

Algebraic Expressions

Exercise 8A

Q1

Answer :

(i) x increased by 12 is $(x+12)$.

(ii) y decreased by 7 is $(y-7)$.

(iii) The difference of a and b, when $a > b$ is $(a-b)$.

(iv) The product of x and y is xy .

The sum of x and y is $(x+y)$.

So, product of x and y added to their sum is $xy+(x+y)$.

(v) One third of x is $\frac{x}{3}$.

The sum of a and b is $(a+b)$.

\therefore One-third of x multiplied by the sum of a and b $= \frac{x}{3} \times (a+b) = \frac{x(a+b)}{3}$

(vi) 5 times x added to 7 times y $= (5 \times x) + (7 \times y)$, which is equal to $5x + 7y$.

(vii) Sum of x and the quotient of y by 5 is $x + \frac{y}{5}$.

(viii) x taken away from 4 is $(4-x)$.

(ix) 2 less than the quotient of x by y is $\frac{x}{y} - 2$.

(x) x multiplied by itself is $x \times x = x^2$.

(xi) Twice x increased by y is $(2 \times x) + y = 2x + y$.

(xii) Thrice x added to y squared is $(3 \times x) + (y \times y) = 3x + y^2$.

(xiii) x minus twice y is $x - (2 \times y) = x - 2y$.

(xiv) x cubed less than y cubed is $(y \times y \times y) - (x \times x \times x) = y^3 - x^3$.

(xv) The quotient of x by 8 is multiplied by y is $\frac{x}{8} \times y = \frac{xy}{8}$.

Q2

Answer :

Ranjit's score in English = 80 marks

Ranjit's score in Hindi = x marks

Total score in the two subjects = (Ranjit's score in English + Ranjit's score in Hindi)

\therefore Total score in the two subjects = $(80 + x)$ marks

Q3

Answer :

(i) $b \times b \times b \times \dots$ 15 times $= b^{15}$

(ii) $y \times y \times y \times \dots$ 20 times $= y^{20}$

(iii) $14 \times a \times a \times a \times a \times a \times b \times b \times b = 14 \times (a \times a \times a \times a \times a) \times (b \times b \times b) = 14a^4b^3$

(iv) $6 \times x \times x \times x \times y \times y = 6 \times (x \times x \times x) \times (y \times y) = 6x^3y^2$

(v) $3 \times z \times z \times z \times y \times y \times x = 3 \times (z \times z \times z) \times (y \times y) \times x = 3z^3y^2x$

Q4

Answer :

(i) $x^2y^4 = (x \times x) \times (y \times y \times y \times y) = x \times x \times y \times y \times y \times y$

(ii) $6y^5 = 6 \times (y \times y \times y \times y \times y) = 6 \times y \times y \times y \times y \times y$

(iii) $9xy^2z = 9 \times x \times (y \times y) \times z = 9 \times x \times y \times y \times z$

(iv) $10a^3b^3c^3 = 10 \times (a \times a \times a) \times (b \times b \times b) \times (c \times c \times c) = 10 \times a \times a \times a \times b \times b \times b \times c \times c \times c$

Algebraic Expressions

Exercise 8B

Q1

Answer :

(i) $a+b$

Substituting $a = 2$ and $b = 3$ in the given expression:

$$2+3 = 5$$

(ii) $a^2 + ab$

Substituting $a = 2$ and $b = 3$ in the given expression:

$$(2)^2 + (2 \times 3) = 4 + 6 \\ = 10$$

(iii) $ab - a^2$

Substituting $a = 2$ and $b = 3$ in the given expression:

$$(2 \times 3) - (2)^2 = 6 - 4 \\ = 2$$

(iv) $2a-3b$

Substituting $a = 2$ and $b = 3$ in the given expression:

$$(2 \times 2) - (3 \times 3) = 4 - 9 \\ = -5$$

(v) $5a^2 - 2ab$

Substituting $a=2$ and $b=3$ in the given expression:

$$5 \times (2)^2 - 2 \times 2 \times 3 = 5 \times 4 - 12 = 20 - 12 \\ = 8$$

(vi) $a^3 - b^3$

Substituting $a=2$ and $b=3$ in the given expression:

$$2^3 - 3^3 = 2 \times 2 \times 2 - 3 \times 3 \times 3 = 8 - 27 \\ = -19$$

Q2

Answer :

(i) $3x-2y+4z$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$3 \times (1) - 2 \times (2) + 4 \times (5) = 3 - 4 + 20 \\ = 19$$

(ii) $x^2 + y^2 + z^2$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$1^2 + 2^2 + 5^2 = (1 \times 1) + (2 \times 2) + (5 \times 5) = 1 + 4 + 25 \\ = 30$$

(iii) $2x^2 - 3y^2 + z^2$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$2 \times (1)^2 - 3 \times (2)^2 + 5^2 = 2 \times (1 \times 1) - 3 \times (2 \times 2) + (5 \times 5) = 2 - 12 + 25 \\ = 15$$

(iv) $xy + yz - zx$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$(1 \times 2) + (2 \times 5) - (5 \times 1) = 2 + 10 - 5 \\ = 7$$

