Exercise 1.5

Chapter 1 Functions and Limits Exercise 1.5 1E

$\lim_{x\to 2} f(x)=5$ Means that as x approaches 2, $f(x)$ approaches 5

Yes, it is possible that $\lim_{x\to 2} f(x)=5$ and still $f(2)=3$, because $\lim_{x\to 2} f(x)=5$ does not say anything about the value of $f(x)$ at $x=2$.

Chapter 1 Functions and Limits Exercise 1.5 2E

Consider the following limit of the functions:

$$\lim_{x\to 1^-} f(x)=3 \quad \text{and} \quad \lim_{x\to 1^+} f(x)=7$$

The objective is to explain the meaning of the above limits and also determine the limit $\lim_{x\to 1} f(x)$ exists or not.

The limit of the function $\lim_{x\to 1^-} f(x)=3$ means that the value of $f(x)$ equals to 3 when x approaches 1 from the left. The symbol " $x\to 1^-$ " indicates that consider only values of *x* that are less than 1.

The limit of the function $\lim_{x\to 1^+} f(x)=7$ means that the value of $f(x)$ equals to 7 when x approaches 1 from the right. The symbol " $x\to 1^+$ " indicates that consider only values of *x* that are greater than 1.

Again, $\lim_{x\to 1} f(x)$ exists and equal to *L* if and only if $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=L$.

But here $\lim_{x\to 1^-} f(x)=3$, and $\lim_{x\to 1^+} f(x)=7$, so $\lim_{x\to 1^-} f(x)\neq \lim_{x\to 1^+} f(x)$.

Hence, $\lim_{x\to 1} f(x)$ **does not exist**.

Chapter 1 Functions and Limits Exercise 1.5 3E

(A)

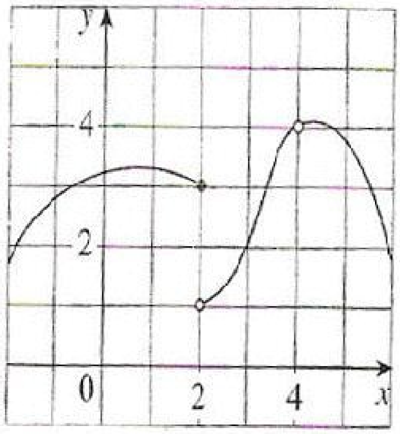
$\lim_{x\to -3} f(x)=\infty$ means that we can take the values of f(x) as large(positive) as we please by taking the value of x sufficiently close to -3, but not equal to -3.

(B)

$\lim_{x\to 4^+} f(x)=-\infty$ means that we can take the value of f(x) as large (negative) as we please by taking sufficiently close to 4 but larger than 4.

Chapter 1 Functions and Limits Exercise 1.5 4E

Given graph of function f



(a) From the given graph, we observe that

$\lim_{x \to 2^-} f(x) = 3$

(b) From the given graph, we observe that

$\lim_{x \to 2^+} f(x) = 1$

(c) Left hand limit and right hand limit are different. From the graph we observe that

$\lim_{x \to 2^-} f(x) = 3$ and $\lim_{x \to 2^+} f(x) = 1$.

Therefore, $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$, and hence $\lim_{x \to 2} f(x)$ does not exist.

(d) From the graph, we observe that

$f(2) = 3$

(e) From graph, we observe that

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = 4$

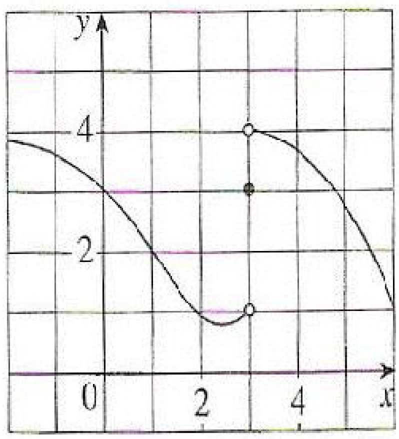
Therefore, $\lim_{x \to 4} f(x) = 4$

(f) From graph, we observe that

$f(4)$ does not exist, as there is no point on the graph corresponding to $x = 4$.

Chapter 1 Functions and Limits Exercise 1.5 5E

Given graph of function f



(a) From the given graph, we observe that

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$

Therefore

$\lim_{x \to 1} f(x) = 2$

(b) From the given graph, we observe that

$\lim_{x\to 3^-} f(x) = 1$

(c) From the given graph, we observe that

$\lim_{x\to 3^+} f(x) = 4$

(d) Left hand limit and right hand limit are different. From the graph we observe that

$\lim_{x\to 3^-} f(x) = 1$ and $\lim_{x\to 3^+} f(x) = 4$.

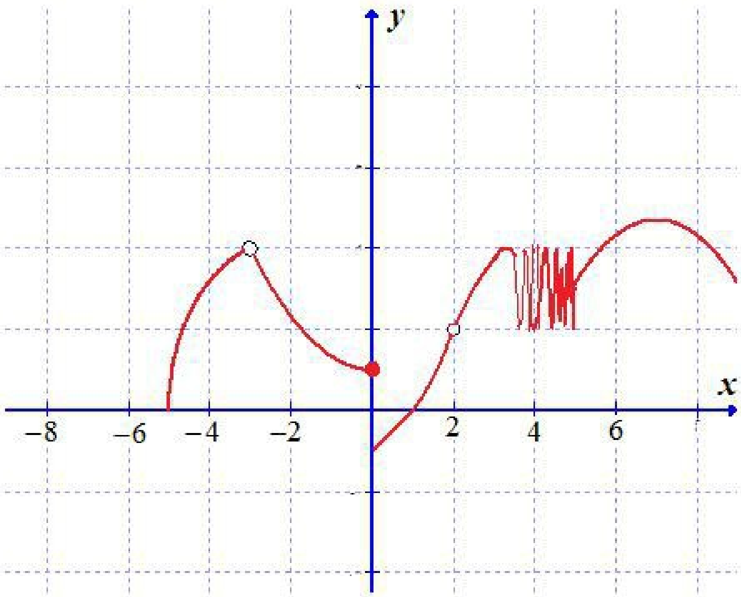
Therefore, $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$, and hence $\lim_{x\to 3} f(x)$ does not exist.

(e) From the given graph, we observe that

$f(3) = 3$

Chapter 1 Functions and Limits Exercise 1.5 6E

Consider the graph:



(a)

To find $\lim_{x\to -3^-} h(x)$

From the graph, the value of $h(x)$ approaches to 4 as x approaches to -3 from the left.

Therefore $\boxed{\lim_{x\to -3^-} h(x) = 4}$.

(b)

To find $\lim_{x\to -3^+} h(x)$

From the graph, see that the value of $h(x)$ approaches to 4 as x approaches to -3 from the right.

Therefore $\boxed{\lim_{x\to -3^+} h(x) = 4}$.

(c)

To find $\lim_{x\to -3} h(x)$

Let $\lim_{x\to a} f(x) = l$ if and only if $\lim_{x\to a^-} f(x) = l$ and $\lim_{x\to a^-} f(x) = l$

From (a) and (b), $\lim_{x\to -3^-} h(x) = 4$ and $\lim_{x\to -3^+} h(x) = 4$

Therefore $\boxed{\lim_{x\to -3} h(x) = 4}$.

(d)

To find $h(-3)$.

From the graph, it cannot be getting any value of $h(x)$ corresponding to the value of $x=-3$.

Therefore $h(-3)$ does not exist.

(e)

To find $\lim_{x\to 0^-} h(x)$

From the graph, the value of $h(x)$ approaches to 1 as x approaches to 0 from the left.

Therefore $\boxed{\lim_{x\to 0^-} h(x)=1}$.

(f)

To find $\lim_{x\to 0^+} h(x)$

From the graph, the value of $h(x)$ approaches to -1 as x approaches to 0 from the right.

Therefore, $\boxed{\lim_{x\to -0^+} h(x)=-1}$.

(g)

To find $\lim_{x\to 0} h(x)$

Let $\lim_{x\to a} f(x)=l$ if and only if $\lim_{x\to a^-} f(x)=l$ and $\lim_{x\to a^-} f(x)=l$

From (a) and (b),

$\lim_{x\to 0^-} h(x)=1$ and $\lim_{x\to 0^+} h(x)=-1$

Therefore $\lim_{x\to 0} h(x)$ does not exist.

(h)

To find $h(0)$

From the graph, $h(x)=1$ corresponding to the value $x=0$.

(i)

To find $\lim_{x\to 2} h(x)$

Let $\lim_{x\to a} f(x)=l$ if and only if $\lim_{x\to a^-} f(x)=l$ and $\lim_{x\to a^-} f(x)=l$

From the graph, $\lim_{x\to 2^-} h(x)=2$ and $\lim_{x\to 2^+} h(x)=2$

Therefore $\boxed{\lim_{x\to 2} h(x)=2}$.

(j)

To find $h(2)$

From the graph, it cannot get any value of $h(x)$ corresponding to the value of $x=2$.

Therefore $h(2)$ does not exist.

(k)

To find $\lim_{x\to 5^+} h(x)$

From the graph, the value of $h(x)$ approaches to 3 as x approaches to 5 from the right.

Therefore $\boxed{\lim_{x\to 5^+} h(x)=3}$.

(l)

To find $\lim_{x\to 5^-} h(x)$

From the graph, the value of $h(x)$ approaches to an interval $[2,4]$ as x approaches to 5 from the left.

Therefore $\lim_{x\to 5^-} h(x)$ does not exist.

Chapter 1 Functions and Limits Exercise 1.5 7E

(A)

$$\lim_{t\to 0^-} g(t) = -1$$

(B)

$$\lim_{t\to 0^+} g(t) = -2$$

(C)

$\lim_{t\to 0} g(t)$ done not exist, because $\lim_{t\to 0^-} g(t) \neq \lim_{t\to 0^+} g(t)$

(D)

$$\lim_{t\to 2^-} g(t) = 2$$

(E)

$$\lim_{t\to 2^+} g(t) = 0$$

(F)

$\lim_{t\to 2} g(t)$ does not exist because $\lim_{t\to 2^-} g(t) \neq \lim_{t\to 2^+} g(t)$

(G)

$$g(2) = 1$$

(H)

$$\lim_{t\to 4} g(t) = 3$$

Chapter 1 Functions and Limits Exercise 1.5 8E

(A)

$$\lim_{x\to 2} R(x) = -\infty$$

(B)

$$\lim_{x\to 5} R(x) = \infty$$

(C)

$$\lim_{x\to -3^-} R(x) = -\infty$$

(D)

$$\lim_{x\to -3^+} R(x) = +\infty$$

(E)

The vertical asymptotes for $R(x)$ are $x = -3$, $x = 2$, and $x = 5$

Chapter 1 Functions and Limits Exercise 1.5 9E

(A)

$$\lim_{x\to -7} f(x) = -\infty$$

(B)

$$\lim_{x\to -3} f(x) = \infty$$

(C)

$$\lim_{x\to 0} f(x) = \infty$$

(D)

$$\lim_{x\to 6^-} f(x) = -\infty$$

(E)

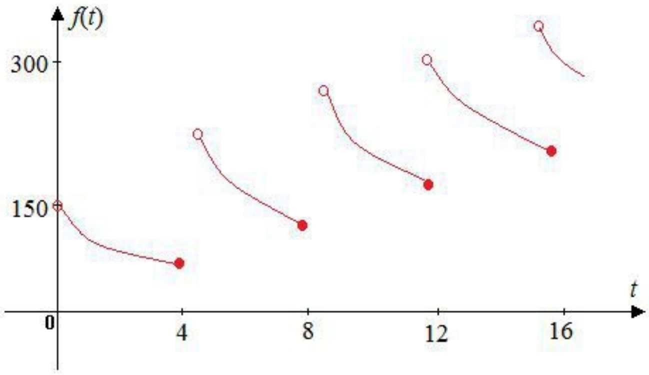
$$\lim_{x\to 6^+} f(x) = \infty$$

(F)

The vertical asymptotes for $f(x)$ are $x = -7$, $x = -3$, $x = 0$ and $x = 6$

Chapter 1 Functions and Limits Exercise 1.5 10E

Consider the following graph:



Considering that a patient receives a 150-mg injection of a drug every 4 hours.

The above graph shows the amount *f*(*t*) of the drug in the bloodstream after *t* hours.

$\lim_{t\to 12^-} f(t)$

From the graph, the value of $f(t)$ when $t \to 12^-$ is $\boxed{150}$.

$\lim_{t\to 12^+} f(t)$

From the graph, the value of $f(t)$ when $t \to 12^+$ is $\boxed{300}$.

In the graph the significant is that, at the beginning the initial quantity of drug is 150-mg, and after 4 hours it will decreases to a certain amount.

At the same time the next amount of drug is injected.

The total amount of drug will also be equal to 150-mg.

