

CBSE Board
Class IX Mathematics

Time: 3 hrs

Total Marks: 80

General Instructions:

1. All questions are **compulsory**.
 2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
 3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
 4. Use of calculator is **not** permitted.
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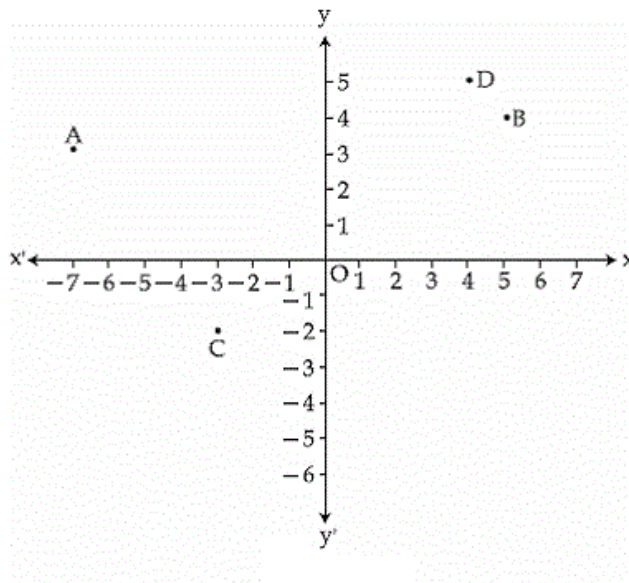
Section A
(Questions 1 to 6 carry 1 mark each)

1. Simplify the given expression: $(3 + \sqrt{3})(2 + \sqrt{2})$
2. Write the equation $7x = 3$ in the Standard form?
3. If a line l intersects two concentric circles at P, Q, R and S, then state whether $PQ = RS$ is true.
4. For a line whose equation is $2x + y = 5$, does point $(2, 1)$ lie on it?
5. Find the Mode of the given data 1, 1, 2, 2, 2, 2, 4, 4, 4, 4, 4, 3, 3, 3, 3, 1, 1, 2, 2, 2, 3, 3, 3.
6. Three angles of a quadrilateral are 60° , 110° and 86° . What is the measure of the fourth angle of the quadrilateral?

Section B

(Questions 7 to 12 carry 2 marks each)

7. Express $\overline{0.975}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
8. See the given figure and answer the following:

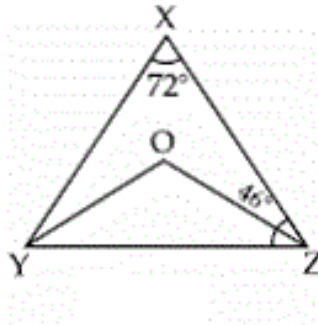


- (i) Coordinates of point A
- (ii) Abscissa of point D
- (iii) The point identified by the coordinates (5, 4)
- (iv) Coordinates of point C
9. Factorise: $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$
10. What is the area of the triangle having sides of lengths 7 cm, 8 cm and 9 cm?
11. A rectangular sheet of card paper, 44 cm \times 20 cm in size, is rolled along its length and a cylinder is formed. Find the volume of the cylinder.
12. Two angles are complementary. The larger angle is 3° less than twice the measure of the smaller angle. Find the measure of each angle

Section C

(Questions 13 to 22 carry 3 marks each)

13. Express $5.\overline{347}$ in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
14. $(x + 2)$ is one of the factors of the polynomial $x^3 + 13x^2 + 32x + 20$. Find its remaining factors.
15. In the figure, $\angle X = 72^\circ$, $\angle XZY = 46^\circ$. If YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$, respectively of $\triangle XYZ$; find $\angle OYZ$ and $\angle YOZ$.



