CHAPTER 21
TRIANGLE AND ITS PROPERTIES

TRIANGLES

A triangle is a convex polygon having three sides.
A triangle is represented by the symbol Δ.
Triangles can be classified on the basis of their sides or angles.

On the basis of sides, triangles are of the following types

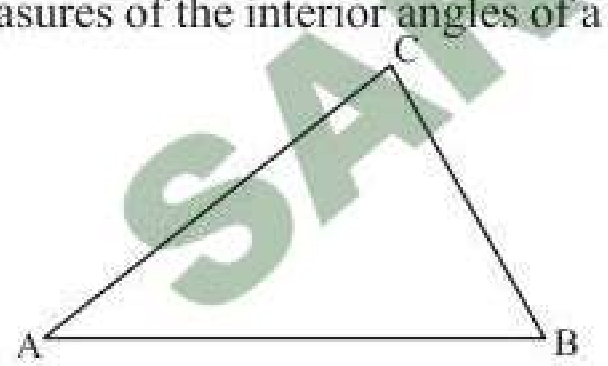
(a) Equilateral triangle: All the three sides are equal
(b) Isosceles triangle: Two sides are equal
(c) Scalene triangle: All the three sides are unequal.

On the basis of angles, triangles are of the following types

(a) Acute angled triangle: Each interior angle is less than 90°.
(b) Right angled triangle: One of the interior angle is equal 90°.
(c) Obtuse angled triangle: One of the interior angle is more than 90°.

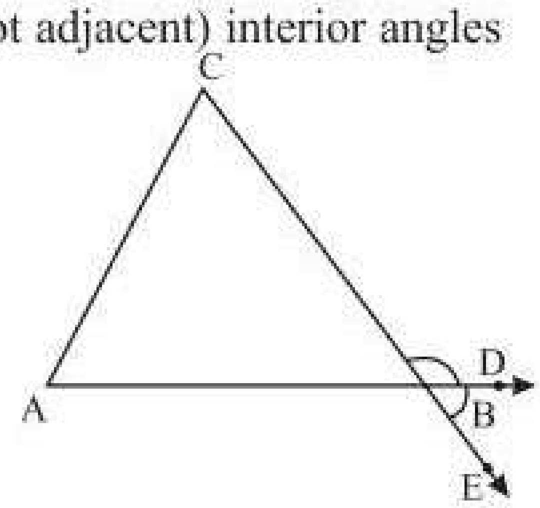
BASIC PROPERTIES AND SOME IMPORTANT THEOREMS OF TRIANGLES

- Sum of measures of the interior angles of a triangle is 180°.



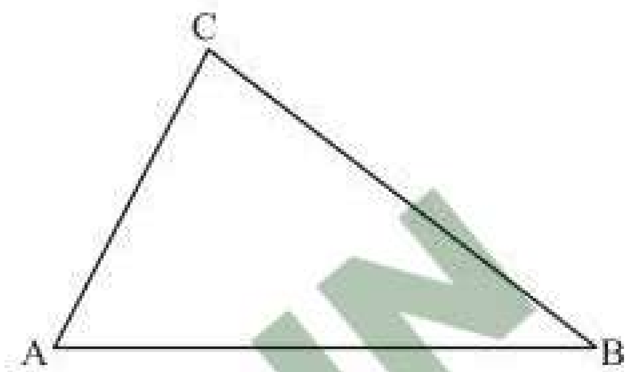
In ΔABC, $\angle CAB + \angle ABC + \angle ACB = 180°$
or $\angle A + \angle B + \angle C = 180°$

- The exterior angle of a triangle is equal to the sum of the opposite (not adjacent) interior angles



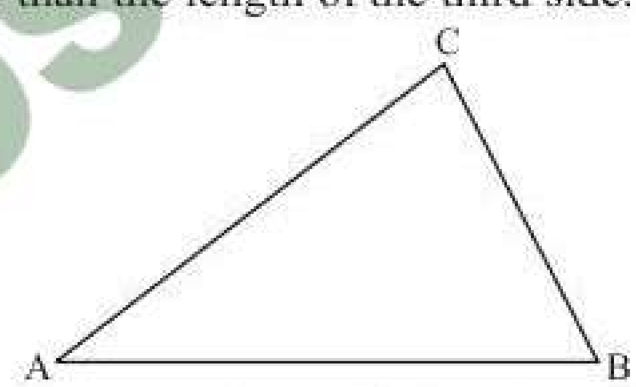
In ΔABC, $\angle CBD = \angle A + \angle C = \angle ABE$

- Sum of the lengths of any two sides of a triangle is greater than the length of the third side.



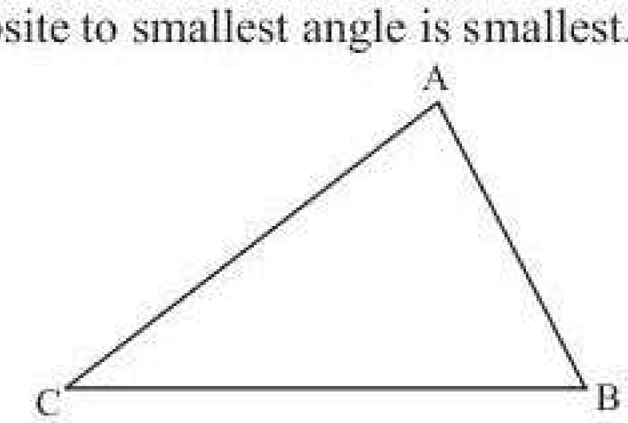
(i) $AB + AC > BC$ (ii) $AC + BC > AB$
(iii) $AB + BC > AC$

- Difference between the lengths of any two sides of a triangle is smaller than the length of the third side.



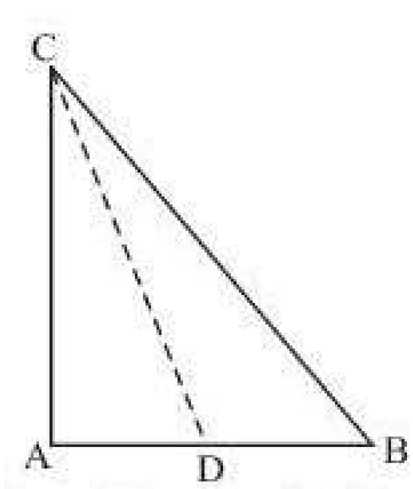
(i) $|AB - BC| < AC$ (ii) $|AC - AB| < BC$
(iii) $|AC - BC| < AB$

- In any triangle, side opposite to greatest angle is largest and side opposite to smallest angle is smallest.



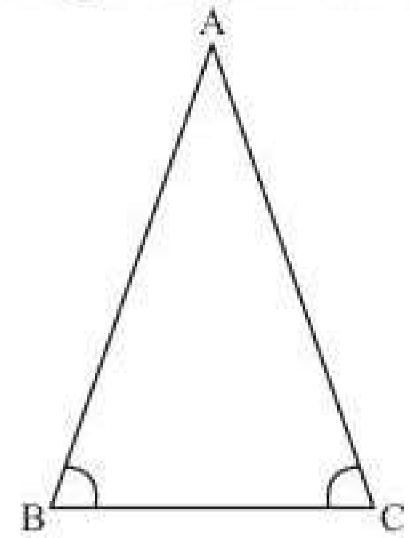
In ΔABC, if $\angle A > \angle B > \angle C$, then BC is the largest side and AB is the smallest side.

- In any triangle line joining any vertex to the mid point of its opposite side is called a median of the opposite side of the triangle.



In ΔABC, D is the mid point of AB
Hence CD is a median of ΔABC.
A triangle can have 3 medians.
Any median of a triangle divides the triangle into two triangles of equal areas.

- Sides opposite to equal angles in a triangle are equal.

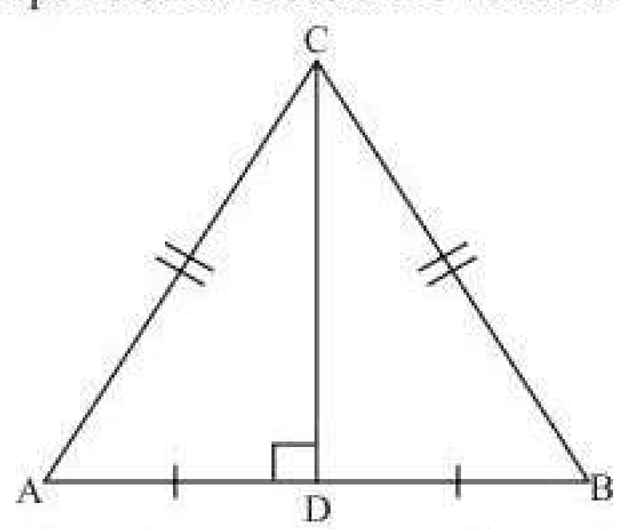


In ΔABC, $\angle B = \angle C$

$\therefore$ $AB = AC$

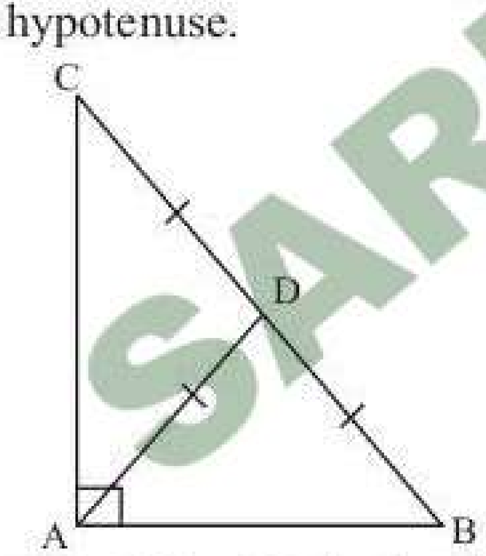
Converse of this property is also true.

- In an isosceles triangle, if a perpendicular is drawn to unequal side from its opposite vertex, then
  (a) The perpendicular is the median
  (b) The perpendicular bisects the vertex angle.



ΔABC is an isosceles triangle in which $AC = BC$.
CD is perpendicular to AB, hence CD is a median and $\angle ACD = \angle BCD$

- In a right angled triangle, the line joining the vertex of the right angle to the mid point of the hypotenuse is half the length of the hypotenuse.

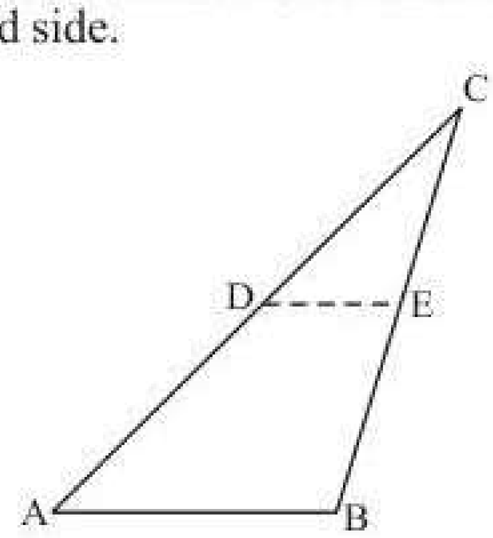


In ΔABC, $\angle BAC = 90°$ and D is the mid point of BC, then

$$AD = \frac{1}{2}BC = BD = CD$$

Mid-point theorem

In any triangle, line segment joining the mid points of any two sides is parallel to the third side and equal to half of the length of third side.

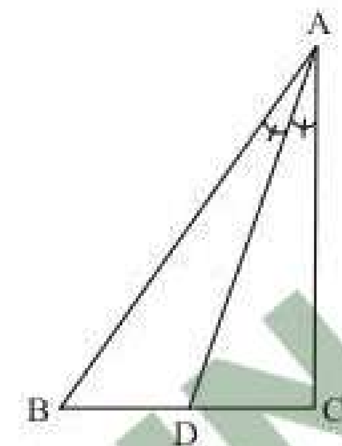


In ΔABC, D and E are mid points of sides AC and BC, then DE is parallel to AB i.e. $DE \parallel AB$ and $DE = \frac{1}{2} AB$

Angle Bisector Theorem

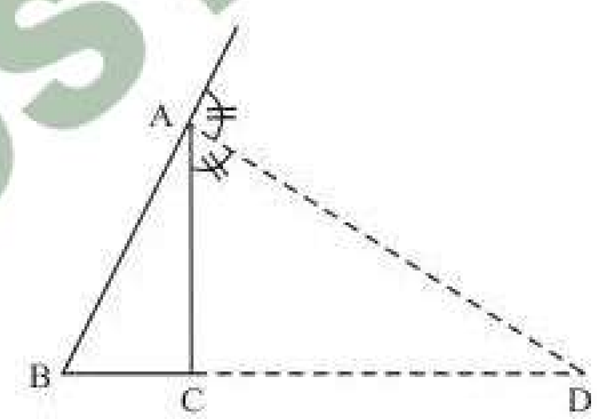
Bisector of an angle (internal or external) of a triangle divides the opposite side (internally or externally) in the ratio of the sides containing the angle.

For example:



In figure AD is the bisector of exterior $\angle BAC$

$\therefore$ $\frac{AB}{AC} = \frac{BD}{DC}$



In figure AD is the bisector of exterior $\angle BAC$.

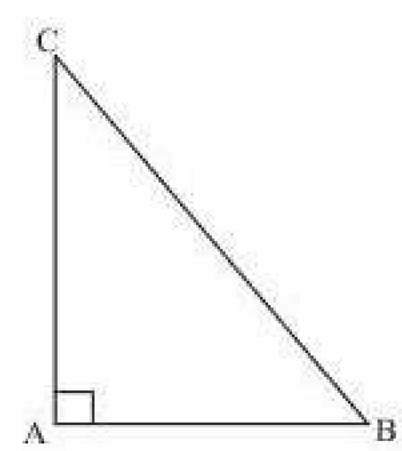
$\therefore$ $\frac{AB}{AC} = \frac{BD}{DC}$

Converse of the angle bisector theorem is also true.

Pythagoras Theorem

In a right angled triangle.

Square of longest or hypotenuse = Sum of square of other two sides.



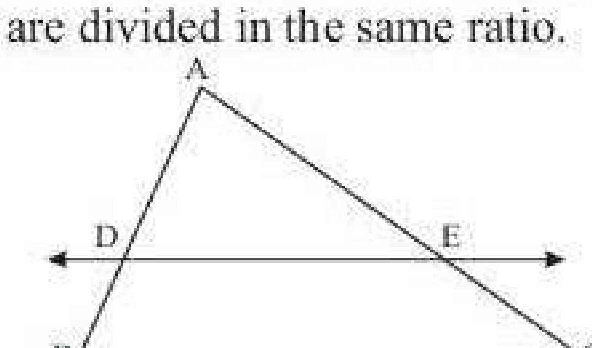
In figure ΔABC is a triangle right angled at A.

$\therefore$ $(BC)^2 = (AB)^2 + (AC)^2$

Converse of this theorem is also true.

Basic Proportionality Theorem (BPT)

If a line is drawn parallel to one side of a triangle which intersects the other two sides in distinct points, the other two sides are divided in the same ratio.



In ΔABC, $DE \parallel BC$,

Then, $\frac{AD}{DB} = \frac{AE}{EC}$

This theorem is also known as Thalse theorem.

Converse of this theorem is also true.

Example 1 : In a triangle ABC, $\angle A = x$, $\angle B = y$, and $\angle C = y + 20$.

If $4x - y = 10$, then the triangle is __________.

Solution:

We have, $x + y + (y + 20) = 180$

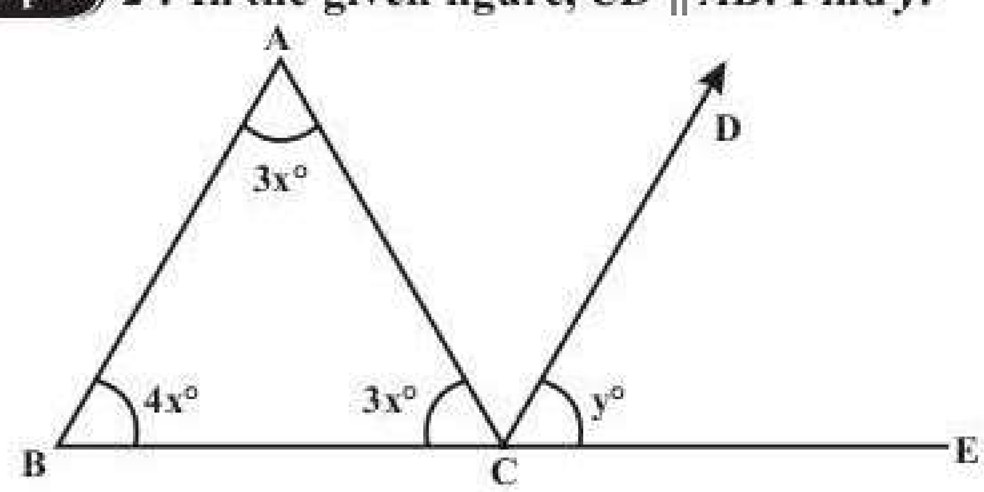
or $\quad x + 2y = 160 \quad ...(1)$

and $\quad 4x - y = 10 \quad ...(2)$

From (i) and (ii), $y = 70$, $x = 20$

Angles of the triangles are 20°, 70°, 90°. Hence the triangle is a right angled.

Example 2 : In the given figure, $CD \parallel AB$. Find y.



(a) 79° **(b) 72°**
(c) 74° **(d) 77°**

Solution :

In ΔABC,

$\angle ABC + \angle BCA + \angle CAB = 180°$

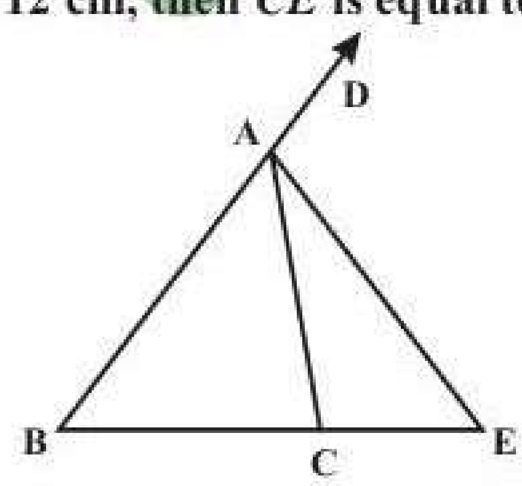
$\Rightarrow \quad 4x + 3x + 3x = 180° \Rightarrow 10°x = 180° \Rightarrow x = 18°$

Now, $\quad \angle ABC = \angle DCE$

(Corresponding angles are equal)

$\Rightarrow \quad \angle DCE = 4x° \Rightarrow y = 4 \times 18° = 72°$

Example 3 : In the adjoining figure, AE is the bisector of exterior $\angle CAD$ meeting BC produced in E. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then CE is equal to



Solution :

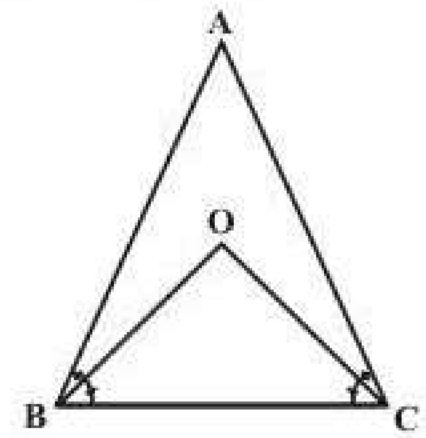
$\frac{BE}{CE} = \frac{AB}{AC}$ as AE is an exterior angle bisector.