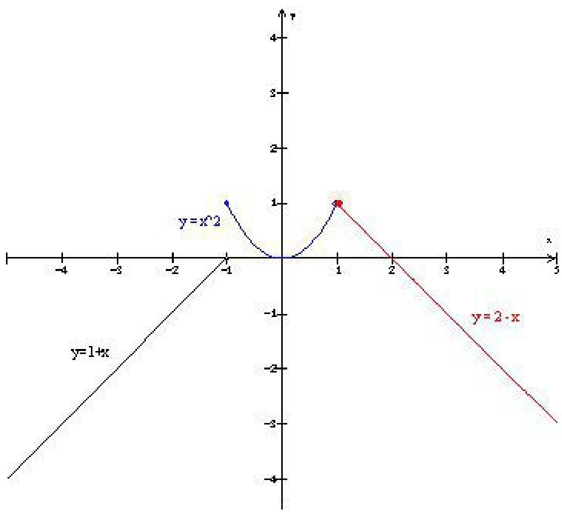
This is calculated by subtracting the remaining amount from the injected amount.

Chapter 1 Functions and Limits Exercise 1.5 11E

Given function

$$f(x)=\begin{cases} 1+x & \text{if } x<-1 \\ x^2 & \text{if } -1\le x<1 \\ 2-x & \text{if } x\ge 1 \end{cases}$$

Graph of the given function is



From the given graph, we observe that limit of function *f* exists at all the points except at $x=-1$

So, values of a for which $\lim_{x\to a} f(x)$ exists are

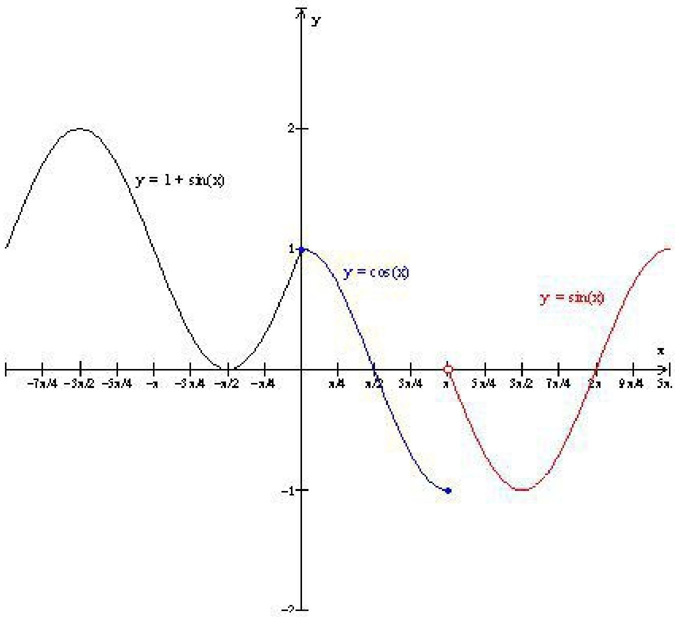
$(-\infty,-1)\cup(-1,\infty)$

Chapter 1 Functions and Limits Exercise 1.5 12E

Given function

$$f(x)=\begin{cases}1+\sin x & \text{if } x<0\\ \cos x & \text{if } 0\le x\le \pi\\ \sin x & \text{if } x>\pi\end{cases}$$

Graph of the given function is



From the given graph, we observe that limit of function f exists at all the points except at $x=\pi$

So, values of a for which $\lim_{x\to a} f(x)$ exists are

$(-\infty,\pi)\cup(\pi,\infty)$

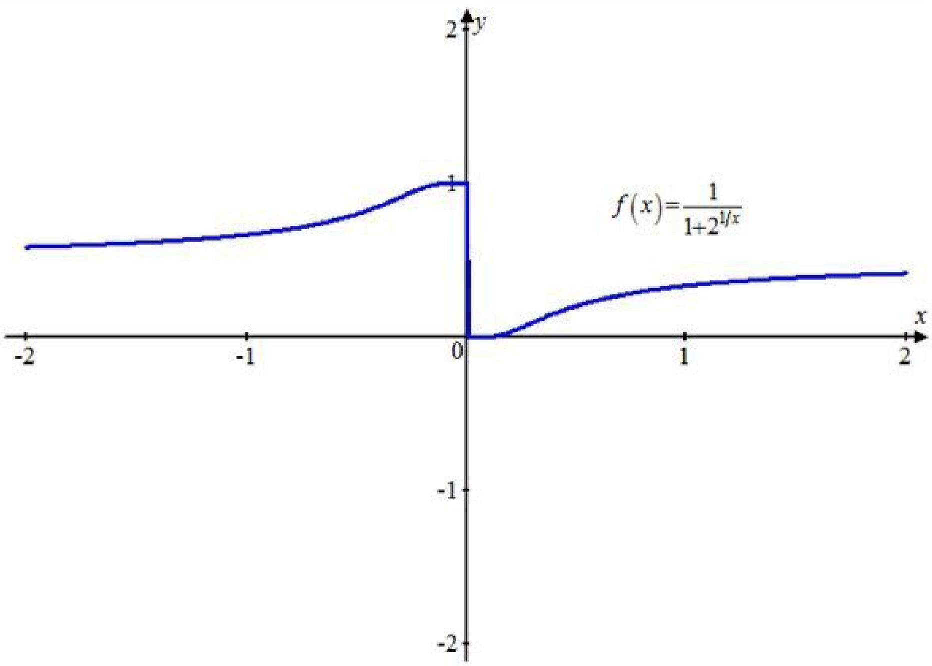
Chapter 1 Functions and Limits Exercise 1.5 13E

Consider the following function:

$$f(x)=\frac{1}{1+2^{1/x}}.$$

The objective is to find the limits using the graph of the above function.

Use a graphing utility, sketch the graph of the function $f(x)=\frac{1}{1+2^{1/x}}$ as shown below:



(a)

The objective is to state the value of the limit $\lim_{x \to 0^-} f(x)$.

The symbol " $x \to 0^-$ " indicates that consider only values of *x* that are less than 0.

See above graph of the function, the values of $f(x)$ approach 1 as x approaches 0 from the left.

Therefore, the value of the limit is $\lim_{x \to 0^-} f(x) = \boxed{1}$.

(b)

The objective is to state the value of the limit $\lim_{x \to 0^+} f(x)$.

The symbol " $x \to 0^+$ " indicates that consider only values of *x* that are greater than 0.

See above graph of the function, the values of $f(x)$ approach 0 as x approaches 0 from the right.

Therefore, the value of the limit is $\lim_{x \to 0^+} f(x) = \boxed{0}$.

(c)

The objective is to state the value of the limit $\lim_{x \to 0} f(x)$.

Recollect that, $\lim_{x \to 1} f(x)$ exists and equal to *L* if and only if $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = L$.

But here $\lim_{x \to 0^-} f(x) = 1$, and $\lim_{x \to 0^+} f(x) = 0$, so $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

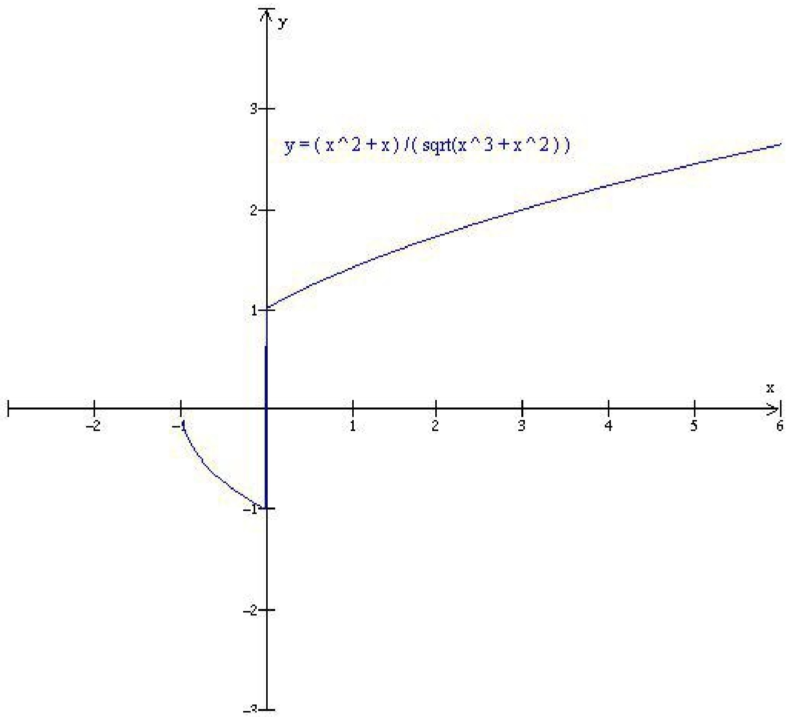
Hence, $\lim_{x \to 0} f(x)$ **does not exist**.

Chapter 1 Functions and Limits Exercise 1.5 14E

Given function

$$f(x) = \frac{x^2 + x}{\sqrt{x^3 + x^2}}$$

Graph of the given function is



(a) From the graph we observe that

$\lim_{x \to 0^-} f(x) = -1$

(b) From the graph we observe that

$\lim_{x \to 0^+} f(x) = 1$

(c) From the graph we observe that

$$\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$$

So, $\lim_{x\to 0} f(x)$ does not exist.

Chapter 1 Functions and Limits Exercise 1.5 15E

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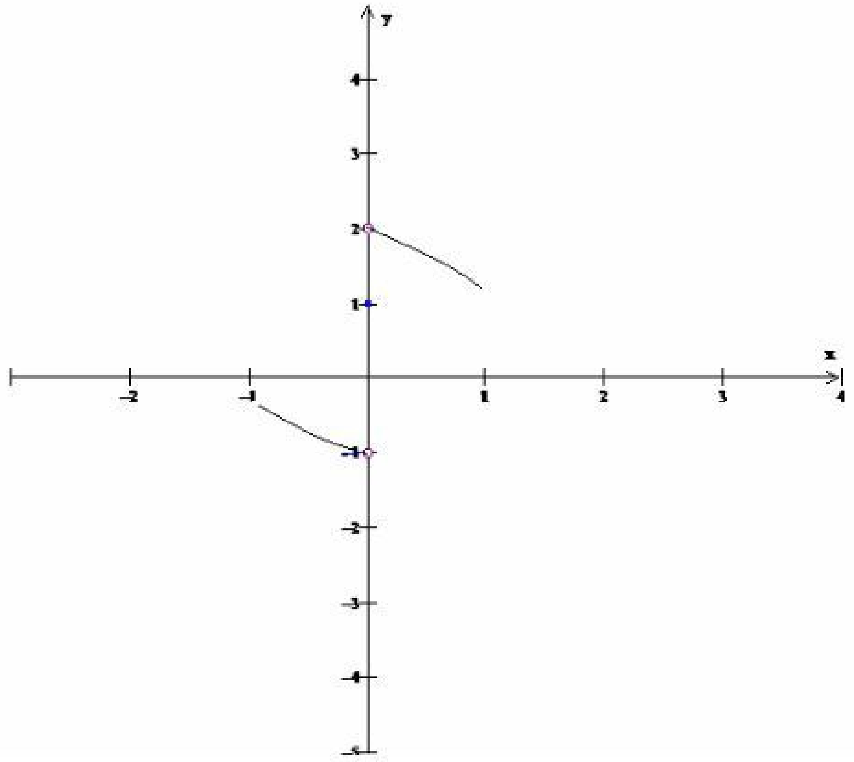
Given conditions

$\lim_{x\to 0^-} f(x) = -1$
$\lim_{x\to 0^+} f(x) = 2$
$f(0) = 1$

Graph of a function which satisfy all the above conditions is



Chapter 1 Functions and Limits Exercise 1.5 16E

Consider the following conditions:

$\lim_{x\to 0} f(x) = 1$

$\lim_{x\to 3^-} f(x) = -2$

$\lim_{x\to 3^+} f(x) = 2$

$f(0) = -1$

$f(3) = 1$

Sketch the graph of the function f.

Consider the condition $\lim_{x\to 0} f(x)=1$.

This means that the function $f(x)$ approaches to 1 as x approaches 0 from left and from right.

Consider the condition $\lim_{x\to 3^-} f(x)=-2$.