16. Find ab, if $a + b = \sqrt{11}$, $a^2 + b^2 = 5$
17. In $\triangle ABC$, BE and CF are altitudes on the sides AC and AB, respectively, such that $BE = CF$. Using RHS congruency rule, prove that $AB = AC$.
18. A survey was undertaken in 30 classes at a school to find the total number of left-handed students in each class. The table below shows the results:

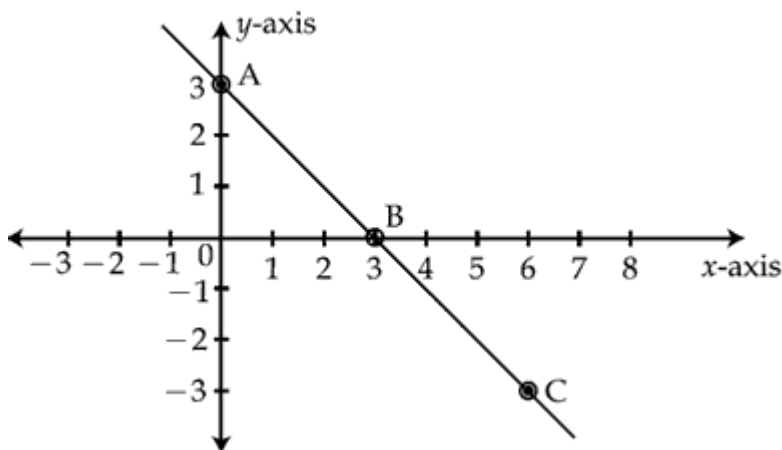
No. of left-handed students	0	1	2	3	4	5
Frequency (no. of classes)	1	2	5	12	8	2

A class was selected at random.

- (a) Find the probability that the class has 2 left-handed students.
- (b) What is the probability that the class has at least 3 left-handed students?
- (c) Given that the total number of students in the 30 classes is 960, find the probability that a student randomly chosen from these 30 classes is left-handed.

19. Observe the graph and answer the following questions:

- Write the co-ordinates of points B and C.
- Find one more solution of the line passing through A and B.
- Write equations of the x-axis and y-axis.



20. Solve:

$$x - \frac{2}{3}y = \frac{8}{3}, \frac{2x}{5} - y = \frac{7}{5}.$$

- The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m².
- The distance (in km) of 40 engineers from their residence to place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
2	19	10	12	17	18	11	32	17	16
3	7	9	7	8	3	5	12	15	18
12	12	14	2	9	6	15	15	7	6

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0 - 5 (5 not included). What main feature do you observe from this tabular representation?

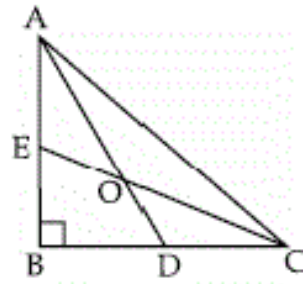
Section D

(Questions 23 to 30 carry 4 marks each)

23. Prove that: $\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$

24. In the figure, AD and CE are the bisectors of $\angle A$ and $\angle C$, respectively.

If $\angle ABC = 90^\circ$, then find $\angle AOC$.



25. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $(x + 1)$ leaves the remainder 19. Find the value of a . Also find the remainder when $p(x)$ is divided by $x + 2$.
26. Prove that: If two lines intersect each other, then the vertically opposite angles are equal.
27. Sonu and Monu had adjacent triangular fields with a common boundary of 25 m. The other two sides of Sonu's field were 52 m and 63 m, while Monu's were 114 m and 101 m. If the cost of fertilization is Rs 20 per sq m, then find the total cost of fertilization for both of Sonu and Monu together.
28. Prove that the medians of an equilateral triangle are equal.
29. There is a triangular field PQR whose corner angles P, Q and R have been measured as 50° , 60° and 70° , respectively. Three friends Anuja, Nikita and Raghav daily go on morning walk and walk along AB, BC and AC, respectively. Who walk the maximum distance among these three? Who walks the least? What value is indicated from this action?
30. (i) Multiply $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y - 4$ using suitable identities.
(ii) Factorise: $a^2 + b^2 - 2(ab - ac + bc)$.