Let $\quad CE = x$, $BE = BC + EC = 12 + x$

$\Rightarrow \quad \frac{12 + x}{x} = \frac{10}{6} \quad \Rightarrow (12 + x)\,6 = 10x$

$\Rightarrow \quad 72 + 6x = 10x \Rightarrow 4x = 72 \Rightarrow x = 18$ cm

Example 4 : OB and OC are respectively the bisectors of $\angle ABC$ and $\angle ACB$. Then, $\angle BOC$ is equal to

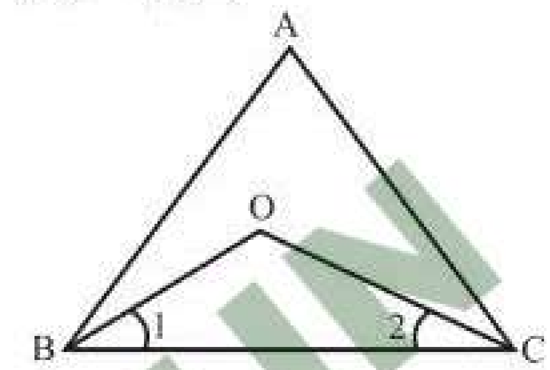


Solution :

In ΔBOC,

$\angle 1 + \angle 2 + \angle BOC = 180°$

$\angle A + \angle B + \angle C = 180°. \quad .... (1)$



$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90°$

$\Rightarrow \quad \frac{1}{2}(\angle A) + \angle 1 + \angle 2 = 90° \Rightarrow \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$

Put $\angle 1 + \angle 2$ in Eq. (1), we get

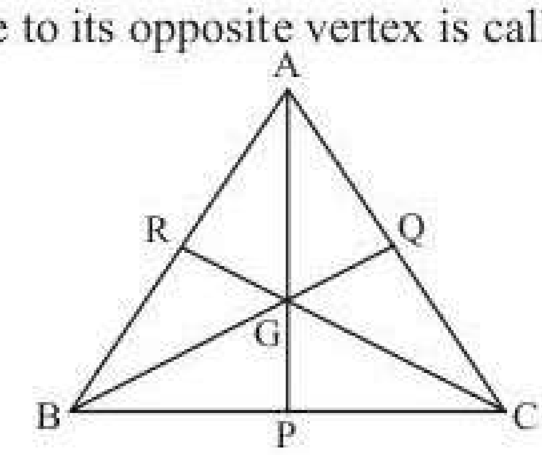
$\angle BOC = 180° - \left(90° - \frac{1}{2}\angle A\right)$

$= 90° + \frac{1}{2}\angle A$

IMPORTANT TERMS RELATED TO A TRIANGLE

Medians and Centroid

We know that a line segment joining the mid point of a side of a triangle to its opposite vertex is called a median.



AP, BQ and CR are medians of ΔABC where P, Q and R are mid points of sides BC, CA and AB respectively.

(i) Three medians of a triangle on concurrent. The point of concurrent of three medians is called Centroid of the triangle denoted by G.

(ii) Centroid of the triangle divides each median in the ratio 2 : 1

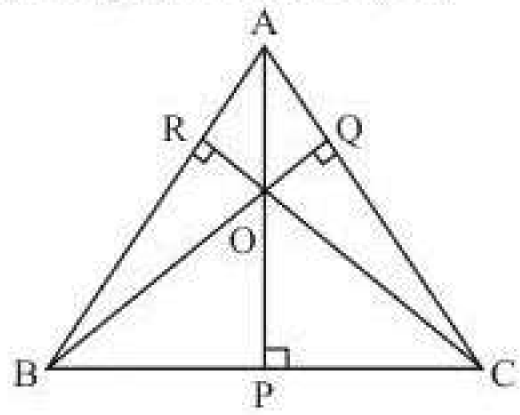
i.e. $AG : GP = BG : GQ = CG : GR = 2 : 1$, where G is the centroid of ΔABC.

Altitudes and Orthocentre

A perpendicular drawn from any vertex of a triangle to its opposite side is called altitude of the triangle. There are three altitudes of a triangle.

In the figure, AP, BQ and CR are altitudes of ΔABC.

The altitudes of a triangle are concurrent (meet at a point) and the point of concurrency of altitudes is called Orthocentre of the triangle, denoted by O.



In figure, AP, BQ and CR meet at O, hence O is the orthocentre of the triangle ABC.

Note: The angle made by any side at the orthocentre and at the vertex opposite to the side are supplementary angle.

Hence, $\angle BAC + \angle BOC = \angle ABC + \angle AOC = \angle ACB + \angle AOB = 180°$.

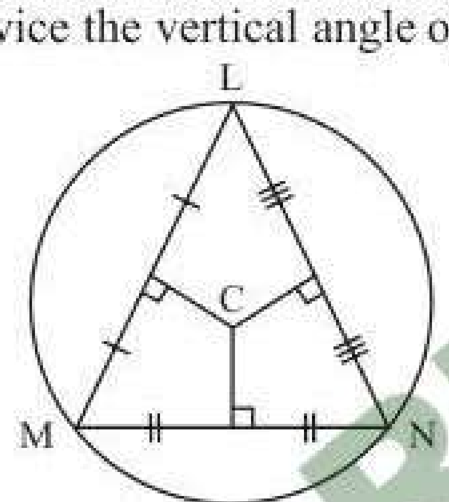
Perpendicular Bisectors and Circumcenter

A line which is perpendicular to a side of a triangle and also bisects the side is called a perpendicular bisector of the side.

(i) Perpendicular bisectors of sides of a triangle are concurrent and the point of concurrency is called circumcentre of the triangle, denoted by 'C'.

(ii) The circumcentre of a triangle is centre of the circle that circumscribes the triangle.

(iii) Angle formed by any side of the triangle at the circumcentre is twice the vertical angle opposite to the side.



In figure, perpendicular bisectors of sides LM, MN and NL of ΔLMN meets at C. Hence C is the circumcentre of the triangle LMN.

$\angle MCN = 2\angle MLN$.

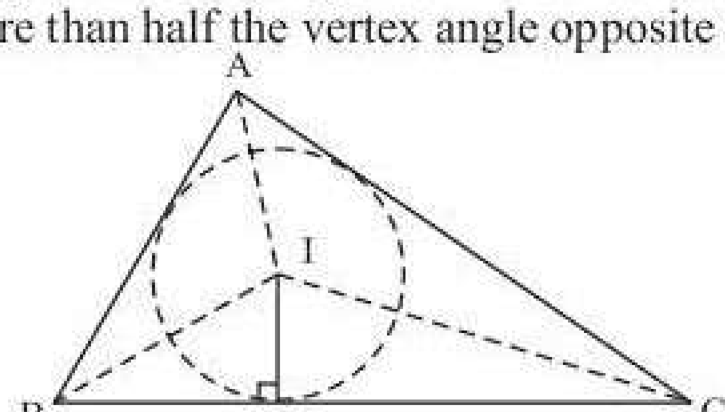
Angle Bisectors and Incentre

Lines bisecting the interior angles of a triangle are called angle bisectors of triangle.

(i) Angle bisectors of a triangle are concurrent and the point of concurrency is called Incentre of the triangle, denoted by I.

(ii) With I as centre and radius equal to length of the perpendicular drawn from I to any side, a circle can be drawn touching the three sides of the triangle. So this is called incircle of the triangle. Incentre is equidistant from all the sides of the triangle.

(iii) Angle formed by any side at the incentre is always 90° more than half the vertex angle opposite to the side.

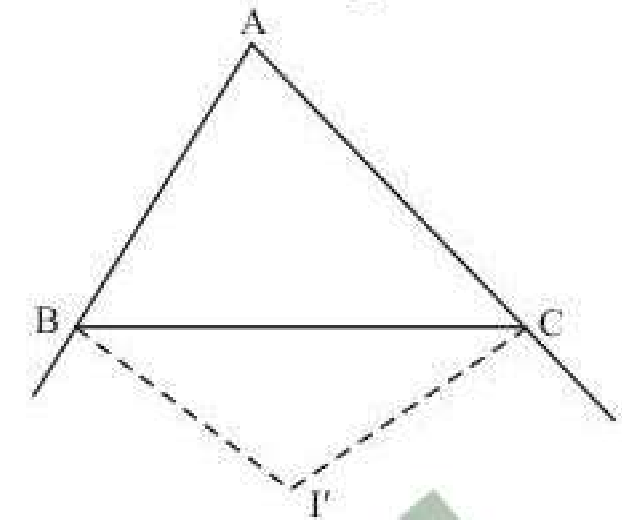


In figure AI, BI, CI are angle bisectors of ΔABC.

Hence I is the incentre of the ΔABC and

$$\angle BIC = 90° + \frac{1}{2}\angle A,\ \angle AIC = 90° + \frac{1}{2}\angle B$$

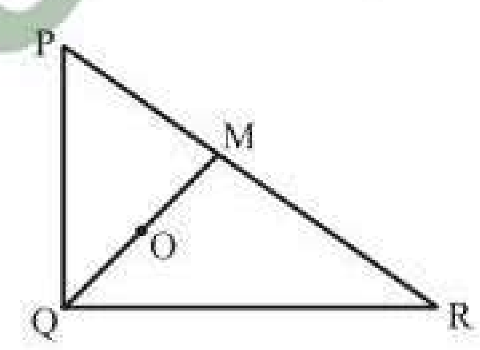
and $$\angle AIB = 90° + \frac{1}{2}\angle C$$



If BI' and CI' be the angle bisectors of exterior angles at B and C, then

$$\angle BI'C = 90° - \frac{1}{2}\angle A.$$

Example 5 : If in the given figure $\angle PQR = 90°$, O is the centroid of ΔPQR, $PQ = 5$ cm and $QR = 12$ cm, then OQ is equal to



***Solution*:**

By Pythagoras theorem,

$$PR = \sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

$\therefore$ O is centroid $\Rightarrow QM$ is median and M is mid-point of PR.

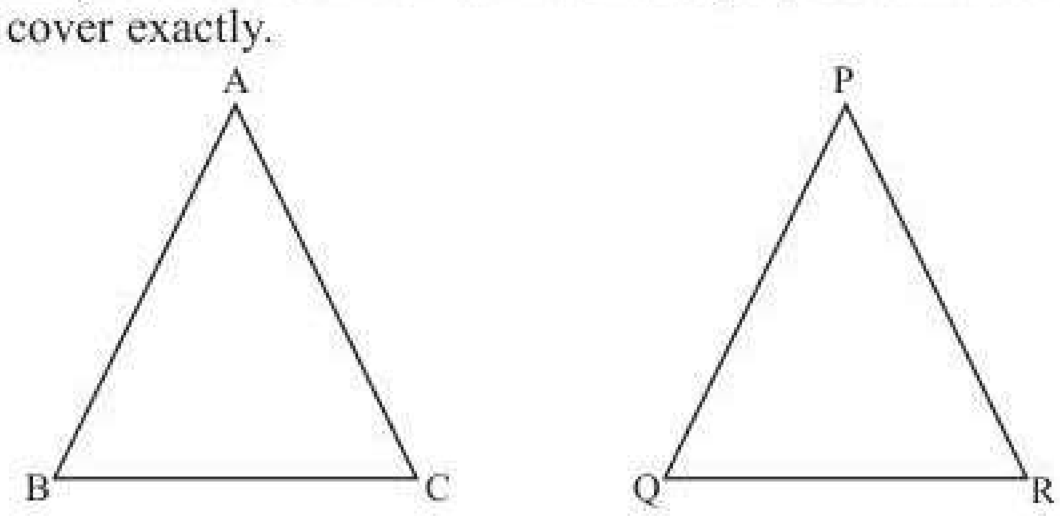
$$QM = PM = \frac{13}{2}$$

$\therefore$ Centroid divides median in ratio 2 : 1.

$$\therefore\ OQ = \frac{2}{3}QM = \frac{2}{3}\times\frac{13}{2} = \frac{13}{3}\ \therefore\ OQ = 4\frac{1}{3}\text{cm}$$

CONGRUENCY OF TWO TRIANGLES

Two triangles are congruent if they are of the same shape and size i.e .if any one of them can be made to superpose on the other it will cover exactly.



If two triangles ABC and PQR are congruent then 6 elements (i.e. three sides and three angles) of one triangle are equal to corresponding 6 elements of other triangle.

(i) $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

(ii) $AB = PQ, BC = QR, AC = PR$

This is symbolically written as $\Delta ABC \cong \Delta PQR$

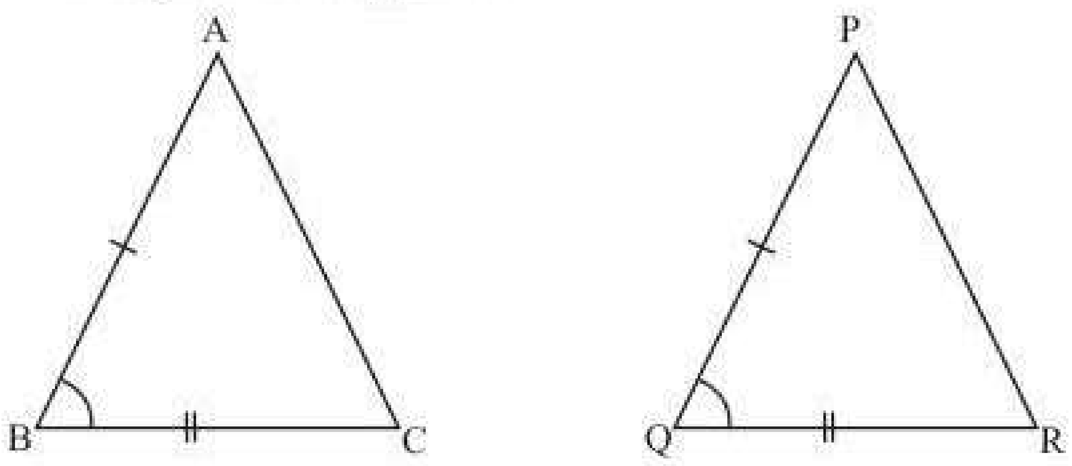
Remember

- ✧ In two congruent triangles, sides opposite to equal angles are corresponding sides and angles opposite to equal sides are corresponding angles.

Conditions of Congruency

There are 4 conditions of congruency of two triangles.

1. **SAS (Side-Angle-Side) Congruency:** If two sides and the included angle between these two sides of one triangle is equal to corresponding two sides and included angle between these two sides of another triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

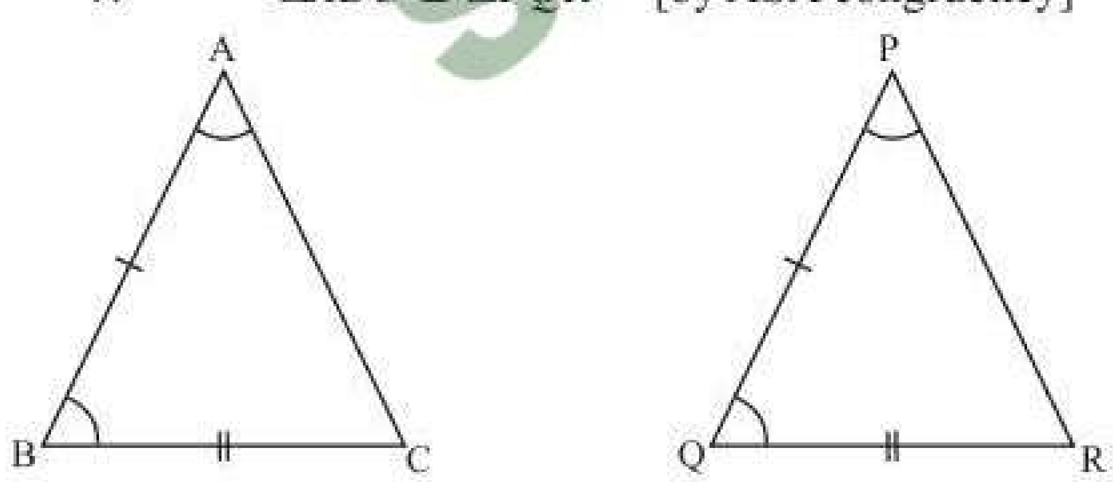
$$AB = PQ,$$
$$BC = QR$$
and $$\angle ABC = \angle PQR$$
$\therefore$ $\Delta ABC \cong \Delta PQR$ [by SAS congruency]

Here $\cong$ is the sign of congruency.

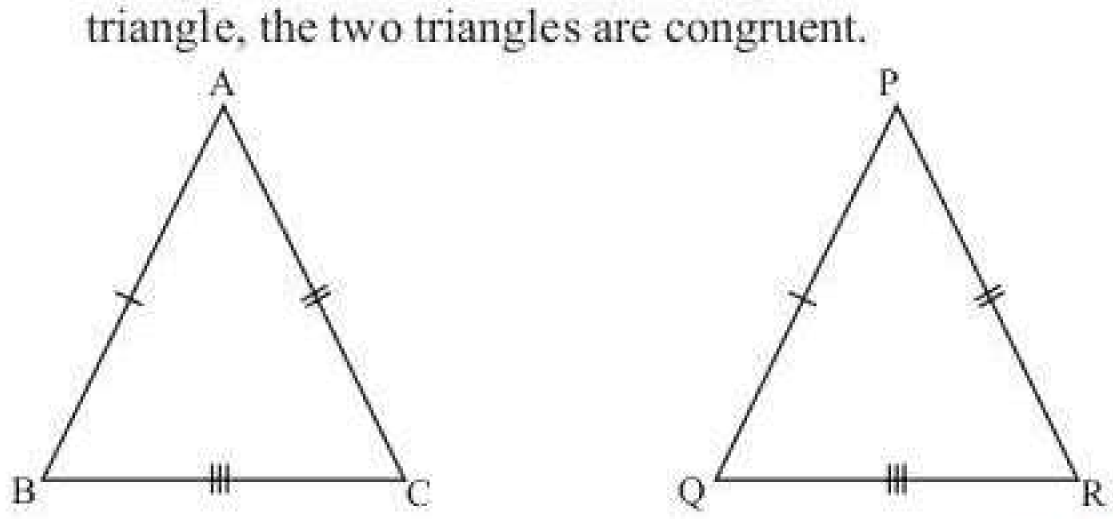
2. **ASA (Angle-Side-Angle) Congruency:** If two angles and included side between these two angles of one triangle are equal to corresponding angles and included side between these two angles of another triangle, then two triangles are congruent.

In ΔABC and ΔPQR

$$\angle A = \angle P$$
$$\angle B = \angle Q$$
$$AB = PQ$$
$\therefore$ $\Delta ABC \cong \Delta PQR$ [by ASA congruency]



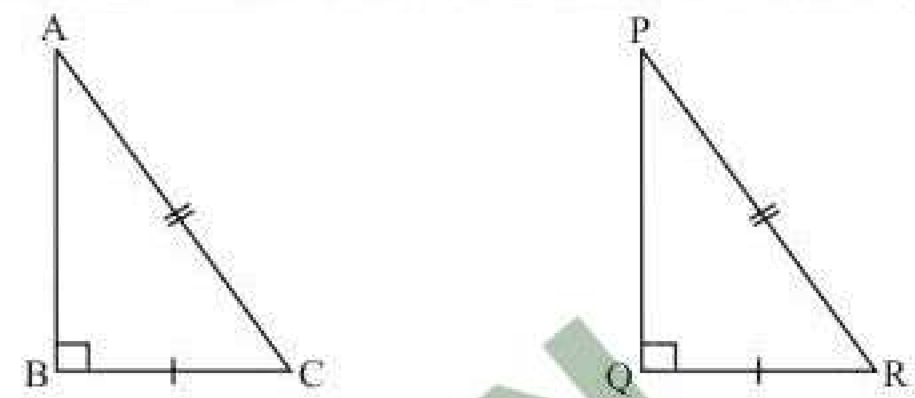
3. **SSS (Side-Side-Side) Congruency:** If three sides of one triangle are equal to corresponding three sides of another triangle, the two triangles are congruent.



In ΔABC and ΔPQR

$$AB = PQ$$
$$BC = QR$$
$$CA = RP$$
$\therefore$ $\Delta ABC \cong \Delta PQR$ [by SSS congruency]

4. **RHS (Rightangle-Hypotenuse-Side) Congruency:** Two right angled triangles are congruent to each other if hypotenuse and one side of one triangle are equal to hypotenuse and corresponding side of another triangle.