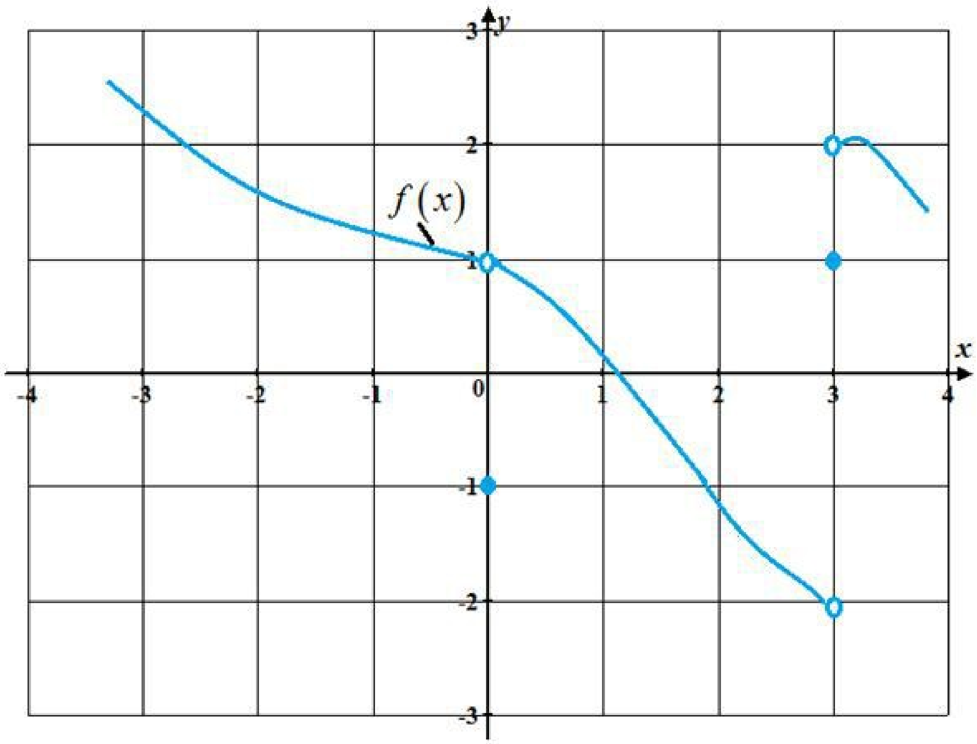
This means that the function $f(x)$ approaches to -2 as x approaches 3 from left.

Consider the condition, $\lim_{x\to 3^+} f(x)=2.$

This means that the function $f(x)$ approaches to 2 as x approaches 3 from right.

The value of the function $f(x)$ at $x=0$ is $f(0)=-1$ and $f(3)=1$.

Sketch of the graph of an example function f that satisfies the conditions $\lim_{x\to 0} f(x)=1, \lim_{x\to 3^-} f(x)=-2, \lim_{x\to 3^+} f(x)=2, f(0)=-1, f(3)=1$ is as follows.



Chapter 1 Functions and Limits Exercise 1.5 17E

Consider the following conditions:

$\lim_{x\to 3^+} f(x)=4, \lim_{x\to 3^-} f(x)=2, \lim_{x\to -2} f(x)=2$ and $f(3)=3, f(-2)=1$

The objective is to sketch the graph of a function f that satisfies all of the given conditions.

Consider the condition, $\lim_{x\to 3^+} f(x)=4.$

This means that the function $f(x)$ approaches to 4 as x approaches 3 from right.

Consider the condition, $\lim_{x\to 3^-} f(x)=2.$

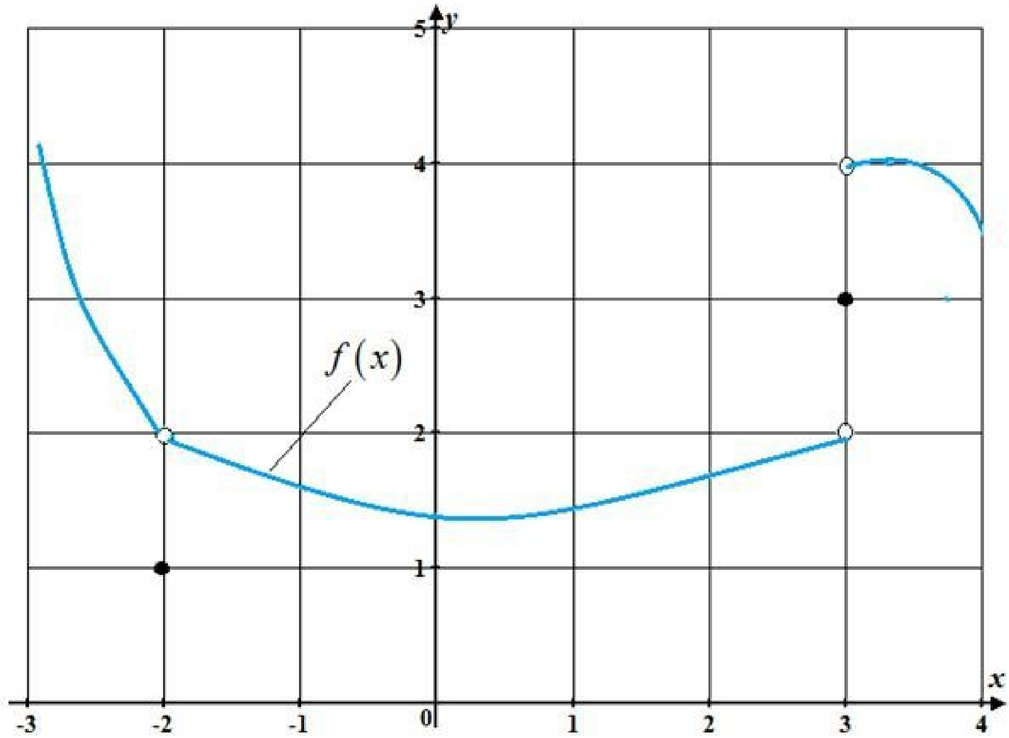
This means that the function $f(x)$ approaches to 2 as x approaches 3 from left.

Consider the condition, $\lim_{x\to -2} f(x)=2.$

This means that the function $f(x)$ approaches to 2 as x approaches -2 from left and from right.

The value of the function $f(x)$ at $x=3$ is $f(3)=3$ and at $x=-2$ is $f(-2)=1$.

Sketch of the graph of an example function f that satisfies the conditions $\lim_{x \to 3^+} f(x) = 4, \lim_{x \to 3^-} f(x) = 2, \lim_{x \to -2} f(x) = 2, f(3) = 3, f(-2) = 1$ is as follows.



Chapter 1 Functions and Limits Exercise 1.5 18E

Given conditions

$\lim_{x \to 0^-} f(x) = 2$

$\lim_{x \to 0^+} f(x) = 0$

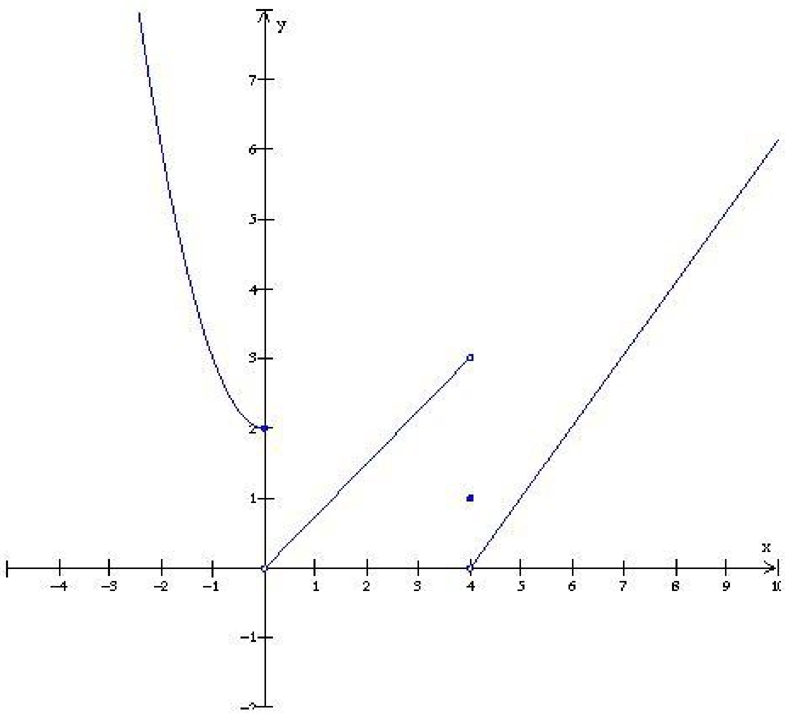
$\lim_{x \to 4^-} f(x) = 3$

$\lim_{x \to 4^+} f(x) = 0$

$f(0) = 2$

$f(4) = 1$

Graph of a function which satisfy all the above conditions is



Chapter 1 Functions and Limits Exercise 1.5 19E

Consider the expression.

$$\lim_{x\to 2}\frac{x^2-2x}{x^2-x-2}.$$

The function $f(x)=\frac{x^2-2x}{x^2-x-2}$ is not defined when $x=2.$

Consider the values of x that are close to 2 but not equal to 2.

Construct a table for $f(x)$ different values of x (correct to six decimal places).

For $x>2.$

x	$f(x)$
2.5	0.714286
2.1	0.677419
2.05	0.672131
2.01	0.667774
2.005	0.667221
2.001	0.666778

For $x<2.$

x	$f(x)$
1.9	0.655172
1.95	0.661017
1.99	0.665552
1.995	0.666110
1.999	0.666556

Observe that the values of x approach to 2 but not equal to 2.

For $f(x)>2,$ the value of $f(x)$ tends to 0.666778.

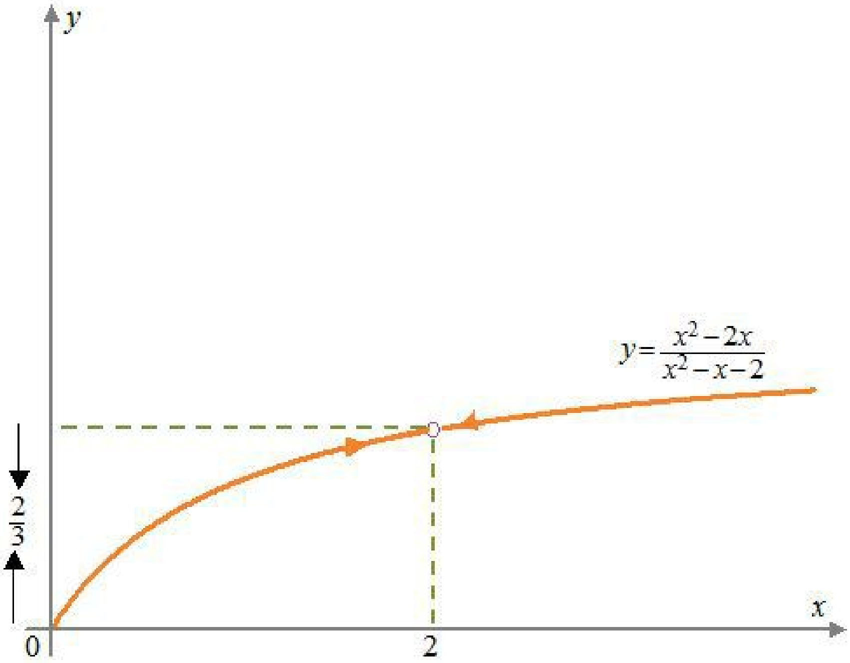
$$\lim_{x\to 2^+} f(x)=0.667$$
$$\approx\frac{2}{3}$$

For $f(x)<2,$ the value of $f(x)$ tends to 0.666556.

$$\lim_{x\to 2^-} f(x)=0.667$$
$$\approx\frac{2}{3}$$

Therefore, the limit of $f(x)$ at $x\to 2$ is $\boxed{\frac{2}{3}}$.

Sketch the graph of $y=\dfrac{x^2-2x}{x^2-x-2}$.



Chapter 1 Functions and Limits Exercise 1.5 20E

First we make a table of values of the function $f(x)=\dfrac{x^2-2x}{x^2-x-2}$,

For the given values of x

x	f(x)
0	0.0000
-0.5	-1.00000
-0.9	-9.0000
-0.95	-19.0000
-0.99	-99.0000
-0.999	-999.0000
Table 1	

x	f(x)
-2	2.000
-1.5	3.0000
-1.1	11.0000
-1.01	101.0000
-1.001	1001.000
Table 2	

Here we can see that (in table 1 and table 2), as we take the values from 0 to (close to) -1, the value of f(x) approaches large negative number and tends to $-\infty$

So $\lim_{x\to -1^-} f(x)=-\infty$

Similarly if we take the values from -2 to (close to) -1, then value of *f(x)* is being large positive number and tends to $+\infty$

So $\lim_{x\to -1^+} f(x)=\infty$

Therefore both sides' limits are not same.

Hence $\lim_{x\to -1} f(x) \Rightarrow \lim_{x\to -1}\dfrac{x^2-2x}{x^2-x-2}$ does not exist

Chapter 1 Functions and Limits Exercise 1.5 21E

Consider the expression, $\lim_{x\to 0}\dfrac{\sin x}{x+\tan x}$.

