7.3 Straight Line in Plane

Point coordinates: X, Y, x, x_0, x_1, y_0, y_1, a_1, a_2, ...
Real numbers: k, a, b, p, t, A, B, C, A_1, A_2, ...
Angles: α, β
Angle between two lines: φ
Normal vector: $\vec{n}$
Position vectors: $\vec{r}$, $\vec{a}$, $\vec{b}$

622. General Equation of a Straight Line
$Ax + By + C = 0$

623. Normal Vector to a Straight Line
The vector $\vec{n}(A, B)$ is normal to the line $Ax + By + C = 0$.

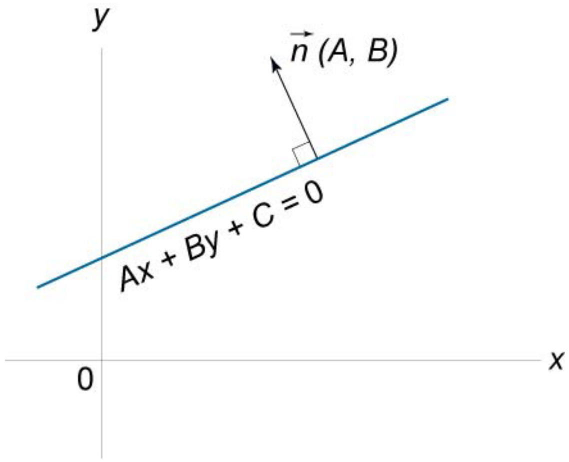

Figure 98.

624. Explicit Equation of a Straight Line (Slope-Intercept Form)
$y = kx + b$.

The gradient of the line is $k = \tan\alpha$.

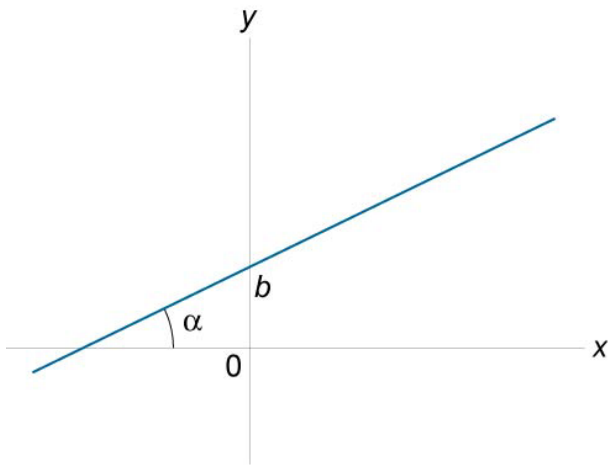

Figure 99.

625. Gradient of a Line

$$k = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

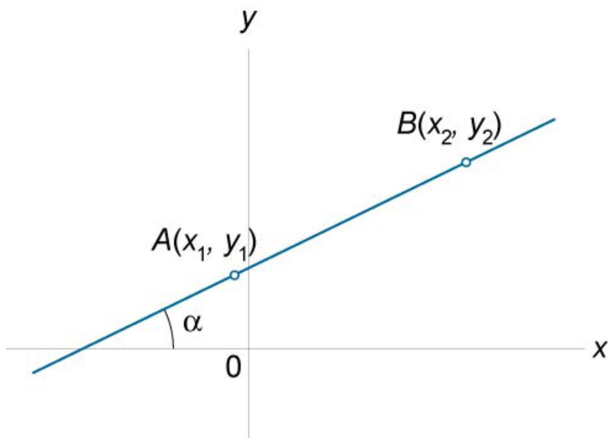

Figure 100.

626. **Equation of a Line Given a Point and the Gradient**

$y = y_0 + k(x - x_0)$,

where k is the gradient, $P(x_0, y_0)$ is a point on the line.

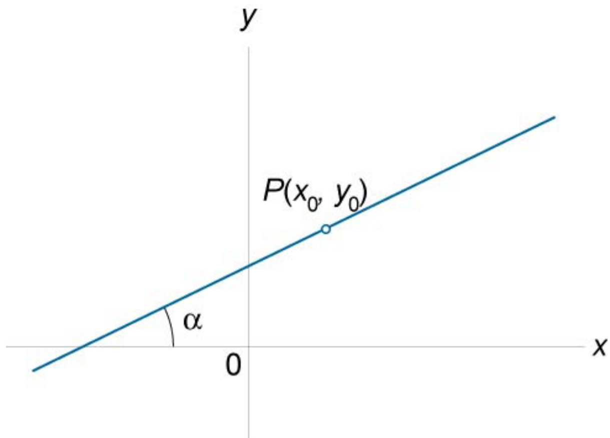

Figure 101.