In ΔABC and ΔPQR

$$\angle ABC = \angle PQR = 90°$$
$$AC = PR$$
$$BC = QR$$
$\therefore$ $\Delta ABC \cong \Delta PQR$ [by RHS congruency]

SIMILARITY OF TWO TRIANGLES

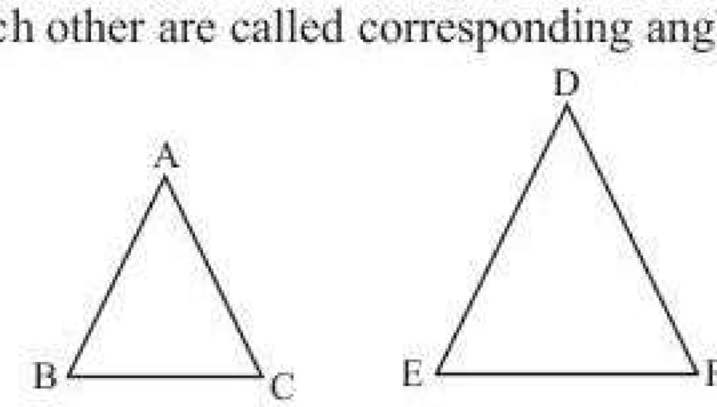
Two triangles are said to be similar, if their shapes are the same but their size may or may not be equal.

When two triangles are similar, then

(i) all the corresponding angles are equal and

(ii) all the corresponding sides are in the same ratio (or proportion)

Note: In two similar triangles, sides opposite to equal angles are called corresponding sides. And angles opposite to side proportional to each other are called corresponding angles.



If ΔABC and ΔDEF are similar, then

$$\angle A = \angle D$$
$$\angle B = \angle E$$
$$\angle C = \angle F$$
and $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$\Delta ABC \sim \Delta DEF$, read as triangle ABC is similar to triangle DEF.

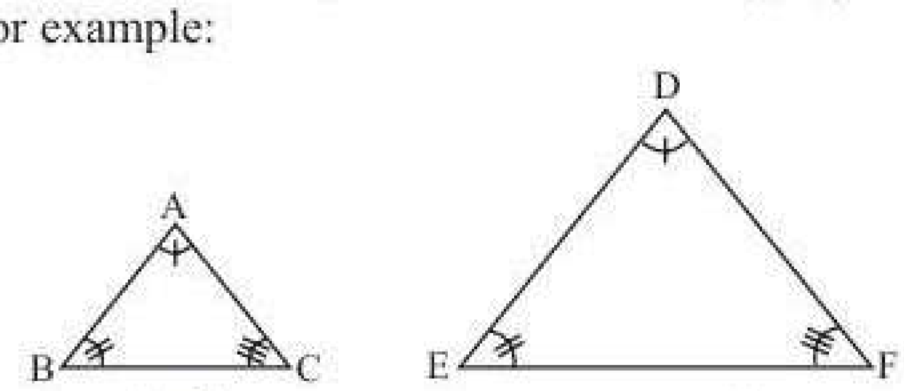
Here ~ is the sign of similarity.

Conditions of Similarity

There are 4 conditions of similarity.

1. **AAA (Angle–Angle–Angle) Similarity:** Two triangles are said to be similar, if their all corresponding angles are equal.

For example:



In ΔABC and ΔDEF, if

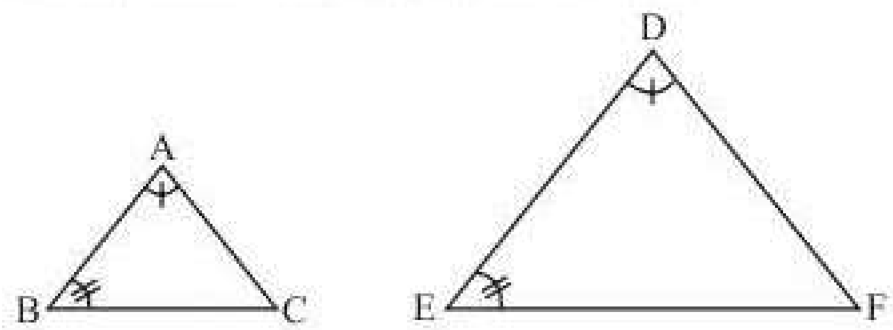
$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

Then $\Delta ABC \sim \Delta DEF$ [By AAA Similarity]

Corollary AA (Angle-Angle) Similarity: If two angles of one triangle are respectively equal to two angles of another triangles, then two triangles are similar.



In ΔABC and ΔDEF, if

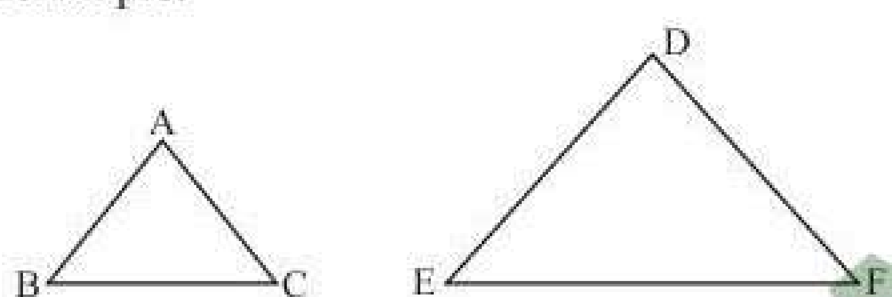
$\angle A = \angle D$

$\angle B = \angle E$

then $\Delta ABC \sim \Delta DEF$ [By AA Similarity]

2. **SSS (Side–Side–Side) Similarity:** Two triangles are said to be similar, if sides of one triangle are proportional (or in the same ratio of) to the sides of the other triangle:

For example:



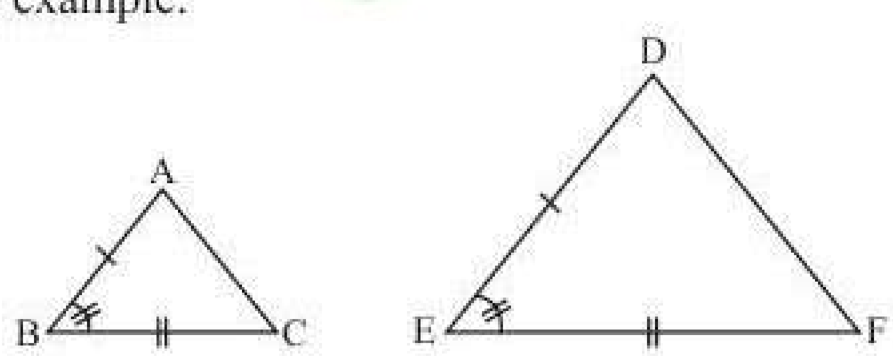
In ΔABC and ΔDEF, if

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Then $\Delta ABC \sim \Delta DEF$ [By SSS Similarity]

3. **SAS (Side–Angle–Side) Similarity:** Two triangles are said to be similar if two sides of a triangle are proportional to the two sides of the other triangle and the angles included between these sides of two triangles are equal.

For example:



In ΔABC and ΔDEF, if

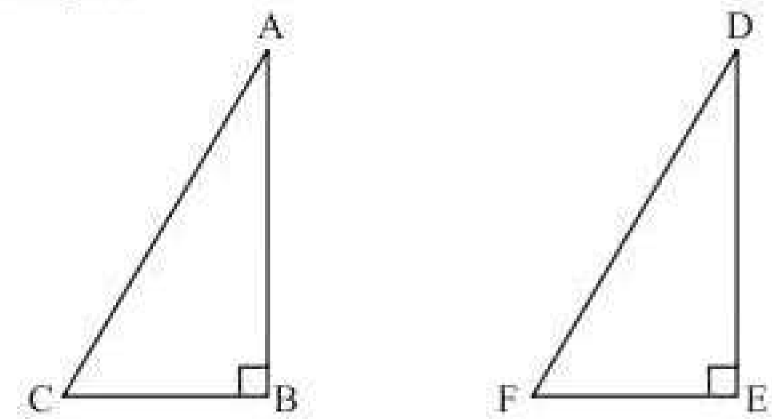
$$\frac{AB}{DE} = \frac{BC}{EF}$$

and $\angle B = \angle E$

Then, $\Delta ABC \sim \Delta DEF$ [By SAS Similarity]

4. **RHS (Rightangle-Hypotenuse-Side) Similarity:** Two triangles are said to be similar if one angle of both triangle is right angle and hypotenuse of both triangles are proportional to any one other side of both triangles respectively.

For example:



In ΔABC and ΔDEF, if

$\angle B = \angle E$ [= 90°]

$$\frac{AC}{DF} = \frac{AB}{DE}$$

Then $\Delta ABC \sim \Delta DEF$ [By RHS similarity]

Note: In similar triangles,

Ratio of medians = Ratio of corresponding heights
= Ratio of circumradii
= Ratio of inradii

Remember

✧ If two triangles are similar, then ratio of areas of two similar triangle is equal to the ratio of square of corresponding sides.

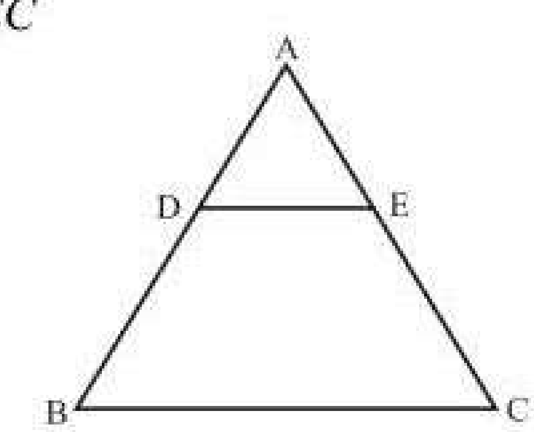
Example 6 : ***D* and *E* are the points on the sides *AB* and *AC* respectively of a ΔABC and *AD* = 8 cm, *DB* = 12 cm, *AE* = 6 cm and *EC* = 9 cm, then *BC* is equal to**

Solution :

As in ΔADE and ΔABC

$$\frac{AD}{AB} = \frac{8}{20} = \frac{2}{5}, \frac{AE}{AC} = \frac{6}{15} = \frac{2}{5}$$

So, $\frac{AD}{AB} = \frac{AE}{EC}$



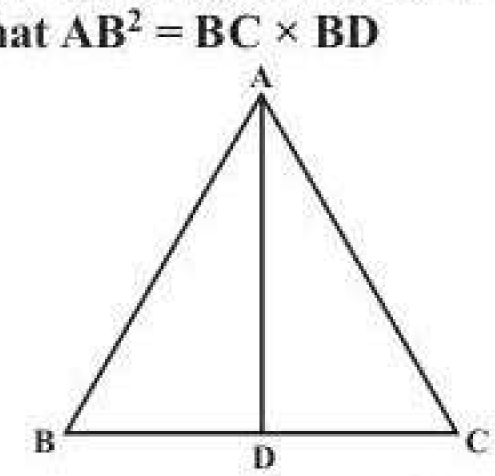
and $\angle A = \angle A$ (common)

$\Delta ADE \sim \Delta ABC$

$$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB} \Rightarrow \frac{DE}{BC} = \frac{2}{5}$$

$$\Rightarrow \quad BC = \frac{5}{2} DE$$

Example 7 : In a right angled ΔABC in which $\angle A = 90°$. If $AD \perp BC$, show that $AB^2 = BC \times BD$



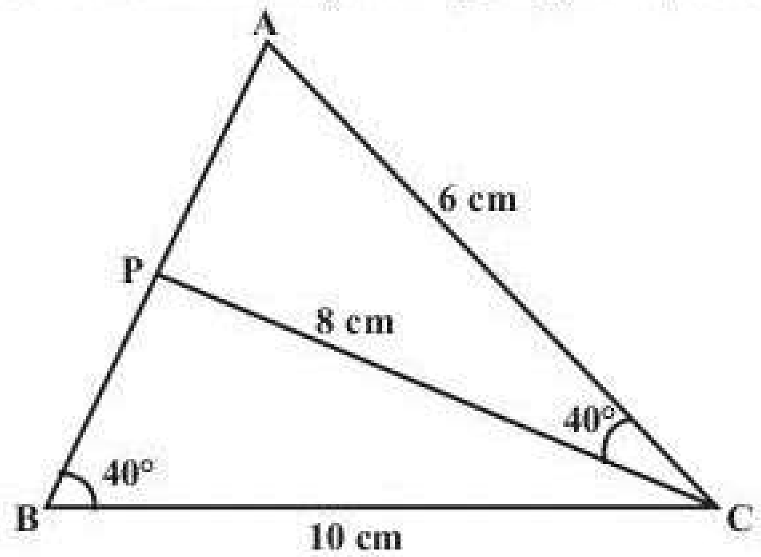
(a) $AB^2 = BD \times DC$ **(b)** $AB^2 = BD \times AD$
(c) $AB^2 = BC \times DC$ **(d)** $AB^2 = BC \times BD$

Solution : Clearly, $\Delta ABD \sim \Delta CBA$

$$\Rightarrow \frac{AB}{BD} = \frac{CB}{BA}$$

$$\Rightarrow AB^2 = BC \times BD$$

Example 8 : From the adjoining diagram, calculate



Solution: In ΔAPC and ΔABC,

$\angle ACP = \angle ABC$

$\angle A = \angle A$

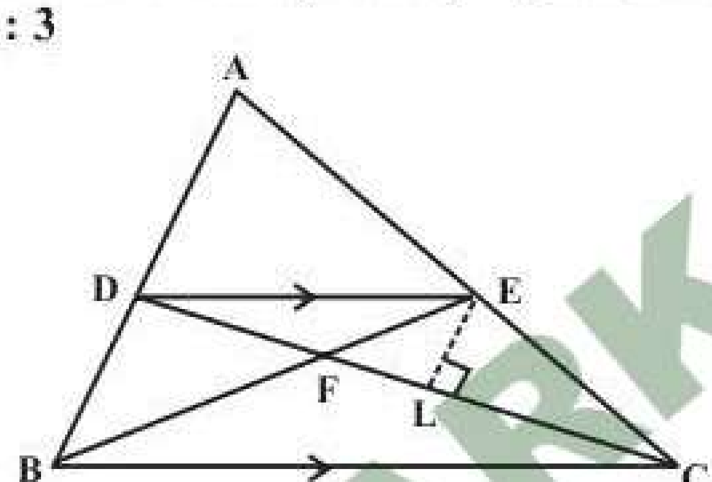
$$\Rightarrow \Delta ACP \sim \Delta ABC \Rightarrow \frac{AP}{AC} = \frac{PC}{BC} = \frac{AC}{AB}$$

$$\therefore \frac{AP}{6} = \frac{8}{10} = \frac{6}{AB}$$

$$\Rightarrow AP = 6 \times \frac{8}{10} = 4.8 \text{ and } AB = \frac{60}{8} = 7.5$$

$$\Rightarrow AP = 4.8 \text{ cm and } AB = 7.5 \text{ cm}$$

Example 9 : In the adjoining figure, $DE \parallel BC$ and $AD : DB = 4 : 3$



Find $\frac{AD}{AB}$ and then $\frac{DE}{BC}$

Solution :

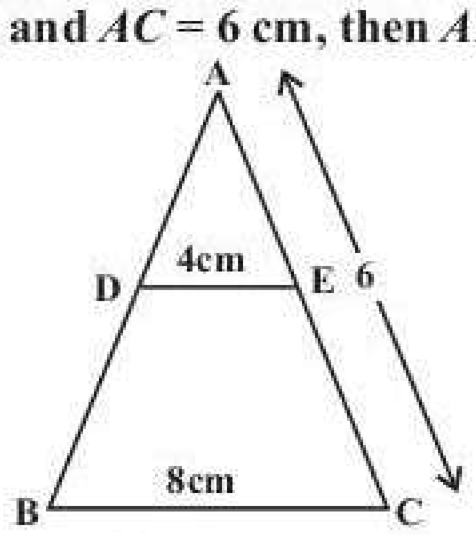
Since the sides of similar triangles are proportional, we have

$$\frac{AD}{AB} = \frac{DE}{BC}$$

But, $\frac{AD}{DB} = \frac{4}{3} \Rightarrow \frac{AD}{AD + DB} = \frac{4}{4+3} \Rightarrow \frac{AD}{AB} = \frac{4}{7}$

$$\therefore \frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{7}$$

Example 10 : In the given figure, DE parallel to BC. If $AD = 2$ cm, $DB = 3$ cm and $AC = 6$ cm, then AE is



Solution :

The triangles ADE and ABC are similar.

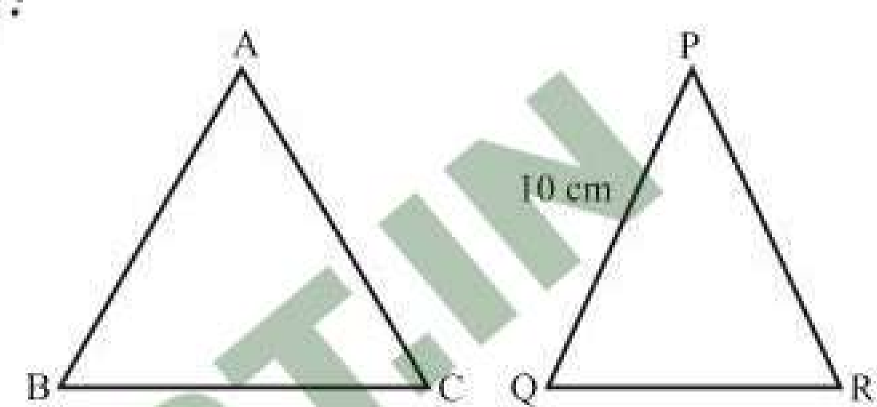
$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$$

or $\frac{2}{5} = \frac{AE}{6}$

$$\therefore AE = \frac{12}{5} = 2.4\text{cm}$$

Example 11 : The perimeters of two similar triangles ABC and PQR are 36 cm, and 24 cm, respectively. If $PQ = 10$ cm, then the length of AB is :

Solution :



Δ ABC and ΔPQR are similar.