The objective is to determine the value of limit by evaluating the function at numbers

$x=\pm 1,\pm 0.5,\pm 0.2,\pm 0.1,\pm 0.05,\pm 0.01$

Let $f(x)=\dfrac{\sin x}{x+\tan x}$

Evaluate the function $f(x)$ at each value.

Find the functional values.

Substitute $x=1$ in the function $f(x)=\frac{\sin x}{x+\tan x}$.

$f(1)=\frac{\sin 1}{1+\tan 1}$

$=0.329033$

Substitute $x=-1$ in the function $f(x)=\frac{\sin x}{x+\tan x}$.

$f(-1)=\frac{\sin(-1)}{-1+\tan(-1)}$

$=0.329033$ Since $f(-x)=f(x)$

Substitute $x=0.5$ in the function $f(x)=\frac{\sin x}{x+\tan x}$.

$f(0.5)=\frac{\sin 0.5}{0.5+\tan 0.5}$

$=0.458210$

This implies $f(-0.5)=0.458210$

The functional values at other points are,

$f(0.2)=\frac{\sin 0.2}{0.2+\tan 0.2}$

$=0.493330$

The value of $f(-0.2)$ is same as $f(0.2)$.

$f(0.1)=\frac{\sin 0.1}{0.1+\tan 0.1}$

$=0.498330$

The value of $f(-0.1)$ is same as $f(0.1)$.

$f(0.05)=\frac{\sin 0.05}{0.05+\tan 0.05}$

$=0.499590$

The value of $f(-0.05)$ is same as $f(0.05)$

Continue further,

$f(0.01)=\frac{\sin 0.01}{0.01+\tan 0.01}$

$=0.5$

The value of $f(-0.01)$ is same as $f(0.01)$.

Construct a table with functional values.

x	$f(x)=\frac{\sin x}{x+\tan x}$
± 1	0.329033
± 0.5	0.458210
± 0.2	0.493330
± 0.1	0.498330
± 0.05	0.499590
± 0.01	0.5

As x approaches 0, the value of the function approaches to 0.5

Therefore, the result is $\lim_{x\to 0}\frac{\sin x}{x+\tan x}=\boxed{0.5}$.

Chapter 1 Functions and Limits Exercise 1.5 22E

$$f(h)=\frac{(2+h)^5-32}{h}$$

We have to guess the value of limit by evaluating the function at given numbers $h=\pm0.5,\pm0.1,\pm0.01,\pm0.001,\pm0.0001$

h	$f(h)$		h	$f(h)$
- 0.5	48.8125000		0.5	131.3125000
-0.1	72.3901000		0.1	88.4101000
-0.01	79.2039900		0.01	80.8040100
-0.001	79.9200400		0.001	80.0800400
-0.0001	79.9920000		0.0001	80.0080000

From the values of $f(h)=\dfrac{(2+h)^5-32}{h}$ shown in the table, we observe that when h approaches 0, the value of $f(h)$ approaches 80.

So we guess that the value of the limit

$$\lim_{h\to 0}\frac{(2+h)^5-32}{h}=80$$

Chapter 1 Functions and Limits Exercise 1.5 23E

Find,

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$$

We calculate the value of the function $f(x)=\dfrac{\sqrt{x+4}-2}{x}$ for different values of x near 0 .

x	$f(x)=\dfrac{\sqrt{x+4}-2}{x}$
-0.01	0.250156446
-0.001	0.250015627
-0.0001	0.250001563
+0.0001	0.249998438
+0.001	0.249984377
+0.01	0.249843945

So, from the above table, it is clear that.

$$\boxed{\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=0.25=\frac{1}{4}}$$

Consider the limit $\lim_{x \to 0} \frac{\tan 3x}{\tan 5x}$

To estimate the value of the limit by using table of values.

Now, calculate the values of the function $f(x) = \frac{\tan 3x}{\tan 5x}$ for different values of *x* near 0.

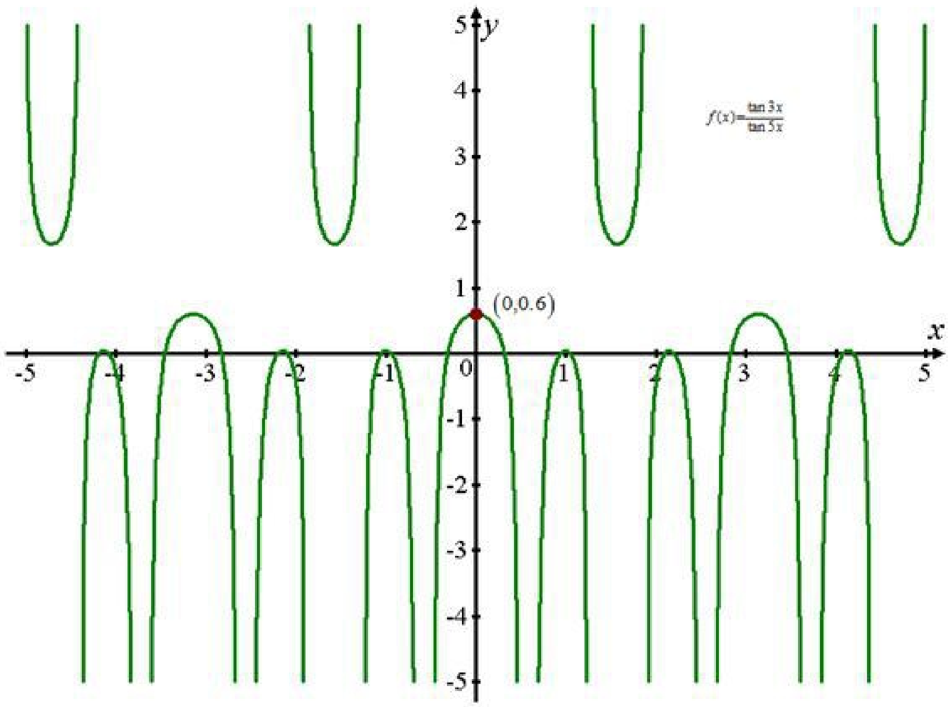
x	$f(x) = \frac{\tan 3x}{\tan 5x}$
± 0.2	0.43928
± 0.1	0.56624
± 0.05	0.59189
± 0.01	0.59968
± 0.001	0.59999

From the table,

As *x* approaches 0, the values of the function seem to approach 0.59999

That is, $\boxed{\lim_{x \to 0} \frac{\tan 3x}{\tan 5x} = 0.6}$

Sketch the graph of $f(x) = \frac{\tan 3x}{\tan 5x}$



From the graph,

As *x* approaches 0, the values of the function seem to approaches 0.6

Therefore, $\boxed{\lim_{x \to 0} \frac{\tan 3x}{\tan 5x} = 0.6}$.

Chapter 1 Functions and Limits Exercise 1.5 25E

Estimate the value of $\lim_{x\to 1}\frac{x^6-1}{x^{10}-1}$.

Let $f(x)=\frac{x^6-1}{x^{10}-1}$.

This function is not defined at $x=1$, but that does not matter because the values of x close to 1 but not equals to 1.

First find the values of the function $f(x)=\frac{x^6-1}{x^{10}-1}$ at values of x near 1 from left.

Construct the following table:

x	$f(x)=\frac{x^6-1}{x^{10}-1}$
0	1
0.5	0.985337
0.9	0.719397
0.99	0.612018
0.999	0.6012
0.9999	0.60012

From the table of values, the left hand limit is $\lim_{x\to 1^-}\frac{x^6-1}{x^{10}-1}=0.6$.

Next choose the values of x approaching 1 from right.

Construct the following table:

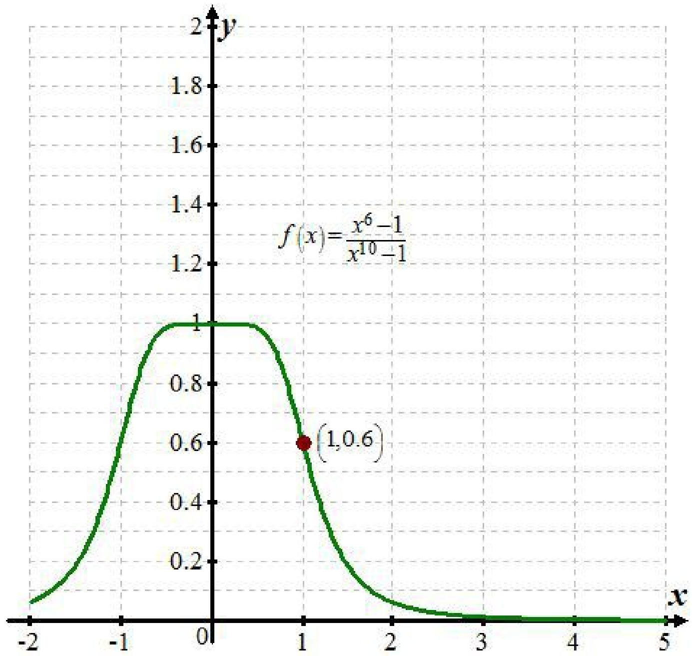
x	$f(x)=\frac{x^6-1}{x^{10}-1}$
1.5	0.183369
1.1	0.484119
1.01	0.588022
1.001	0.5988
1.0001	0.59988

From the table of values, the right hand limit is $\lim_{x\to 1^+}\frac{x^6-1}{x^{10}-1}=0.6$.

Since the left hand limit and right hand limit approach the same value, the limit of the function is 0.6.

Therefore, the estimated value is $\boxed{\lim_{x\to 1}\frac{x^6-1}{x^{10}-1}=0.6}$.

The graph of function $f(x)=\frac{x^6-1}{x^{10}-1}$ is shown below:



From the graph, as the value of x approaches 1, the functional value approaches $\boxed{0.6}$

Chapter 1 Functions and Limits Exercise 1.5 26E

Consider the following limit:

$$\lim_{x\to 0}\frac{9^x-5^x}{x}.$$

Take, $f(x)=\frac{9^x-5^x}{x}$.

To evaluate the limit, construct a table of values for some values of *x* sufficiently near to 0, but not equal to 0 as follows:

Evaluate $f(x)=\frac{9^x-5^x}{x}$ for some *x* values close to 0 from left as shown in the table – 1:

x	$f(x)=\frac{9^x-5^x}{x}$
-1	0.08889
-0.5	0.22776
-0.1	0.48598
-0.05	0.53445
-0.01	0.57670
-0.001	0.58667
-0.0001	0.58767
-0.00001	0.58778

Table – 1

Evaluate $f(x)=\frac{9^x-5^x}{x}$ for some x values close to 0 from right as shown in the table – 2:

x	$f(x)=\frac{9^x-5^x}{x}$
1	4
0.5	1.52786
0.1	0.71112
0.05	0.64649
0.01	0.59908
0.001	0.58890
0.0001	0.58790
0.00001	0.58770

Table – 2

From the table – 1 and table – 2, it can be observed that the value of $f(x)=\frac{9^x-5^x}{x}$ is approximately close to 0.59, when x is close to 0 (on either side of 0).

This implies that, the limit of the function $f(x)=\frac{9^x-5^x}{x}$ as x approaches to 2 is 0.59.

Therefore, $\lim_{x\to 0}\frac{9^x-5^x}{x}=\boxed{0.59}$.

Sketch the graph of the function, $f(x)=\frac{9^x-5^x}{x}$ as shown in figure – 1:

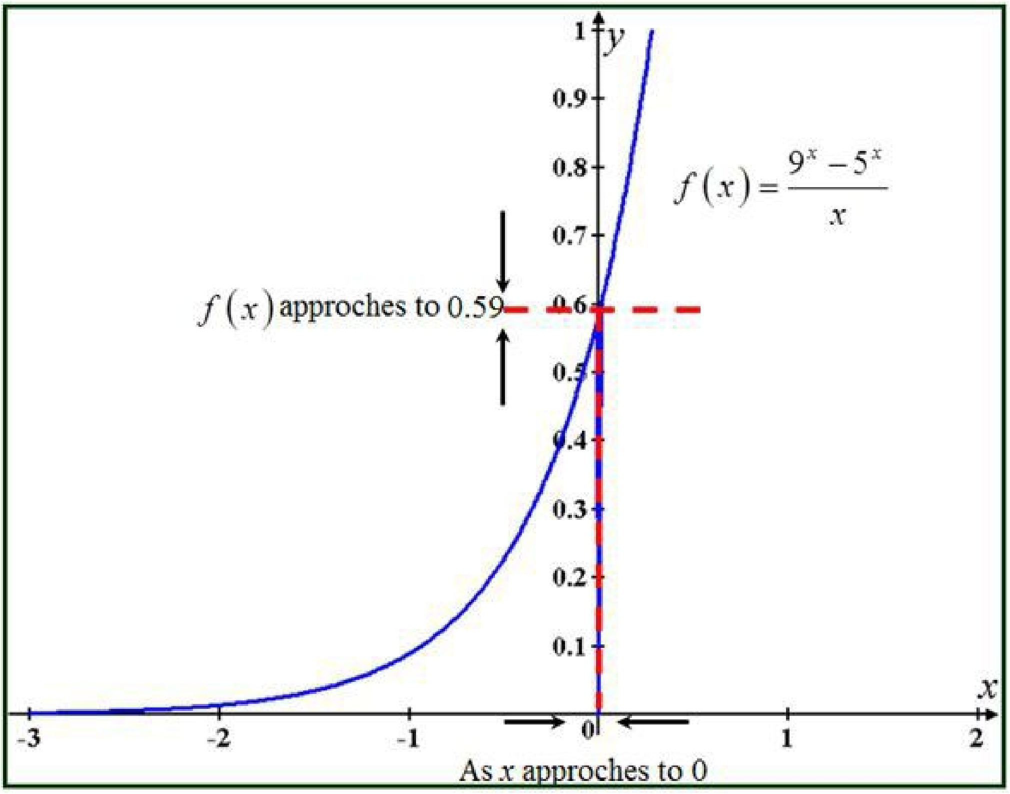


Figure – 1

From the figure – 1, notice that the value of $f(x)=\frac{9^x-5^x}{x}$ is approximately close to 0.59, when x is close to 0 (on either side of 0).