627. **Equation of a Line That Passes Through Two Points**

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

or

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

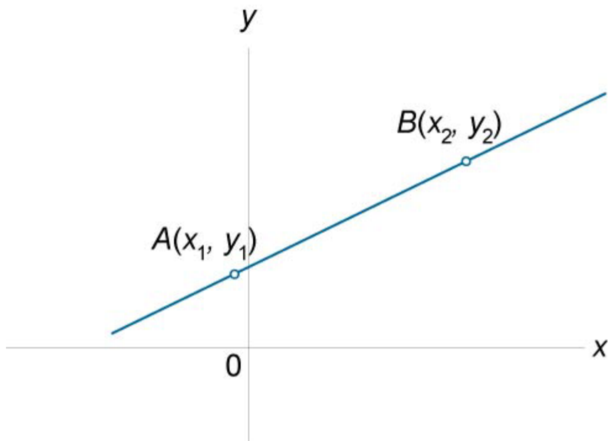

Figure 102.

628. **Intercept Form**

$$\frac{x}{a}+\frac{y}{b}=1$$

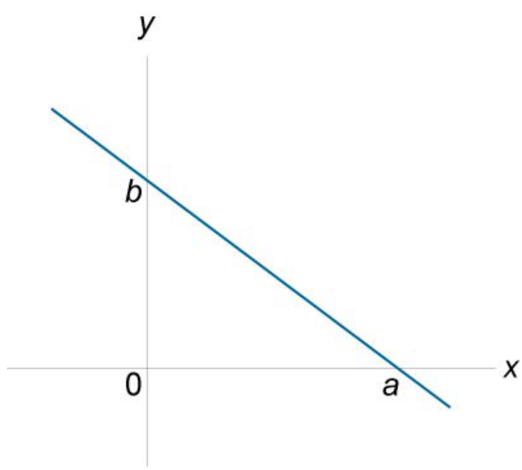

Figure 103.

629. Normal Form

$x \cos\beta + y \sin\beta - p = 0$

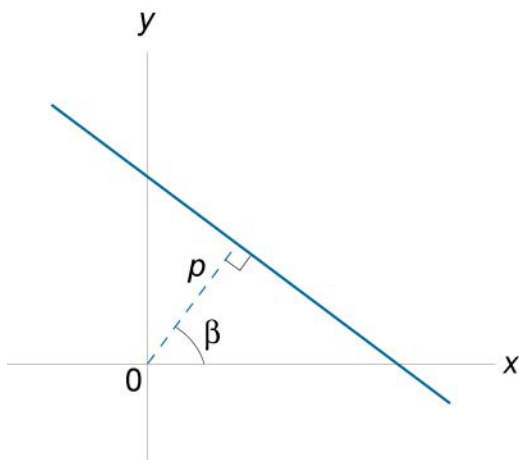

Figure 104.

630. Point Direction Form

$$\frac{x - x_1}{X} = \frac{y - y_1}{Y},$$

where (X, Y) is the direction of the line and $P_1(x_1, y_1)$ lies on the line.

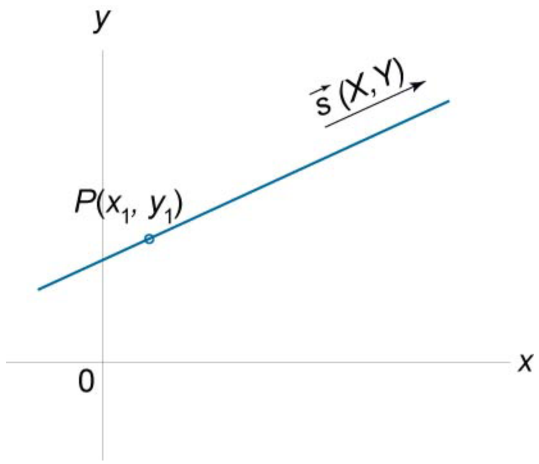

Figure 105.