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Class IX Mathematics
Solution

Time: 3 hrs

Total Marks: 80

Section A

1. We have,

$$\begin{aligned}(3 + \sqrt{3})(2 + \sqrt{2}) &= 3 \times 2 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3} \times \sqrt{2} \\ &= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3 \times 2} \\ &= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}\end{aligned}$$

2. Standard form of the Linear Equation in two variables is $ax + by + c = 0$.

Here, $7x = 3$

$\therefore 7x + 0y - 3 = 0$ is the Standard form.

3. Let $OA \perp l$

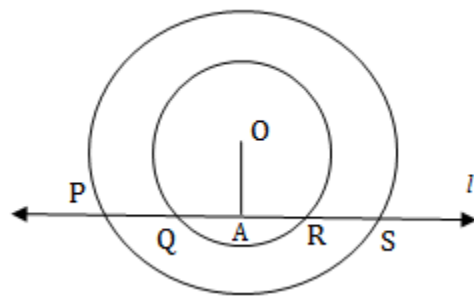
Perpendicular from centre to the chord bisects the chord.

$\therefore PA = AS$ and $AQ = AR$ (i)

$\therefore PA - QA = PQ$ and $AS - AR = RS$

(Since, A-Q-R and A-R-S)

$\therefore PQ = RS$ (from i)



4. Substituting $x = 2$ and $x = 1$ in the equation $2x + y = 5$, we get

L.H.S. = $2(2) + 1 = 5 =$ R.H.S.

\therefore Point $(2, 1)$ lie on the line $2x + y = 5$.

5. Number 1 occurs maximum number of times in the data.

\therefore The Mode of the given data = 1

6. Let the fourth angle of a quadrilateral be x .

Since, the sum of all angles of a quadrilateral = 360°

$\therefore 60^\circ + 110^\circ + 86^\circ + x = 360^\circ$

$\therefore 256^\circ + x = 360^\circ \Rightarrow x = 104^\circ$

Section B

7. Let $x = 0.\overline{975} = 0.975975975 \dots$ (1)

On multiplying both sides of equation (1) by 1000,

$$1000x = 975.975975 \dots \quad (2)$$

On subtracting equation (1) from equation (2),

$$999x = 975$$

$$\Rightarrow x = \frac{975}{999} = \frac{325}{333}$$

8. (i) Coordinates of A are $(-7, 3)$

(ii) Abscissa of point D is 4.

(iii) Point is B.

(iv) Coordinates of C are $(-3, -2)$

9.
$$\begin{aligned} & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\ &= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2} \\ &= 7\sqrt{2}(x - \sqrt{2}) + 4(x - \sqrt{2}) \\ &= (7\sqrt{2} + 4)(x - \sqrt{2}) \end{aligned}$$

10. Let $a = 7$ cm, $b = 8$ cm and $c = 9$ cm.

$$\therefore \text{Semi-perimeter} = s = \frac{a+b+c}{2} = \frac{7 \text{ cm} + 8 \text{ cm} + 9 \text{ cm}}{2} = 12 \text{ cm}$$

Using Heron's formula,

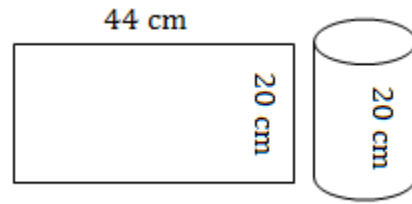
$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{12 \times (12-7) \times (12-8) \times (12-9)} \text{ cm}^2 \\ &= \sqrt{12 \times 5 \times 4 \times 3} \text{ cm}^2 \\ &= 12\sqrt{5} \text{ cm}^2 \end{aligned}$$

11. Height of the cylinder (h) = 20 cm
Let, the radius of the cylinder be r cm.

$$\Rightarrow 2\pi r = 44$$

$$\Rightarrow r = \frac{44}{2\pi} = \frac{44}{2} \times \frac{7}{22} = 7 \text{ cm}$$

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 = 3080 \text{ cm}^3$$



12. Let the measure of the smaller angle be x and that of the larger angle be y.
The larger angle is 3° less than twice the measure of the smaller angle.

$$\Rightarrow y = 2x - 3^\circ \quad \dots(1)$$

Now, $x + y = 180^\circ$ (Two angles are complementary)

$$\Rightarrow x + (2x - 3^\circ) = 180^\circ$$

$$\Rightarrow 3x = 183^\circ$$

$$\Rightarrow x = 61^\circ$$

Substitute value of x in equation (1)

$$y = 2(61) - 3^\circ$$

$$\Rightarrow y = 119^\circ$$

So, the measures of the two angles are 61° and 119° .