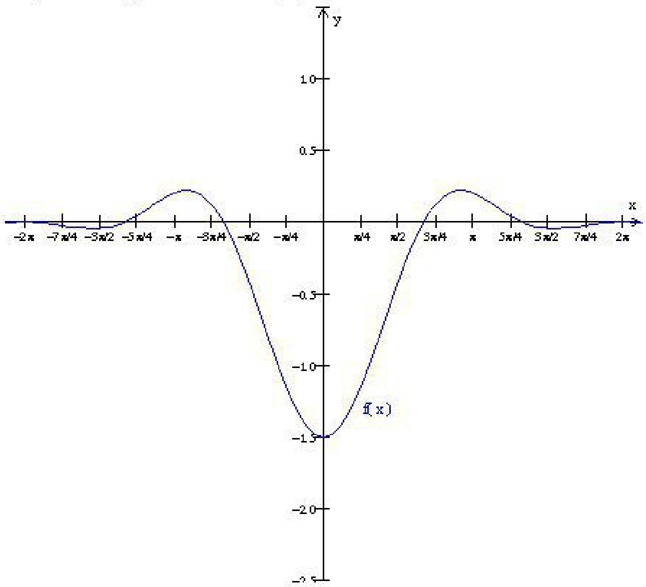
Therefore, $\lim_{x\to 0}\frac{9^x-5^x}{x}=\boxed{0.59}$.

Chapter 1 Functions and Limits Exercise 1.5 27E

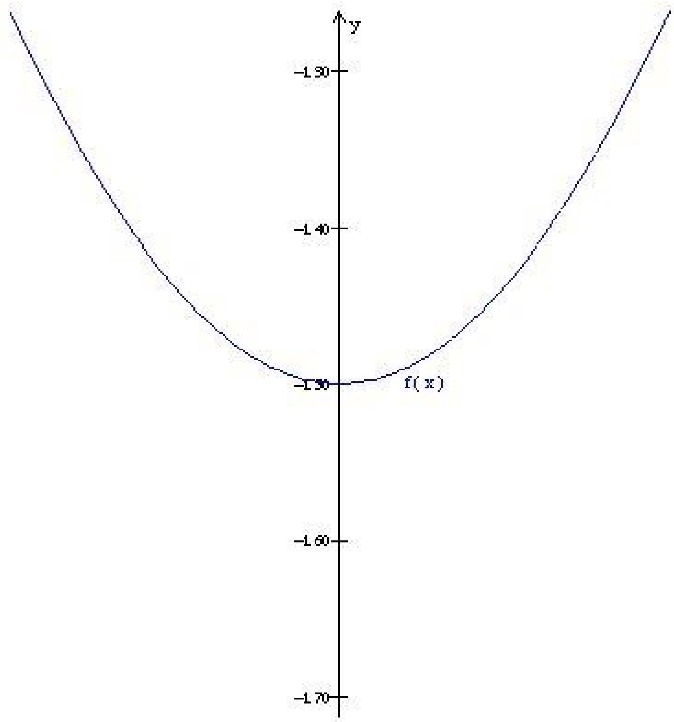
(a) Given function

$$f(x)=\frac{\cos 2x-\cos x}{x^2}$$

Graph of the given function $f(x)$ is



By zooming in towards the point where the graph crosses y-axis, we get



After zooming, we observe that the graph of $f(x)$ crosses y axis at point $(0, -1.5)$. It implies that when x approaches 0, the value of $f(x)$ approaches -1.5.
Therefore, we can say

$$\lim_{x\to 0} \frac{\cos 2x - \cos x}{x^2} = -1.5$$

(b) Now we evaluate the value of the function $f(x)$ for value of x that approach 0.
Let us take the following values of x, approaching zero
$h = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$

x	$f(x)$		x	$f(x)$
- 0.5	-1.3491210		0.5	-1.3491210
-0.1	-1.4937587		0.1	-1.4937587
-0.01	-1.4999375		0.01	-1.4999375
-0.001	-1.4999994		0.001	-1.4999994
-0.0001	-1.5000000		0.0001	-1.5000000

From the values of $f(x) = \dfrac{\cos 2x - \cos x}{x^2}$ shown in the table, we observe that when x approaches 0, the value of $f(x)$ approaches -1.5.

So we see that the value of the limit

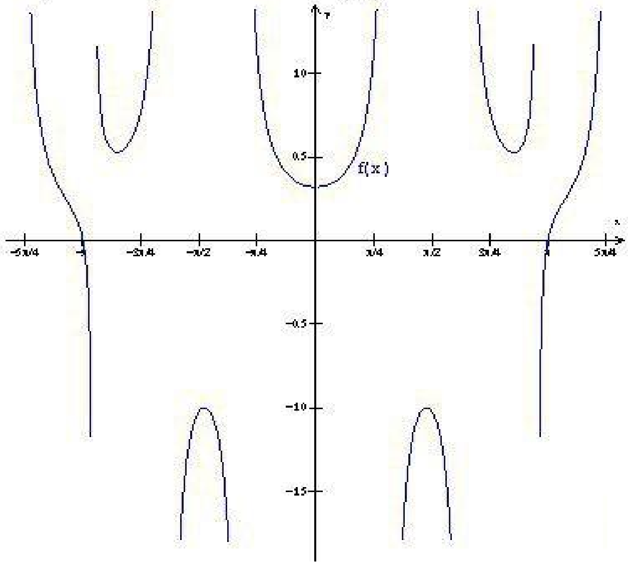
$$\lim_{x\to 0} \frac{\cos 2x - \cos x}{x^2} = -1.5$$

This value matches with the value we got graphically.

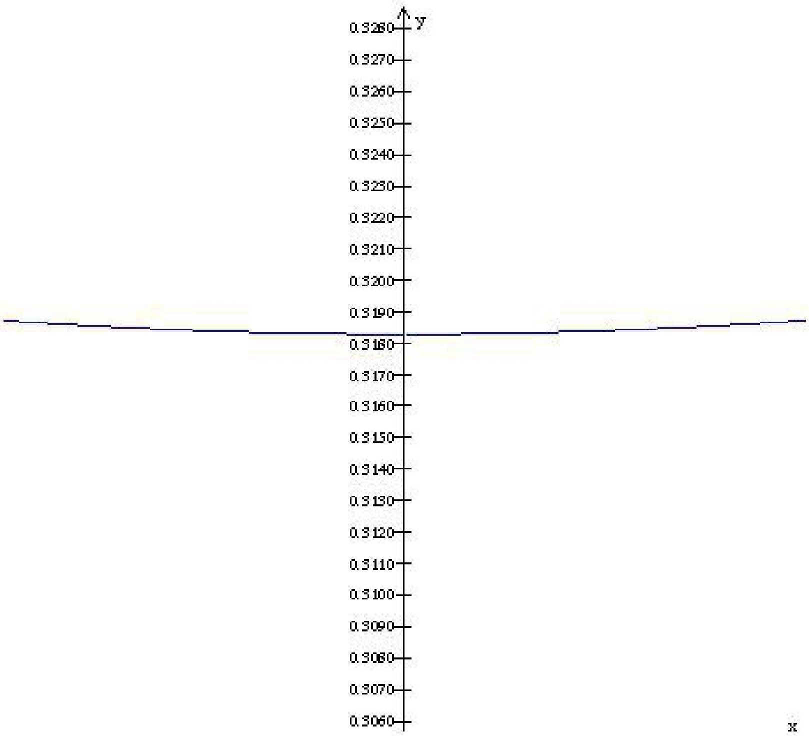
(a) Given function

$$f(x) = \frac{\sin x}{\sin \pi x}$$

Graph of the given function $f(x)$ is



By zooming in towards the point where the graph crosses y-axis, we get



After zooming, we observe that the graph of $f(x)$ crosses y axis at point near to $(0, 0.318)$. It implies that when x approaches 0, the value of $f(x)$ approaches 0.32 (correct to two decimal places).

Therefore, we can say

$$\lim_{x \to 0} \frac{\sin x}{\sin \pi x} = 0.32$$

(b) Now we evaluate the value of the function $f(x)$ for value of x that approach 0.

Let us take the following values of x, approaching zero

$h = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$

x	$f(x)$		x	$f(x)$
- 0.5	0.4794255		0.5	0.4794255
-0.1	0.3230677		0.1	0.3230677
-0.01	0.3183569		0.01	0.3183569
-0.001	0.3183104		0.001	0.3183104
-0.0001	0.3183099		0.0001	0.3183099

From the values of $f(x) = \dfrac{\sin x}{\sin \pi x}$ shown in the table, we observe that when x approaches 0, the value of $f(x)$ approaches $0.318 \approx 0.32$ (correct to two places of decimal)

So we see that the value of the limit

$$\lim_{x \to 0} \frac{\sin x}{\sin \pi x} = 0.32$$

This value matches with the value we got graphically.

Chapter 1 Functions and Limits Exercise 1.5 29E

Consider the limit,

$$\lim_{x \to (-3)^+} \frac{x+2}{x+3}.$$

The objective of the problem is to find the limit $\lim_{x \to -3^+} \frac{x+2}{x+3}$.

Rewrite the limit as,

$$\lim_{x \to (-3)^+} \frac{x+2}{x+3} = \lim_{x \to (-3)^+} \left(1 - \frac{1}{x+3}\right)$$

$$= 1 - \lim_{x \to (-3)^+} \frac{1}{x+3}$$

Since $\frac{1}{x+3} \to \infty$ as $x \to (-3)^+$, and $1 - \frac{1}{x+3} \to -\infty$.

So the value of the limit is,

$$\lim_{x \to (-3)^+} \frac{x+2}{x+3} = \lim_{x \to (-3)^+} \left(1 - \frac{1}{x+3}\right)$$

$$= 1 - \lim_{x \to (-3)^+} \frac{1}{x+3}$$

$$= 1 - \infty$$

$$= -\infty$$

Therefore, $\lim_{x \to (-3)^+} \frac{x+2}{x+3} = \boxed{-\infty}$.

Let $f(x) = \frac{x+2}{x+3}$.

Graphically this is shown below:

The graph of the function $f(x)=\frac{x+2}{x+3}$ and $x=-3$ is shown in the below figure:

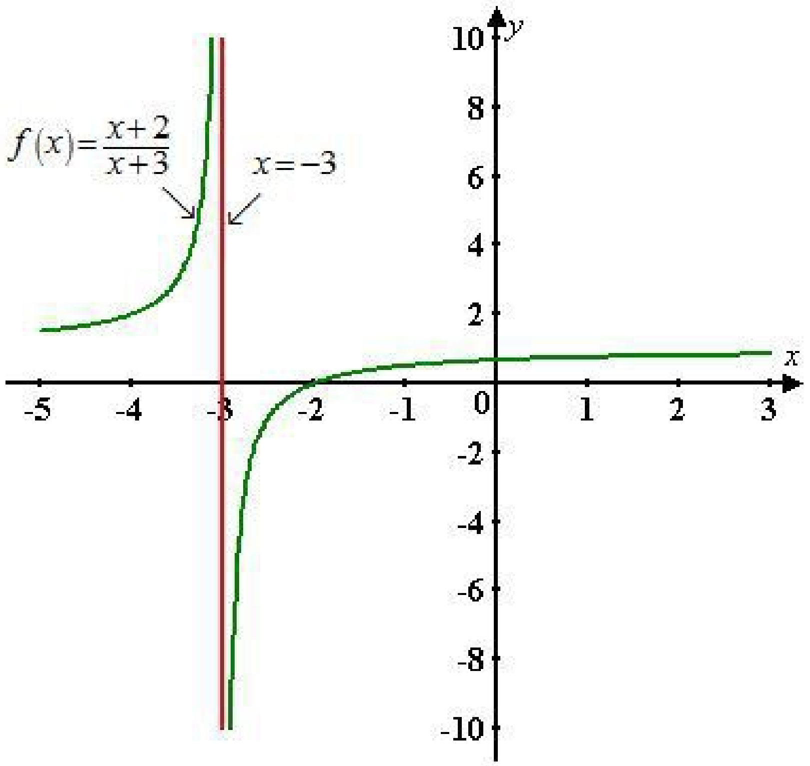


Figure 1

Observe the figure 1, it is seen that the line $x=-3$ is does not touch the graph of $f(x)=\frac{x+2}{x+3}$ from the left and right side and approaches to $-\infty$.

