5. Expansion formulae

- $(x + a)(x + b) = x^2 + (a + b)x + ab$

Example:

Find 206 × 198.

Solution:

We have,

206 × 198 = (200 + 6) (200 – 2)

= $(200)^2 + (6 + (-2)) \times 200 + (6)(-2)$ [Using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$]

= 40000 + 800 – 12

= 40800 – 12

= 40788

- **Factorization by using the identity, $x^2 + (a + b)x + ab = (x + a)(x + b)$.**

  To apply this identity in an expression of the type $x^2 + px + q$, we observe the coefficient of x and the constant term.

  Two numbers, a and b, are chosen such that their product is q and their sum is p.

  i.e., $a + b = p$ and $ab = q$

  Then, the expression, $x^2 + px + q$, becomes $(x + a)(x + b)$.

  Example:

  Factorize $a^2 - 2a - 8$.

  Solution:

  Observe that, $-8 = (-4) \times 2$ and $(-4) + 2 = -2$

  Therefore, $a^2 - 2a - 8 = a^2 - 4a + 2a - 8$

  $= a(a - 4) + 2(a - 4)$

  $= (a - 4)(a + 2)$

- **Identities:** $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ and $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

Other ways to represent these identities are:

- $x^3 + y^3 = (x + y)^3 - 3xy\,(x + y)$
- $x^3 + y^3 = (x + y)\,(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)^3 + 3xy\,(x - y)$
- $x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$

Example:

Expand $(3x + 2y)^3 - (3x - 2y)^3$

Solution:

$(3x + 2y)^3 = (3x)^3 + (2y)^3 + 3\,(3x)\,(2y)\,(3x + 2y)$

$= 27x^3 + 8y^3 + 54x^2y + 36xy^2$ ... (1)

$(3x - 2y)^3 = (3x)^3 - (2y)^3 - 3\,(3x)\,(2y)\,(3x - 2y)$

$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$ ... (2)

From equations (1) and (2) in given expression, we get

$(3x + 2y)^3 - (3x - 2y)^3 = (27x^3 + 8y^3 + 54x^2y + 36xy^2) - (27x^3 - 8y^3 - 54x^2y + 36xy^2)$

$= 27x^3 + 8y^3 + 54x^2y + 36xy^2 - 27x^3 + 8y^3 + 54x^2y - 36xy^2$

$= 16y^3 + 108x^2y$

- **Identity:** $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can use this identity to factorize and expand the polynomials.

For example, the given expression can be factorized as follows:

$$2x^2 + 27y^2 + 25z^2 + 6\sqrt{6}xy - 30\sqrt{3}yz - 10\sqrt{2}xz$$
$$= \left(\sqrt{2}x\right)^2 + \left(3\sqrt{3}y\right)^2 + \left(-5z\right)^2 + 2\cdot\left(\sqrt{2}x\right)\left(3\sqrt{3}y\right) + 2\left(3\sqrt{3}y\right)(-5z) + 2\left(\sqrt{2}x\right)(-5z)$$

On comparing the expression with $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$, we get

$$2x^2 + 27y^2 + 25z^2 + 6\sqrt{6}xy - 30\sqrt{3}yz - 10\sqrt{2}xz$$
$$= \left(\sqrt{2}x + 3\sqrt{3}y - 5z\right)^2$$