631. Vertical Line

$x = a$

632. Horizontal Line

$y = b$

633. Vector Equation of a Straight Line

$\vec{r} = \vec{a} + t\vec{b}$,

where

O is the origin of the coordinates,

X is any variable point on the line,

$\vec{a}$ is the position vector of a known point A on the line,

$\vec{b}$ is a known vector of direction, parallel to the line,

t is a parameter,

$\vec{r} = \overrightarrow{OX}$ is the position vector of any point X on the line.

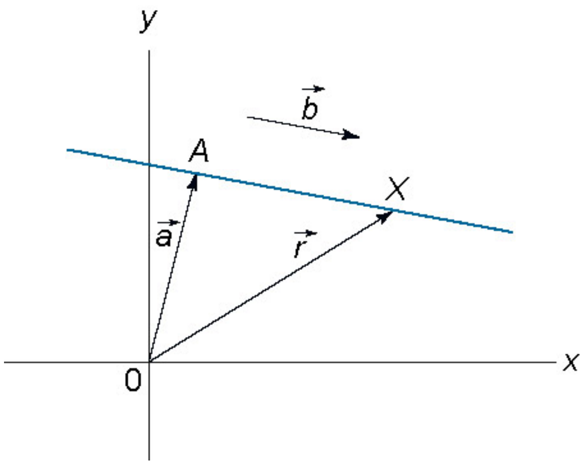

Figure 106.

634. Straight Line in Parametric Form

$$\begin{cases} x = a_1 + tb_1 \\ y = a_2 + tb_2 \end{cases},$$

where

(x, y) are the coordinates of any unknown point on the line,

(a_1, a_2) are the coordinates of a known point on the line,

(b_1, b_2) are the coordinates of a vector parallel to the line,

t is a parameter.

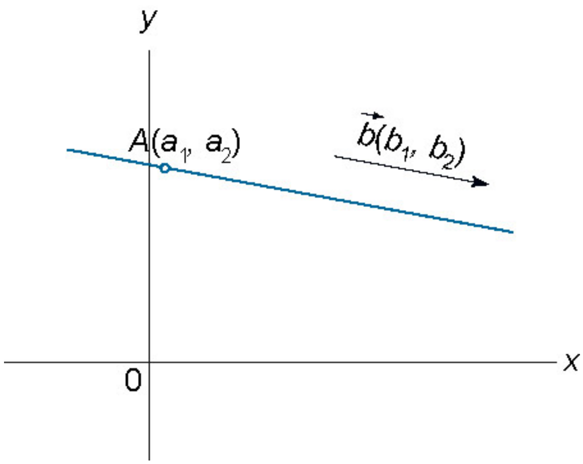

Figure 107.

635. Distance From a Point To a Line
The distance from the point $P(a, b)$ to the line $Ax + By + C = 0$ is

$$d = \frac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}}.$$

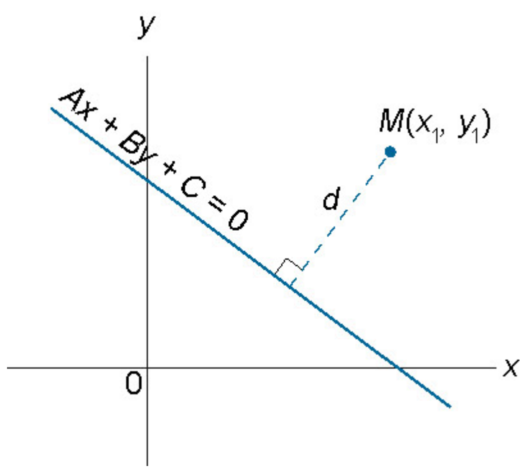

Figure 108.

636. Parallel Lines

Two lines $y = k_1 x + b_1$ and $y = k_2 x + b_2$ are parallel if $k_1 = k_2$.