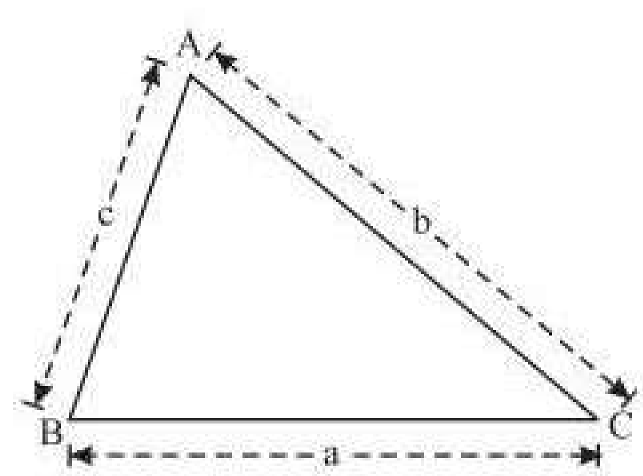
$$\frac{AB}{PQ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} \Rightarrow \frac{AB}{PQ} = \frac{36}{24}$$

or $AB = \frac{36}{24} \times 10 = 15$

SINE AND COSINE RULE

If in a Δ ABC; a, b and c are the length of the sides opposite to vertices A, B and C respectively, then

(i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (sine rule)



(ii) $a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = c^2 + a^2 - 2ca \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$ (Cosine rule)

Note that $\sin 0° = 0$, $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{1}{\sqrt{2}}$,

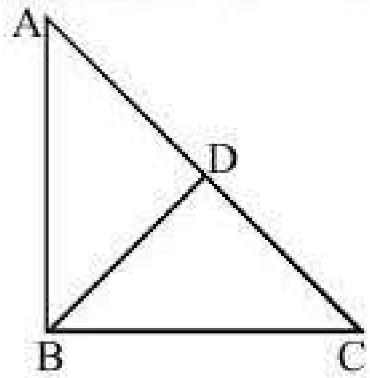
$\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 90° = 1$

$\cos 0° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$,

$\cos 60° = \frac{1}{2}$, $\cos 90° = 0$

EXERCISE

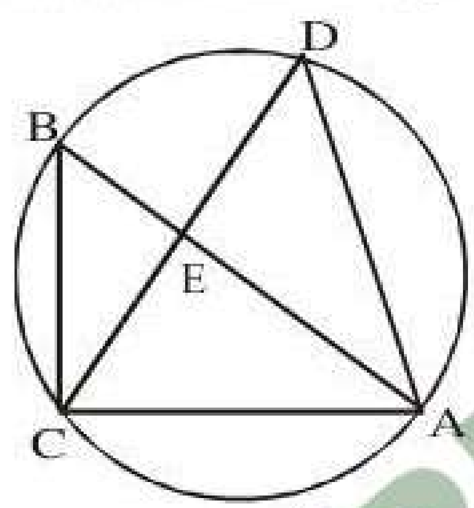
1. In triangle ABC, angle B is a right angle. If (AC) is 6 cm, and D is the mid-point of side AC. The length of BD is



(a) 4 cm (b) $\sqrt{6}$cm
(c) 3 cm (d) 3.5 cm

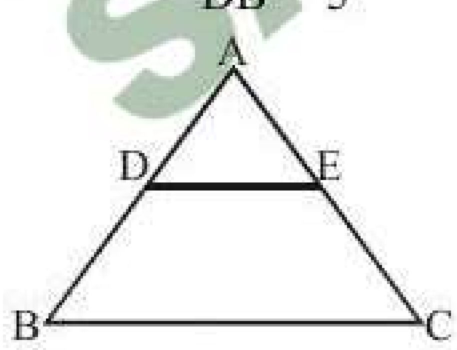
2. In a triangle ABC, points P, Q and R are the mid-points of the sides AB, BC and CA respectively. If the area of the triangle ABC is 20 sq. units, find the area of the triangle PQR
(a) 10 sq. units (b) 5.3 sq. units
(c) 5 sq. units (d) None of these

3. In the adjoining the figure, points A, B, C and D lie on the circle. AD = 24 and BC = 12. What is the ratio of the area of the triangle CBE to that of the triangle ADE



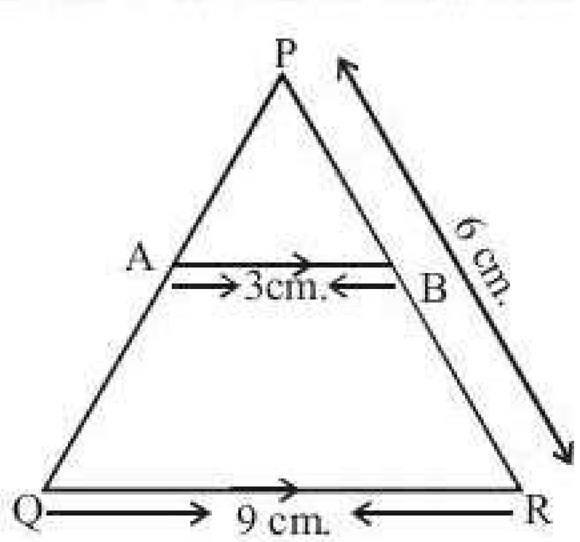
(a) 1 : 4 (b) 1 : 2
(c) 1 : 3 (d) Insufficient data

4. In ΔABC, DE || BC and $\frac{AD}{DB}=\frac{3}{5}$. If AC = 5.6 cm, find AE.



(a) 2.1 cm (b) 3.1 cm
(c) 1.2 cm (d) 2.3 cm

5. In the given fig. AB || QR, find the length of PB.



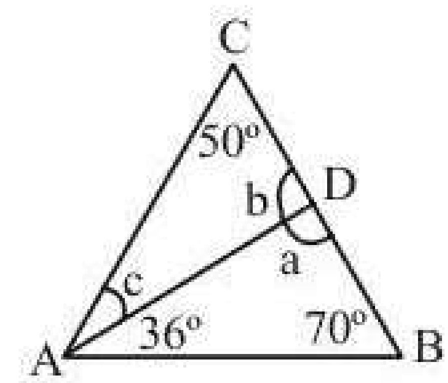
(a) 3 cm (b) 2 cm
(c) 4 cm (d) 6 cm

6. In ΔABC, AD is the bisector of $\angle A$ if AC = 4.2 cm., DC = 6 cm., BC = 10 cm., find AB.
(a) 2.8 cm (b) 2.7 cm
(c) 3.4 cm (d) 2.6 cm

7. In a triangle ABC, the internal bisector of the angle A meets BC at D. If AB = 4, AC = 3 and $\angle A = 60°$, then the length of AD is
(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$
(c) $15\sqrt{\frac{3}{8}}$ (d) $6\sqrt{\frac{3}{7}}$

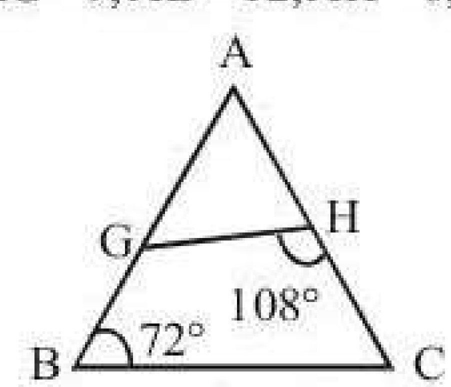
8. The centroid of a triangle, whose vertices are (2, 1), (5, 2) and (3, 4) is
(a) $\left(\frac{8}{3},\frac{7}{3}\right)$ (b) $\left(\frac{10}{3},\frac{7}{3}\right)$
(c) $\left(-\frac{10}{3},\frac{7}{3}\right)$ (d) $\left(\frac{10}{3},-\frac{7}{3}\right)$

9. Given the adjoining figure. Find a, b, c



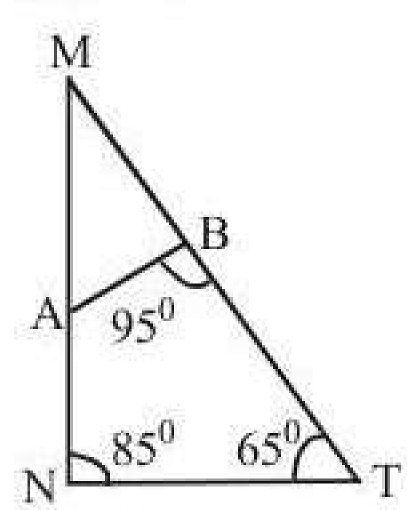
(a) 74°, 106°, 240° (b) 90°, 20°, 24°
(c) 60°, 30°, 24° (d) 106°, 24°, 74°

10. In the figure AG = 9, AB = 12, AH = 6, Find HC.



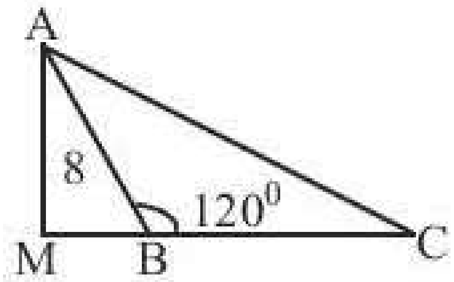
(a) 18 (b) 12
(c) 16 (d) 6

11. In the figure, if $\frac{NT}{AB}=\frac{9}{5}$ and if MB = 10, find MN.



(a) 5 (b) 4
(c) 28 (d) 18

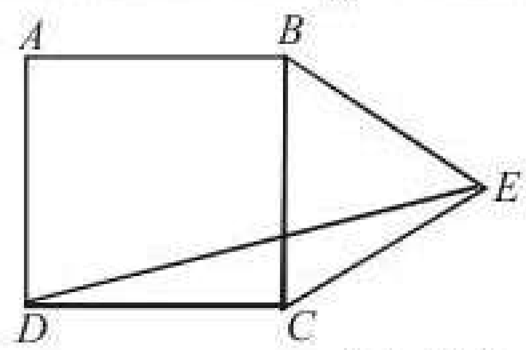
12. In the figure, AB = 8, BC = 7 m, $\angle ABC = 120^0$. Find AC.



(a) 11 (b) 12
(c) 13 (d) 14

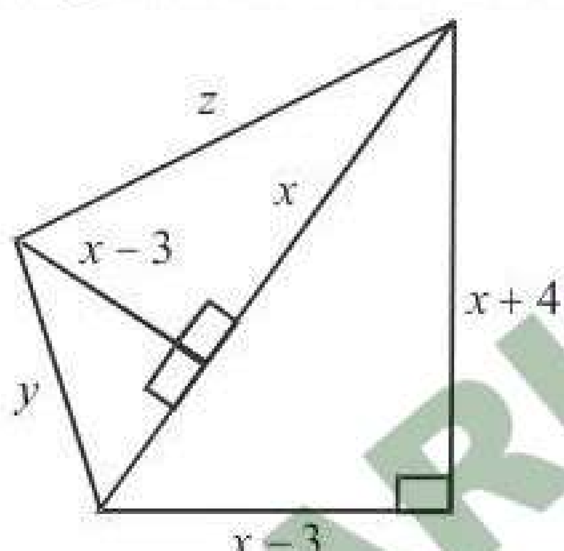
13. In a Δ ABC, $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$ and $\angle C = 50°$, then $\angle BAD = ?$
(a) 60° (b) 20°
(c) 30° (d) 50°

14. If *ABCD* is a square and *BCE* is an equilateral triangle, what is the measure of the angle *DEC*?



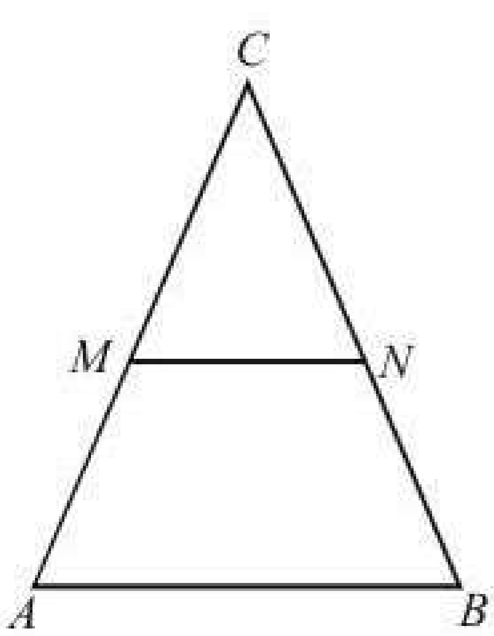
(a) 15° (b) 30°
(c) 20° (d) 45°

15. Based on the figure below, what is the value of *x*, if *y* = 10



(a) 10 (b) 11
(c) 12 (d) None of these

16. In the triangle *ABC*, *MN* is parallel to *AB*. Area of trapezium *ABNM* is twice the area of triangle *CMN*. What is ratio of *CM* : *AM* ?

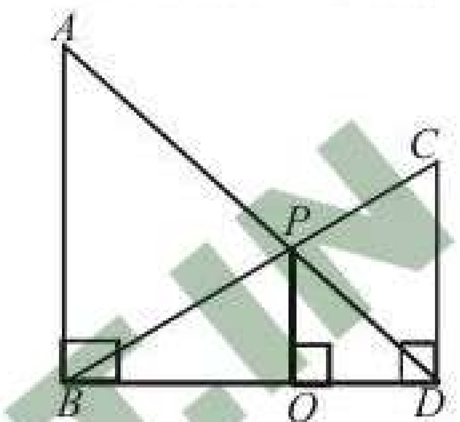


(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) None of these

17. In a ΔABC, angle *C* is 68°, the perpendicular bisector of *AB* at *R* meets *BC* at *P*. If $\angle PAC = 42°$ then $\angle ABC$ is equal to
(a) 45° (b) 42°
(c) 35° (d) 34°

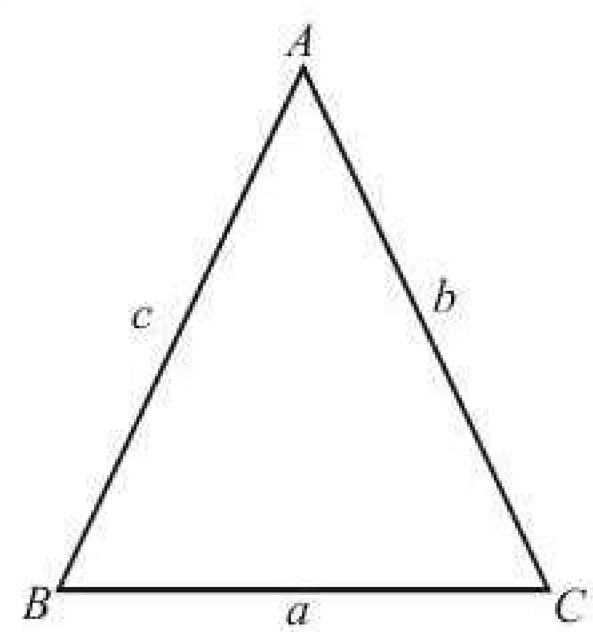
18. If in a ΔABC, $\angle B = 120°$, then which of the following is true? [cos 120° = $-\frac{1}{2}$]
(a) $a^2 + c^2 = b^2 + ac$ (b) $a^2 + c^2 = b^2 - ac$
(c) $a^2 + c^2 = b^2 + 2ac$ (d) $a^2 + c^2 = b^2 - 2ac$

19. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90°$. If $AB : CD = 3 : 1$, the ratio of $CD : PQ$ is



(a) 1 : 0.69 (b) 1 : 0.75
(c) 1 : 072 (d) None of these

20. In the given triangle *ABC*, the length of sides *AB* and *AC* is same (i.e., *b* = *c*) and $60° < A < 90°$, then the possible length of *BC* is

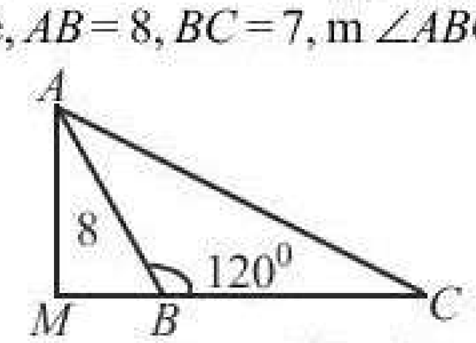


(a) $b < a < 2b$ (b) $\frac{c}{3} < a < 3a$
(c) $b < a < b\sqrt{3}$ (d) $c < a < c\sqrt{2}$

21. *a*, *b* and *c* are sides of a triangle. If $a^2 + b^2 + c^2 = ab + bc + ac$ then the triangle will be
(a) equilateral (b) isosceles
(c) right angled (d) obtuse angle

22. In an isosceles right angled triangle *ABC*, $\angle B$ is right angle. Angle bisector of $\angle BAC$ is *AN* cut at *M* to the median *BO*. Point '*O*' lies on the hypotenuse, *OM* is 20 cm, then the value of *AB* is:
(a) 38.96 cm (b) 24.18 cm
(c) 34.134 cm (d) None of these

23. In the figure, *AB* = 8, *BC* = 7, m $\angle ABC = 120°$. Find *AC*.

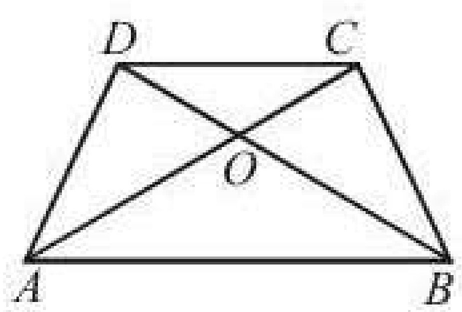


(a) 11 (b) 12
(c) 13 (d) 14

24. An equilateral triangle BPC is drawn inside a square $ABCD$. What is the value of the angle APD in degrees?
(a) 75 (b) 90
(c) 120 (d) 150

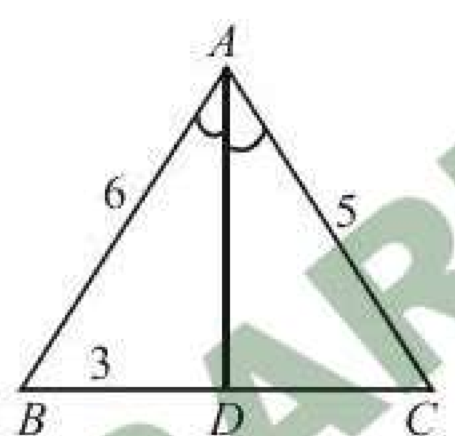
25. Let S be an arbitrary point on the side PQ of an a cute angle ΔPQR. Let T be the point of intersection of QR extended with the straight line PT drawn parallel to SR through P. Let U be the point of intersection of PR extended with the straight line QU drawn parallel to SR through Q. If $PT = a$ and $QU = b$, then the length of SR is
(a) $\frac{a+b}{ab}$ (b) $\frac{a-b}{ab}$
(c) $\frac{ab}{a+b}$ (d) $\frac{ab}{a-b}$

26. In the adjoining figure, $ABCD$ is a trapezium in which $AB \,||\, DC$ and $AB = 3\,DC$. Determine the ratio of the areas of (ΔAOB and ΔCOD).



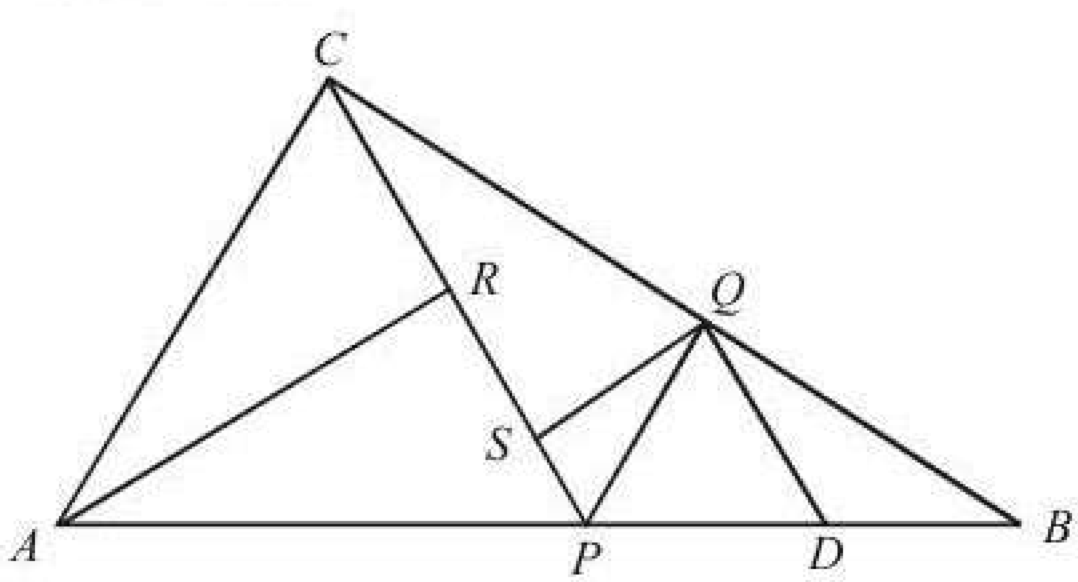
(a) 9 : 1 (b) 1 : 9
(c) 3 : 1 (d) 1 : 3

27. In the given figrue, AD is the bisector of $\angle BAC$, $AB = 6$ cm, $AC = 5$ cm and $BD = 3$ cm. Find DC.



(a) 11.3 cm (b) 2.5 cm
(c) 3.5 cm (d) 4 cm

28. In the figure (not drawn to scale) given below, P is a point on AB such that $AP : PB = 4 : 3$. PQ is parallel to AC and QD is parallel to CP. In ΔARC, $\angle ARC = 90°$, and in ΔPQS, $\angle PSQ = 90°$. The length of QS is 6 cms. What is ratio $AP : PD$?



