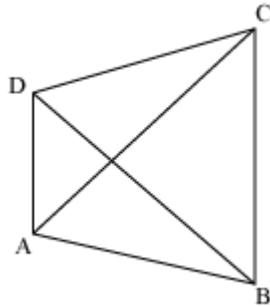


Construction of Quadrilaterals

Construction of Quadrilateral when One Diagonal and Four Sides Are Given

Look at the following quadrilateral ABCD.



It has ten elements - four sides (AB, BC, CD, and DA), four angles ($\angle ABC$, $\angle BCD$, $\angle CDA$, and $\angle DAB$), and two diagonals (AC and BD).

If we want to construct a unique quadrilateral, then how many elements are required?

To construct a unique quadrilateral, at least five elements are necessary.

Now, suppose we want to construct a quadrilateral whose one diagonal and four sides are given, then how will we construct it?

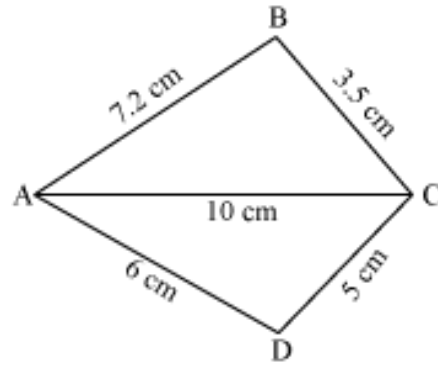
Let us now look at some more examples.

Example 1:

Construct a quadrilateral ABCD with $AC = 10$ cm, $AD = 6$ cm, $DC = 5$ cm, $AB = 7.2$ cm, and $BC = 3.5$ cm.

Solution:

First, we draw a rough sketch of quadrilateral ABCD and indicate the given lengths as shown below.

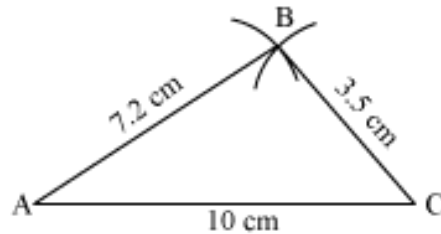


The steps of construction are as follows.

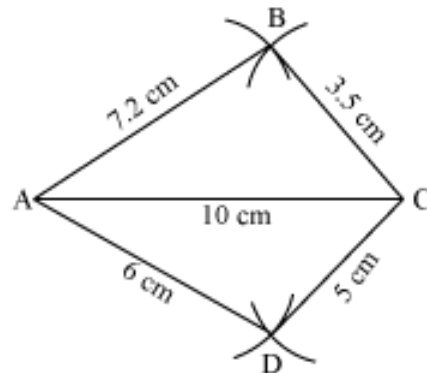
1. Draw $AC = 10$ cm.



2. Then, draw an arc taking A as centre and 7.2 cm as the radius. Now, draw another arc with C as the centre and 3.5 cm as the radius cutting the previous arc at B. Then, join AB and BC.



3. Draw an arc on the opposite side of B by taking A as centre and 6 cm as the radius. Then, draw another arc with C as the centre and 5 cm as the radius cutting the previous arc at D. Then, join AD and CD.



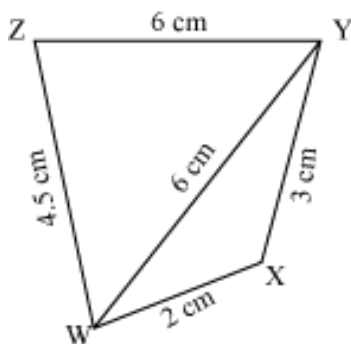
Thus, ABCD is the required quadrilateral.

Example 2:

Can you construct a quadrilateral WXYZ with $WY = 6$ cm, $WX = 2$ cm, $XY = 3$ cm, $YZ = 6$ cm, and $WZ = 4.5$ cm? Justify your answer.

Solution:

First, we draw a rough sketch of the quadrilateral and indicate the given lengths as shown below.