Section C

13. Let $x = 5.\overline{347} = 5.34747 \dots$

Multiplying by 10, we get

$$10x = 53.4747 \dots \quad (i)$$

Multiplying by 100, we get

$$100x = 5347.47 \dots \quad (ii)$$

Subtracting (i) from (ii), we get

$$100x - 10x = 5347.47 \dots - 53.47 \dots$$

$$\Rightarrow 990x = 5294$$

$$\Rightarrow x = \frac{5294}{990} = \frac{2647}{495}$$

$$\therefore 5.\overline{347} = \frac{2647}{495}$$

- $$\begin{array}{r}
 x^2 + 11x + 10 \\
 x + 2 \overline{) x^3 + 13x^2 + 32x + 20} \\
 \underline{x^3 + 2x^2} \\
 11x^2 + 32x \\
 \underline{11x^2 + 22x} \\
 (-) (-) \\
 \underline{ 10x + 20} \\
 10x + 20 \\
 \underline{(-) (-)} \\
 0
 \end{array}$$

$$\begin{aligned}\text{Now, } x^2 + 11x + 10 &= x^2 + 10x + x + 10 \\ &= (x + 1)(x + 10) \\ \Rightarrow x^3 + 13x^2 + 32x + 20 &= (x + 1)(x + 2)(x + 10)\end{aligned}$$

- $$\angle YOZ = 126^\circ$$

16.

$$a + b = \sqrt{11}$$

$$\Rightarrow (a + b)^2 = (\sqrt{11})^2$$

$$\Rightarrow a^2 + b^2 + 2ab = 11$$

$$\Rightarrow 5 + 2ab = 11$$

$$\Rightarrow 2ab = 6$$

$$\Rightarrow ab = 3$$

17. In right angled triangles, BEC and CFB

$$\angle BEC = \angle CFB \text{ (} 90^\circ \text{ each)}$$

$$BE = CF \text{ (given)}$$

$$BC = BC \text{ (Common)}$$

$$\therefore \triangle BEC \cong \triangle CFB \quad (\text{RHS congruence})$$

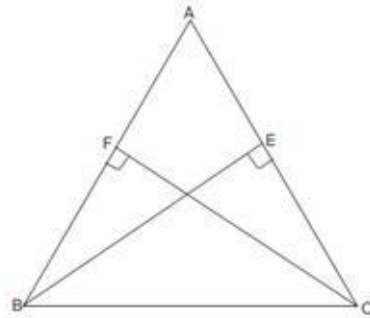
$$\angle BCE = \angle CBF \text{ (CPCT)}$$

$$\Rightarrow \angle BCA = \angle CBA \text{ (same angle)}$$

In $\triangle ABC$, we have

$$\angle BCA = \angle CBA \text{ (Proved above)}$$

$$AB = AC \text{ [sides opposite equal angles are equal]}$$



18. Let S be the sample space.

Thus, $n(S) = 30$

(a) Let A be the event of a class having 2 left-handed students.

$$\therefore n(A) = 5$$

$$\therefore P(A) = \frac{5}{30} = \frac{1}{6}$$

(b) Let B be the event of a class having at least 3 left-handed students.

$$\therefore n(B) = 12 + 8 + 2 = 22$$

$$\therefore P(B) = \frac{22}{30} = \frac{11}{15}$$

(c) First find the total number of left-handed students:

No. of left-handed students, x	0	1	2	3	4	5
Frequency, f (no. of classes)	1	2	5	12	8	2
fx	0	2	10	36	32	10

Total number of left-handed students = $2 + 10 + 36 + 32 + 10 = 90$

Here, the sample space is the total number of students in the 30 classes, which was given as 960.