(a) 10 : 3 (b) 2 : 1
(c) 7 : 3 (d) 8 : 3

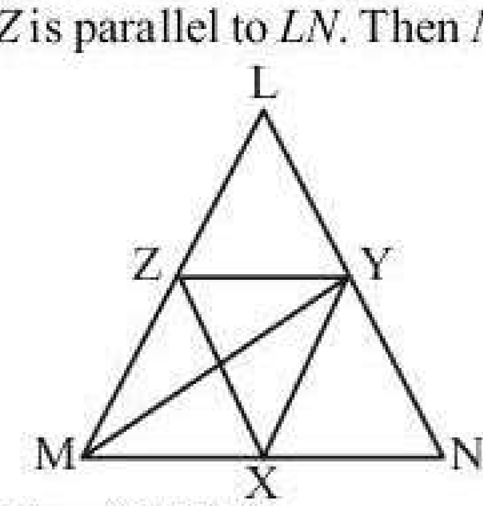
29. The medians of ΔABC intersect at G. Which one of the following is correct?
(a) Five times the area of ΔAGB is equal to four times the area of ΔABC
(b) Four times the area of ΔAGB is equal to three times the area of ΔABC
(c) Three times the area of ΔAGB is equal to the area of ΔABC
(d) None of the above

30. ABC is a right angled triangle such that $AB = a - b$, $BC = a$ and $CA = a + b$. D is a point on BC such that $BD = AB$. The ratio of $BD : DC$ for any value of a and b is given by
(a) 3 : 2 (b) 4 : 3
(c) 5 : 4 (d) 3 : 1

31. In a right angled ΔABC, $\angle C = 90°$ and CD is perpendicular to AB. If $AB \times CD = CA \times CB$, then $\frac{1}{CD^2}$ is equal to
(a) $\frac{1}{AB^2} - \frac{1}{CA^2}$ (b) $\frac{1}{AB^2} - \frac{1}{CB^2}$
(c) $\frac{1}{BC^2} + \frac{1}{CA^2}$ (d) $\frac{1}{BC^2} - \frac{1}{CA^2}$, if $CA > CB$

32. Let ABC be a triangle with $AB = 3$ cm and $AC = 5$ cm. If AD is a median drawn from the vertex A to the side BC, then which one of the following is correct?
(a) AD is always greater than 4 cm but less than 5 cm
(b) AD is always greater than 5 cm
(c) AD is always less than 4 cm
(d) None of the above

33. In the figure given below, YZ is parallel to MN, XY is parallel to LM and XZ is parallel to LN. Then MY is



(a) The median of ΔLMN
(b) the angular bisector of $\angle LMN$
(c) perpendicular to LN
(d) perpendicular bisector of LN

34. A ΔDEF is formed by joining the mid-points of the sides of ΔABC. Similarly, a ΔPQR is formed by joining the mid-points of the sides of the ΔDEF. If the sides of the ΔPQR are of lengths 1, 2 and 3 units, what is the perimeter of the ΔABC?
(a) 18 units (b) 24 units
(c) 48 units (d) Cannot be determined

35. Consider the following statements
I. If the diagonals of a parallelogram $ABCD$ are perpendicular, then $ABCD$ may be a rhombus.
II. If the diagonals of a quadrilateral $ABCD$ are equal and perpendicular, then $ABCD$ is a square.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

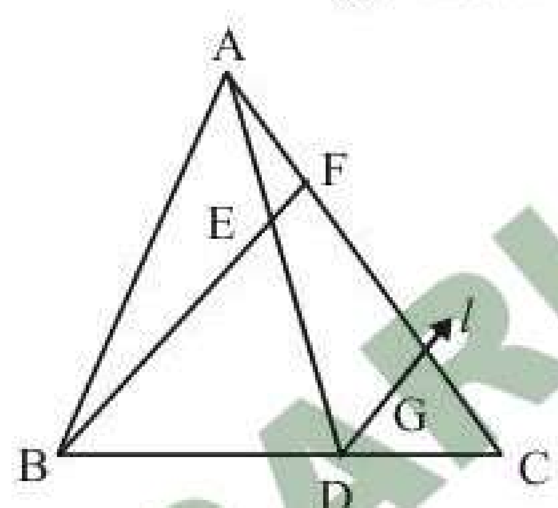
36. ABC and XYZ are two similar triangles with $\angle C = \angle Z$, whose areas are respectively 32 cm^2 and 60.5 cm^2. If XY = 7.7 cm, then what is AB equal to?
(a) 5.6 cm (b) 5.8 cm
(c) 6.0 cm (d) 6.2 cm

37. The three sides of a triangle are 15, 25, x units. Which one of the following is correct?
(a) $10 < x < 40$ (b) $10 \le x \le 40$
(c) $10 \le x < 40$ (d) $10 < x \le 40$

38. Which one of the following is a Pythagorean triple in which one side differs from the hypotenuse by two units?
(a) $(2n+1, 4n, 2n^2+2n)$ (b) $(2n, 4n, n^2+1)$
(c) $(2n^2, 2n, 2n+1)$ (d) $(2n, n^2-1, n^2+1)$
Where, n is a positive real number.

39. The sides of a triangle are in geometric progression with common ratio $r < 1$. If the triangle is a right angled triangle, the square of common ratio is given by
(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{5}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$

40. In a ΔABC, AD is the median through A and E is the mid–point of AD and BE produced meets AC at F. Then, AF is equal to
(a) $AC/5$ (b) $AC/4$
(c) $AC/3$ (d) $AC/2$



41. Three straight lines are drawn through the three vertices of a ΔABC, the line through each vertex being parallel to the opposite side. The ΔDEF is bounded by these parallel lines.
Consider the following statements in respect of the ΔDEF.
1. Each side of ΔDEF is double the side of ΔABC to which it is parallel.
2. Area of ΔDEF is four times the area of ΔABC.
Which of the above statements is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

42. In a ΔABC, if $\angle B = 2\angle C = 2\angle A$. Then, what is the ratio of AC to AB ?
(a) $\sqrt{2}:1$ (b) $\sqrt{3}:1$
(c) $1:1$ (d) $1:\sqrt{2}$

43. Consider the following :
ABC and DEF are triangles in a plane such that AB is parallel to DE, BC is parallel to EF and CA is parallel to FD.
Statement I If $\angle ABC$ is a right angle, then $\angle DEF$ is also a right angle.
Statement II Triangles of the type ABC and DEF are always congruent.
Which one of the following is correct in respect of the above statements ?
(a) Statements I and II are correct and Statement II is the correct explanation of Statement I
(b) Statements I and II are correct and Statement II is not the correct explanation of Statement I
(c) Statement I is correct and Statement II is incorrect
(d) Statement I is incorrect and Statement II is correct

44. Consider the following statements in respect of an equilateral triangle :
1. The altitudes are congruent.
2. The three medians are congruent.
3. The centroid bisects the altitude.
Which of the above statements are correct ?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1,2 and 3

45. If every side of an equilateral triangle is doubled, then the area of new triangle becomes k times the area of the old one. What is k equal to?
(a) $\sqrt{3}$ (b) 2
(c) 4 (d) 8

DIRECTIONS: For the next three (3) items that follow.
Consider the triangle ABC with vertices A (–2, 3), B (2, 1) and C (1, 2).

46. What is the circumcentre of the triangle ABC ?
(a) (–2,–2) (b) (2, 2)
(c) (–2, 2) (d) (2, –2)

47. What is the centroid of the tirnalge ABC ?
(a) $\left(\frac{1}{3},1\right)$ (b) $\left(\frac{1}{3},2\right)$
(c) $\left(1,\frac{2}{3}\right)$ (d) $\left(\frac{1}{2},3\right)$

48. What is the foot of the altitude from the vertex A of the triangle ABC?
(a) (1, 4) (b) (–1, 3)
(c) (–2, 4) (d) (–1, 4)

49. The angles of a triangle are in the ratio 4 : 1 : 1. Then the ratio of the largest side to the perimeter is
(a) $\frac{2}{3}$ (b) $\frac{1}{2+\sqrt{3}}$
(c) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (d) $\frac{2}{1+\sqrt{3}}$

50. Let a, b, c be the sides of a right triangle, where c is the hypotenuse. The radius of the circle which touches the sides of the triangle is **(CDS)**
(a) $(a+b-c)/2$ (b) $(a+b+c)/2$
(c) $(a+2b+2c)/2$ (d) $(2a+2b-c)/2$

51. Consider the following statements : **(CDS)**
1. Let D be a point on the side BC of a triangle ABC. If area of triangle ABD = area of triangle ACD, then for all points O on' AD, area of triangle ABO = area of triangle ACO.

2. If G is the point of concurrence of the medians of a triangle ABC, then area of triangle ABG = area of triangle BCG = area of triangle ACG.

Which of the above statements is /are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

52. Suppose ABC is a triangle with AB of unit length D and E are the points lying on AB and AC respectively such that BC and DE are parallel. If the area of triangle ABC is twice the area of triangle ADE, then the length of AD is **(CDS)**

(a) $\frac{1}{2}$ unit (b) $\frac{1}{3}$ unit
(c) $\frac{1}{\sqrt{2}}$ unit (d) $\frac{1}{\sqrt{3}}$ unit

53. Let the triangles ABC and DEF be such that $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $\angle BAC = \angle EDF$. Let L be the midpoint of BC and M be the midpoint of EF. Consider the following statements:

Statement I. Triangles ABL and DEM are similar.

Statement II. Triangle ALC is congruent to triangle DMF even in $AC \neq DF$

Which one of the following is correct in respect of the above statements? **(CDS)**

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.
(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

54. Let ABC be a triangle in which $AB = AC$. Let L be the locus of points X inside or on the triangle such that $BX = CX$. Which of the following statements are correct? **(CDS)**

1. L is a straight line passing through A and in-centre of triangle ABC is on L.
2. L is a straight line passing through A and orthocentre of triangle ABC is a point on L.
3. L is a straight line passing through A and centroid of triangle ABC is a point on L.

Select the correct answer using the code given below:

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

55. In a triangle PQR, point X is on PQ and point Y is on PR such that $XP = 1\ 5$ units, $XQ = 6$ units, $PY = 2$ units and $YR = 8$ units. Which of the following are correct? **(CDS)**

1. $QR = 5XY$
2. QR is parallel to XY.
3. Triangle PYX is similar to triangle PRQ.

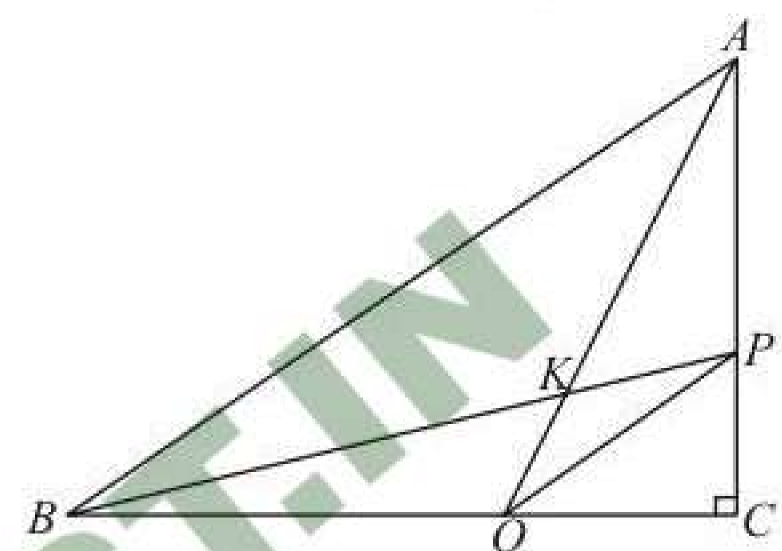
Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

56. A person travels 7 km eastwards and then turns right and travels 3 km and further turns right and travels 13 km. What is the shortest distance of the present position of the person from his starting point? **(CDS)**

(a) 6km (b) $3\sqrt{5}$ km
(c) 7km (d) $4\sqrt{5}$ km

57.



ABC is a triangle right angled at C as shown in the figure above. Which one of the following is correct?

(a) $AQ^2 + AB^2 = BP^2 + PQ^2$
(b) $AQ^2 + PQ^2 = AB^2 + BP^2$
(c) $AQ^2 + BP^2 = AB^2 + PQ^2$
(d) $AQ^2 + AP^2 = BK^2 + KQ^2$

58. ABC is an equilateral triangle and X, Y and Z are the points on BC, CA and AB respectively such that $BX = CY = AZ$. Which of the following is/are correct? **(CDS)**

1. XYZ is an equilateral triangle.
2. Triangle XYZ is similar to triangle ABC.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

59. An equilateral triangle BOC is drawn inside a square ABCD. If angle AOD= 2θ, what is tanθ equal to ? **(CDS)**

(a) $2-\sqrt{3}$ (b) $1+\sqrt{2}$
(c) $4-\sqrt{3}$ (d) $2+\sqrt{3}$

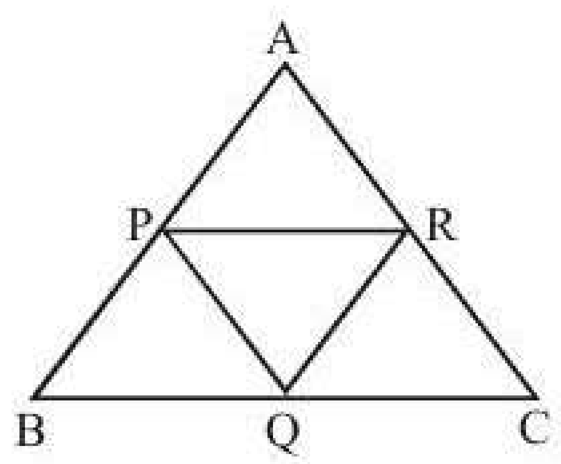
60. A square is inscribed in a right triangle with legs x and y and has common right angle with the triangle. The perimeter of the square is given by **(CDS)**

(a) $\frac{2xy}{x+y}$ (b) $\frac{4xy}{x+y}$
(c) $\frac{2xy}{\sqrt{x^2+y^2}}$ (d) $\frac{4xy}{\sqrt{x^2+y^2}}$

HINTS & SOLUTIONS

1. (c) In a right angled Δ, the length of the median is $\frac{1}{2}$ the length of the hypotenuse. Hence $BD = \frac{1}{2}AC = 3cm.$

2. (c) Consider for an equilateral triangle. Hence ΔABC consists of 4 such triangles with end points on mid points AB, BC and CA



$\Rightarrow \frac{1}{4}$ ar (ΔABC) = ar (ΔPQR)

$\Rightarrow$ ar (ΔPQR) = 5 sq. units

3. (a) AD = 24, BC = 12

In ΔBCE & ΔADE

since $\angle CBA = \angle CDA$ (Angles by same arc)

$\angle BCE = \angle DAE$ (Angles by same arc)

$\angle BEC = \angle DEA$ (Opp. angles)

$\therefore \angle BCE \ \& \angle DAE$ are similar Δs

with sides in the ratio 1 : 2

Ratio of area = 1:4 (i.e square of sides)

4. (a) In ΔABC, DE || BC

By applying basic Proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

But $\frac{AD}{DB} = \frac{3}{5}$ (Given)

$$\therefore \frac{AE}{EC} = \frac{3}{5} \text{ or } \frac{AE}{EC + AE} = \frac{3}{5+3} \text{ or } \frac{AE}{AC} = \frac{3}{8}$$

$$\text{or } \frac{AE}{5.6} = \frac{3}{8} \Rightarrow 8AE = 3 \times 5.6 \Rightarrow AE = 3 \times 5.6/8$$

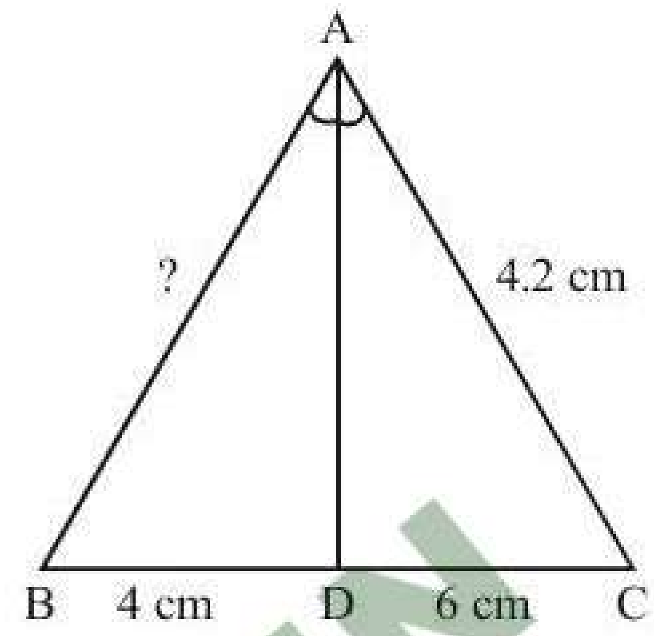
$\therefore AE = 2.1$ cm.

5. (b) ΔPAB ~ ΔPQR

$$\frac{PB}{AB} = \frac{PR}{QR} \Rightarrow \frac{PB}{3} = \frac{6}{9}$$

$\therefore$ PB = 2 cm

6. (a)



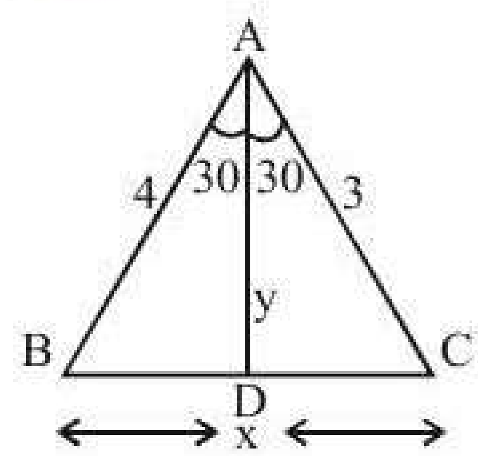
using angle bisector theorem

$$\frac{AC}{AB} = \frac{DC}{BD} \Rightarrow \frac{4.2}{6} = \frac{AB}{4}$$

$\therefore$ AB = 2.8 cm

Height of wall = 12 + 3 = 15 m

7. (b)



Using the theorem of angle of bisector,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3} \Rightarrow BD = \frac{4}{7}x \ \& \ DC = \frac{3}{7}x$$

In ΔABD, by sine rule, $\frac{\sin 30}{4/7x} = \frac{\sin B}{y}$ ......(i)

In ΔABC, by sine rule; $\frac{\sin 60}{x} = \frac{\sin B}{3}$

or $\frac{\sqrt{3}}{2x} = \frac{\sin 30.y}{4/7x \times 3}$ [putting the value of sin B from (i)]

$$\Rightarrow y = \frac{\sqrt{3}}{2x} \times \frac{4}{7}x \times 3 \times \frac{2}{1} = \frac{12\sqrt{3}}{7}$$

8. (b) $x = \frac{2+5+3}{3} = \frac{10}{3}$ and $y = \frac{1+2+4}{3} = \frac{7}{3}$

9. (a) $a + 36^o + 70^o = 180^o$ (sum of angles of triangle)

$\Rightarrow a = 180^o - 36^o - 70^o = 74^o$

$b = 36^o + 70^o$(Ext. angle of triangle) = 106^o

$c = a - 50^o$ (Ext. angle of triangle) $= 74^o - 50^o = 24^o$.

10. (b) $\angle AHG = 180 - 108 = 72^0$

$\therefore \angle AHG = \angle ABC = 720\ (<BAC = \angle GAH)$

$\therefore \Delta AHG - \Delta ABC$ .....(AA test for similarity)

$\frac{AH}{AB} = \frac{AG}{AC}$; $\frac{6}{12} = \frac{9}{AC}$

$\therefore AC = \frac{12 \text{ x } 9}{6} = 18$

$\therefore HC = AC - AH = 18 - 6 = 12$

11. (d) $\angle MBA = 180^o - 95^o = 85^o$

$\angle AMB = \angle TMN$ ...(Same angles with different names)

$\therefore \Delta MBA \sim \Delta MNT$ ......(AA test for similarity)

$\frac{MB}{MN} = \frac{AB}{NT}$ ....(proportional sides)

$\frac{10}{MN} = \frac{5}{9}$ $\quad \therefore MN = \frac{90}{5} = 18.$

12. (c) $m\angle ABM = 180^o - 120^o = 60^o$

$\therefore \Delta AMB$ is a $30^o - 60^o - 90^o$ triangle.

