

Pair of Linear Equations in Two Variables

IIT Foundation Material

SECTION - I

Straight Objective Type

1. $x + y = 7$ and $xy = 12$

$$\Rightarrow (x, y) = (3, 4) \text{ or } (4, 3)$$

$$\Rightarrow x^2 + y^2 = 3^2 + 4^2 = 25$$

Hence (b) is the correct option.

2. If $x + \frac{1}{x} = 3$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 7$$

Hence (b) is the correct option.

3. If $a + b + c = 9$ and $ab + bc + ca = 26$

$$\text{then } (a+b+c)^2 = 9^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$$

$$a^2 + b^2 + c^2 + 2(26) = 81$$

$$\Rightarrow a^2 + b^2 + c^2 = 81 - 52 = 29$$

Hence (c) is the correct option.

4. If $a - b = 2$ and $ab = 15$

$$\text{Then } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

$$= (a - b)(a - b)^2 + 3ab$$

$$= 2(2^2 + 3 \times 15)$$

$$= 2(4 + 45) = 2 \times 49 = 98$$

Hence (c) is the; correct option.

5. If $(x+1)(x+2)(x+3) =$
 $x^3 + kx^2 + 11x + 6$
 $(x+a)(x+b)(x+c)$
 $= x^3 + x^2(a+b+c) + x(ab+bc+ca) + abc$
 $\Rightarrow (x+1)(x+2)(x+3)$
 $= x^3 + x^2(1+2+3) + x$
 $(1 \times 2 + 2 \times 3 + 3 \times 1) + 1 \times 2 \times 3$
 $= x^3 + 6x^2 + 11x + 6$
 $= x^3 + kx^2 + 11x + 6$

$\Rightarrow k = 6$

Hence (d) is the correct option.

6. Let $f(x) = x^3 - 3x^2 + 2x - 5$

If $f(x)$ is divided by $x - a$ then the remainder is $f(a)$

If $f(x)$ is divided by $x - 2$ then the remainder is $f(2)$

$$\begin{aligned}f(2) &= 2^3 - 3 \cdot 2^2 + 2 \cdot 2 - 5 \\&= 8 - 12 + 4 - 5 \\&= 8 - 8 - 5 = -5\end{aligned}$$

Hence (b) is the correct option.

7. If $a+b+c=0$ then $a^3 + b^3 + c^3$

$$a+b+c=0 \Rightarrow a+b=-c$$

$$\Rightarrow (a+b)^3 = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(a+b) = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(-c) = -c^3$$

$$\Rightarrow a^3 - b^3 - 3abc = -c^3$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

Hence (b) is the correct option.

8.

$$\begin{aligned}
 & \frac{1.3 \times 1.3 + 2.6 \times 2.7 + 2.7 \times 2.7}{(2.7 \times 2.7) - (1.3 \times 1.3)} \\
 &= \frac{(1.3)^2 + 2 \times 1.3 \times 1.7 + (2.7)^2}{(2.7)^2 - (1.3)^2} \\
 &= \frac{(1.3 + 2.7)^2}{(2.7 - 1.3)(2.7 + 1.3)} \\
 &= \frac{2.7 + 1.3}{2.7 - 1.3} = \frac{4}{1.4} = \frac{40}{14} = \frac{20}{7} = \frac{10}{3.5}
 \end{aligned}$$

Hence (b) is the correct option.

9. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ and $ab \neq 0$

Then the numerical value of $\frac{a}{b} + \frac{b}{a}$

$$\begin{aligned}
 (a^2 + b^2)^3 &= (a^3 + b^3)^2 \\
 a^6 + b^6 + 3a^2b^2(a^2 + b^2) &= a^6 + b^6 + 2a^3b^3 \\
 \Rightarrow \frac{a^2b^2(a^2 + b^2)}{a^3b^3} &= \frac{2}{3} \\
 \Rightarrow \frac{a}{b} + \frac{b}{a} &= \frac{2}{3}
 \end{aligned}$$

Hence (b) is the correct option.

10. If $x = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ $y = y = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\begin{aligned}
 x &= \frac{(\sqrt{3}-1)^2}{2} & y &= \frac{(\sqrt{3}+1)^2}{2} \\
 &= \frac{3+1-2\sqrt{3}}{2} & &= \frac{3+1+2\sqrt{3}}{2} \\
 &= 2-\sqrt{3} & &= 2+\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 x^2 + y^2 &= (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2 \\
 &= 2(4 + 3) = 14 \\
 x^2 + y^2 - xy &= 14 + (2 + \sqrt{3})(2 - \sqrt{3}) \\
 &= 14 - 1 = 13 \\
 x^2 + y^2 + xy &= 14 + (2 + \sqrt{3})(2 - \sqrt{3}) \\
 &= 14 + = 15
 \end{aligned}$$

then $\frac{x^2 - xy + y^2}{x^2 + xy + y^2} = \frac{13}{15}$

Hence (b) is the correct option.

