Chapter – 3
Pair of Linear Equations in Two Variables

Exercise 3.5

Q. 1 Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0

3x – 9y – 2 = 0

(ii) 2x + y = 5

3x + 2y = 8

(iii) 3x – 5y = 20

6x – 10y = 40

(iv) x – 3y – 7 = 0

3x – 3y – 15 = 0

For two linear equations: $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations have exactly one solution

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations has infinitely many solutions

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations has no solution

(i) Linear equations:

x – 3y – 3 = 0

3x – 9y – 2 = 0

$\frac{a_1}{a_2} = \frac{1}{3}$,

$\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) Linear Equations:

2x + y = 5

3x + 2y = 8

$\frac{a_1}{a_2} = \frac{2}{3}$,

$\frac{b_1}{b_2} = \frac{1}{2}$,

$\frac{c_1}{c_2} = \frac{-5}{-8}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2-b_2c_1} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{-8-(-10)} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$$\frac{x}{2} = 1$$

$$\frac{y}{1} = 1$$

$\therefore$ x = 2, y = 1

(iii) Linear Equations:

3x – 5y = 20

6x – 10y = 40

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$,

$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$,

$\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) Linear Equations:

x – 3y – 7 = 0

3x – 3y – 15 = 0

$\frac{a_1}{a_2} = \frac{1}{3}$,

$\frac{b_1}{b_2} = \frac{-3}{-3} = 1$

$\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$\frac{x}{45-(21)} = \frac{y}{-21-(-15)} = \frac{1}{-3-(-9)}$

$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\frac{x}{24} = \frac{1}{6}$ *and* $\frac{y}{-6} = \frac{1}{6}$

x = 4 and y = -1

∴ x = 4, y = -1

Q. 2 (A) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

2x + 3y = 7

(a – b)x + (a + b)y = 3a + b – 2

Answer: (i) 2x + 3y - 7 =0

(a - b)x + (a + b)y - (3a + b - 2) = 0

we know, a pair of linear equations (say $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$) have infinite solution,

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore from the given equations,

$\frac{a_1}{a_2} = \frac{2}{a-b}$

$\frac{b_1}{b_2} = \frac{3}{a+b}$

$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$

For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a-b} = \frac{7}{3a+b-2}$

6a + 2b – 4 = 7a – b

a – 9b = -4.............. (i)

$\frac{2}{a-b} = \frac{3}{a+b}$

2a + 2b = 3a – 3b

a – 5b = 0 ....(ii)

Subtracting (i) from (ii), we obtain

4b = 4

b = 1

Substituting this in equation (ii), we obtain

a - 5 × 1 = 0

a = 5

Hence, a = 5 and b = 1 are the values for which the given equations give infinitely many solutions.

Q. 2 (B) For which value of k will the following pair of linear equations have no solution?

3x + y = 1

(2k – 1)x + (k – 1)y = 2k + 1

Answer: 3x + y - 1 = 0

(2k - 1)x + (k - 1)y - 2k - 1 = 0

$\frac{a_1}{a_2} = \frac{3}{2k-1}$

$\frac{b_1}{b_2} = \frac{1}{k-1}$

$\frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$

For no solution,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(This means that the coefficients of variables bear the same ratio, due to which they will eliminate together leaving no value for a variable)

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

⇒ 3k - 3 = 2k – 1

⇒ k = 2

Therefore, for k = 2, the given equation has no solution.

Also, k - 1 ≠ 2k + 12k - k ≠ -1 - 1k ≠ -2Hence, for k = 2 and k ≠ -2 the equation has no solution.

Q. 3 Solve the following pair of linear equations by the substitution and cross-multiplication methods:

8x + 5y = 9

3x + 2y = 4

Answer: 8x + 5y = 9 .... (i)

3x + 2y = 4 .... (ii)

From equation (ii), we get

$x = \frac{4-2y}{3}$ .... (iii)

Substituting this value in equation (i), we obtain

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

32 - 16y + 15y = 27

-y = -5

y = 5............ (iv)

Substituting this value in equation (ii), we obtain

3x + 10 = 4

x = -2

Hence, x = -2, y =5

Again, by cross-multiplication method, we obtain

8 x + 5 y - 9 =0

3 x + 2 y - 4 = 0

$$\frac{x}{-20-(-18)} = \frac{y}{-27-(-32)} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\frac{x}{-2} = 1 \ and \ \frac{y}{5} = 1$$

x = -2 and y = 5

Q. 4 Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer: (i) Let x be the fixed charge of the food and y be the charge for food per day. According to the given information,

x + 20y = 1000 .... (1)

x + 26y = 1180 .... (2)

Subtracting equation (1) from equation (2), we obtain

6y = 180

y = 30

Substituting this value in equation (1), we obtain

X + 20 × 30 = 1000

x = 1000 - 600 = 400

x = 400

Hence, fixed charge = Rs 400 And charge per day = Rs 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x-1}{y} = \frac{1}{3} \rightarrow 3x - y = 3$ …. (i)

$\frac{x}{y+8} = \frac{1}{4} \rightarrow 4x - y = 8$ … (ii)

Subtracting equation (i) from equation (ii), we obtain

x = 5 .... (iii)

Putting this value in equation (i), we obtain

$15 - y = 3$

$y = 12$

Hence, the fraction is $\frac{5}{12}$.

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the given information,

Case I

$3x - y = 40$......(i)

Case II

$4x - 2y = 50$

$2x - y = 25$ .... (ii)

Subtracting equation (ii) from equation (i),

we obtain $x = 15$ (iii)

Substituting this in equation (ii), we obtain

$30 - y = 25$

$y = 5$

Therefore,

number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of car from A be 'a' and of car from B be 'b'

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a – b

Given, places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour

⇒ $a - b = 100/5 = 20$ (1)

Also, $a + b = 100/1 = 100$ (2)

Adding (1) and (2)

$a - b + a + b = 20 + 100$

⇒ $2a = 120$

⇒ a = 60 km/hr

Putting value of a in (1) we get,

Thus, b = 60 – 20 = 40 km/hr

Speed of two cars are 60 km/h and 40 km/h

(v) Let length and breadth of rectangle be x unit and y unit respectively.

Area = xy

According to the question,

The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units.

$(x - 5)(y + 3) = xy - 9$

$3x - 5y - 6 = 0$ .... (i)

and if we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units

$(x + 3)(y + 2) = xy + 67$

$2x + 3y - 61 = 0$ .... (ii)

By cross-multiplication method, we obtain

$$\frac{x}{305-18} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

x = 17, y = 9