(v) $2x^2y - 5yz + xy^2$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$2 \times (1)^2 \times 2 - 5 \times 2 \times 5 + 1 \times (2)^2 = 4 - 50 + 4 \\ = -42$$

(vi) $x^3 - y^3 - z^3$

Substituting $x = 1$, $y = 2$ and $z = 5$ in the given expression:

$$1^3 - 2^3 - 5^3 = (1 \times 1 \times 1) - (2 \times 2 \times 2) - (5 \times 5 \times 5) = 1 - 8 - 125 \\ = -132$$

Q3

Answer :

(i) $p^2 + q^2 - r^2$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$(-2)^2 + (-1)^2 - (3)^2 = (-2 \times -2) + (-1 \times -1) - (3 \times 3) \\ \Rightarrow 4 + 1 - 9 = -4$$

(ii) $2p^2 - q^2 + 3r^2$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$2 \times (-2)^2 - (-1)^2 + 3 \times (3)^2 = 2 \times (-2 \times -2) - (-1 \times -1) + 3 \times (3 \times 3) \\ \Rightarrow 8 - 1 + 27 = 34$$

(iii) $p - q - r$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$(-2) - (-1) - (3) = -2 + 1 - 3 \\ = -4$$

(iv) $p^3 + q^3 + r^3 + 3pqr$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$(-2)^3 + (-1)^3 + (3)^3 + 3 \times (-2 \times -1 \times 3) \\ = (-2 \times -2 \times -2) + (-1 \times -1 \times -1) + (3 \times 3 \times 3) + 3 \times (6) \\ = (-8) + (-1) + (27) + 18 \\ = 36$$

(v) $3p^2q + 5pq^2 + 2pqr$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$3 \times (-2)^2 \times (-1) + 5 \times (-2) \times (-1)^2 + 2 \times (-2 \times -1 \times 3) \\ = 3 \times (-2 \times -2) \times (-1) + 5 \times (-2) \times (-1 \times -1) + 2 \times (-2 \times -1 \times 3) \\ = -12 - 10 + 12 \\ = -10$$

(vi) $p^4 + q^4 - r^4$

Substituting $p = -2$, $q = -1$ and $r = 3$ in the given expression:

$$(-2)^4 + (-1)^4 - (3)^4 \\ = (-2 \times -2 \times -2 \times -2) + (-1 \times -1 \times -1 \times -1) - (3 \times 3 \times 3 \times 3) \\ = 16 + 1 - 81 \\ = -64$$

Q4

Answer :

- (i) Coefficient of x in $13x$ is 13.
- (ii) Coefficient of y in $-5y$ is -5.
- (iii) Coefficient of a in $6ab$ is 6b.
- (iv) Coefficient of z in $-7xz$ is $-7x$.
- (v) Coefficient of p in $-2pqr$ is $-2qr$.
- (vi) Coefficient of y^2 in $8xy^2z$ is $8xz$.
- (vii) Coefficient of x^3 in x^3 is 1.
- (viii) Coefficient of x^2 in $-x^2$ is -1.

Q5

Answer :

- (i) Numerical coefficient of ab is 1.
- (ii) Numerical coefficient of $-6bc$ is -6.
- (iii) Numerical coefficient of $7xyz$ is 7.
- (iv) Numerical coefficient of $-2x^3y^2z$ is -2.

Q6

Answer :

A term of expression having no literal factors is called a constant term.

- (i) In the expression $3x^2 + 5x + 8$, the constant term is 8.
- (ii) In the expression $2x^2 - 9$, the constant term is -9.
- (iii) In the expression $4y^2 - 5y + \frac{3}{5}$, the constant term is $\frac{3}{5}$.
- (iv) In the expression $z^3 - 2z^2 + z - \frac{8}{3}$, the constant term is $-\frac{8}{3}$.

Q7

Answer :

The expressions given in (i), (iii), (vi) and (viii) contain only one term. So, each one of them is monomial.
The expressions given in (ii) and (ix) contain two terms. So, both of them are binomial.
The expressions given in (iv) and (v) contain three terms. So, both of them are trinomial.
The expression given in (vii) contains four terms. So, it does not represent any of the given types.

Q8

Answer :

- (i) Expression $4x^5 - 6y^4 + 7x^2y - 9$ has four terms, namely $4x^5$, $-6y^4$, $7x^2y$ and -9.
- (ii) Expression $9x^3 - 5z^4 + 7z^3y - xyz$ has four terms, namely $9x^3$, $-5z^4$, $7z^3y$ and $-xyz$.

Q9

Answer :

The terms that have same literals are called like terms.

- (i) a^2 and $2a^2$ are like terms.
- (ii) $-yz$ and $\frac{1}{2}zy$ are like terms.
- (iii) $-2xy^2$ and $5y^2x$ are like terms.
- (iv) ab^2c , acb^2 , b^2ac and cab^2 are like terms.