Let T be the sample space and C be the event that a student is left-handed.

$$n(T) = 960$$

$$n(C) = 90$$

$$\therefore P(C) = \frac{90}{960} = \frac{3}{32}$$

19.

i. Co-ordinates of points B and C are (3, 0) and (6, -3) respectively.

ii. (1, 2) is a solution of line passing through A and B.

iii. Equation of the x-axis is $y = 0$ and the y-axis is $x = 0$.

20. We have,

$$x - \frac{2}{3}y = \frac{8}{3}, \frac{2x}{5} - y = \frac{7}{5}$$

$$x - \frac{2}{3}y = \frac{8}{3} \quad \text{.....(1)}$$

$$\frac{2x}{5} - y = \frac{7}{5} \quad \text{.....(2)}$$

From (1)

$$x = \frac{8}{3} + \frac{2}{3}y = \frac{8+2y}{3} \quad \text{.....(3)}$$

Substituting the value of x in (2),

$$\frac{2}{5} \left(\frac{8+2y}{3} \right) - y = \frac{7}{5}$$

$$\Rightarrow \frac{16+4y}{15} - y = \frac{7}{5}$$

$$\Rightarrow 16 + 4y - 15y = 21$$

$$\Rightarrow -11y = 5$$

$$y = \frac{-5}{11}$$

Substituting the value of y in (3),

$$x = \frac{8 + 2 \left(\frac{-5}{11} \right)}{3} = \frac{8 - \frac{10}{11}}{3}$$

$$= \frac{88 - 10}{11 \times 3} = \frac{78}{11 \times 3}$$

$$x = \frac{26}{11}$$

$$x = \frac{26}{11}, y = \frac{-5}{11}$$

21. Slant height (l) of conical tomb = 25 m

$$\text{Base radius (r) of tomb} = \frac{14}{2} \text{ m} = 7 \text{ m}$$

$$\text{C.S.A. of conical tomb} = \pi r l = \left(\frac{22}{7} \times 7 \times 25 \right) \text{ m}^2 = 550 \text{ m}^2$$

$$\text{Cost of white-washing } 100 \text{ m}^2 = \text{Rs. } 210$$

$$\text{Cost of white-washing } 550 \text{ m}^2 = \text{Rs. } \left(\frac{210 \times 550}{100} \right) = \text{Rs. } 1155$$

Thus, the cost of white washing the conical tomb is Rs. 1155.

22. Given that we have to construct a grouped frequency distribution table of class size 5. So, the class intervals will be as 0 – 5, 5 – 10, 10 – 15, 15 – 20, and so on.

Required grouped frequency distribution table is as follows:

Distance (in km)	Tally marks	Number of engineers
0 – 5		5
5 – 10		11
10 – 15		11
15 – 20		9
20 – 25		1
25 – 30		1
30 – 35		2
Total		40

Only 4 engineers have homes at a distance of more than or equal to 20 km from their work place.

Most of the engineers have their workplace at a distance of upto 15 km from their homes.

Section D

23. We have

$$\begin{aligned}& \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} \\&= \left(\frac{1}{2^2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{\frac{1}{2}} \\&= \left(2^{-2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^{2 \times \frac{-1}{2}}}{4^{2 \times \frac{-1}{2}}}\right) \\&= 2^{(-2) \times (-2)} - 3 \times 8^{\frac{2}{3}} + \left(\frac{3^{-1}}{4^{-1}}\right) \\&= 2^4 - 3 \times 2^{3 \times \frac{2}{3}} + \frac{4}{3} \\&= 2^4 - 3 \times 2^2 + \frac{4}{3} \\&= 2^4 - 3 \times 4 + \frac{4}{3} \\&= 16 - 12 + \frac{4}{3} \\&= 4 + \frac{4}{3} = \frac{12 + 4}{3} \\&= \frac{16}{3} \\&\Rightarrow \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = 16\end{aligned}$$

24. In fig., AD and CE are the bisectors of $\angle A$ and $\angle C$ respectively. If $\angle ABC = 90^\circ$ then find $\angle AOC$.