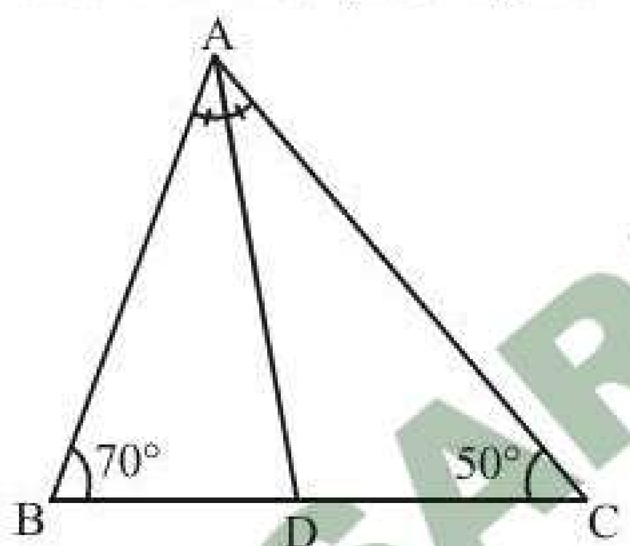
$\therefore AM = \frac{\sqrt{3}}{2} AB = \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}$

$MB = \frac{1}{2} AB = \frac{1}{2} \text{ x } 8 = 4$

$(AC)^2 = (AM)^2 + (MC)^2 = (4\sqrt{3})^2 + (4+7)^2$

$= 48 + 121 = 169$; $AC = \sqrt{169} = 13.$

13. (c)



Given, $\frac{AB}{AC} = \frac{BD}{DC}$

According to angle bisector theorem which states that the angle bisector, like segment AO, divides the sides of the triangle proportionally. Therefore, $\angle A$ being the bisector of triangle.

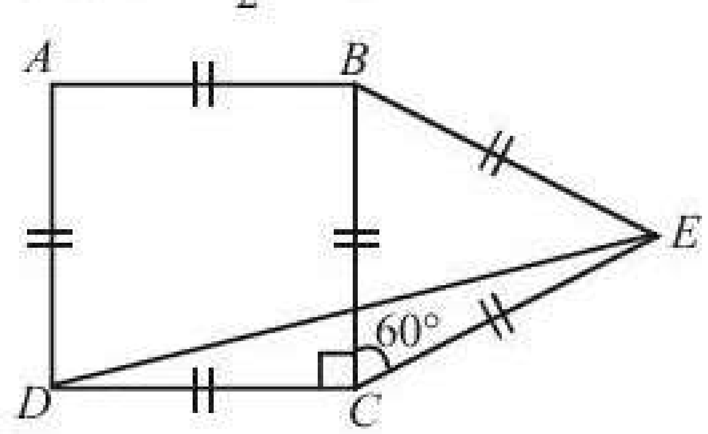
In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - 70° = 60°$

$\angle BAD = \frac{60°}{2} = 30°$

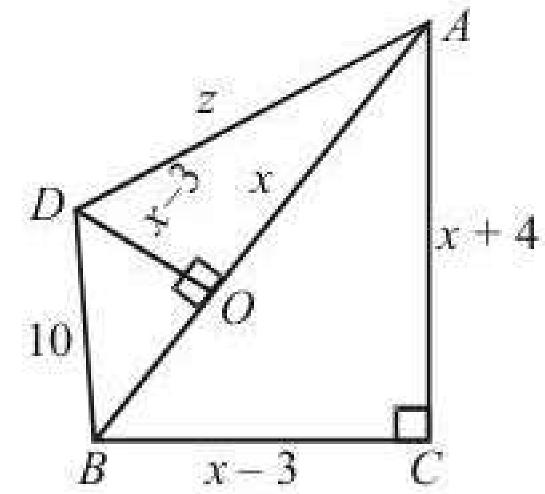
14. (a)



In $\Delta DEC, \angle DCE = 90 + 60 = 150°$

$\angle CDE = \angle DEC = \frac{180 - 150}{2} = 15°$

15. (b)



$AB^2 = (x+4)^2 + (x-3)^2 = 2x^2 + 25 + 2x$

Since solving this equation is very difficult. So, it is a better approach (Time saving) to put the values given in the options and try to find out a solution.

Hence, trying out we get 11 as the value of x.

16. (c) $\frac{ar(\Delta CMN)}{ar(ABNM)} = \frac{1}{2}$

$\therefore \quad \frac{ar(\Delta CMN)}{ar(\Delta CAB)} = \frac{1}{3}$

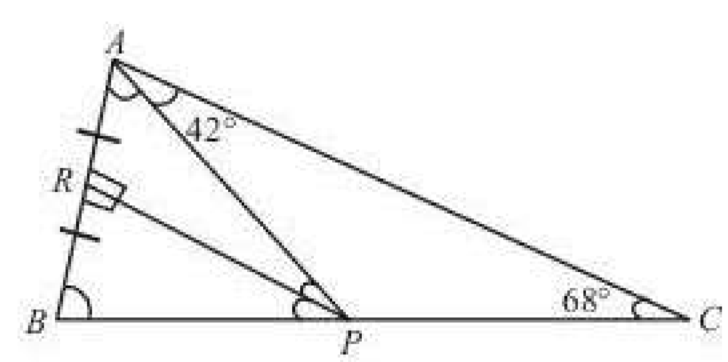
$\Rightarrow \quad \frac{MN}{AB} = \frac{CM}{CA} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{CM}{MA} = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2}$

$MA = (CA - CM)$

17. (c) $\angle APB = 42° + 68° = 110°$

(Exterior angle of a triangle is equal to sum of opposite interior $\angle s$).



$\Delta APR \cong \Delta BPR$ [SAS condition]

$\therefore \angle RPB = \angle RPA = \frac{110°}{2} = 55°$

$\therefore$ In $\Delta BRP, \angle ABC = 90° - 55° = 35°.$

18. (b) $\cos B = (a^2 + c^2 - b^2)/2ac$

$\angle B = 120°$ and $\cos 120° = -\frac{1}{2}$,

$-\frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow -ac = a^2 + c^2 - b^2$

$\Rightarrow a^2 + c^2 = b^2 - ac$

19. (b) Using the Properties of similar triangles,

$$\frac{CP}{PB}=\frac{CD}{AB}=\frac{1}{3};$$

In ΔBPQ and BCD, $\frac{CD}{PQ}=\frac{BC}{BP}=\frac{4}{3}=1:0.75$

20. (d) At $\angle A=60°$, $BC=b=c$

and at $\angle A=90°$, $BC=\sqrt{2}b=\sqrt{2}c$

$\therefore 60°<A<90°$, $BC=c<a<c\sqrt{2}$

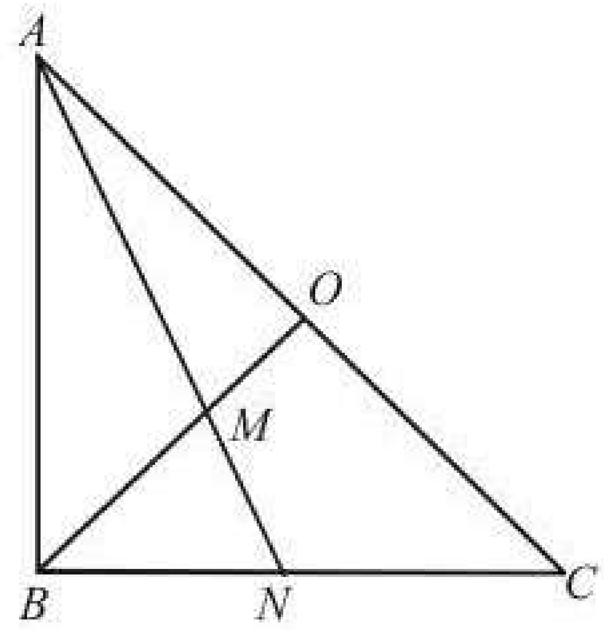
21. (a) $a^2+b^2+c^2=ab+bc+ac$

Put $a=b=c=k$ we get $3k^2=3k^2$, which satisfies the above equation. Thus the triangle is equilateral.

22. (d) Let $AB=BC=$ a

then $AC=\sqrt{2}a$

$$\therefore AO=OC=BO=\frac{\sqrt{2}a}{2}=\frac{a}{\sqrt{2}}$$



Now, by angle bisector theorem

$$\frac{AB}{AO}=\frac{BM}{MO}\Rightarrow\frac{BM}{MO}=\frac{a}{a\sqrt{2}}=\frac{\sqrt{2}}{1}$$

$\therefore$ $MO=20$ cm

$\therefore$ $BM=20\sqrt{2}$ cm

$\therefore$ $BO=20+20\sqrt{2}=20\left(1+\sqrt{2}\right)$cm

Now, since $BO=\frac{a}{\sqrt{2}}=\frac{AB}{\sqrt{2}}$

$\therefore AB=\sqrt{2}(BO)=1.414[20(1+1.414)]$

$=68.2679=68.27$ cm

23. (c) $m\angle ABM=180°-120°=60°$

$\therefore$ ΔAMB is a $30°-60°-90°$ triangle.

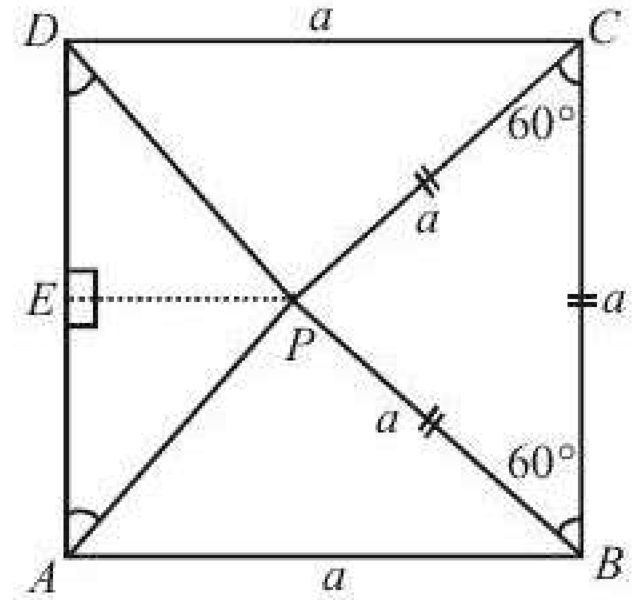
$\therefore$ $AM\ \frac{\sqrt{3}}{2}AB=\frac{\sqrt{3}}{2}\times 8=4\sqrt{3}$

$MB=\frac{1}{2}AB=\frac{1}{2}\times 8=4$

$(AC)^2=(AM)^2+(MC)^2=(4\sqrt{3})^2+(4+7)^2$

$=48+121=169;\ AC=\sqrt{169}=13.$

24. (d)



$\angle PBA=\angle ABC-\angle PBC=90°-60°=30°$

Further in ΔABP

$PB=AB=a\Rightarrow\ \angle BPA=\angle BAP$

Further $2(\angle BPA)+\angle PBA=180°$

$\Rightarrow\ 2\angle BPA=180°-30°=150°$

$\Rightarrow\ \angle BPA=75°=\angle BAP$

Similarly

$\angle PAD=90°-\angle PAB=90°-75°=15°$

Again in right angled ΔAPE,

$\angle EPA=90°-\angle PAE=90°-15°=75°$

Similarly we can calculate that $\angle DPE=75°$

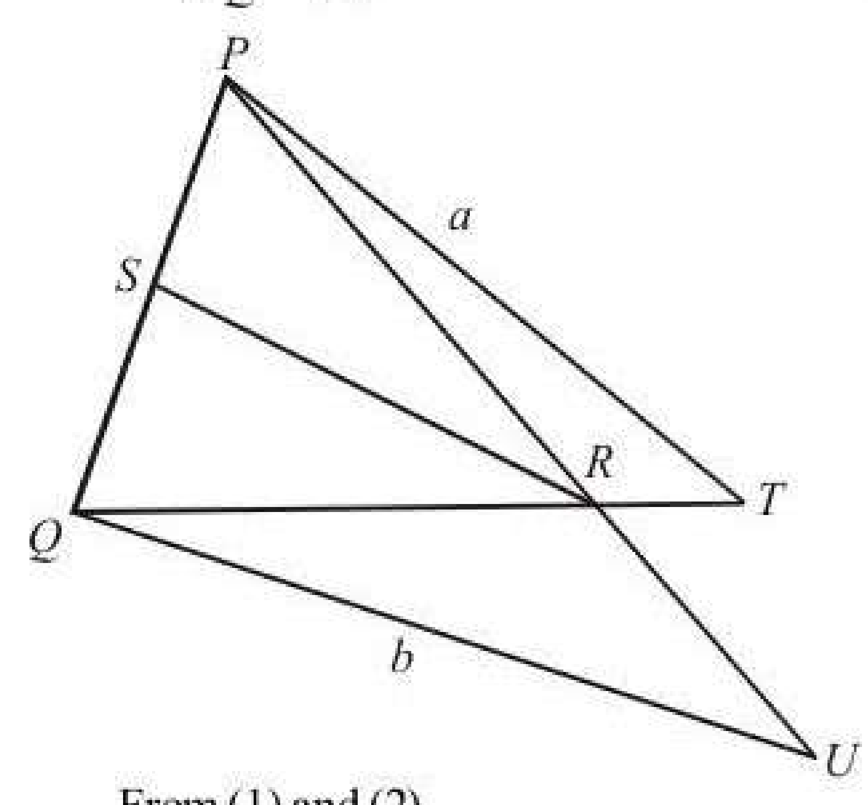
$\therefore\ \angle DPA=75°+75°=150°$

25. (c) $\Delta PQU\sim PSR$

$$\Rightarrow\frac{PS}{PQ}=\frac{SR}{QU}\quad\ldots(1)$$

$\Delta PQT\sim SQR$

$$\Rightarrow\frac{SQ}{PQ}=\frac{SR}{PT}\quad\ldots(2)$$



From (1) and (2)

$PQ\times SR=PS\times QU=SQ\times PT\Rightarrow=\frac{SQ}{PS}=\frac{b}{a}$

Now use componendo and equation (1) to obtain

$$SR=\frac{ab}{a+b}$$

26. (a) ΔDOC and ΔAOB are similar (by AAA property)

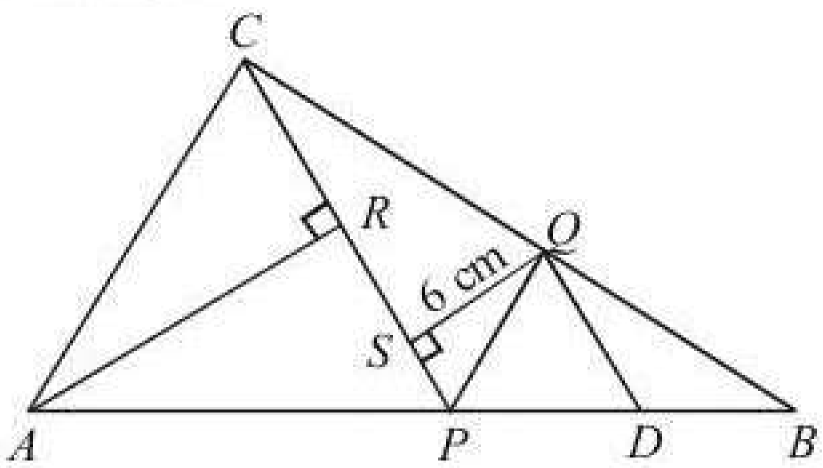
$$\frac{ar\,\Delta AOB}{ar\,\Delta DOC} = \frac{(AB)^2}{(DC)^2} = \frac{9}{1}$$

So area of AOB : Area of $DOC = (3:1)^2 \Rightarrow 9:1$

27. (b) In the given figure, ΔABD is similar to ΔACD

Then $\frac{AB}{BD} = \frac{AC}{DC} \Rightarrow \frac{6}{3} = \frac{5}{DC} \Rightarrow DC = 2.5$ cm

28. (c) From figure



Given that,

$PQ \,||\, AC,$

$$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$$

Again, $QD \,||\, CP,$

$$\therefore \frac{PD}{BD} = \frac{CQ}{QB} = \frac{4}{3}$$

$$\text{As } \frac{PD}{DB} = \frac{4}{3} \Rightarrow \frac{PD}{DB+PD} = \frac{4}{3+4} \Rightarrow \frac{PD}{PB} = \frac{4}{7}$$

$$\Rightarrow PD = \frac{4}{7}PB$$

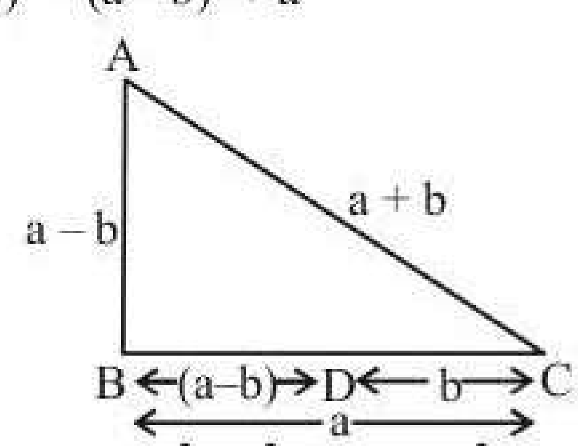
$$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4} \times \frac{AP}{PB} = \frac{7}{4} \times \frac{4}{3} = 7:3$$

29. (c) Suppose ΔABC is an equilateral triangle.
A median divides an equilateral triangle into the three equal area of triangles.

$$\Delta AGB = \text{ar}\,\frac{(\Delta ABC)}{3} = \text{ar}\,BGC = ar\,\Delta AGC$$

$$\therefore \text{ar}\,\Delta AGB = \frac{1}{3}\Delta ABC.$$

30. (d) In ΔABC
Using Pythagoras theorem
$(a+b)^2 = (a-b)^2 + a^2$

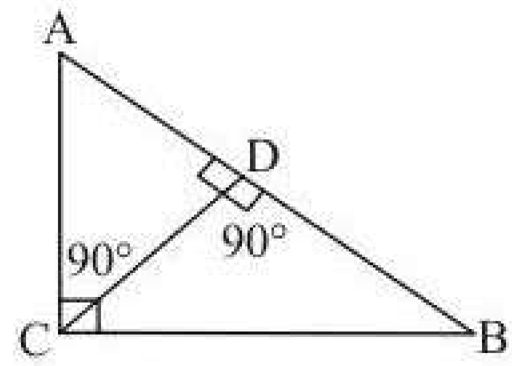


$\Rightarrow \quad a^2 + b^2 + 2ab = a^2 + b^2 - 2ab + a^2$

$\Rightarrow \quad 4ab = a^2 \Rightarrow 4b = a$

So, $\frac{BD}{DC} = \frac{a-b}{b} = \frac{4b-b}{b} = \frac{3b}{b} = \frac{3}{1}$

31. (c)



In ΔABC

$CD \perp AB$

and $AB \times CD = CA \times CB$ .... (i)

In ΔABC

$AB^2 = CA^2 + CB^2$

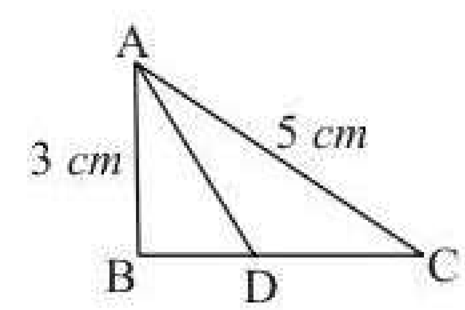
$$CD^2 = \frac{CA^2 \times CB^2}{AB^2} \text{ from ...(i)}$$

$$\frac{1}{CD^2} = \frac{AB^2}{CA^2 \times CB^2}$$

$$= \frac{CA^2 + CB^2}{CA^2 \times CB^2}$$

$$\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{CA^2}$$

32. (c) According to theorem:– the sum of any two sides of a triangle is greater than twice the median drawn to the third side.



$(AB + AC) > 2\,AD$

$(3+5) > 2AD$

$AD < 4$

Thus, AD is always less than 4 cm.

33. (a) Given that, $YZ \,\|\, MN$ and $XZ \,\|\, LN$

$\therefore XNYZ$ is a parallelogram.

$\Rightarrow ZX = YN$ ...(i)

Also, $ZX \,||\, YN$ and $XY \,||\, ZL$

Hence, $XYLZ$ is a parallelogram.