$$\begin{aligned}
 \textbf{11.} \quad \text{If } 3^{x-y} &= 27 & 3^{y+1} &= 27^x \\
 3^{x-y} &= 3^3 & 3^{y+1} &= 3^{3x} \\
 \Rightarrow x - y &= 3 & y + 1 &= 3x \\
 x - 3 &= y & x - 3 + 1 &= 3x \\
 \Rightarrow 2x & & = -2 & \\
 & & x &= -1 \\
 y &= -1 - 3 = -4
 \end{aligned}$$

$$\begin{aligned}
 \text{Then } 2^{2x-y} &= 2^{2(-1)+4} \\
 &= 2^{-2+4} = 2^2 = 4
 \end{aligned}$$

Hence (b) is the correct option.

$$\begin{aligned}
 \textbf{12.} \quad \text{If } zx + y &= 7z, yz + x = 8z \\
 x + y + z &= 12 \\
 (x, y, z) &= (4, 6, 2)
 \end{aligned}$$

Hence (b) is the correct option.

$$\frac{119^2 + 119 \times 111 + 111^2}{119^3 - 111^3}$$

$$= \frac{119^2 + 119 \times 111 \times 111^2}{(119 - 111)(119^2 + 119 \times 111 + 111^2)}$$

$$= \frac{1}{8}$$

Hence (b) is the correct option.

- 14.** If $a^x = b^y = c^z$ and $\frac{b}{a} = \frac{c}{b}$

$$a^x = k \Rightarrow a = K^{\frac{1}{x}}$$

$$b^y = k \Rightarrow b = K^{\frac{1}{y}}$$

$$c^z = k \Rightarrow c = K^{\frac{1}{z}}$$

$$\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$$

$$\Rightarrow \left(K^{\frac{1}{y}} \right)^2 = K^{\frac{1}{x}} \cdot K^{\frac{1}{z}}$$

$$\Rightarrow K^{\frac{2}{y}} = K^{\frac{1}{x} + \frac{1}{z}}$$

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\frac{2}{y} = \frac{z+x}{zx}$$

$$\Rightarrow \frac{2z}{x+z} = \frac{y}{x}$$

Hence (b) is the correct option.

- 15.** Factors of $4x^2 - y^2 + 2x - 2y - 3xy$

$$4x^2 - y^2 + 2x - 2y - 3xy$$

$$= (x-y)(4x+y+2)$$

Hence (b) is the correct option.

16. If $x^4 + ax^3 + bx^2 + cx + d = (x^2 + px + q)^2$

$$x^4 + ax^3 + bx^2 + cx + d$$

$$= x^4 + p^2 x^2 + q^2 + 2px^3 + 2pqx + 2pqr^2$$

$$= x^4 + 2px^3 + (p^2 + 2q)x^2 + 2pqr + q^2$$

$$\Rightarrow a = 2p, \quad p^2 + 2q = b$$

$$\left(\frac{\alpha}{2}\right)^2 + 2q = b$$

$$\Rightarrow 2q = b - \frac{\alpha^2}{4}$$

$$a = 2p, \quad 2pq = C$$

$$\Rightarrow 2q = \frac{C}{P}$$

$$2q = \frac{C}{\cancel{\alpha}/2} = \frac{2C}{\alpha}$$

Hence (b) is the correct option.

17. If $1 + 3^{x-1} = 4^{x-2}$

$$1 + 3^{x-1} = (2^2)^{x-2}$$

$$1 + 3^{x-1} = 2^{2x-4}$$

$$\Rightarrow x = (-4, 1)$$

Hence (b) is the correct option.

18. If $x = \frac{\sqrt{3}}{2}$

$$1 + x = 1 + \frac{\sqrt{3}}{2} = \frac{2 + \sqrt{3}}{2}$$

$$\begin{aligned}
\sqrt{1+x} &= \sqrt{\frac{2+\sqrt{3}}{2}} = \sqrt{\frac{4+2\sqrt{3}}{4}} \\
&= \sqrt{\frac{(\sqrt{3}+1)^2}{2^2}} = \frac{\sqrt{3}+1}{2} \\
\sqrt{1-x} &= \frac{\sqrt{3}-1}{2} \\
\Rightarrow \frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}} & \\
&= \frac{2+\sqrt{3}}{1+\frac{\sqrt{3}+1}{2}} + \frac{2-\sqrt{3}}{1-\frac{\sqrt{3}-1}{2}} \\
&= \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{3-\sqrt{3}} \\
&= \frac{6+3\sqrt{3}-2\sqrt{3}-3+6-3\sqrt{3}+2\sqrt{3}-3}{9-3} \\
&= \frac{6}{6} = 1
\end{aligned}$$

Hence (b) is the correct option.