$$\angle DAC = \frac{1}{2} \angle A \quad [\because AD \text{ is bisector of } \angle A]$$

$$\Rightarrow \angle OAC = \frac{1}{2} \angle A \quad \text{--- (i)} \quad [\because \angle OAC = \angle DAC]$$

$$\angle ECA = \frac{1}{2} \angle C \quad [\because CE \text{ is angle bisector of } \angle C]$$

$$\Rightarrow \angle OCA = \frac{1}{2} \angle C \quad \text{---(ii)} \quad [\because \angle OCA = \angle ECA]$$

In ΔABC

$$\angle A + \angle B + \angle C = 180^\circ \quad (\text{Angle sum property})$$

$$\angle A + \angle C + 90^\circ = 180^\circ$$

$$\angle A + \angle C = 90^\circ$$

$$\frac{1}{2} \angle A + \frac{1}{2} \angle C = 45^\circ \quad \text{--- (iii)}$$

$$\Rightarrow \angle OAC + \angle OCA = 45^\circ \quad \text{--- (iv)}$$

In ΔOAC ,

$$\angle AOC + \angle OAC + \angle OCA = 180^\circ \quad (\text{Angle sum property})$$

$$\angle AOC + 45^\circ = 180^\circ$$

$$\angle AOC = 135^\circ$$

25. $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$

$$\text{Now, } p(-1) = 19$$

$$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) - 7 = 19$$

$$\Rightarrow 1 + 2 + 3 + a - 7 = 19$$

$$\Rightarrow a - 1 = 19$$

$$\Rightarrow a = 18$$

$$\therefore \text{ The given polynomial is } x^4 - 2x^3 + 3x^2 - 18x + 3 \times 18 - 7,$$

$$\text{i.e. } p(x) = x^4 - 2x^3 + 3x^2 - 18x + 47$$

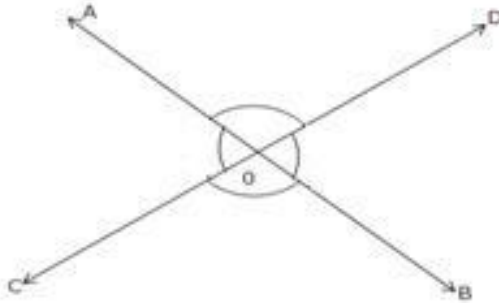
The remainder when $p(x)$ is divided by $(x + 2) = p(-2)$

$$\therefore p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 18(-2) + 47$$

$$= 16 + 16 + 12 + 36 + 47$$

$$= 127$$

26. Given: Two lines AB and CD which intersect each other at O.
To prove: $\angle AOC = \angle BOD$ and $\angle AOD = \angle BOC$



Proof: Since AB is line and ray OD stands on it

$$\therefore \angle AOD + \angle BOD = 180^\circ \quad \dots(1) \text{ (Linear pair axiom)}$$

Since CD is a line and ray OA stands on it

$$\therefore \angle AOC + \angle AOD = 180^\circ \quad \dots(2) \text{ (Linear pair axiom)}$$

From (i) and (ii)

$$\angle AOC + \angle AOD = \angle AOD + \angle BOD$$

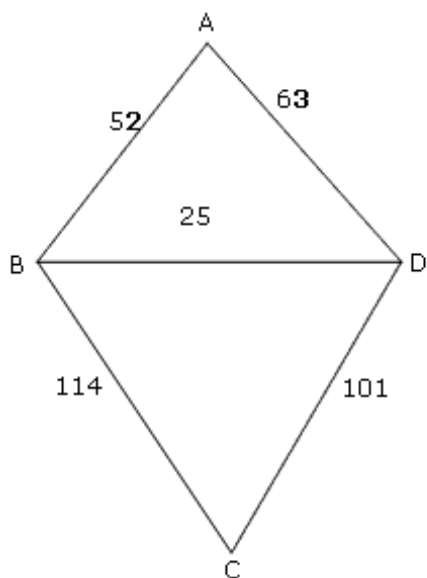
$$\Rightarrow \angle AOC = \angle BOD$$

Similarly, we can prove that

$$\angle BOC = \angle AOD$$

27.