Algebraic Expressions

Exercise 8C

Q1

Answer :

$$\begin{aligned}\text{(i) Required sum} &= 3x + 7x \\ &= (3+7)x = 10x\end{aligned}$$

$$\begin{aligned}\text{(ii) Required sum} &= 7y + (-9y) \\ &= (7-9)y = -2y\end{aligned}$$

$$\begin{aligned}\text{(iii) Required sum} &= 2xy + 5xy + (-xy) \\ &= (2+5-1)xy = 6xy\end{aligned}$$

$$\text{(iv) Required sum} = 3x + 2y$$

$$\begin{aligned}\text{(v) Required sum} &= 2x^2 + (-3x^2) + 7x^2 \\ &= (2-3+7)x^2 = 6x^2\end{aligned}$$

$$\begin{aligned}\text{(vi) Required sum} &= 7xyz + (-5xyz) + 9xyz + (-8xyz) \\ &= (7-5+9-8)xyz = 3xyz\end{aligned}$$

$$\begin{aligned}\text{(vii) Required sum} &= 6a^3 + (-4a^3) + 10a^3 + (-8a^3) \\ &= (6-4+10-8)a^3 = 4a^3\end{aligned}$$

$$\begin{aligned}\text{(viii) Required sum} &= x^2 - a^2 + (-5x^2 + 2a^2) + (-4x^2 + 4a^2) \\ \text{Rearranging and collecting the like terms} &= x^2 - 5x^2 - 4x^2 - a^2 + 2a^2 + 4a^2 \\ &= (1-5-4)x^2 + (-1+2+4)a^2 \\ &= -8x^2 + 5a^2\end{aligned}$$

Q2

Answer :

(i)

$$\begin{array}{r} x - 3y - 2z \\ 5x + 7y - z \\ -7x - 2y + 4z \\ \hline -x + 2y + z \end{array}$$

(ii)

$$\begin{array}{r} m^2 - 4m + 5 \\ - 2m^2 + 6m - 6 \\ \hline - m^2 - 2m - 7 \\ - 2m^2 + 0 \times m - 8 \\ \hline = - 2m^2 + 0 - 8 = - 2m^2 - 8 \end{array}$$

(iii)

$$\begin{array}{r} 2x^2 - 3xy + y^2 \\ - 7x^2 - 5xy - 2y^2 \\ \hline 4x^2 + xy - 6y^2 \\ \hline - x^2 - 7xy - 7y^2 \end{array}$$

(iv)

$$\begin{array}{r} 4xy - 5yz - 7zx \\ - 5xy + 2yz + zx \\ \hline - 2xy - 3yz + 3zx \\ - 3xy - 6yz - 3zx \end{array}$$

Q3

Answer :

(i) Sum of the given expressions

$$= (3a - 2b + 5c) + (2a + 5b - 7c) + (-a - b + c)$$

Rearranging and collecting the like terms

$$= 3a + 2a - a - 2b + 5b - b + 5c - 7c + c$$

$$= (3+2-1)a + (-2+5-1)b + (5-7+1)c$$

$$= 4a + 2b - c$$

(ii) Sum of the given expressions

$$= (8a - 6ab + 5b) + (-6a - ab - 8b) + (-4a + 2ab + 3b)$$

Rearranging and collecting the like terms

$$= (8-6-4)a + (-6-1+2)ab + (5-8+3)b$$

$$= -2a - 5ab + 0 = -2a - 5ab$$

(iii) Sum of the given expressions

$$= (2x^3 - 3x^2 + 7x - 8) + (-5x^3 + 2x^2 - 4x + 1) + (3 - 6x + 5x^2 - x^3)$$

Rearranging and collecting the like terms

$$= 2x^3 - 5x^3 - x^3 - 3x^2 + 2x^2 + 5x^2 + 7x - 4x - 6x - 8 + 1 + 3$$

$$= (2-5-1)x^3 + (-3+2+5)x^2 + (7-4-6)x - 4$$

$$= -4x^3 + 4x^2 - 3x - 4$$

(iv) Sum of the given expressions

$$= (2x^2 - 8xy + 7y^2 - 8xy^2) + (2xy^2 + 6xy - y^2 + 3x^2) + (4y^2 - xy - x^2 + xy^2)$$

Rearranging and collecting the like terms

$$= 2x^2 + 3x^2 - x^2 + 7y^2 - y^2 + 4y^2 - 8xy + 6xy - xy - 8xy^2 + 2xy^2 + xy^2$$

$$= (2+3-1)x^2 + (7-1+4)y^2 + (-8+6-1)xy + (-8+2+1)xy^2$$

$$= 4x^2 + 10y^2 - 3xy - 5xy^2$$

(v) Sum of the given expressions

$$= (x^3 + y^3 - z^3 + 3xyz) + (-x^3 + y^3 + z^3 - 6xyz) + (x^3 - y^3 - z^3 - 8xyz)$$

Rearranging and collecting the like terms

$$= x^3 - x^3 + x^3 + y^3 + y^3 - y^3 - z^3 + z^3 - z^3 + 3xyz - 6xyz - 8xyz$$

$$= (1-1+1)x^3 + (1+1-1)y^3 + (-1+1-1)z^3 + (3-6-8)xyz$$

$$= x^3 + y^3 - z^3 - 11xyz$$

(vi) Sum of the given expressions

$$= (2 + x - x^2 + 6x^3) + (-6 - 2x + 4x^2 - 3x^3) + (2 + x^2) + (3 - x^3 + 4x - 2x^2)$$

Rearranging and collecting the like terms

$$= 6x^3 - 3x^3 - x^3 - x^2 + 4x^2 + x^2 - 2x^2 + x - 2x + 4x + 2 - 6 + 2 + 3$$

$$= (6-3-1)x^3 + (-1+4+1-2)x^2 + (1-2+4)x + 1$$

$$= 2x^3 + 2x^2 + 3x + 1$$

Q4

Answer :

Change the sign of each term of the expression that is to be subtracted and then add.

