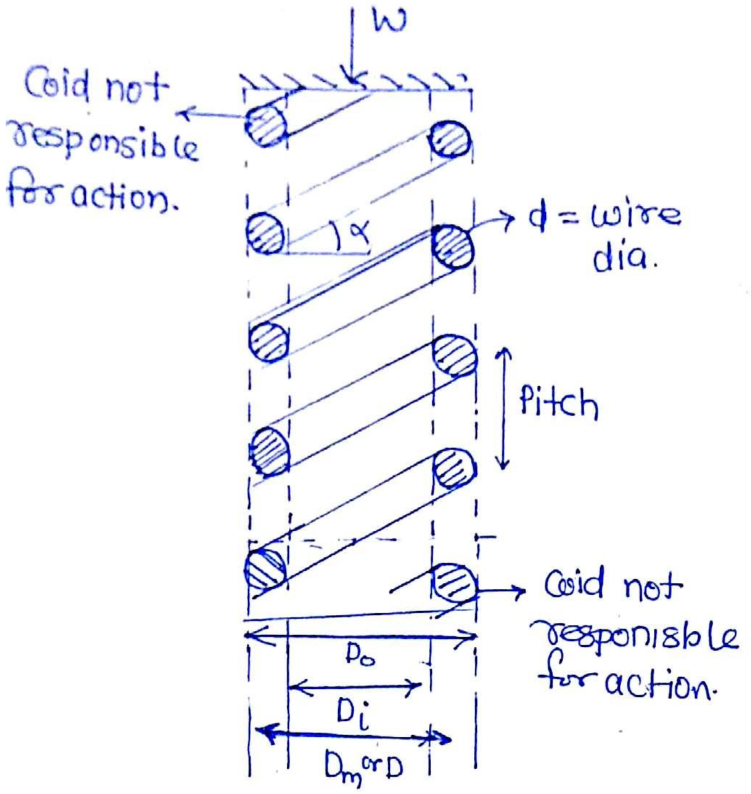
Helical spring



n = no. of active Coil

α = Helical angle

D_o = outer dia

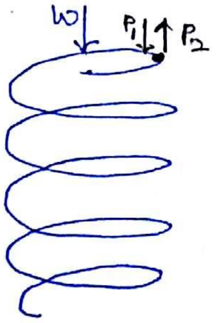
D_i = inner dia

D_m/D = mean dia

$D = D_o - d = D_i + d$

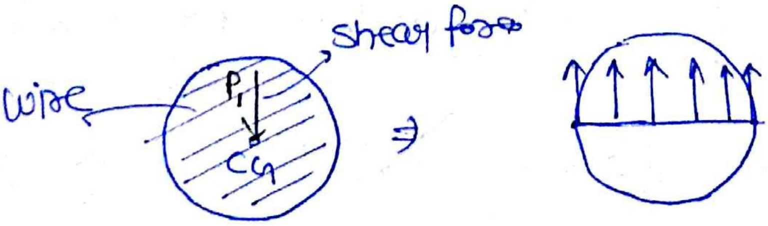
$$\boxed{C = \frac{D}{d} =} \text{ Spring ratio.}$$

$4 \leq C \leq 12.$



* wire will wear load

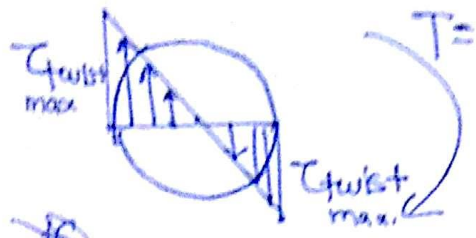
Step ① Apply two equal and opposite load at CG of CG of wire



Direct shear stress

$$\tau_{\substack{dir.\\avg}} = \frac{W}{\frac{\pi}{4}d^2} = \frac{4W}{\pi d^2}$$

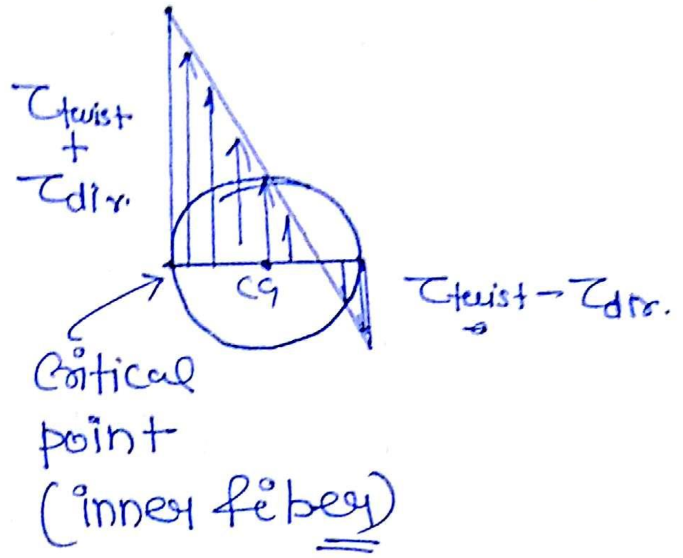
effect of P_2 & W (twisting in wire about CG)