Here, in ΔWXY ,

$$WX + XY = 2 + 3 = 5 < WY = 6 \text{ cm}$$

Therefore, $WX + XY < WY$

\Rightarrow Sum of two sides $<$ Third side

Therefore, ΔWXY is not possible. Thus, we cannot construct a quadrilateral with the given lengths.

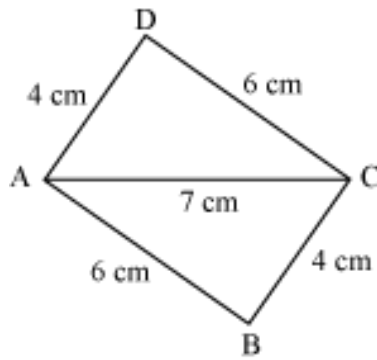
Example 3:

Construct a parallelogram whose adjacent sides are 6 cm and 4 cm and one of its diagonals is 7 cm.

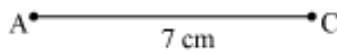
Solution:

We know that opposite sides of a parallelogram are equal. If ABCD is a parallelogram where the adjacent sides AB and BC are of lengths 6 cm and 4 cm respectively and the diagonal AC is of length 7 cm, then $CD = 6$ cm and $AD = 4$ cm

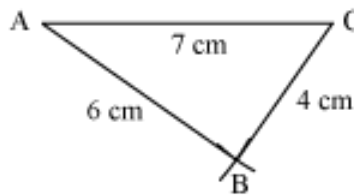
Now, we have the parallelogram ABCD, where $AB = CD = 6$ cm, $BC = AD = 4$ cm, and $AC = 7$ cm. For this, we have to follow the below given steps.



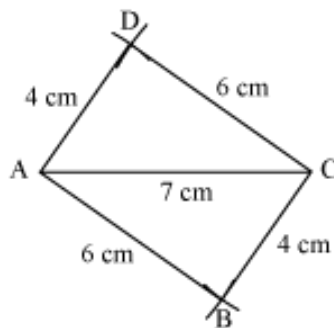
1. Draw $AC = 7$ cm



2. Draw an arc taking A as centre and 6 cm as the radius. Now, draw another arc with C as the centre and 4 cm as the radius cutting the previous arc at B. Then, join AB and BC.



3. Now, draw an arc taking A as centre and 4 cm as the radius. Then, draw another arc with C as the centre and 6 cm as the radius cutting the previous arc at D. Then, join AD and CD.



Thus, ABCD is the required parallelogram.

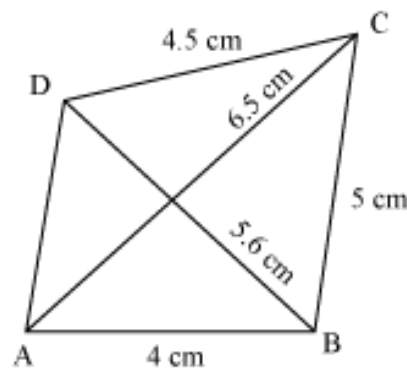
Construction of a Quadrilateral When Three Sides and Both Diagonals Are Given

We know that to construct a quadrilateral, we require five elements. Now suppose three sides and the two diagonals of a quadrilateral are given.

Then, **how will we construct the quadrilateral with these five elements?**

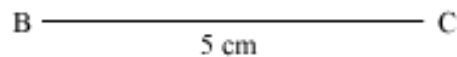
Let us construct a quadrilateral ABCD in which $AB = 4\text{ cm}$, $BC = 5\text{ cm}$, $CD = 4.5\text{ cm}$, $AC = 6.5\text{ cm}$, and $BD = 5.6\text{ cm}$.

First we draw a rough sketch of the quadrilateral ABCD and indicate the given lengths in the figure as shown below.

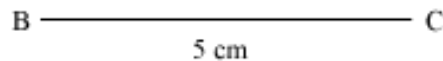


To construct the quadrilateral, we follow the below given steps.

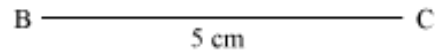
1. Draw $BC = 5\text{ cm}$ (the side opposite to the unknown side).



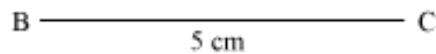
2. Then, draw an arc taking B as centre and AB (4 cm) as radius.