(i) Term to be subtracted = $5x$

Changing the sign of each term of the expression gives $-5x$.

On adding:

$$2x + (-5x) = 2x - 5x$$

$$= (2-5)x$$

$$= -3x$$

(ii) Term to be subtracted = $-xy$

Changing the sign of each term of the expression gives xy .

On adding:

$$6xy + xy$$

$$= (6+1)xy$$

$$= 7xy$$

(iii) Term to be subtracted = $3a$

Changing the sign of each term of the expression gives $-3a$.

On adding:

$$5b + (-3a)$$

$$= 5b - 3a$$

(iv) Term to be subtracted = $-7x$

Changing the sign of each term of the expression gives $7x$.

On adding:

$$9y + 7x$$

(v) Term to be subtracted = $10x^2$

Changing the sign of each term of the expression gives $-10x^2$.

On adding:

$$-7x^2 + (-10x^2) = -7x^2 - 10x^2$$

$$= (-7-10)x^2$$

$$= -17x^2$$

(vi) Term to be subtracted = $a^2 - b^2$

Changing the sign of each term of the expression gives $-a^2 + b^2$.

On adding:

$$b^2 - a^2 + (-a^2 + b^2) = b^2 - a^2 - a^2 + b^2$$

$$= (1+1)b^2 + (-1-1)a^2$$

$$= 2b^2 - 2a^2$$

Q5

Answer :

Change the sign of each term of the expression that is to be subtracted and then add.

(i) Term to be subtracted = $5a + 7b - 2c$

Changing the sign of each term of the expression gives $-5a - 7b + 2c$.

On adding:

$$\begin{aligned}(3a - 7b + 4c) + (-5a - 7b + 2c) &= 3a - 7b + 4c - 5a - 7b + 2c \\ &= (3-5)a + (-7-7)b + (4+2)c \\ &= -2a - 14b + 6c\end{aligned}$$

(ii) Term to be subtracted = $a - 2b - 3c$

Changing the sign of each term of the expression gives $-a + 2b + 3c$.

On adding:

$$\begin{aligned}(-2a + 5b - 4c) + (-a + 2b + 3c) &= -2a + 5b - 4c - a + 2b + 3c \\ &= (-2-1)a + (5+2)b + (-4+3)c \\ &= -3a + 7b - c\end{aligned}$$

(iii) Term to be subtracted = $5x^2 - 3xy + y^2$

Changing the sign of each term of the expression gives $-5x^2 + 3xy - y^2$.

On adding:

$$\begin{aligned}(7x^2 - 2xy - 4y^2) + (-5x^2 + 3xy - y^2) &= 7x^2 - 2xy - 4y^2 - 5x^2 + 3xy - y^2 \\ &= (7-5)x^2 + (-2+3)xy + (-4-1)y^2 \\ &= 2x^2 + xy - 5y^2\end{aligned}$$

(iv) Term to be subtracted = $6x^3 - 7x^2 + 5x - 3$

Changing the sign of each term of the expression gives $-6x^3 + 7x^2 - 5x + 3$.

On adding:

$$\begin{aligned}(4 - 5x + 6x^2 - 8x^3) + (-6x^3 + 7x^2 - 5x + 3) &= 4 - 5x + 6x^2 - 8x^3 - 6x^3 + 7x^2 - 5x + 3 \\ &= (-8-6)x^3 + (6+7)x^2 + (-5-5)x + 7 \\ &= -14x^3 + 13x^2 - 10x + 7\end{aligned}$$

(v) Term to be subtracted = $x^3 + 2x^2y + 6xy^2 - y^3$

Changing the sign of each term of the expression gives $-x^3 - 2x^2y - 6xy^2 + y^3$.

On adding:

$$\begin{aligned}(y^3 - 3xy^2 - 4x^2y) + (-x^3 - 2x^2y - 6xy^2 + y^3) &= y^3 - 3xy^2 - 4x^2y - x^3 - 2x^2y - 6xy^2 + y^3 \\ &= -x^3 + (-2-4)x^2y + (-6-3)xy^2 + (1+1)y^3 \\ &= -x^3 - 6x^2y - 9xy^2 + 2y^3\end{aligned}$$

(vi) Term to be subtracted = $-11x^2y^2 + 7xy - 6$

Changing the sign of each term of the expression gives $11x^2y^2 - 7xy + 6$.

On adding:

$$\begin{aligned}(9x^2y^2 - 6xy + 9) + (11x^2y^2 - 7xy + 6) &= 9x^2y^2 - 6xy + 9 + 11x^2y^2 - 7xy + 6 \\ &= (9+11)x^2y^2 + (-6-7)xy + 15 \\ &= 20x^2y^2 - 13xy + 15\end{aligned}$$

(vii) Term to be subtracted = $-2a + b + 6d$

Changing the sign of each term of the expression gives $2a - b - 6d$.