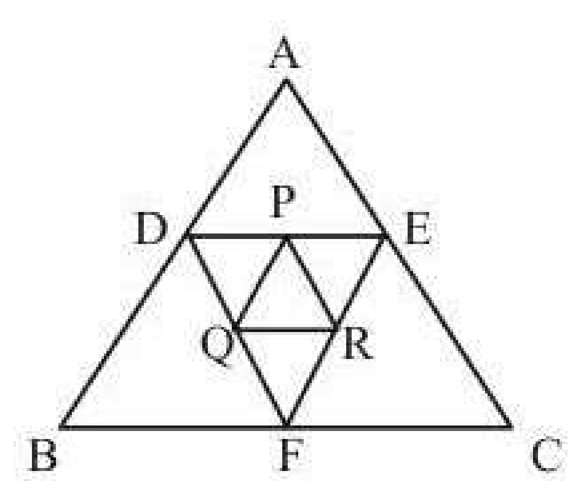
$\therefore \quad XZ = LY$ ...(ii)

Now, From Eqs. (i) and (ii),

$\therefore \quad YN = LY$

So, MY is a median of ΔLMN.

34. (b) Perimeter of $\Delta PQR = 1 + 2 + 3 = 6$ units



Now, in ΔDEF,

$$\frac{DQ}{DF}=\frac{1}{2}=\frac{PQ}{FE}$$

So, $2PQ = FE$

Similarly, $DF = 2PR$ and $DE = 2QR$

$\therefore$ perimeter of $\Delta DEF = 2 \times 6 = 12$ units

Similarly, perimeter of $\Delta ABC = 2 \times$ Perimeter of ΔDEF

$= 2 \times 12$

$= 24$ units

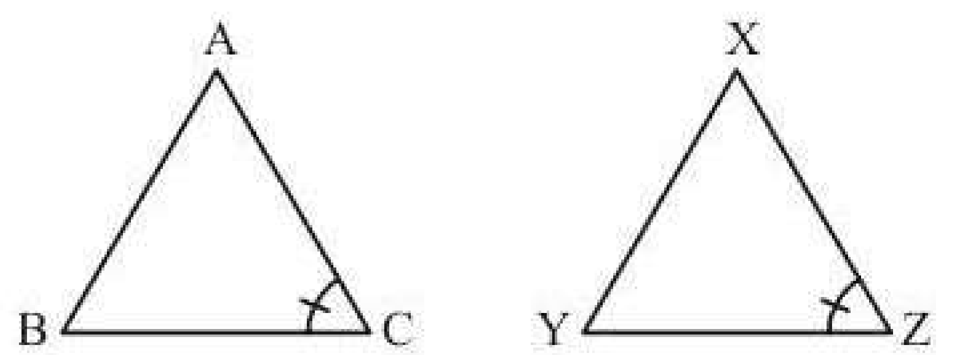
35. (c) **Statement-1**

If the diagonal of a parallelogram ABCD are perpendicular then ABCD may Rectangle or Rhombus. So it is true.

Statement-II

If the diagonal of quadrilateral $ABCD$ are equal and perpendicular then it is square.
So it is also true.

36. (a) We know that when two triangles are similar then ratio of their areas is equal to square of corresponding sides.

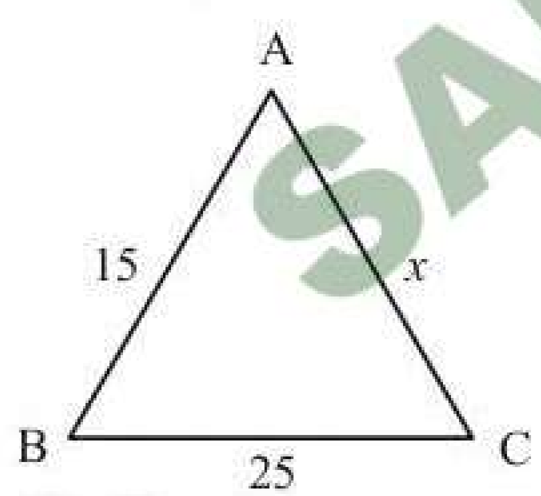


$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta XYZ}=\frac{AB^2}{XY^2} \Rightarrow \frac{32}{60.5}=\frac{AB^2}{(7.7)^2}$$

$$\frac{32\times 59.29}{60.5}=AB^2 \Rightarrow 31.36=AB^2$$

$$\therefore \ AB=\sqrt{31.36}=5.6\,cm$$

37. (a)



$AB = 15$

$BC = 25$

$AC = x$, then

We know that the sum of two sides of a triangle is always greater than third side.

$\Rightarrow \quad AB + BC > x$

$\Rightarrow \quad 15 + 25 > x \Rightarrow \quad 40 > x$ ... (i)

Also, the differences of two sides is always less than third side.

$BC - AB < AC$

$25 - 15 < x$

$10 < x$ ...(ii)

From eq. (i) and (ii)

$10 < x < 40$

38. (d) According to Pythagorean triplet.

The sum of square of base and perpendicular equal to square of hypotenuse.

By hit and trial method:—

$(2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2$

$4n^2 + n^4 + 1 - 2n^2 = n^4 + 2n^2 + 1$

$n^4 + 2n^2 + 1 = n^4 + 2n^2 + 1$

$LHS = RHS$

39. (b) The sides of a triangle in geometric progression are a, ar, ar^2

Triangle is right angled. Therefore, we use Pythagoras theorem.

$(a)^2 + (ar)^2 = (ar^2)^2$

$a^2 + a^2r^2 = a^2r^4$

$1 + r^2 = r^4$ or $r^4 - r^2 - 1 = 0$

$$\therefore r^2=\frac{-1\pm\sqrt{1-4(-1)}}{2}$$

$$r^2=\frac{-1\pm\sqrt{5}}{2}$$

$$r=\frac{-1+\sqrt{5}}{2}$$

$$r\neq\frac{-1-\sqrt{5}}{2}$$ (Because Radius is not negative)

So, common ratio $=\frac{\sqrt{5}-1}{2}$.

40. (c) In ΔABC, we draw a line $l \parallel BF$ which intersect AC at G.

In ΔADG and ΔAEF;

given that EA is the mid point of AD and $DL \parallel EF$.

So, concept of similar triangle.

F is also mid point of AG.

$AF = FG$ ...(i)

ΔADG and ΔAEF are similar.

Again

ΔFBC and ΔDCL

$BF \parallel DG$

given that AD is median so thad CD the mid point of BC.

G will be The mid point of CF

$CG = GF$ ...(ii)

From equations (i) and (ii), we get

$AF = FG = CG$ ...(iii)

From figure, $AC = AF + FG + CG$

$= AF + AF + AF + 3AF$

$\Rightarrow \quad AF = \frac{1}{3} AC$

41. (c)

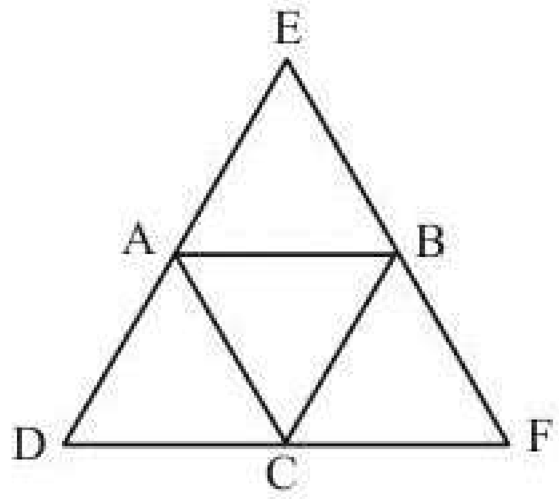
1. On drawing the three straight lines through the three vertices of ΔABC, we get the following figure.

Here, $AB \parallel DF$, $BC \parallel DE$ and $AC \parallel EF$.

Clearly, A, B and C are the mid–points of DE, EF and DF respectively.

By mid–point theorem, $BC = \frac{1}{2} DE$ or $DE = 2BC$

Similarly, $DF = 2AB$ and $EF = 2\,AC$. Hence, Statement 1 is correct.



2. Also, area of $\Delta ABC = \frac{1}{4}$ area of ΔDEF or area of $\Delta DEF = 4$ area of ΔABC. Hence, Statement 2 is also coreect.

42. (a) We know that, sum of angles of a triangle = 180°

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 2\angle A + \angle A = 180°$

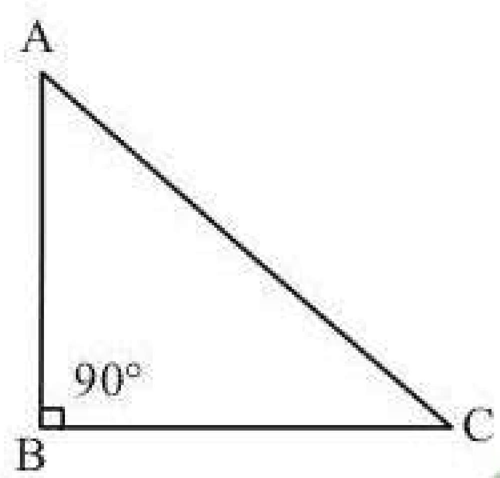
$\Rightarrow 4\angle A = 180°$

$\Rightarrow \angle A = \frac{180°}{4} = 45°$

$\angle B = 90°$ and $\angle C = 45°$ Givan that $2\angle C = 2\angle A = \angle B$]

ΔABC is a right angled triangle,

$\angle B = 90°$, $\angle C = 45°$ and $\angle A = 45°$



Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\Rightarrow AB^2 + AB^2 = AC^2$ $[\because AB = BC]$

$\Rightarrow 2AB^2 = AC^2$

$\Rightarrow \frac{AC^2}{AB^2} = \frac{2}{1}$

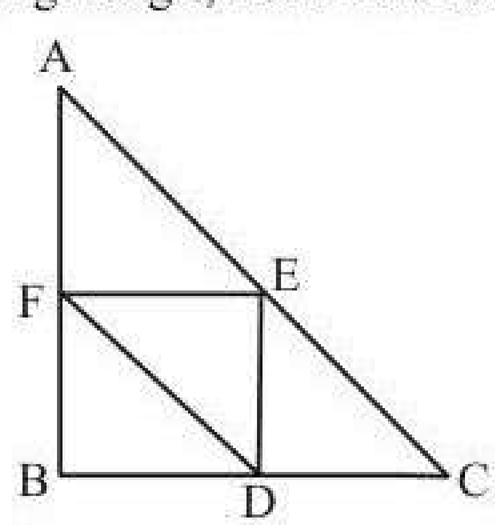
$\Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$

$\therefore AC : AB = \sqrt{2} : 1$

43. (c) In ΔABC and ΔDEF,

$AB \parallel DE$, $BC \parallel EF$ and $CA \parallel FD$

If $\angle ABC$ is right angle, then $\angle DEF$ is also a right angle.



Both triangles are similar but not congruent.

Statement I is correct and Statement I is correct and Statement II is incorrect.

44. (a) The altitude and medians of an equilateral triangle are congruent but centroid divide the altitude in 2 : 1. So, Statements 1 and 2 are correct.

45. (c) Let the sides of an old triangle be a, then area of an old equilateral triangle, $A_{old} = \frac{\sqrt{3}}{4}a^2$

Again, let the sides of a new triangle be 2a, then are of a new equilateral triangle,

$A_{new} = \frac{\sqrt{3}}{4}(2a)^2 = \frac{\sqrt{3}}{4} \times 4a^2$

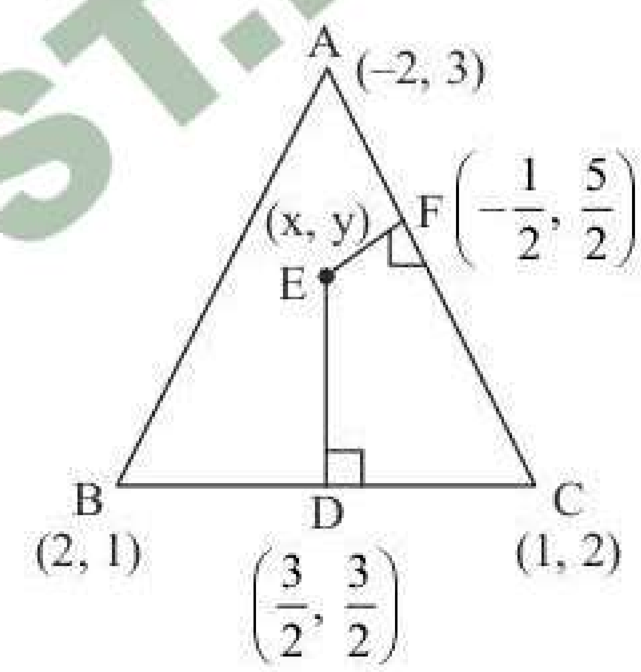
According to question, $A_{new} = KA_{old}$

$\Rightarrow \frac{\sqrt{3}}{4} \times 4a^2 = k \times \frac{\sqrt{3}}{4}a^2$

$\therefore k = 4$

46. (a) A circumcentre is a point at which perpendicular bisectors meet each other.

Here, 'E' represents circumcentre



Mid-point of BC = $\left(\frac{2+1}{2}, \frac{1+2}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

Slope of BC = $\frac{2-1}{1-2} = -1$

$\therefore$ Slope of DE = 1

Now, equation of $\overleftrightarrow{ED}$ is $\left(y - \frac{3}{2}\right) = 1\left(x - \frac{3}{2}\right)$

$\therefore 2y - 3 = 2x - 3$

$\therefore x = y$ ... (i)

Now, mid-point of AC = $\left(\frac{-2+1}{2}, \frac{3+2}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$

Slope of AC = $\frac{3-2}{-2-1} = -\frac{1}{3}$

$\therefore$ Slope of EF = 3

Now, equation of $\overleftrightarrow{EF}$ is $\left(y - \frac{5}{2}\right) = 3\left(x + \frac{1}{2}\right)$

$\therefore 2y - 5 = 6x + 3$ ...(ii)

From equations (i) and (ii),

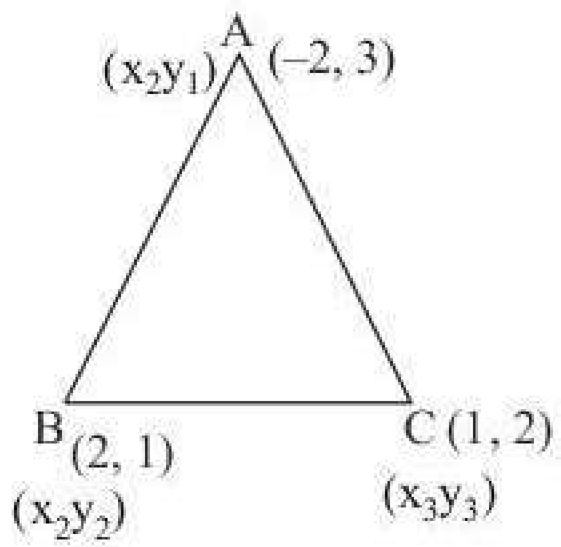
x = –2 and y = –2

Hence, circumcentre of ΔABC is (x, y) = (–2, –2)

$\therefore$ Option (a) is correct.

47. (b) Centroid of the triangle

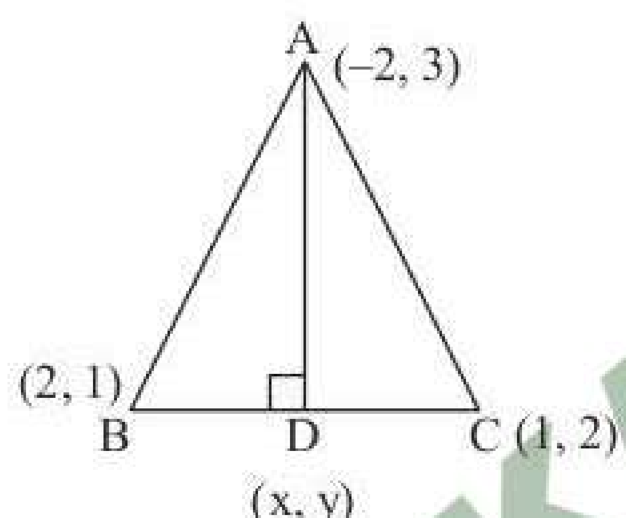
$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$



$$= \left(\frac{-2+2+1}{3}, \frac{3+1+2}{3}\right) = \left(\frac{1}{3}, 2\right)$$

$\therefore$ Option (b) is correct.

48. (d) Slope of BC $= \frac{2-1}{1-2} = -1$



Slope of AD = 1

Now, equation of $\overleftrightarrow{BC}$ is

$y - 2 = -1(x - 1)$

$\therefore$ $y - 2 = -x + 1$

$\therefore$ $x + y - 3 = 0$ ... (i)

and equation of $\overleftrightarrow{AD}$ is

$(y - 3) = 1(x + 2)$

$\therefore$ $x - y + 5 = 0$ ... (ii)

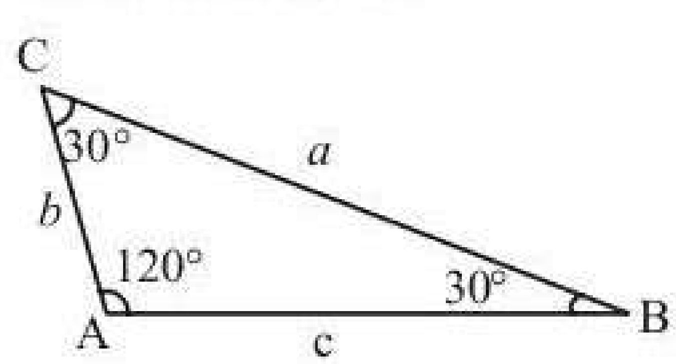
From equations (i) and (ii),

$x = -1$ and $y = 4$

$\therefore$ Foot of altitude from the vertex A of the triangle ABC is (–1, 4)

$\therefore$ Option (d) is correct.

49. (c)



By Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 120°} = \frac{b}{\sin 30°} = \frac{c}{\sin 30°}$$

$$\frac{b}{a} = \frac{\sin 30°}{\sin 120°} \text{ and } \frac{c}{a} = \frac{\sin 30°}{\sin 120°}$$

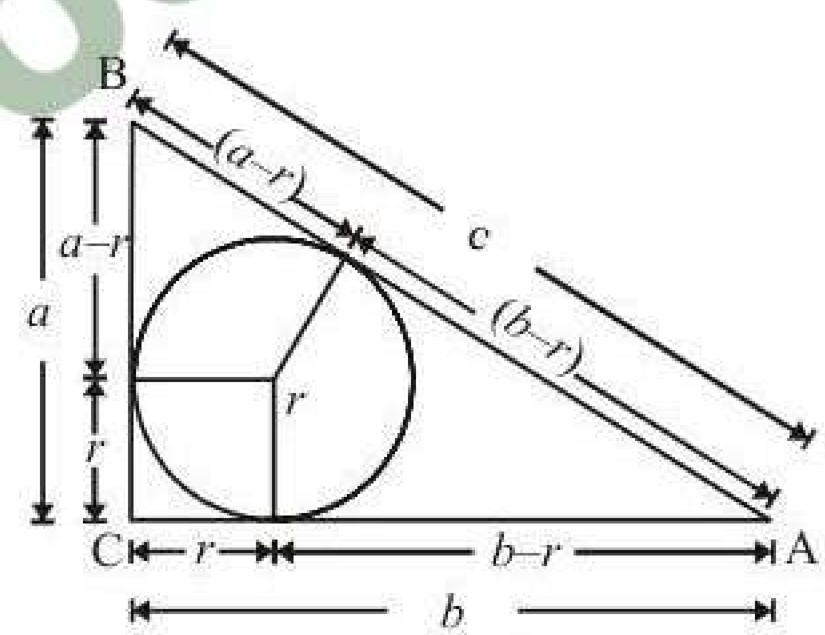
$$\frac{a}{a+b+c} = \frac{1}{1 + \frac{b}{a} + \frac{c}{a}}$$

$$= \frac{1}{1 + \frac{\sin 30°}{\sin 120°} + \frac{\sin 30°}{\sin 120°}}$$

$$= \frac{1}{1 + \frac{1/2}{\sqrt{3}/2} + \frac{1/2}{\sqrt{3}/2}}$$

$$= \frac{1}{1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2+\sqrt{3}}$$

50. (a)



$c = b - r + a - r$

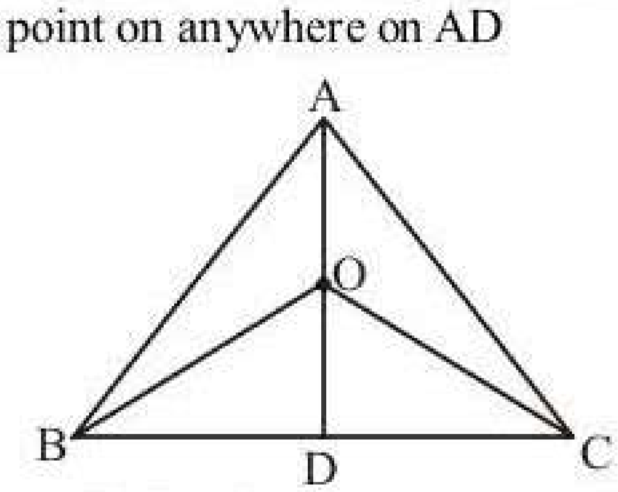
$c = a + b - 2r$

$2r = a + b - c$

$$\therefore \quad r = \frac{a+b-c}{2}$$

51. (c) **Statement 1**

AD divides Δ ABC in equal area of two parts. Then O is point on anywhere on AD



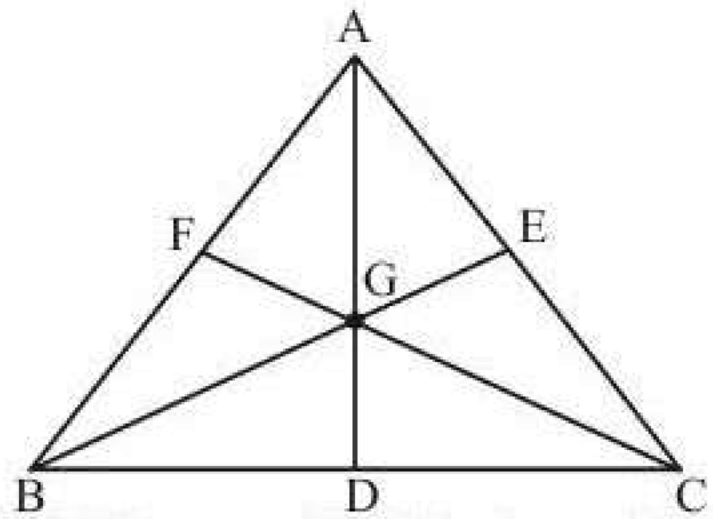
So area of triangle

ΔABO = ΔAOC,

So statement 1 is true.