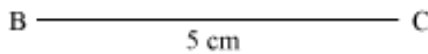
3. Then, draw another arc taking C as centre and AC (6.5 cm) as radius to intersect the previously drawn arc at a point A.



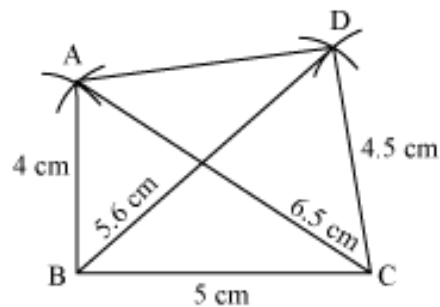
4. Then, draw an arc taking B as centre and BD (5.6 cm) as radius on the same side of BC in which point A lies.



5. Then, draw another arc taking C as centre and CD (4.5 cm) as radius to intersect the arc drawn in step IV at point D.



6. Then, join AB, AD, CD, AC, and BD.



This is the required quadrilateral ABCD.

In this way, we can construct a quadrilateral if we are given three of its sides and the two diagonals.

Let us now solve another example.

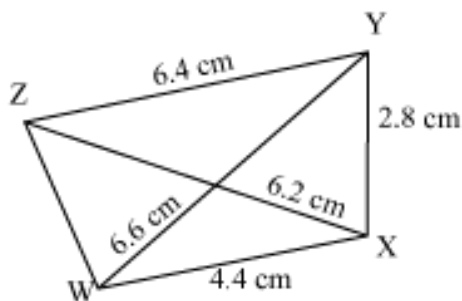
Example 1:

Construct a quadrilateral WXYZ in which $WX = 4.4$ cm, $XY = 2.8$ cm,

$YZ = 6.4$ cm, $XZ = 6.2$ cm, and $WY = 6.6$ cm.

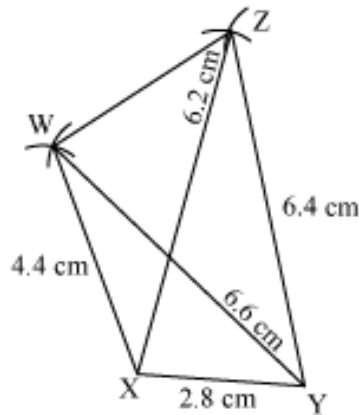
Solution:

First, draw a rough sketch of quadrilateral WXYZ and indicate the given lengths as shown below.



The steps of construction are as follows.

- (i) Draw $XY = 2.8$ cm.
- (ii) Draw an arc taking X as centre and WX (4.4 cm) as radius.
- (iii) Draw another arc taking Y as centre and WY (6.6 cm) as radius to intersect the previous arc at W.
- (iv) Again, draw an arc taking X as centre and XZ (6.2 cm) as radius on the same side of XY in which point W lies.
- (v) Draw another arc taking Y as centre and YZ (6.4 cm) as radius to intersect the arc drawn in previous step at Z.
- (vi) Join XW, WZ, YZ, XZ, and WY.



Thus, WXYZ is the required quadrilateral.

Construction Of Quadrilaterals When Three Sides And Two Included Angles Are Given

We know that to construct a quadrilateral, we require five elements. Suppose three sides and two angles included between them are given. Then how will we construct the quadrilateral?

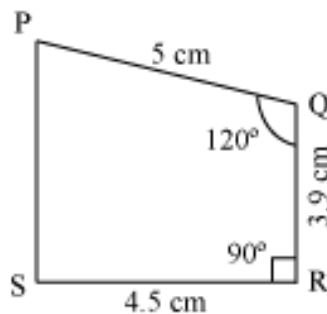
Let us now look at one more example to understand the construction of a quadrilateral better.

Example:

Construct a quadrilateral PQRS with $SR = 4.5$ cm, $QR = 3.9$ cm, $PQ = 5$ cm, $\angle R = 90^\circ$, and $\angle Q = 120^\circ$.

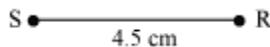
Solution:

First, we draw a rough sketch of the quadrilateral PQRS as shown in the following figure.