On adding:

$$\begin{aligned}(5a - 2b - 3c) + (2a - b - 6d) &= 5a - 2b - 3c + 2a - b - 6d \\ &= (5+2)a + (-2-1)b - 3c - 6d \\ &= 7a - 3b - 3c - 6d\end{aligned}$$

Answer :

(i) $2p^3 - 3p^2 + 4p - 5 - 6p^3 + 2p^2 - 8p - 2 + 6p + 8$

Rearranging and collecting the like terms

$$\begin{aligned} &= (2-6)p^3 + (-3+2)p^2 + (4-8+6)p - 5-2+8 \\ &= -4p^3 - p^2 + 2p + 1 \end{aligned}$$

(ii) $2x^2 - xy + 6x - 4y + 5xy - 4x + 6x^2 + 3y$

Rearranging and collecting the like terms

$$\begin{aligned} &= (2+6)x^2 + (-1+5)xy + (6-4)x + (-4+3)y \\ &= 8x^2 + 4xy + 2x - y \end{aligned}$$

(iii) $x^4 - 6x^3 + 2x - 7 + 7x^3 - x + 5x^2 + 2 - x^4$

Rearranging and collecting the like terms

$$\begin{aligned} &= (1-1)x^4 + (-6+7)x^3 + 5x^2 + (2-1)x - 7+2 \\ &= 0 + x^3 + 5x^2 + x - 5 \\ &= x^3 + 5x^2 + x - 5 \end{aligned}$$

Q7

Answer :

Adding:

$$(3x^2 - 5x + 2) + (-5x^2 - 8x + 6)$$

Rearranging and collecting the like terms:

$$\begin{aligned} &(3-5)x^2 + (-5-8)x + 2+6 \\ &= -2x^2 - 13x + 8 \end{aligned}$$

Subtract $4x^2 - 9x + 7$ from $-2x^2 - 13x + 8$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $4x^2 - 9x + 7$

Changing the sign of each term of the expression gives $-4x^2 + 9x - 7$.

On adding:

$$\begin{aligned} &(-2x^2 - 13x + 8) + (-4x^2 + 9x - 7) = -2x^2 - 13x + 8 - 4x^2 + 9x - 7 \\ &= (-2-4)x^2 + (-13+9)x + 8-7 \\ &= -6x^2 - 4x + 1 \end{aligned}$$

Q8

Answer :

$$A = 7x^2 + 5xy - 9y^2$$

$$B = -4x^2 + xy + 5y^2$$

$$C = 4y^2 - 3x^2 - 6xy$$

Substituting the values of A, B and C in $A+B+C$:

$$\begin{aligned} &= (7x^2 + 5xy - 9y^2) + (-4x^2 + xy + 5y^2) + (4y^2 - 3x^2 - 6xy) \\ &= 7x^2 + 5xy - 9y^2 - 4x^2 + xy + 5y^2 + 4y^2 - 3x^2 - 6xy \end{aligned}$$

Rearranging and collecting the like terms:

$$\begin{aligned} &(7-4-3)x^2 + (5+1-6)xy + (-9+5+4)y^2 \\ &= (0)x^2 + (0)xy + (0)y^2 \\ &= 0 \end{aligned}$$

$$\Rightarrow \mathbf{A + B + C = 0}$$

Q9

Answer :

Let the expression to be added be X.

$$(5x^3 - 2x^2 + 6x + 7) + X = (x^3 + 3x^2 - x + 1)$$

$$X = (x^3 + 3x^2 - x + 1) - (5x^3 - 2x^2 + 6x + 7)$$

Changing the sign of each term of the expression that is to be subtracted and then adding:

$$X = (x^3 + 3x^2 - x + 1) + (-5x^3 + 2x^2 - 6x - 7)$$

$$X = x^3 + 3x^2 - x + 1 - 5x^3 + 2x^2 - 6x - 7$$

Rearranging and collecting the like terms:

$$X = (1-5)x^3 + (3+2)x^2 + (-1-6)x + 1-7$$

$$X = -4x^3 + 5x^2 - 7x - 6$$

So, $-4x^3 + 5x^2 - 7x - 6$ must be added to $5x^3 - 2x^2 + 6x + 7$ to get the sum as $x^3 + 3x^2 - x + 1$.

Q10

Answer :

$$P = a^2 - b^2 + 2ab$$

$$Q = a^2 + 4b^2 - 6ab$$

$$R = b^2 + 6$$

$$S = a^2 - 4ab$$

$$T = -2a^2 + b^2 - ab + a$$

Adding P, Q, R and S:

$$P+Q+R+S$$

$$= (a^2 - b^2 + 2ab) + (a^2 + 4b^2 - 6ab) + (b^2 + 6) + (a^2 - 4ab)$$

$$= a^2 - b^2 + 2ab + a^2 + 4b^2 - 6ab + b^2 + 6 + a^2 - 4ab$$

Rearranging and collecting the like terms:

$$= (1+1+1)a^2 + (-1+4+1)b^2 + (2-6-4)ab + 6$$

$$P+Q+R+S = 3a^2 + 4b^2 - 8ab + 6$$

To find $P + Q + R + S - T$, subtract $T = (-2a^2 + b^2 - ab + a)$ from $P+Q+R+S = (3a^2 + 4b^2 - 8ab + 6)$.

