6.LIMIT OF FUNCTION

1. Limit of a function f(x) is said to exist as $x \to a$ when,

$\underset{h\to 0^+}{\text{Limit}}\ f(a-h) = \underset{h\to 0^+}{\text{Limit}}\ f(a+h)$ = some finite value M.

(Left hand limit) (Right hand limit)

2. Indeterminant Forms:

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, \infty^0, 0^0$, and 1^∞.

3. Standard Limits:

$$\underset{x\to 0}{\text{Limit}} \frac{\sin x}{x} = \underset{x\to 0}{\text{Limit}} \frac{\tan x}{x} = \underset{x\to 0}{\text{Limit}} \frac{\tan^{-1}x}{x} = \underset{x\to 0}{\text{Limit}} \frac{\sin^{-1}x}{x} = \underset{x\to 0}{\text{Limit}} \frac{e^x-1}{x} = \underset{x\to 0}{\text{Limit}} \frac{\ell n(1+x)}{x} = 1$$

$$\underset{x\to 0}{\text{Limit}} (1+x)^{1/x} = \underset{x\to\infty}{\text{Limit}} \left(1+\frac{1}{x}\right)^x = e, \quad \underset{x\to 0}{\text{Limit}} \frac{a^x-1}{x} = \log_e a,\ a>0, \quad \underset{x\to a}{\text{Limit}} \frac{x^n-a^n}{x-a} = na^{n-1}.$$

4. Limits Using Expansion

(i) $a^x = 1 + \frac{x\ln a}{1!} + \frac{x^2\ln^2 a}{2!} + \frac{x^3\ln^3 a}{3!} + \ldots\ldots\ldots a>0$

(ii) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(iii) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots \text{for } -1 < x \le 1$

(iv) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$

(v) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$

(vi) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\ldots$

(vii) $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$

(viii) $\sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2.3^2}{5!}x^5 + \frac{1^2.3^2.5^2}{7!}x^7 + \ldots\ldots$

(x) for $|x| < 1$, $n \in R$ $(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + \ldots\ldots\ldots\ldots\infty$

5. Limits of form $1^\infty, 0^0, \infty^0$

Also for $(1)^\infty$ type of problems we can use following rules.

$\lim_{x\to 0} (1+x)^{1/x} = e$, $\lim_{x\to a} [f(x)]^{g(x)}$, where $f(x) \to 1$; $g(x) \to \infty$ as $x \to a = \lim_{x\to a} = e^{\lim_{x\to a}[f(x)-1]\,g(x)}$

6. Sandwich Theorem or Squeeze Play Theorem:

If $f(x) \le g(x) \le h(x)\ \forall x$ & $\underset{x\to a}{\text{Limit}} f(x) = \ell = \underset{x\to a}{\text{Limit}} h(x)$ then $\underset{x\to a}{\text{Limit}} g(x) = \ell$.