$T = \frac{WD}{2}$

$$\tau_{torsional\ max} = \frac{16T}{\pi d^3} = \frac{8WD}{\pi d^3}$$

Combined effect (only shear)



Conclusion

(1) Hence the inner fiber of spring wire is subject to max stress

(2) Helical spring under comp. / tensile load it only subjected to shear (Direct + torsional)

$$\tau_{max} = \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

*

$$\boxed{\tau_{max} = \frac{8WD}{\pi d^3}\left[1 + \frac{1}{2C}\right]}$$

$C = \frac{D}{d}$.

$\Downarrow$

K_{sh} = Direct shear stress factor.

*

$$\boxed{\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_{sh}}$$

*

K_c = curvature effect factor.

$$\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_{sh} \cdot K_c$$

*

$$\boxed{\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_w}$$

$K_{sh} \cdot K_c = K_w$

K_w = wahl's factor.

*

$$\boxed{K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

* wahl's factor include direct shear and curvature both.

safe Cond^n

$$\tau_{max} \le \tau_{per.}$$

$$\frac{8WD}{\pi d^3} \cdot K_w \le \tau_{per.}$$

Com

$$\boxed{W_{max} = \frac{\pi d^3}{8DK_w} \cdot \tau_{per.}}$$

(tension/(comp)

max. value of load that a spring wire bear without any shear.

Expression for deflection and stiffness :-

Strain energy $SE = \frac{1}{2} T\theta = \frac{T^2 L}{2GJ}$

$$U = \frac{\frac{1}{2}\left(\frac{WD}{2}\right)^2 (\pi D n)}{G \cdot \frac{\pi}{32} \cdot d^4}$$

$$\boxed{U = \frac{4W^2 D^3 n}{G d^4}}$$

Castiglians theorem

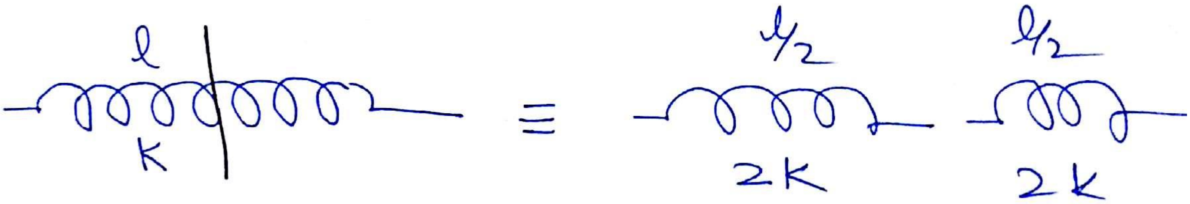
$$\delta = \frac{dU}{dW} = \text{deflection.}$$

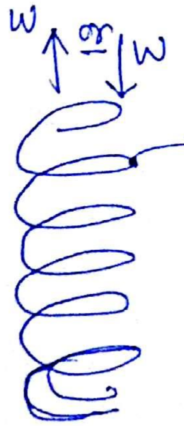
$$\delta = \frac{8WD^3 n}{Gd^4}$$

$$k = \frac{W}{\delta}$$

** $$\boxed{k = \frac{Gd^4}{8D^3 n}}$$

$$k \propto \frac{1}{n}$$

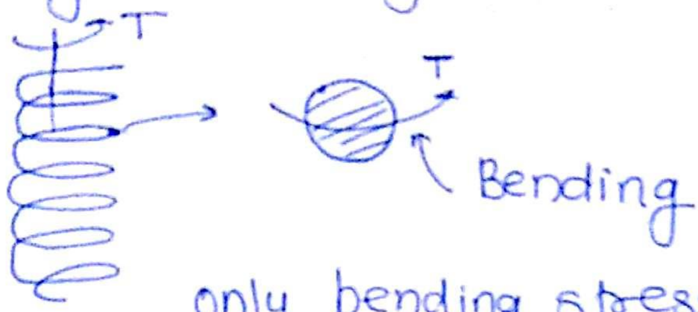
* 

* 

→ only shear → direct shear

→ torsional shear.

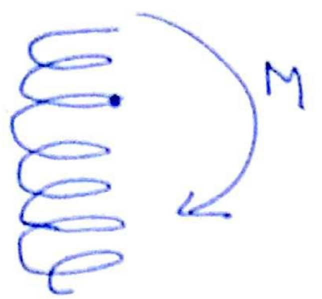
* Twisting in spring



only bending stress in wire.

$$\sigma_{b_{max}} = \frac{32T}{\pi d^3}$$

* bending



only twisting in wire

$$\tau_{max} = \frac{16M}{\pi d^3}$$