On changing the sign of each term of the expression that is to be subtracted and then adding:

$$\text{Term to be subtracted} = -2a^2 + b^2 - ab + a$$

Changing the sign of each term of the expression gives $2a^2 - b^2 + ab - a$.

Now add:

$$(3a^2 + 4b^2 - 8ab + 6) + (2a^2 - b^2 + ab - a) = 3a^2 + 4b^2 - 8ab + 6 + 2a^2 - b^2 + ab - a$$

$$= (3+2)a^2 + (4-1)b^2 + (-8+1)ab - a + 6$$

$$\mathbf{P + Q + R + S - T = 5a^2 + 3b^2 - 7ab - a + 6}$$

Q11

Answer :

Let the expression to be subtracted be X.

$$(a^3 - 4a^2 + 5a - 6) - X = (a^2 - 2a + 1)$$

$$X = (a^3 - 4a^2 + 5a - 6) - (a^2 - 2a + 1)$$

Since '-' sign precedes the parenthesis, we remove it and change the sign of each term within the parenthesis.

$$X = a^3 - 4a^2 + 5a - 6 - a^2 + 2a - 1$$

Rearranging and collecting the like terms:

$$X = a^3 + (-4-1)a^2 + (5+2)a - 6 - 1$$

$$X = a^3 - 5a^2 + 7a - 7$$

So, $a^3 - 5a^2 + 7a - 7$ must be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$.

Q12

Answer :

To calculate how much is $a + 2b - 3c$ greater than $2a - 3b + c$, we have to subtract $2a - 3b + c$ from $a + 2b - 3c$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $2a - 3b + c$

Changing the sign of each term of the expression gives $-2a + 3b - c$.

On adding:

$$\begin{aligned} & (a + 2b - 3c) + (-2a + 3b - c) \\ &= a + 2b - 3c - 2a + 3b - c \\ &= (1-2)a + (2+3)b + (-3-1)c \\ &= -a + 5b - 4c \end{aligned}$$

Q13

Answer :

To calculate how much less than $x - 2y + 3z$ is $2x - 4y - z$, we have to subtract $2x - 4y - z$ from $x - 2y + 3z$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $2x - 4y - z$

Changing the sign of each term of the expression gives $-2x + 4y + z$.

On adding:

$$\begin{aligned} & (x - 2y + 3z) + (-2x + 4y + z) \\ &= x - 2y + 3z - 2x + 4y + z \\ &= (1-2)x + (-2+4)y + (3+1)z \\ &= -x + 2y + 4z \end{aligned}$$

Q14

Answer :

To calculate how much does $3x^2 - 5x + 6$ exceed $x^3 - x^2 + 4x - 1$, we have to subtract $x^3 - x^2 + 4x - 1$ from $3x^2 - 5x + 6$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $x^3 - x^2 + 4x - 1$

Changing the sign of each term of the expression gives $-x^3 + x^2 - 4x + 1$.

On adding:

$$\begin{aligned} & (3x^2 - 5x + 6) + (-x^3 + x^2 - 4x + 1) \\ &= 3x^2 - 5x + 6 - x^3 + x^2 - 4x + 1 \\ &= -x^3 + (3+1)x^2 + (-5-4)x + 6 + 1 \\ &= -x^3 + 4x^2 - 9x + 7 \end{aligned}$$

Q15

Answer :

Add $5x - 4y + 6z$ and $-8x + y - 2z$.

$$\begin{aligned} & (5x - 4y + 6z) + (-8x + y - 2z) \\ &= 5x - 4y + 6z - 8x + y - 2z \\ &= (5-8)x + (-4+1)y + (6-2)z \\ &= \mathbf{-3x - 3y + 4z} \end{aligned}$$

Adding $12x - y + 3z$ and $-3x + 5y - 8z$:

$$\begin{aligned} & (12x - y + 3z) + (-3x + 5y - 8z) \\ &= 12x - y + 3z - 3x + 5y - 8z \\ &= (12-3)x + (-1+5)y + (3-8)z \\ &= \mathbf{9x + 4y - 5z} \end{aligned}$$

Subtract $-3x - 3y + 4z$ from $9x + 4y - 5z$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $-3x - 3y + 4z$

Changing the sign of each term of the expression gives $3x + 3y - 4z$.

On adding:

$$\begin{aligned} & (9x + 4y - 5z) + (3x + 3y - 4z) \\ &= 9x + 4y - 5z + 3x + 3y - 4z \\ &= (9+3)x + (4+3)y + (-5-4)z \\ &= \mathbf{12x + 7y - 9z} \end{aligned}$$

Q16

Answer :

To calculate how much is $2x - 3y + 4z$ greater than $2x + 5y - 6z + 2$, we have to subtract $2x + 5y - 6z + 2$ from $2x - 3y + 4z$.

Change the sign of each term of the expression that is to be subtracted and then add.

Term to be subtracted = $2x + 5y - 6z + 2$

Changing the sign of each term of the expression gives $-2x - 5y + 6z - 2$.

