow \quad 32 - 30 = \frac{15 - x}{3} \quad \Rightarrow \quad 15 - x = 6 \Rightarrow x = 9$$

Also, $75 + x + y = 100 \Rightarrow 75 + 9 + y = 100$

$\Rightarrow y = 100 - 84 = 16$

**11.** Let $O$ be the centre and $r$ be the radius of the circle.




Join $OF$, $OE$ and $OD$.

Since tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OFA = \angle OEA = 90°$

Also, $\angle A = 90°$ and $OE = OF$

$\therefore \quad OEAF$ is a square

$\Rightarrow AF = OE = r$

Since tangents drawn from an external point to a circle are equal in length.

$\therefore$ $BE = BD$, $CD = CF$ and $AF = AE$

$AB = AE + BE = r + BD = r + 30$

$AC = AF + CF = r + CD = r + 7$

Now, $BC^2 = AB^2 + AC^2$ [By Pythagoras theorem]

$\Rightarrow$ $(BD + DC)^2 = (r + 30)^2 + (r + 7)^2$

$\Rightarrow$ $(30 + 7)^2 = 2r^2 + 74r + 949$

$\Rightarrow$ $1369 = 2r^2 + 74r + 949 \Rightarrow 2r^2 + 74r - 420 = 0$

$\Rightarrow$ $r^2 + 37r - 210 = 0 \Rightarrow r^2 + 42r - 5r - 210 = 0$

$\Rightarrow$ $(r + 42)(r - 5) = 0 \Rightarrow r = -42$ or $5$

$\Rightarrow$ $r = 5$ cm [$\because$ $r \neq -42$]

(i) Radius of circle is 5 cm

(ii) $AF = r = 5$ cm

(iii) $AB = r + 30 = 5 + 30 = 35$ cm

(iv) $AC = r + 7 = 5 + 7 = 12$ cm

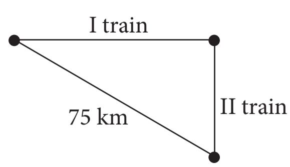
**12.** Let speed of second train be $x$ km/hr, then speed of first train will be $(x + 30)$ km/hr.

Distance covered by first train in 30 minutes

$= (x + 30)\dfrac{30}{60} = \dfrac{x + 30}{2}$ km

Distance covered by second train in 30 minutes

$= x \times \dfrac{30}{60} = \dfrac{x}{2}$ km




According to question,

$$\left(\frac{x + 30}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = (75)^2$$

(By using Pythagoras theorem)

$\Rightarrow$ $x^2 + 900 + 60x + x^2 = 4 \times 5625$

$\Rightarrow$ $2x^2 + 60x + 900 - 22500 = 0$

$\Rightarrow$ $2x^2 + 60x - 21600 = 0$

$\Rightarrow$ $x^2 + 30x - 10800 = 0$

$\Rightarrow$ $x^2 + 120x - 90x - 10800 = 0$

$\Rightarrow$ $x(x + 120) - 90(x + 120) = 0$

$\Rightarrow$ $(x + 120)(x - 90) = 0$

$\Rightarrow$ $x = -120$ or $x = 90$

But speed can't be negative.

$\therefore$ $x = 90$

Thus, speed of first train is 120 km/hr and that of second train is 90 km/hr.

**OR**

Let smaller tap fill the tank in $x$ hours.

Then larger tap will fill the tank in $(x - 2)$ hours.

Since, both the taps can fill the tank in $1\frac{7}{8}$ hours.

$\therefore$ $\dfrac{1}{x} + \dfrac{1}{x - 2} = \dfrac{8}{15} \Rightarrow \dfrac{x - 2 + x}{x(x - 2)} = \dfrac{8}{15}$

$\Rightarrow$ $\dfrac{2x - 2}{x^2 - 2x} = \dfrac{8}{15} \Rightarrow 30x - 30 = 8x^2 - 16x$

$\Rightarrow$ $8x^2 - 46x + 30 = 0 \Rightarrow 4x^2 - 23x + 15 = 0$

$\Rightarrow$ $4x^2 - 20x - 3x + 15 = 0$

$\Rightarrow$ $(4x - 3)(x - 5) = 0 \Rightarrow x = \dfrac{3}{4}$ or 5

But $x \neq \dfrac{3}{4}$ $\therefore$ $x = 5$

When $x = 5$, $x - 2 = 5 - 2 = 3$

$\therefore$ Smaller tap will fill the tank in 5 hours and larger tap will fill the tank in 3 hours.

**13.** Given, side of square top = 2 m

$\therefore$ $AB = HT = QR = CD = 2$ m

Also, $AC$ and $BD$ are perpendicular to the ground.

So, $AH = HQ = QC$. (By B.P.T. Theorem)

(i) In $\Delta AEC$,

$\sin 60° = \dfrac{AC}{AE} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{6}{AE} \Rightarrow AE = 6.93$ m

$\therefore$ Length of each leg *i.e.*, $AE = BF = 6.93$ m.

(ii) In $\Delta AGH$, $\tan 60° = \dfrac{AH}{GH} \Rightarrow \sqrt{3} = \dfrac{2}{GH}$

$\Rightarrow$ $GH = 1.15$ m

$\therefore$ Length of second step $= GH + HT + TU$

$= 1.15 + 2 + 1.15 = 4.3$ m

**14.** (i) Slant height of conical cavity, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ cm

$\therefore$ Curved surface area of conical cavity $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25 = 550$ cm$^2$

(ii) Volume of conical cavity $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 24 = 1232$ cm$^3$