Sonu and Monu's field together form a quadrilateral ABCD.



Sonu's field is $\triangle ABD$,

$$s = \frac{a+b+c}{2} = \frac{52+25+63}{2} = 70$$

$$s-a = 70-52 = 18, \quad s-b = 70-25 = 45 \quad \text{and} \quad s-c = 70-63 = 7$$

Area of $\triangle ABD$ =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{70 \cdot 18 \cdot 45 \cdot 7} = 630 \text{ sq m}$$

Monu's field is $\triangle BCD$,

$$s = \frac{a+b+c}{2} = \frac{114+25+101}{2} = 120$$

$$s-a = 120-114 = 6, \quad s-b = 120-25 = 95 \quad \text{and} \quad s-c = 120-101 = 19$$

Area of $\triangle BCD$ =

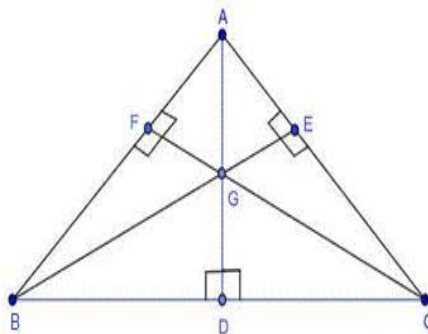
$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{120 \cdot 6 \cdot 95 \cdot 19} = 1140 \text{ sq m}$$

$$\text{Total area is} = 630 + 1140 = 1770 \text{ sq m}$$

The cost of fertilization is Rs 20 per sq m.

$$\text{Therefore the total cost is} = 1770 \times 20 = \text{Rs } 35,400.$$

28.



Given: ABC is an equilateral triangle. AD, BE and CF are medians of triangle ABC.

To prove: $AD = BE = CF$

Proof:

In $\triangle CBF$ and $\triangle BCE$

$\angle B = \angle C = 60^\circ$ [Angles of an equilateral triangle]

$BC = BC$ [common]

$BF = EC$ $\left[\because AB = AC \therefore \frac{1}{2} AB = \frac{1}{2} AC \right]$

By SAS, $\triangle CBF \cong \triangle BCE$

$\therefore BE = CF$ [c.p.c.t.]

Similarly $AD = BE$

$\therefore AD = BE = CF$.

Hence proved.

29. Given: Three friends Anuja, Nikita and Raghav daily go on morning walks and walk along PQ, QR and PR of triangular field PQR, respectively.

Here, $50^\circ < 60^\circ < 70^\circ$

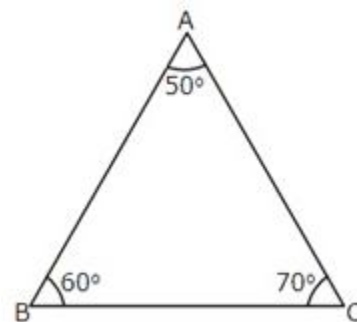
i.e. $\angle A < \angle B < \angle C$

We know that side opposite to greater angle is longer.

Hence, $BC < CA < AB$

So, distance AB is the maximum. Thus, Anuja walks the maximum distance. Further, distance QR is the least. Thus, Nikita walks the least distance.

Value: Awareness for a daily walk for better health without gender and religion bias.



30.

(i) $(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$

$$= (3x + (-5y) + (-4)) [(3x)^2 + (-5y)^2 + (-4)^2 - (3x)(-5y) - (-5y)(-4) - (3x)(-4)]$$

$$= (3x)^3 + (-5y)^3 + (-4)^3 - 3(3x)(-5y)(-4)$$

$$[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

$$= 27x^3 - 125y^3 - 64 - 180xy$$

(ii) $a^2 + b^2 - 2(ab - ac + bc)$

$$= a^2 + b^2 - 2ab + 2ac - 2bc$$

$$= (a^2 + b^2 - 2ab) + 2c(a - b)$$

$$= (a - b)^2 + 2c(a - b)$$

$$= (a - b)(a - b + 2c)$$