On adding:

$$\begin{aligned} & (2x - 3y + 4z) + (-2x - 5y + 6z - 2) \\ &= 2x - 3y + 4z - 2x - 5y + 6z - 2 \\ &= (2-2)x + (-3-5)y + (4+6)z - 2 \\ &= 0x - 8y + 10z - 2 \\ &= \mathbf{-8y + 10z - 2} \end{aligned}$$

Q17

Answer :

To calculate how much does 1 exceed $2x - 3y - 4$, we have to subtract $2x - 3y - 4$ from 1.

Change the sign of each term of the expression to be subtracted and then add.

Term to be subtracted = $2x - 3y - 4$

Changing the sign of each term of the expression gives $-2x + 3y + 4$.

On adding:

$$\begin{aligned} & (1) + (-2x + 3y + 4) \\ &= 1 - 2x + 3y + 4 \\ &= \mathbf{5 - 2x + 3y} \end{aligned}$$

Algebraic Expressions

Exercise 8D

Q1

Answer :

$$a - (b - 2a)$$

Here, '-' sign precedes the parenthesis. So, we will remove it and change the sign of each term within the parenthesis.

$$= a - b + 2a$$

$$= 3a - b$$

Q2

Answer :

$$4x - (3y - x + 2z)$$

Here, '-' sign precedes the parenthesis. So, we will remove it and change the sign of each term within the parenthesis.

$$= 4x - 3y + x - 2z$$

$$= 5x - 3y - 2z$$

Q3

Answer :

$$(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$$

Here, '-' sign precedes the second parenthesis. So, we will remove it and change the sign of each term within the parenthesis.

$$a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$$

Rearranging and collecting the like terms:

$$a^2 - a^2 + b^2 - b^2 + 2ab + 2ab$$

$$= (1 - 1)a^2 + (1 - 1)b^2 + (2 + 2)ab$$

$$= 0 + 0 + 4ab$$

$$= 4ab$$

Q4

Answer :

$$-3(a + b) + 4(2a - 3b) - (2a - b)$$

Here, '-' sign precedes the first and the third parenthesis. So, we will remove them and change the sign of each term within the two parenthesis.

$$= -3a - 3b + (4 \times 2a) - (4 \times 3b) - 2a + b$$

$$= -3a - 3b + 8a - 12b - 2a + b$$

Rearranging and collecting the like terms:

$$-3a + 8a - 2a - 3b - 12b + b$$

$$= (-3 + 8 - 2)a + (-3 - 12 + 1)b$$

$$= 3a - 14b$$

Q5

Answer :

$$-4x^2 + \{(2x^2 - 3) - (4 - 3x^2)\}$$

We will first remove the innermost grouping symbol () and then { }.

$$\therefore -4x^2 + \{(2x^2 - 3) - (4 - 3x^2)\}$$

$$= -4x^2 + \{2x^2 - 3 - 4 + 3x^2\}$$

$$= -4x^2 + \{5x^2 - 7\}$$

$$= -4x^2 + 5x^2 - 7$$

$$= x^2 - 7$$

Q6

Answer :

$$-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$$

Here a '-' sign precedes both the parenthesis. So, we will remove them and change the sign of each term within the two parenthesis.

$$= -2x^2 + 2y^2 - 2xy - 3x^2 - 3y^2 + 3xy$$

$$= (-2 - 3)x^2 + (2 - 3)y^2 + (-2 + 3)xy$$

$$= -5x^2 - y^2 + xy$$

Q7

Answer :

$$a - [2b - \{3a - (2b - 3c)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\therefore a - [2b - \{3a - (2b - 3c)\}]$$

$$= a - [2b - \{3a - 2b + 3c\}]$$

$$= a - [2b - 3a + 2b - 3c]$$

$$= a - [4b - 3a - 3c]$$

$$= a - 4b + 3a + 3c$$

$$= 4a - 4b + 3c$$

Q8

Answer :

$$-x + [5y - \{x - (5y - 2x)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\therefore -x + [5y - \{x - (5y - 2x)\}]$$

$$= -x + [5y - \{x - 5y + 2x\}]$$

$$= -x + [5y - \{3x - 5y\}]$$

$$= -x + [5y - 3x + 5y]$$

$$= -x + [10y - 3x]$$

$$= -x + 10y - 3x$$

$$= -4x + 10y$$

Q9

Answer :

$$86 - [15x - 7(6x - 9) - 2\{10x - 5(2 - 3x)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 86 - [15x - 7(6x - 9) - 2\{10x - 5(2 - 3x)\}] \\ = 86 - [15x - 42x + 63 - 2\{10x - 10 + 15x\}] \\ = 86 - [15x - 42x + 63 - 2\{25x - 10\}] \\ = 86 - [15x - 42x + 63 - 50x + 20] \\ = 86 - [-77x + 83] \\ = 86 + 77x - 83 \\ = 77x + 3 \end{aligned}$$