Two lines $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$ are parallel if

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}.$$

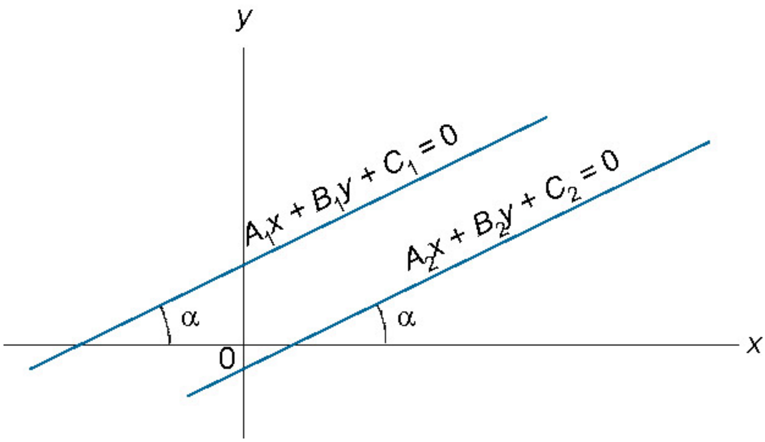

Figure 109.

637. Perpendicular Lines

Two lines $y = k_1 x + b_1$ and $y = k_2 x + b_2$ are perpendicular if

$k_2 = -\frac{1}{k_1}$ or, equivalently, $k_1 k_2 = -1$.

Two lines $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$ are perpendicular if

$A_1 A_2 + B_1 B_2 = 0$.

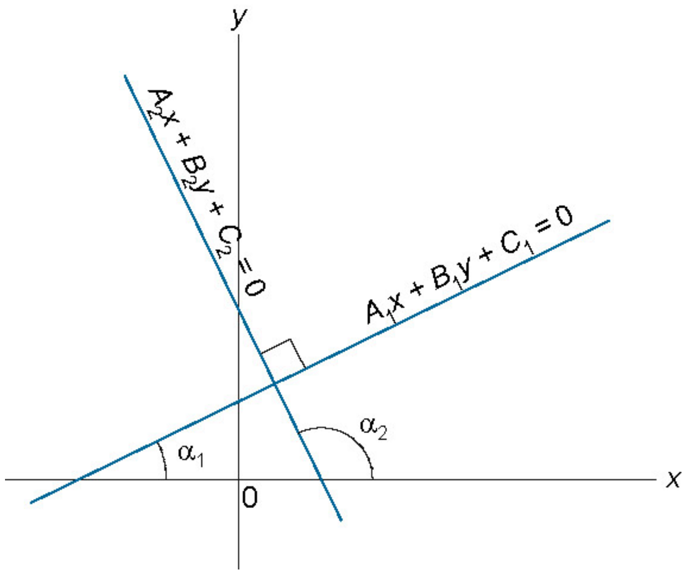

Figure 110.

638. Angle Between Two Lines

$$\tan\varphi = \frac{k_2 - k_1}{1 + k_1 k_2},$$

$$\cos\varphi = \frac{A_1A_2 + B_1B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}}.$$

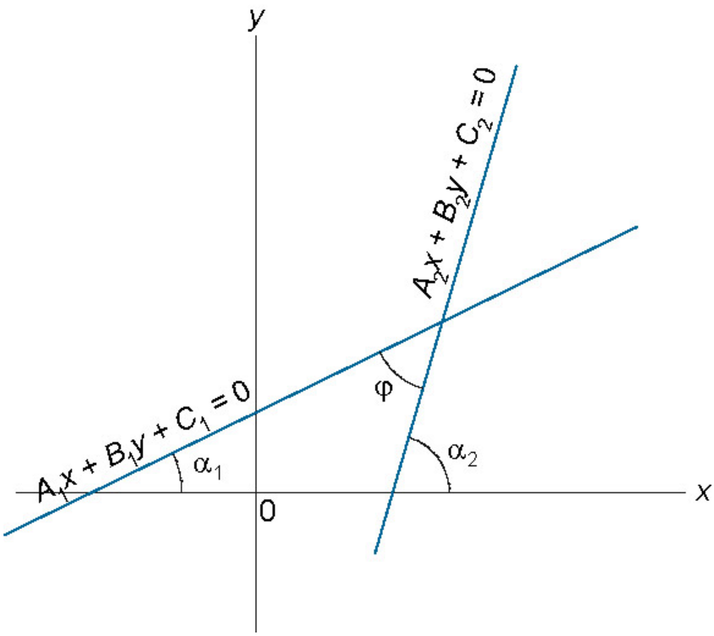

Figure 111.

639. Intersection of Two Lines

If two lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ intersect, the intersection point has coordinates

$$x_0 = \frac{-C_1B_2 + C_2B_1}{A_1B_2 - A_2B_1},\ y_0 = \frac{-A_1C_2 + A_2C_1}{A_1B_2 - A_2B_1}.$$