At $x\to -3^{+}$ the limit of the function approaches to $-\infty$.

Hence, $\lim\limits_{x\to(-3)^{+}}\frac{x+2}{x+3}=\boxed{-\infty}$.

Chapter 1 Functions and Limits Exercise 1.5 30E

Recollect that:

Write the limit $\lim\limits_{x\to a^{-}} f(x)=L$ and say the left-hand limit of $f(x)$ as x approaches a is equal to L if we can make the value of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a and x less than a.

Consider the limit,

$$\lim_{x\to-3^{-}}\frac{x+2}{x+3}$$

If x is close to -3 but smaller than -3, then the denominator $x+3$ is a small negative number and $x+2$ is close to -1.

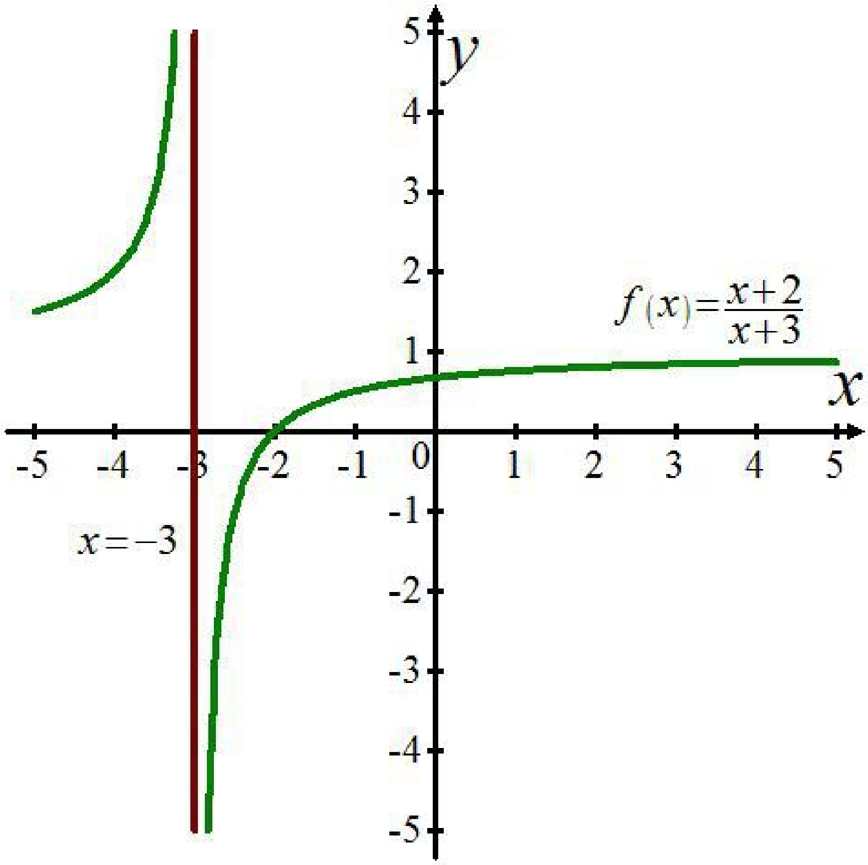
So the quotient $\frac{x+2}{x+3}$ is numerically large positive number.

Thus,

$$\lim_{x\to-3^{-}}\frac{x+2}{x+3}=\infty$$

The graph of the curve $y = \frac{x+2}{x+3}$ as in the below figure.

The line $x = -3$ is a vertical asymptote.



Chapter 1 Functions and Limits Exercise 1.5 31E

Consider the infinite limit,

$$\lim_{x \to 1} \frac{2-x}{(x-1)^2}$$

If *x* is close to 1 but larger than 1, then the expression $x-1$ is a small positive number and so denominator $(x-1)^2$ will be very small positive number. And the numerator $2-x$ will be close to 1.

So, the quotient $\frac{2-x}{(x-1)^2}$ will be a large positive number.

For example, if $x = 1.01$ then

$$\begin{aligned}\frac{2-x}{(x-1)^2} &= \frac{2-1.01}{(1.01-1)^2} \\ &= \frac{0.99}{(0.01)^2} \\ &= \frac{0.99}{0.0001} \\ &= 9900\end{aligned}$$

Thus, $\lim_{x \to 1^+} \frac{2-x}{(x-1)^2} = \infty$ ...... (1)

If x is close to 1 but smaller than 1, then the expression $x-1$ is a small negative number and so denominator $(x-1)^2$ will be very small positive number. And the numerator $2-x$ will be close to 1.

So, the quotient $\frac{2-x}{(x-1)^2}$ will be a large positive number.

For example, if $x=0.99$ then

$$\begin{aligned}\frac{2-x}{(x-1)^2} &= \frac{2-0.99}{(0.99-1)^2} \\ &= \frac{1.01}{(-0.01)^2} \\ &= \frac{1.01}{0.0001} \\ &= 10,100\end{aligned}$$

Thus, $\lim_{x\to 1^-} \frac{2-x}{(x-1)^2} = \infty$ ...... (2)

From equations (1) and (2),

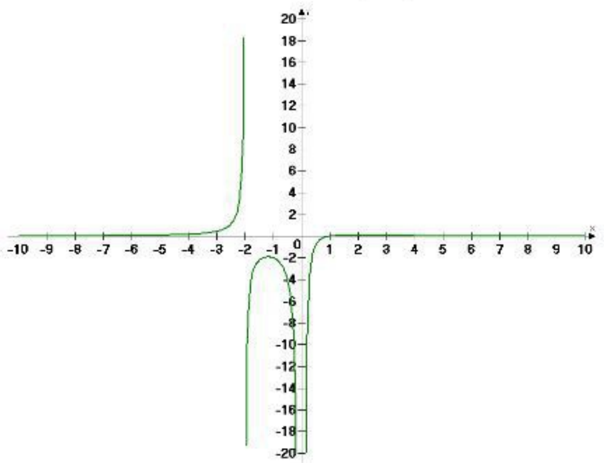
$$\boxed{\lim_{x\to 1} \frac{2-x}{(x-1)^2} = \infty}$$

Chapter 1 Functions and Limits Exercise 1.5 32E

$$\lim_{x\to 0} \frac{x-1}{x^2(x+2)}$$

x	$\frac{x-1}{x^2(x+2)}$
-0.2	-16.6666
-0.1	-57.8947
-0.05	-215.3846
-0.01	-5075.3768
-0.001	-500750.357
0.001	-499250.374
0.01	-4925.373
0.05	-185.3658
0.1	-42.85714
0.2	-9.090909

$$f(x) = \frac{x-1}{x^2(x+2)}.$$



As x becomes close to 0, x^2 also becomes close to zero, and $\frac{x-1}{x^2(x+2)}$ becomes

Very large.(see from the table.) from the above graph values of the $f(x)$ can be made arbitrarily very large taking x close enough to 0.Values of $f(x)$ do not approach the number. from the graph

$x \to 0^-, f(x) \to -\infty.$

$x \to 0^+, f(x) \to -\infty.$

Hence

$$\lim_{x\to 0} \frac{x-1}{x^2(x+2)} = -\infty$$

Chapter 1 Functions and Limits Exercise 1.5 33E

1679-1.5-33E RID: 1411| 21/04/2016

Consider the limit,

$$\lim_{x\to 2^+} \frac{x-1}{x^2(x+2)}.$$

The object is to determine the limit.

If x is close to 2 but larger than 2, then the denominator $x+2$ is a small positive number and $x-1$ is a small negative number and $x-1<0$.

So the quotient $\frac{x-1}{x^2(x+2)}$ is a large negative number.

Therefore, $\lim_{x\to 2^+} \frac{x-1}{x^2(x+2)} = \boxed{-\infty}$.

The sketch of the curve $y = \frac{x-1}{x^2(x+2)}$ is shown in the below figure 1.

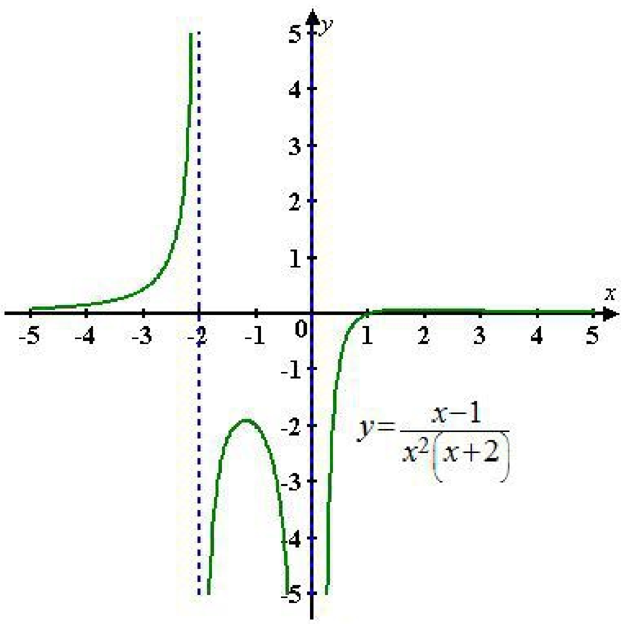


Figure 1

Observe the figure 1, it seen that the lines $x=-2$ and $x=0$ are vertical asymptotes.

The values of the curve go negatively.

Therefore, $\lim_{x\to 2^+} \frac{x-1}{x^2(x+2)} = \boxed{-\infty}$.

Chapter 1 Functions and Limits Exercise 1.5 34E

1679-1.5-34E RID: 1411| 12/03/2016

Consider the limit,

$\lim_{x \to \pi^-} \cot x$.

Need to determine the infinite limit.

Let $f(x) = \cot x$.

Substitute $x = 3.1, 3.01, 3.001 (x < \pi = 3.1416)$ into the function $f(x) = \ln(x^2 - 9)$, we have

$$f(3.1) = \cot 3.1$$
$$\approx -24.0288$$
$$f(3.01) = \cot 3.01$$
$$\approx -7.5552$$
$$f(3.001) = \cot(3.001)$$
$$\approx -7.06582$$

Observe that the function values are different and negative when $x \to \pi^-$.

So it confirms that as $x \to \pi^-$ then $\lim_{x \to \pi^-} \cot x$ approaches to $-\infty$.

Hence, $\lim_{x \to \pi^-} \cot x = \boxed{-\infty}$.

Let $f(x) = \cot x$.

Evaluate the above limit from the graph of the function.

The sketch of the function $f(x) = \cot x$ is shown below:

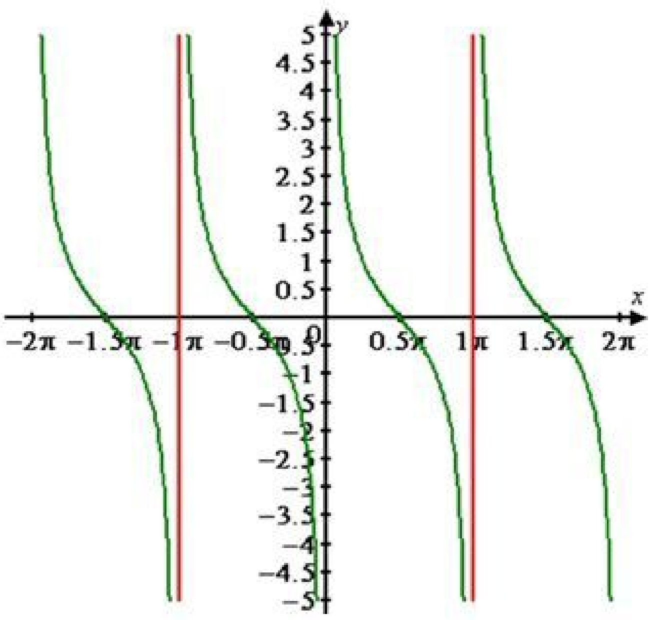


Figure 1

Observe the figure 1, it seen that the line $x = \pi$ and $x = -\pi$ are does not touch the graph of $f(x) = \cot x$ from the left and right side and approaches to $-\infty$.

Here, $x = \pi$ behaves as an asymptote.