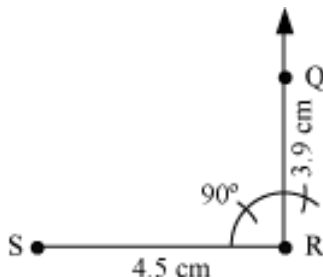


The steps of construction are as follows.

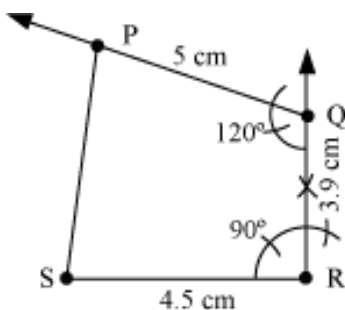
1. We draw a line segment SR of length 4.5 cm.



2. Then we construct an angle of 90° at R and locate the point Q such that RQ = 3.9 cm.



3. Now, we construct an angle of 120° at Q and locate the point P such that QP = 5 cm. Then we join PS.



Thus, PQRS is the required quadrilateral of given measures.

Construction of Quadrilateral when Two Adjacent Sides and Three Angles Are Given

Suppose we have to construct a quadrilateral ABCD and the following elements of the quadrilateral are given.

AB = 5.5 cm, BC = 4.9 cm, $\angle B = 120^\circ$, $\angle C = 90^\circ$, $\angle A = 75^\circ$.

How will we construct the quadrilateral with this information?

To understand the method of construction, look at the following video.

In this way, we can construct a quadrilateral if two adjacent sides and three angles are given.

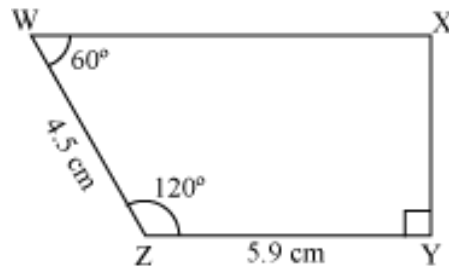
Let us now look at some more examples to understand this concept better.

Example 1:

Construct a quadrilateral WXYZ with $ZY = 5.9$ cm, $WZ = 4.5$ cm, $\angle Y = 90^\circ$, $\angle Z = 120^\circ$, and $\angle W = 60^\circ$.

Solution:

First, we draw a rough sketch of the quadrilateral WXYZ and indicate the given lengths as shown in the following figure.

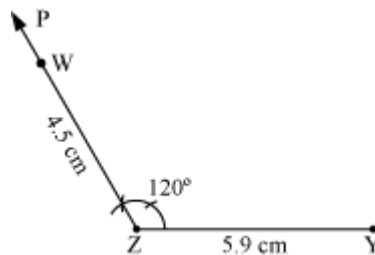


The steps of construction are as follows.

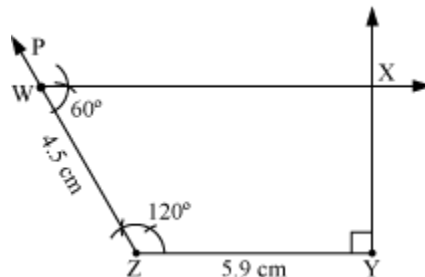
1. First, we draw a line segment $ZY = 5.9$ cm.



2. Then we construct an angle of 120° at Z. Mark the point W, such that $WZ = 4.5$ cm.



3. Now, we construct an angle of 60° at W and an angle of 90° at Y to meet the two line segments at X.



Thus, WXYZ is the required quadrilateral.

Example 2:

Can you construct a quadrilateral ABCD with $AB = 5.5$ cm, $BC = 4.9$ cm, $\angle A = 130^\circ$, $\angle B = 145^\circ$, and $\angle C = 95^\circ$?

Solution:

We know that the sum of all the angles of a quadrilateral is 360° .

Here, we are given that $\angle A = 130^\circ$, $\angle B = 145^\circ$, and $\angle C = 95^\circ$.

$$\begin{aligned}\therefore \angle A + \angle B + \angle C &= 130^\circ + 145^\circ + 95^\circ \\ &= 370^\circ\end{aligned}$$

This is not possible.

Thus, we cannot construct a quadrilateral with the given measurements.