- 19.** If $2x - 3y - x = 0, x + 3y - 14z = 0$

The numerical value of $\frac{x^2 + 3xy}{y^2 + z^2} = 2$

Hence (b) is the correct option.

- 20.** If a, b, c are three positive integers

$$\frac{a+b}{2} = \sqrt{ab}$$

$$\frac{b+c}{2} = \sqrt{bc}$$

$$\frac{c+a}{2} = \sqrt{ac}$$

$$\left(\frac{a+b}{2}\right)\left(\frac{b+c}{2}\right)\left(\frac{c+a}{2}\right) \geq \sqrt{a^2b^2c^2}$$

$$\frac{(a+b)(b+c)(c+a)}{8} \geq abc$$

$$\frac{(a+b)(b+c)(c+a)}{8} \geq abc$$

Hence (b) is the correct option.

21. If $xy = -42$ $\Rightarrow xy = 7 \times -6$

$$yz = 30 \quad yz = -6 \times -5$$

$$zx = -35 \quad zx = -5 \times 7$$

$$\Rightarrow x = +7, y = -6, z = -5$$

Hence (b) is the correct option.

22. $a^x = (x+y+z)^y,$

$$a^y = (x+y+z)^z$$

$$a^z = (x+y+z)^x$$

$$a^x = (x+y+z)^y$$

$$a = (x+y+z)^{\frac{y}{x}}$$

$$a^y = (x+y+z)^z \Rightarrow a = (x+y+z)^{\frac{z}{y}}$$

$$a^z = (x+y+z)^x$$

$$a = (x+y+z)^{\frac{x}{z}} \Rightarrow a = (x+y+z)^{\frac{x}{2}}$$

$$\Rightarrow x = y = z = \frac{\alpha}{3}$$

Hence (b) is the correct option.

23. If $P = \sqrt{7} - \sqrt{5}$, $q = \sqrt{13} - \sqrt{11}$

$$\frac{1}{P} = \frac{\sqrt{7} + \sqrt{5}}{2} \cdot \frac{1}{q} = \frac{\sqrt{13} + \sqrt{11}}{2}$$

$$\frac{1}{q} > \frac{1}{p}$$

$$\Rightarrow q < p \text{ or } p > q$$

Hence (b) is the correct option.

24. $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

The equation has infinite number of roots

Hence (d) is the correct option.

25. LCM of $2x+2, 3x^2 - 2x, 4x^2 + 2x + 8$

$$2x+2 = 2(x+1)$$

$$3x^2 - 2x = x(3x-2)$$

$$4x^2 + 2x + 8 =$$

$$\text{LCM of } 2x+2, 3x^2 - 2x, 4x^2 + 2x + 8$$

= no root.

Hence (a) is the correct option.

26. $x^2 - 2mx + m^2 - 1 = 0$

$$x^2 - (m+1)x - (m-1)x + m^2 - 1 = 0$$

$$x(x-m-1) - (m-1)(x-m-1) = 0$$

$$(x-m-1)(x-m+1) = 0$$

$$[x-(m+1)][x-(m-1)] = 0$$

$$m-1 \leq x \leq m+1$$

$x^2 - 2mx + m^2 - 1 = 0$ lies between -2 and 4 then $-1 \leq m \leq 3$

Hence (d) is the correct option.

27. If $x = 7 + 4\sqrt{3}$

$$\begin{aligned}
 xy = 1 &\Rightarrow y = \frac{1}{x} \\
 y = \frac{1}{7+4\sqrt{3}} &= 7 - 4\sqrt{3} \\
 \frac{1}{x^2} + \frac{1}{y^2} &= \frac{x^2 + y^2}{(xy)^2} \\
 &= \frac{(7+4\sqrt{3})^2 + (7-4\sqrt{3})^2}{(7+4\sqrt{3})(7-4\sqrt{3})} \\
 &= \frac{2(49+48)}{1} \\
 &= 194
 \end{aligned}$$

Hence (c) is the correct option.

28. If $2x - 2 = 52 - x$
 $\Rightarrow x - c = 0$ or $2 - x = 0$
 $\Rightarrow x = 2$

Hence (b) is the correct option.

29. (d) is the correct option.

30. $y = \frac{1-t^2}{1+t^2}$
 $\frac{y}{1} = \frac{1-t^2}{1+t^2}$
 (Components and dividends)
 $\frac{y+1}{g-1} = \frac{1}{t^2}$
 $\frac{\sqrt{y+1}}{\sqrt{y-1}} = \frac{1}{t}$

$$\Rightarrow t = \sqrt{\frac{y-1}{y+1}}$$

Hence (b) is the correct option.

SECTION - II

Assertion - Reason Questions

31. $(a+2b)^2 = a^2 + 4b^2 + 4ab$

Hence (a) is the correct option.