At $x \to \pi^-$ the limit of the function approaches to $-\infty$.

Hence, $\lim_{x \to \pi^-} \cot x = \boxed{-\infty}$.

Chapter 1 Functions and Limits Exercise 1.5 35E

Consider the limit $\lim_{x \to 2\pi^-} x \csc x$

Determine the infinite limit:

Recall that

$$\csc x = \frac{1}{\sin x}$$

So,

$$\lim_{x \to 2\pi^-} x \csc x = \lim_{x \to 2\pi^-} \frac{x}{\sin x}$$

If x is close to 2π but smaller than 2π, then the denominator $\sin x$ approaches 0 through negative values and x is close to 2π. So the quotient $\frac{x}{\sin x}$ is a large negative number.

Thus, intuitively, observe that

$$\begin{aligned} \lim_{x \to 2\pi^-} x \csc x &= \lim_{x \to 2\pi^-} \frac{x}{\sin x} \\ &= \boxed{-\infty} \end{aligned}$$

Chapter 1 Functions and Limits Exercise 1.5 36E

We have to evaluate $\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$

First we simplify the expression by factoring the denominator and numerator then cancelling the like terms

$$\begin{aligned} \lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4} &= \lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2} \\ &= \lim_{x \to 2^-} \frac{x}{(x-2)} \end{aligned}$$

Now

As $x \to 2^-$, $(x-2) \to 0^-$

Therefore,

$$\lim_{x \to 2^-} \frac{x}{(x-2)} = -\infty$$

So

$$\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4} = -\infty$$

We can verify it by a numeric table

x	$\frac{x^2 - 2x}{x^2 - 4x + 4}$
1.5	-3
1.9	-19
1.99	-199
1.999	-1999
1.9999	-19999.00012
1.99999	-199998.9835

This table verifies our result

Chapter 1 Functions and Limits Exercise 1.5 37E

$$\lim_{x\to2^+}\frac{x^2-2x-8}{x^2-5x+6}=\lim_{x\to2^+}\frac{x^2-4x+2x-8}{x^2-2x-3x+6}$$

$$=\lim_{x\to2^+}\frac{x(x-4)+2(x-4)}{x(x-2)-3(x-2)}$$

$$=\lim_{x\to2^+}\frac{(x-4)(x+2)}{(x-2)(x-3)}$$

When x approaches 2 from right then numerator is negative and denominator is very near to 0 and is negative.

$$\lim_{x\to2^+}\frac{(x-4)(x+2)}{(x-2)(x-3)}=\infty$$

Chapter 1 Functions and Limits Exercise 1.5 38E

(A) We have to find vertical asymptotes of the function.

$$y=\frac{x^2+1}{3x-2x^2}$$

Or

$$y=\frac{x^2+1}{x(3-2x)}$$

Denominator will be equal to 0, when $x=0$ or $x=3/2$

Now

$$\lim_{x\to0^-}f(x)=\lim_{x\to0^-}\frac{x^2+1}{3x-2x^2}$$

Since $(3x-2x^2)\to0^-$ and $(x^2+1)\to1^+$ as $x\to0^-$

Thus $\lim_{x\to0^-}f(x)=-\infty$

Similarly $\lim_{x\to0^+}f(x)=\infty$

Now $\lim_{x\to(3/2)^-}f(x)=\lim_{x\to(3/2)^-}\frac{x^2+1}{3x-2x^2}$

Since $(3x-2x^2)\to0^+$ and $(x^2+1)\to3.25$ as $x\to(3/2)^-$

Thus $\lim_{x\to(3/2)^-}f(x)=\infty$

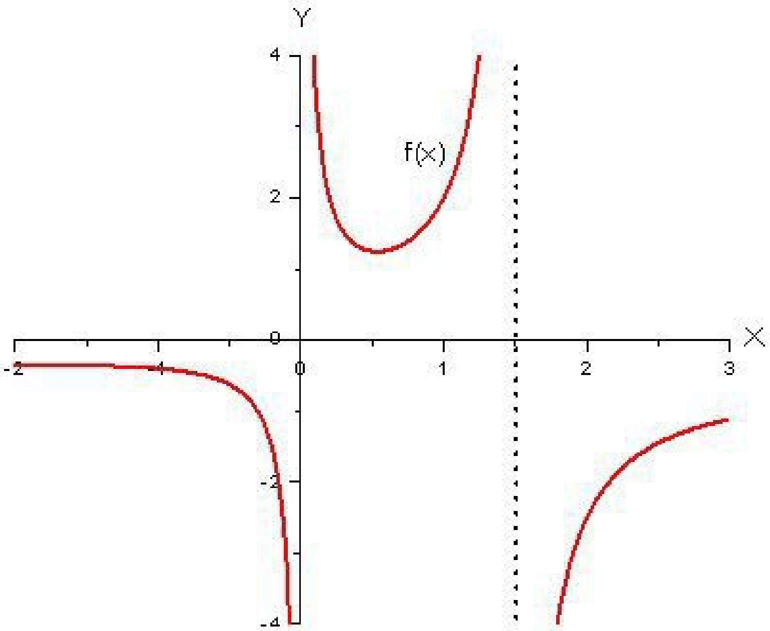
Similarly

$$\lim_{x\to(3/2)^+}f(x)=-\infty.$$

Therefore,

The vertical asymptotes are $\boxed{x=0 \text{ and } x=3/2}$

(B) We graph the function and see that the vertical asymptotes are $x = 0, 3/2$



Chapter 1 Functions and Limits Exercise 1.5 39E

(A)

First we make a table of values of $f(x)$ for different values of x, close to 1.

x	$f(x)$
2	0.14285
1.5	0.421053
1.2	1.37362
1.1	3.02115
1.05	6.34417
1.01	33.0022
1.001	333.00022

$x > 1$ (table 1)

x	$f(x)$
0	-1
0.1	-1.001
0.5	-1.142857
0.9	-3.690037
0.95	-7.011394
0.995	-33.6689
0.999	-333.666889

$x < 1$ (table 2)

Here we see that in table 1, we start with 2 and decrease the value of x (Approaches 1) just close to 1, then the value of f(x) appears higher and higher. It means when $x \to 1$ when $(x > 1)$,

$$\Rightarrow f(x) \to \infty$$

It means $\boxed{\lim_{x \to 1^+} f(x) = \infty}$

Again if we start from 0 and increase the value of x, just close to 1, then value of $f(x) \to -\infty$

So that $\boxed{\lim_{x \to 1^-} f(x) = -\infty}$

(B)

Now we find $\lim_{x \to 1^+} \frac{1}{(x^3 - 1)}$ & $\lim_{x \to 1^-} \frac{1}{(x^3 - 1)}$.

If x is close to 1 but larger than 1, then $(x^3 - 1)$ is a very small positive number

and so that the function $f(x) = \frac{1}{(x^3 - 1)}$ is a large positive number

Thus we see that $\lim_{x \to 1^+} \frac{1}{(x^3 - 1)} = \infty$

Similarly if x is close to 1 but smaller than 1 then $\left(x^3-1\right)$ is a very small negative number, so that the function $f\left(x\right)=\frac{1}{\left(x^3-1\right)}$ is a very large negative number

Thus $\lim_{x\to1^-} f\left(x\right)=\lim_{x\to1^-}\frac{1}{\left(x^3-1\right)}=-\infty$

(C)

Graph of the function $f\left(x\right)=\frac{1}{\left(x^3-1\right)}$ is in the figure 1. The vertical asymptote is the line $\boxed{x=1}$

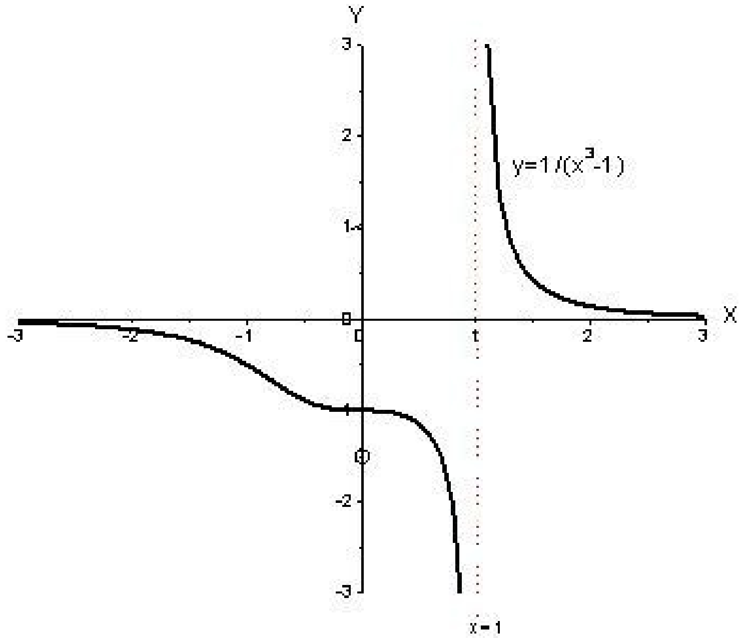
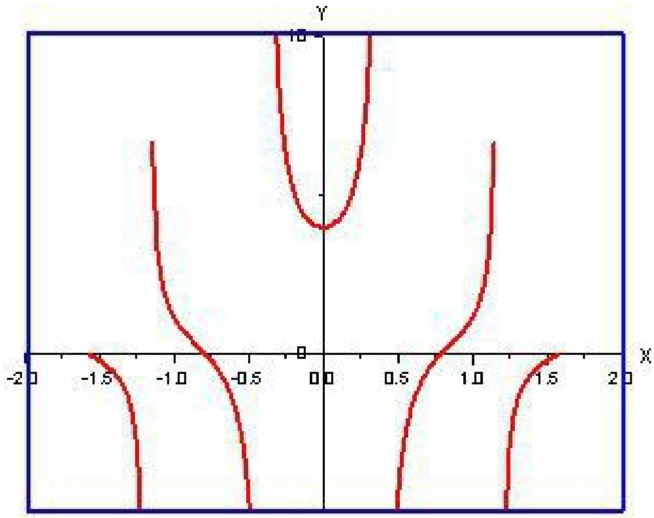


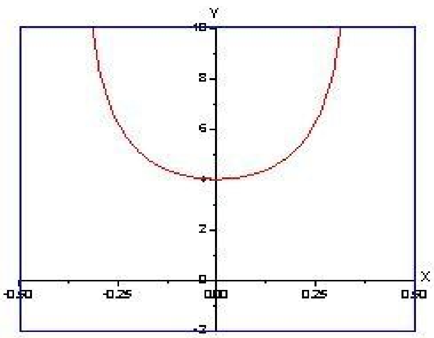
Fig 1

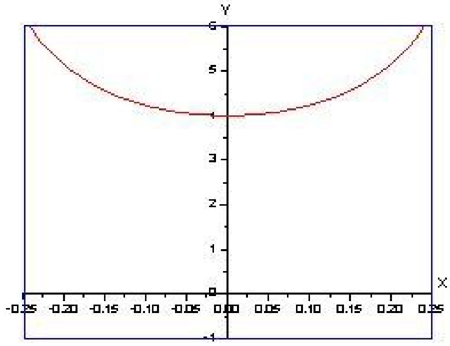
Chapter 1 Functions and Limits Exercise 1.5 40E

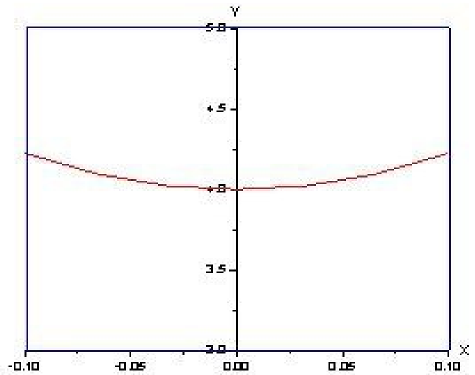
(A) We have $f\left(x\right)=\frac{\tan 4x}{x}$

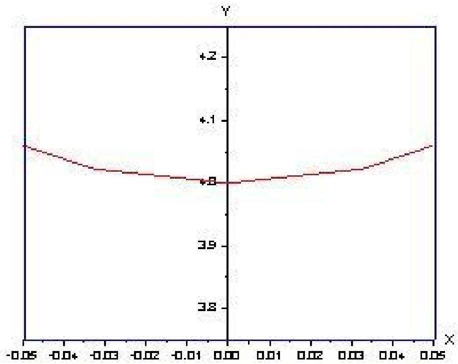


Now we zoom in the scale toward the point where the graph crosses y - axis









From the graph we see that

$$\lim_{x \to 0} f(x) \approx \boxed{4}$$

(B) Now we calculate the value of the functions for different values of x close to 0

$$f(x) = \frac{\tan 4x}{x}$$

x	***f(x)***		**x**	***f(x)***
-0.1	4.227932187		0.1	4.227932187
-0.05	4.05420071		0.05	4.05420071
-0.01	4.0021347		0.01	4.0021347
-0.005	4.000533419		0.005	4.000533419
-0.001	4.000021333		0.001	4.000021333
-0.0005	4.000005333		0.0005	4.000005333
-0.0001	4.000000213		0.0001	4.000000213