Q10

Answer :

$$12x - [3x^3 + 5x^2 - \{7x^2 - (4 - 3x - x^3) + 6x^3\} - 3x]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 12x - [3x^3 + 5x^2 - \{7x^2 - (4 - 3x - x^3) + 6x^3\} - 3x] \\ = 12x - [3x^3 + 5x^2 - \{7x^2 - 4 + 3x + x^3 + 6x^3\} - 3x] \\ = 12x - [3x^3 + 5x^2 - \{7x^2 - 4 + 3x + 7x^3\} - 3x] \\ = 12x - [3x^3 + 5x^2 - 7x^2 + 4 - 3x - 7x^3 - 3x] \\ = 12x - [-2x^2 + 4 - 4x^3 - 6x] \\ = 12x + 2x^2 - 4 + 4x^3 + 6x \\ = 4x^3 + 2x^2 + 18x - 4 \end{aligned}$$

Q11

Answer :

$$5a - [a^2 - \{2a(1 - a + 4a^2) - 3a(a^2 - 5a - 3)\}] - 8a$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 5a - [a^2 - \{2a(1 - a + 4a^2) - 3a(a^2 - 5a - 3)\}] - 8a \\ = 5a - [a^2 - \{2a - 2a^2 + 8a^3 - 3a^3 + 15a^2 + 9a\}] - 8a \\ = 5a - [a^2 - \{5a^3 + 13a^2 + 11a\}] - 8a \\ = 5a - [a^2 - 5a^3 - 13a^2 - 11a] - 8a \\ = 5a - [-5a^3 - 12a^2 - 11a] - 8a \\ = 5a + 5a^3 + 12a^2 + 11a - 8a \\ = 5a^3 + 12a^2 + 8a \end{aligned}$$

Q12

Answer :

$$3 - [x - \{2y - (5x + y - 3) + 2x^2\} - (x^2 - 3y)]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 3 - [x - \{2y - (5x + y - 3) + 2x^2\} - (x^2 - 3y)] \\ = 3 - [x - \{2y - 5x - y + 3 + 2x^2\} - x^2 + 3y] \\ = 3 - [x - \{y - 5x + 3 + 2x^2\} - x^2 + 3y] \\ = 3 - [x - y + 5x - 3 - 2x^2 - x^2 + 3y] \\ = 3 - [6x - 3 - 3x^2 + 2y] \\ = 3 - 6x + 3 + 3x^2 - 2y \\ = 3x^2 - 2y - 6x + 6 \end{aligned}$$

Q13

Answer :

$$xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}] \\ = xy - [yz - zx - \{yx - 3y + xz - xy + zy\}] \\ = xy - [yz - zx - \{-3y + xz + zy\}] \quad (\because xy = yx) \\ = xy - [yz - zx + 3y - xz - zy] \\ = xy - [-2zx + 3y] \quad (\because yz = zy, zx = xz) \\ = xy + 2zx - 3y \end{aligned}$$

Q14

Answer :

$$2a - 3b - [3a - 2b - \{a - c - (a - 2b)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 2a - 3b - [3a - 2b - \{a - c - (a - 2b)\}] \\ = 2a - 3b - [3a - 2b - \{a - c - a + 2b\}] \\ = 2a - 3b - [3a - 2b - \{-c + 2b\}] \\ = 2a - 3b - [3a - 2b + c - 2b] \\ = 2a - 3b - [3a - 4b + c] \\ = 2a - 3b - 3a + 4b - c \\ = -a + b - c \end{aligned}$$

Q15

Answer :

$$-a - [a + \{a + b - 2a - (a - 2b)\} - b]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore -a - [a + \{a + b - 2a - (a - 2b)\} - b] \\ = -a - [a + \{a + b - 2a - a + 2b\} - b] \\ = -a - [a + \{3b - 2a\} - b] \\ = -a - [a + 3b - 2a - b] \\ = -a - [2b - a] \\ = -a - 2b + a \\ = -2b \end{aligned}$$

Q16

Answer :

$$2a - [4b - \{4a - (3b - \overline{2a + 2b})\}]$$

We will first remove the innermost grouping symbol bar bracket. Next, we will remove (), followed by { } and then [].

$$\begin{aligned} \therefore 2a - [4b - \{4a - (3b - \overline{2a + 2b})\}] \\ = 2a - [4b - \{4a - (3b - 2a - 2b)\}] \\ = 2a - [4b - \{4a - (b - 2a)\}] \\ = 2a - [4b - \{4a - b + 2a\}] \\ = 2a - [4b - \{6a - b\}] \\ = 2a - [4b - 6a + b] \\ = 2a - [5b - 6a] \\ = 2a - 5b + 6a \\ = 8a - 5b \end{aligned}$$

Q17

Answer :

$$5x - [4y - \{7x - (3z - 2y) + 4z - 3(x + 3y - 2z)\}]$$

We will first remove the innermost grouping symbol (), followed by { } and then [].

$$\begin{aligned} \therefore 5x - [4y - \{7x - (3z - 2y) + 4z - 3(x + 3y - 2z)\}] \\ = 5x - [4y - \{7x - 3z + 2y + 4z - 3x - 9y + 6z\}] \\ = 5x - [4y - \{4x + 7z - 7y\}] \\ = 5x - [4y - 4x - 7z + 7y] \\ = 5x - [11y - 4x - 7z] \\ = 5x - 11y + 4x + 7z \\ = 9x - 11y + 7z \end{aligned}$$