32. $\sum a^2 b = a^2 b + b^2 a$

Hence (b) is the correct option.

33. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$$(a+b)^2 - (a-b)^2 = 4ab$$

The roots of $5x^2 - 7x - 36 = 0$

are $\frac{9}{5}$ and $-\frac{4}{3}$

Hence (b) is the correct option.

34. $x+y=7$; $xy=12$

$$\Rightarrow (x+y)^2 = 49$$

$$\Rightarrow x^2 + y^2 + 2(12) = 49$$

$$\Rightarrow x^2 + y^2 = 25$$

Hence (a) is the correct option.

35. $(x-3)(x-4) = x^2 - 7x + 12$

$$(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 - 3abc$$

Hence (b) is the correct option.

36. $(x+9)(x+3) = x^2 + 12x + 27$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

Hence (a) is the correct option.

37. $a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc$

$$(a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$$

Hence (a) is the correct option.

38. $a^3+b^3=(a+b)(a^2-ab+b^2)$

Hence (a) is the correct option.

39. $x+\frac{1}{x}=3 \Rightarrow x^3+\frac{1}{x^3}=18$

Hence (b) is the correct option.

40. $(a+b)^2+(a-b)^2=2(a^2+b^2)$

Hence (b) is the correct option.

SECTION - III
Linked Comprehension Type

Paragraph 41 - 43

41. $2x^2+7x+6=(2x+3)(x+2)$

Hence (a) is the correct option.

42. $3x^2-11x+6=(3x-2)(x-3)$

Hence (c) is the correct option.

43. $12x^2-17x+6=12x^2-8x-9x+6$

$$= 4x(3x-2) - 3(3x-2)$$

$$=(4x-3)(3x-2)$$

Hence (a) is the correct option.

Paragraph 44 - 46

44. $a^3 - b^3 + c^3 - 3abc \div (a - b - c)$

$$= a^2 + b^2 + c^2 + ab - bc + ca$$

Hence (a) is the correct option.

45. $(2x+3)(3x+5) = 6x^2 + 19x + 15$

Hence (a) is the correct option.

46. $(x+2)(x+3)(x+4)$

$$= x^3 + x^2(2+3+4) + (2 \times 3 + 3 \times 4 + 2 \times 4) + 24$$

$$= x^3 + 9x^2 + 26x + 24$$

$$K+l = 9+24 = 33$$

Hence (b) is the correct option.

Paragraph 47 - 49

47. $(3x-5)(x^2 + 3x - 1)$

$$= 3x^3 - 5x^2 + 9x^2 - 15x + 5 - 3x$$

$$= 3x^3 + 4x^2 - 18x + 5$$

Hence (b) is the correct option.

48. $(2x+3)(3x-5)$

$$= 6x^2 + 9x - 10x - 15$$

$$= 6x^2 - x - 15$$

Hence (a) is the correct option.

49. $(A \times B) + C =$

$$6x^2 - x - 15 + x^2 + 3x - 1$$

$$= 7x^2 + 2x - 16$$

Hence (c) is the correct option.

Paragraph 50 - 82

- 50.** Let $f(x) = x^3 + 3x^2 - 2x - 5$

$$\begin{aligned}f(-2) &= (-2)^3 + 3(-2)^2 - 2(-2) - 5 \\&= -8 + 12 - 4 - 5 \\&= -5\end{aligned}$$

Hence (b) is the correct option.

- 51.** Let $f(x) = x^3 - 5x^2 + 11x - 10$

$$\begin{aligned}f(2) &= 2^3 - 5(2)^2 + 11(2) - 10 \\&= 8 - 20 + 22 - 10 = 0\end{aligned}$$

Hence (a) is the correct option.

- 52.** $x^2 - 2x + 3 \left| \begin{array}{c} x^4 - 4x^2 - 13x - 4 \\ x^4 - 2x^3 \end{array} \right| x^2 + 2x$

$$\begin{array}{r} 2x^3 - 4x^2 - 13x \\ 2x^3 - 4x^2 - 6x \\ \hline - \quad + \quad + \\ -7x - 4 \end{array}$$

Hence (d) is the correct option.

SECTION - IV

Matrix - Match Type

- 53.**

	p	q	r	s
A	●	○	○	○
B	○	●	○	○
C	○	○	●	○
D	○	○	○	●

54.

	p	q	r	s
A	●	○	○	○
B	○	●	○	○
C	○	○	●	○
D	○	○	○	●

55.

	p	q	r	s
A	●	○	○	○
B	○	●	○	○
C	○	○	●	○
D	○	○	○	●

56.

	p	q	r	s
A	●	○	○	○
B	○	●	○	○
C	○	○	●	○
D	○	○	○	●

57.

	p	q	r	s
A	●	○	○	○
B	○	●	○	○
C	○	○	●	○
D	○	○	○	●