From the table, we see that

$$\lim_{x \to 0} f(x) = \boxed{4}$$

Chapter 1 Functions and Limits Exercise 1.5 41E

(A)

x	$f(x) = \left(x^2 - \frac{2^x}{1000}\right)$
1	0.998000
0.8	0.638259
0.6	0.358484
0.4	0.158680
0.2	0.038851
0.1	0.008928
0.05	0.001474

As we take the value close to 0, $f(x)$ tends to 0

So the limit = 0

$$\lim_{x \to 0}\left(x^2 - \frac{2^x}{1000}\right) = 0$$

(B)

x	$f(x)$
0.04	0.0005719
0.02	-0.000614
0.01	-0.000907
0.005	-0.0009785
0.003	-0.0009930
0.001	-0.000999

Here when we are decreasing the value of x from 0.04 to close to 0

Then $f(x) \to -0.001$

So $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(x^2 - \frac{2^x}{1000}\right) = -0.001$

Chapter 1 Functions and Limits Exercise 1.5 42E

(A) We have $h(x) = \frac{\tan x - x}{x^3}$

x	$h(x)$
1	0.55740
0.5	0.37042
0.1	0.33467
0.05	0.33367
0.01	0.33335
0.005	0.33334

(B) From the table of part (A), we can guess

$$\lim_{x\to 0}\frac{\tan x - x}{x^3} \approx 0.3333 \approx \boxed{1/3}$$

(C)

x	$h(x)$
0.001	0.333333467
0.0005	0.333333367
0.0001	0.333333337
0.00005	0.33333335
0.00001	0.333332876
0.00000001	0.000000000

We are still confident that our guess in part (B) is correct. The values depend on the calculator and calculator will eventually give the false values

(D) We graph the function in the viewing rectangle [-1, 1] by [0, 1]

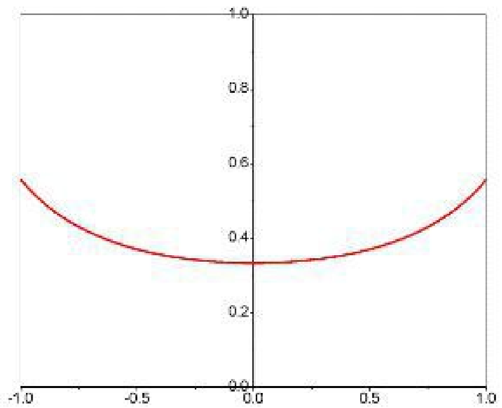


Fig.1

Now we zoom in the scale of the graph

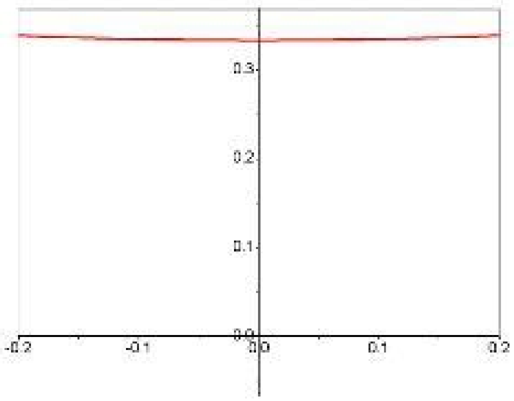


Fig.2

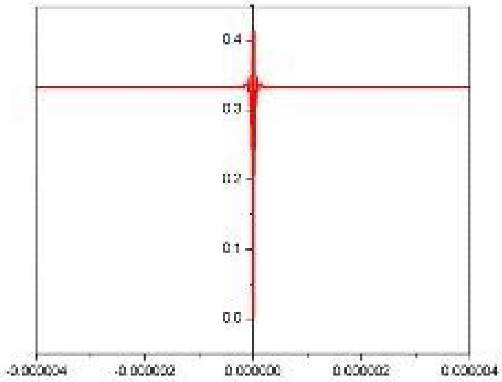


Fig.3

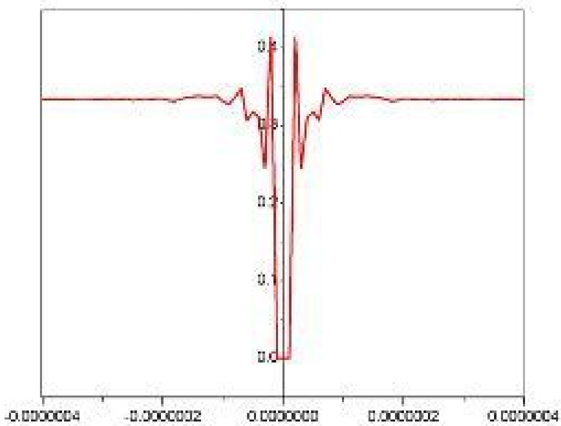


Fig.4

We see that when we take very small viewing rectangle, we get the incorrect results.

Chapter 1 Functions and Limits Exercise 1.5 43E

We graph the function $f(x) = \sin(\pi / x)$ in the viewing rectangle [-1, 1] by [-1, 1]

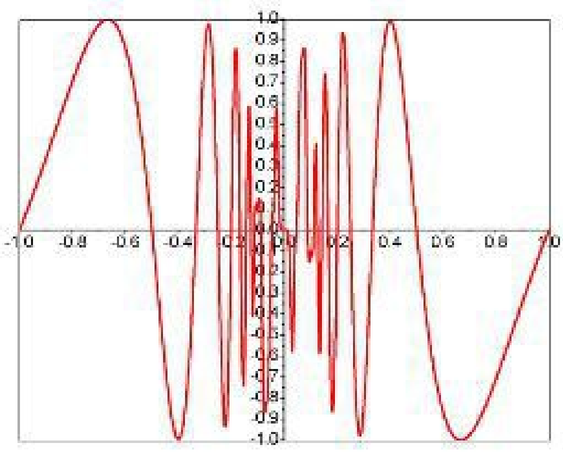


Fig.1

Now we zoom in the scale

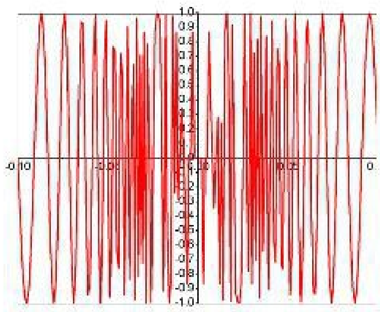


Fig.2

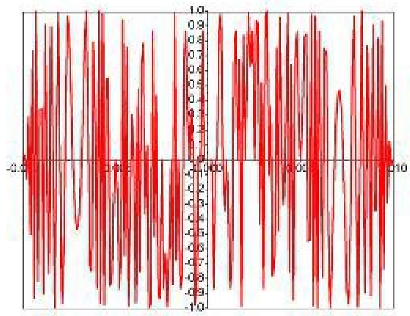


Fig.3

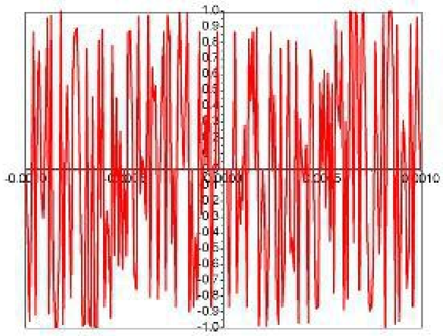


Fig.4

$f(x) = \sin(\pi / x)$

We know that $-1 \leq \sin \frac{\pi}{x} \leq 1$ for all x

It is clear that value of the function oscillates in the interval $[-1,\ 1]$

When the value of x approaches 0, the value of the function oscillates more frequently.

Chapter 1 Functions and Limits Exercise 1.5 44E

The mass of the particle m, which varies with the velocity v, which is expressed as follows:

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

Here m_0 is the mass of the particle at rest and c is the speed of light.

$$\lim_{v \to c^-} \sqrt{1 - v^2 / c^2} = 0$$

This is computed simply by plugging in c for v.

Thus the limit either goes to $+\infty$ or $-\infty$.

Notice that $v < c$ will cause $v^2 / c^2 < 1$, so when $v < c$, since m_o is mass and must be positive and the following result can be obtained:

$$\frac{m_0}{\sqrt{1 - v^2 / c^2}} > 0.$$

So this means the as the limit as v goes to c, from below must be non-negative, the value is as follows:

$$\lim_{v \to c^-} \frac{m_0}{\sqrt{1 - v^2 / c^2}} = +\infty.$$

This means that as a particle moves at high speeds its mass will increase, where it's mass can get arbitrarily large when the particles speed gets close to the speed of light.

Chapter 1 Functions and Limits Exercise 1.5 45E

First we graph the curve $y = \tan(2\sin x)$ $\quad -\pi \le x \le \pi$

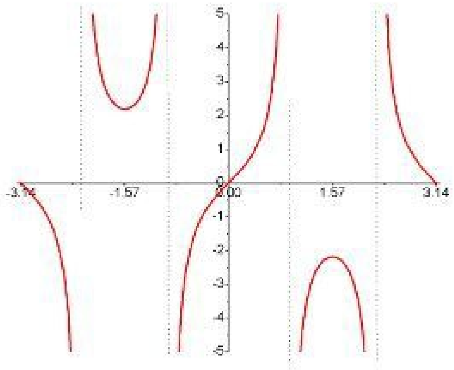


Fig.1

From the graph we see that the vertical asymptotes are $x \approx \pm 0.903$ and $x \approx \pm 2.238$

The given equation is ;

$y = \tan(2\sin x)$ $\quad -\pi \le x \le \pi$

We know that the tangent function has the vertical asymptotes at $x = \frac{\pi}{2} + n\pi$,

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots.$

So for the vertical asymptotes, we must have

$$2\sin x = \frac{\pi}{2} + n\pi$$

Or $$\sin x = \frac{\pi}{4} + \frac{n\pi}{2}$$

Since we know that $-1 \le \sin x \le 1$ for all x

So we take the values of n , 0 and -1 only

Therefore, we must have $\sin x = \pm\frac{\pi}{4}$

And then $$x = \pm\left(\sin^{-1}\frac{\pi}{4}\right) \approx \pm 0.903$$

Now we know that $\sin x = \sin(\pi - x)$

So we can also take $x = \pm\left(\pi - \sin^{-1}\frac{\pi}{4}\right) \approx \pm 2.238$

The equations of the vertical asymptotes are

$$\boxed{x = \pm\left(\sin^{-1}\frac{\pi}{4}\right) \text{ and } x = \pm\left(\pi - \sin^{-1}\frac{\pi}{4}\right)}$$

Chapter 1 Functions and Limits Exercise 1.5 46E

(A) From the graph the function $f(x) = \frac{x^3 - 1}{\sqrt{x} - 1}$, and make a table of values the function for different values of x near 1.

x	$f(x) = \frac{x^3 - 1}{\sqrt{x} - 1}$
0.9	5.280931738
0.99	5.925312187
0.999	5.992503125
0.9999	5.999250031
1.0001	6.000750031
1.001	6.007503125
1.01	6.075312812
1.1	6.781557287

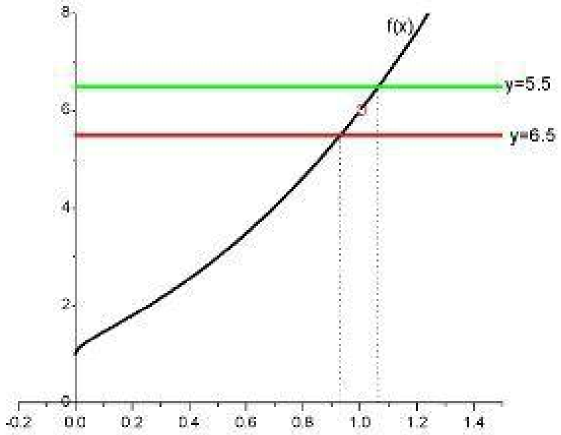


Fig.1

From figure and the table, we can guess the value of $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x-1}} \approx 6$

(B) We need to have the function within the distance 0.5 of its limit 6.
It means we must have

$$5.5 < \frac{x^3-1}{\sqrt{x-1}} < 6.5$$

We graph the function and the lines $y = 5.5$ and $y = 6.5$ (see figure 1 in part (a))
We see that the curve intersects these lines at $x \approx 0.9314$ and $x \approx 1.0649$ respectively.

Now $|1-0.9314| = 0.0686$

And $|1-1.0649| = 0.0649$

So we need to choose x within $\boxed{0.0649}$ of 1.