Statement 2

G is the point of concurrence of the medians then



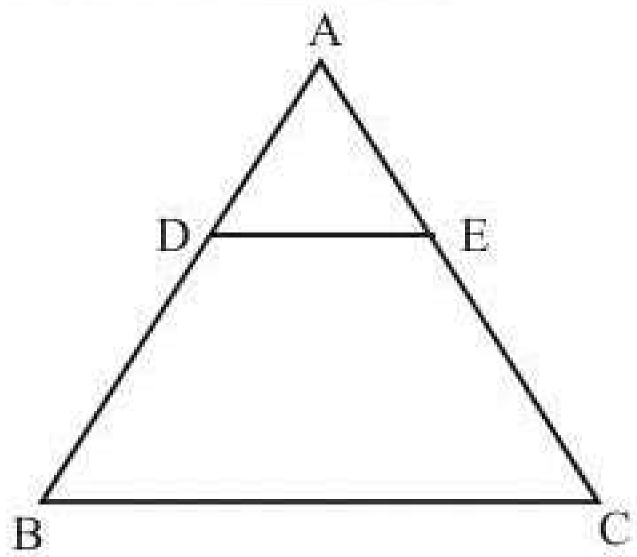
area of Δ ABG = area of Δ BCG = Area of ΔACG

Both are true.

52. (c) We have DE ∥ BC (Given)

So ∠ADE = ∠B and ∠AED = ∠C

(corresponding angles)



Therefore ΔABC ~ ΔADE by AA similarity criterion.

Also given area of ΔABC = 2 area of ΔADE ...(i)

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(ABC)}{ar(ADE)} = \left(\frac{AB}{AD}\right)^2 \quad ...(ii)$$

From (i) we get

$$\frac{ar(ABC)}{ar(ADE)} = \frac{2}{1} \quad ...(iii)$$

Therefore from (2) and (3)

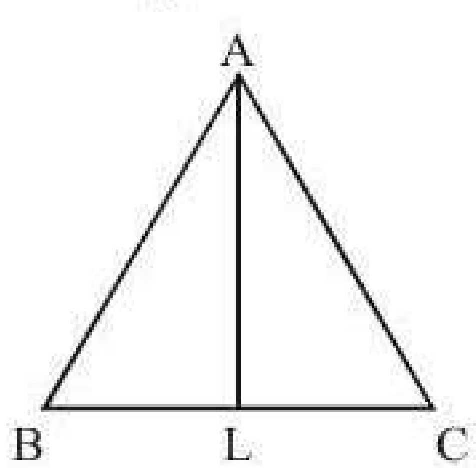
$$\left(\frac{AB}{AD}\right)^2 = \frac{2}{1}$$

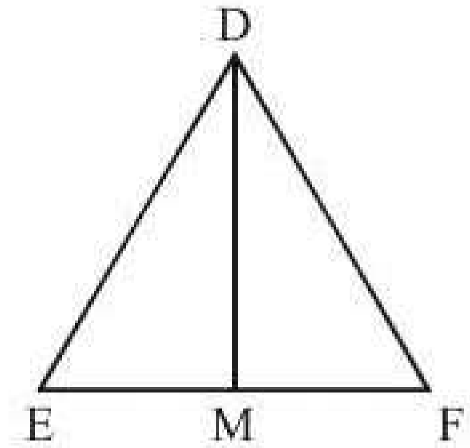
$$\Rightarrow \frac{AB}{AD} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow \frac{1}{AD} = \sqrt{2}\text{unit} \qquad (\because AB = 1 \text{ unit})$$

$$AD = \frac{1}{\sqrt{2}} \text{units}$$

53. (b)





Here ABC and DEF be two triangles such that

∠A = ∠D, ∠B = ∠E and ∠C = ∠F

(Given)

Also ∠L = ∠M = 90°

⇒ ∠ALB = ∠ALC = ∠DME = ∠DMF = 45°

(∵ M and L are mid points of EF and BC respectively)

⇒ ΔABC ~ ΔDEF by ΔAA similarity

Also ΔABC ≅ ΔDEF by ΔAAA Similarity

In ΔABL and ΔDEM

∠ALB = ∠DME = 90°

and ∠B = ∠E (Given)

⇒ ΔABL ~ ΔDEM by AA similarity criterion.

∴ Statement I is true.

In ΔALC and ΔDMF

Given AC ≠ DF

But ∠ALC = ∠DMF = 90°

and ∠C = ∠F (Given)

⇒ ΔALC ~ ΔDMF by AA similarity.

∠CAL = ∠FDM

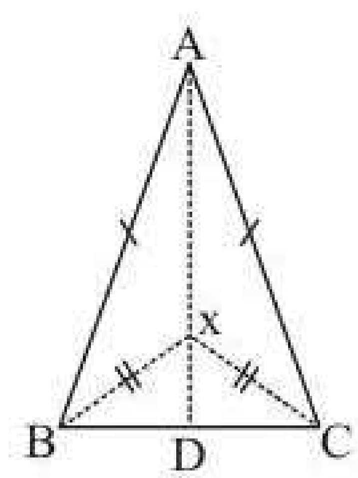
⇒ ΔALC ≅ ΔDMF by AAA similarity.

∴ Statement II is true.

But II is not the correct explanation of Statement-I

∵ these are different triangles.

54. (d) Locus of the point X is L.



Here L is line segment AD.

Now, Δ AXB ≅ Δ AXC (By SSS)

∴ ∠BAX = ∠CAX (Corr. Angles)

⇒ AX and hence AD is the bisector of ∠BAC.

Hence incentre of the ΔABC lies on AD i.e., L (statement 1 correct)

Since AB = AC and AD is the bisector of ΔABC

$$\therefore \frac{BD}{DC} = \frac{AB}{AC} = 1 \Rightarrow BD = DC$$

Now ΔXBD ≅ ΔXCD

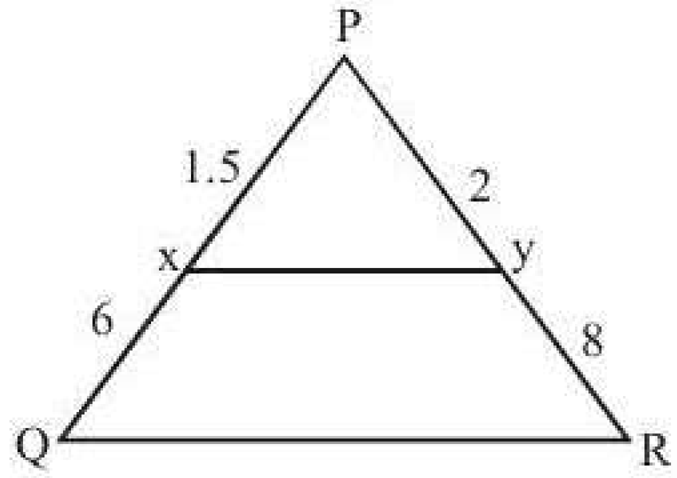
(By SSS)

∴ ∠XDB = ∠XDC = 90°

Hence AD is the perpendicular bisector of BC.

Therefore, orthocentre of the ΔABC lies on AD i.e, L.
(Statement 2 is correct)
Since D is the mid-point of BC, therefore AD is the mediam.
Hence centroid of the ΔABC lies ons AD i.e., L.
(Statement 3 is correct)

55. (d) In ΔPYX and ΔPRQ



$$\frac{PX}{PQ}=\frac{PY}{PR}$$

$$\Rightarrow \frac{1.5}{7.5}=\frac{2}{10}$$

$$\Rightarrow \frac{1}{5}=\frac{1}{5}$$

Now corresponding ratios of two triangles are equal.
Also $\angle P = \angle P$ (Common)

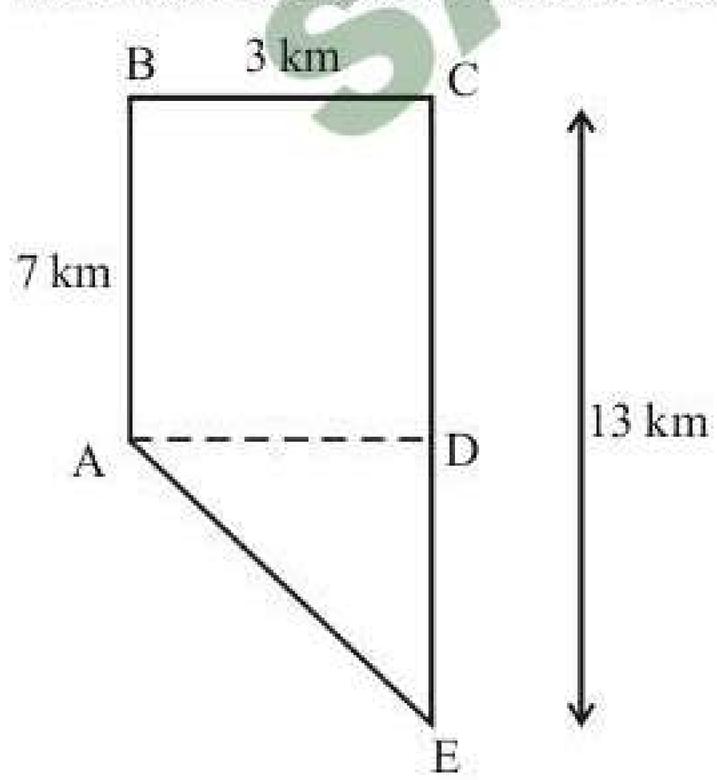
$\Rightarrow$ $\Delta PXY \sim \Delta PQR$ by SAS similarity

$$\Rightarrow \frac{PX}{PQ}=\frac{PY}{PR}=\frac{XY}{QR}$$

$$\Rightarrow \frac{1}{5}=\frac{1}{5}=\frac{XY}{QR}$$

$\Rightarrow$ QR = 5XY
Also QR || XY (By B.P.T)

56. (b) Let the position of person = A
We have to find out the distance between A and E.



AB = 7 km then CD = 7 km
BC = 3 m then AD = 3 km
DE = CE – CD = 13 – 7 = 6 km
Draw $AD \perp CE$
In right angled triangle AED, we get
$(AE)^2 = AD^2 = DE^2$
$= (3)^2 + (6)^2$

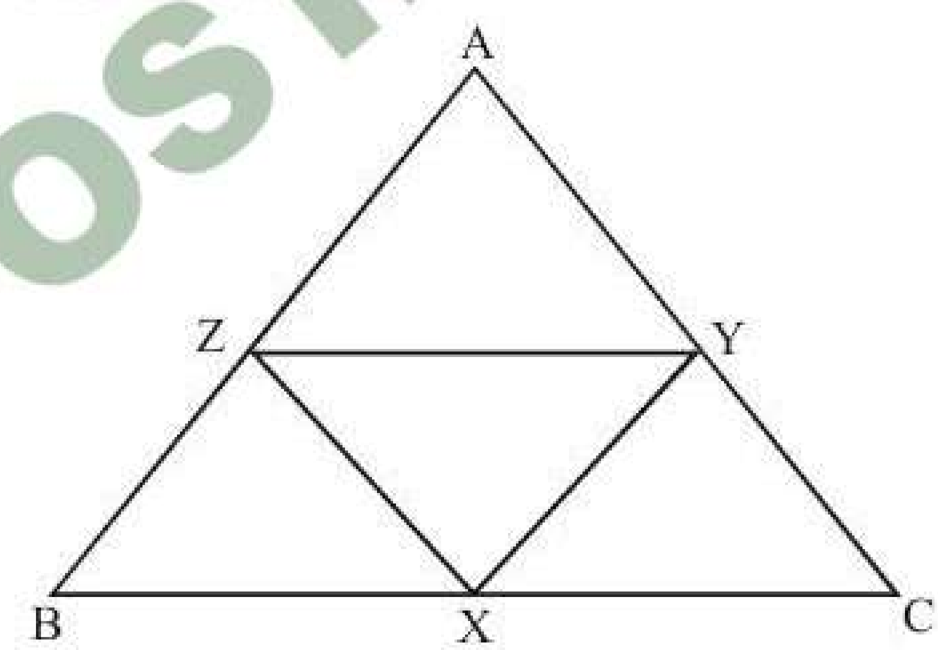
$AE = \sqrt{9+36} = \sqrt{45} = \sqrt{5\times 9}$

$AE = 3\sqrt{5}$km

$\therefore$ Option (b) is correct.

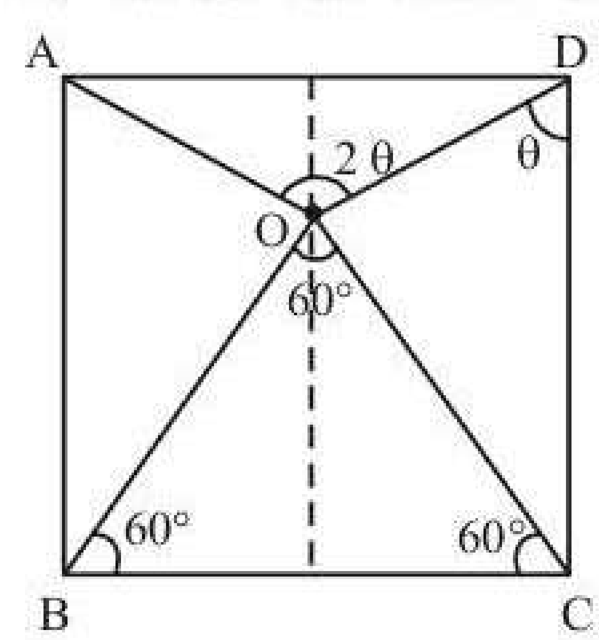
57. (c) Since ABC is a right angled triangle at C.
In ΔABC, we have by pythagorus theorem,
$AB^2 = AC^2 + BC^2$ ...(i)
Also In ΔBPC, we get
$BP^2 = BC^2 + CP^2$ ...(ii)
In ΔAQC, we get
$AQ^2 = AC^2 + CQ^2$ ...(iii)
In ΔPQC, we get
$PQ^2 = PC^2 + QC^2$ ...(iv)
Adding (i) and (iii) we get
$BP^2 + AQ^2 = BC^2 + CP^2 + AC^2 + CQ^2$
$= (BC^2 + AC^2) + (CP^2 + CQ^2)$
Using (i) and (iv) we get
$BP^2 + AQ^2 = AB^2 + PQ^2$

58. (c) Let ABC be an equilateral triangle and x, y, z are points on BC, CA and AB.



Also given
BX = CY = AZ
Since $\angle A = \angle B = \angle C = 60°$ (Equilateral triangle)
$\Rightarrow$ If BX = CY
$\Rightarrow$ $\angle X = \angle Y$
BX = AZ
$\Rightarrow$ $\angle X = \angle Z$
Also If $\angle Y$ = AZ
$\Rightarrow$ $\angle Y = \angle Z$
$\Rightarrow$ $\angle X = \angle Y = \angle Z = 60°$
ΔXYZ is an equilateral triangle.
Consider triangle ΔABC and ΔXYZ
Since $\angle X = \angle Y = \angle Z = 60°$
and $\angle A = \angle B = \angle C = 60°$
$\Rightarrow$ $\Delta ABC \sim \Delta XYZ$ by AA similarity criterion

59. (d)



In ΔOCD
$CD = CO$
$\angle ODC = \angle DOC = \theta$
$\theta + \theta + 30^\circ = 180^\circ$
$\theta = 75^\circ$

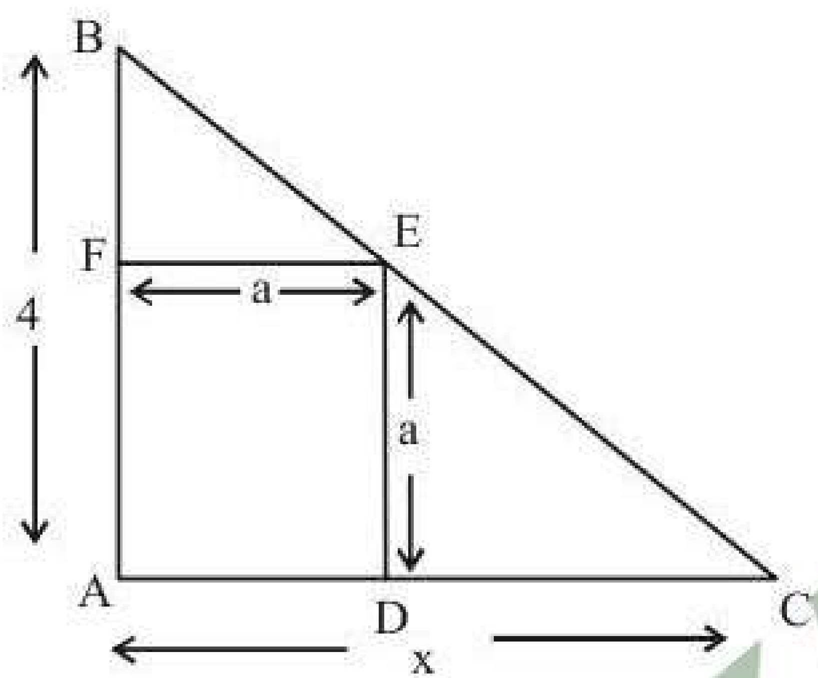
$$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-1.\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3}+1)^2}{2} = \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

60. (b)



Let the side of square be 'a'.
$AD = ED = EF = FA = a$
$CD = x - a$
as ΔCED and ΔCBA are similar

$$\frac{CD}{CA} = \frac{ED}{BA}$$

$$\Rightarrow \frac{x-a}{x} = \frac{a}{y}$$

$\Rightarrow xy - ay = ax$
$\Rightarrow ax + ay = xy$

$$\Rightarrow a = \frac{xy}{(x+y)}$$

Perimeter of square $4a = \frac{4xy}{(x+y)}$

So, option (b) is correct.