JEE Advanced 2024
Sample Paper - 1

Time Allowed: 3 hours   **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

Mathematics (MRQ)

1. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k? **[4]**

      a) p = 2, h = 3, k = -4   b) p = -1, h = 1, k = -3

      c) p = 5, h = 4 ,k = -3   d) p = -2, h = 2, k = -4

2. Let f: R $\rightarrow$ R given by **[4]**

$$f(x) \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x - 2)\log_e(x - 2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

   Then which of the following options is/are correct?

      a) f ' is NOT differentiable at x = 1   b) f is onto

      c) f is increasing on (-∞, 0)   d) f' has a local maximum at x = 1

3. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as **[4]**

$$g(x) = \begin{cases} 0, & \text{if } x < a, \\ \int\limits_a^x f(t)dt, & \text{if } a \le x \le b; \\ \int\limits_a^b f(t)dt, & \text{if } x > b. \end{cases} \quad \text{then}$$

a) g(x) is continuous but not differentiable at b

b) g(x) is differentiable on R

c) g(x) is continuous and differentiable at either (a) or (b) but not both

d) g(x) is continuous but not differentiable at a

Mathematics (MCQ)

4. The expression $\left[x + (x^3 - 1)^{\frac{1}{2}}\right]^5 + \left[x - (x^3 - 1)^{\frac{1}{2}}\right]^5$ is a polynomial of degree **[3]**

a) 7

b) 8

c) 6

d) 5

5. For all x ∈ (0, 1) **[3]**

a) sin x > x

b) $e^x < 1 + x$

c) $\log_e (1 + x) < x$

d) log x > x

6. The graph of the function cosx cos (x + 2) - cos^2 (x + 1) is **[3]**

a) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the X-axis

b) a straight line passing through (0, 0)

c) a straight line passing through (0, - sin^2 1) with slope 2

d) a parabola with vertex (1, - sin^2 1)

7. Let f(x) = (x + 1)^2 - 1, x ≥ -1, Then the set {x : f(x) = f^{-1}(x)} is **[3]**

a) {0, 1, -1}

b) {0, -1}

c) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

d) empty

Mathematics (NUM)

8. Let a$_1$, a$_2$, a$_3$, ..., a$_{11}$ be real numbers satisfying a$_1$ = 15, 27 - 2a$_2$ > 0 and a$_k$ = 2a$_{k-r}$ - a$_{k-2}$ **[4]** for k = 3, 4,..., 11. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11}$ = 90, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is .....

9. Let the point (p, p + 1) lie inside the region E = {(x, y): 3 - x ≤ y ≤ $\sqrt{9 - x^2}$, 0 ≤ x ≤ 3. If **[4]** the set of all values of p is the interval (a, b), then b^2 + b - a^2 is equal to _____.

10. Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1. Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is **[4]**

11. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1, \vec{w} \cdot \vec{w} = 4$ **[4]**
If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is _____.

12. Let f : R → R be a continuous function which satisfies f(x) = $\int\limits_{0}^{x}$ f(t)dt. Then the value of f(ln 5) is: **[4]**

13. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is **[4]**
[**Note:** [k] denotes the largest integer less than or equal to k]

Mathematics (MATCH)

14. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non-zero. Match each entry in **List-I** to the correct entries in **List-II.** **[3]**

List-I	List-II
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z - \bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2 + |z + \bar{z}|^2$ is equal to	(3) 8
(S) $|z + 1|^2$ is equal to	(4) 10
	(5) 7

    a) (P) → (2), (Q) → (3), (R) → (5), (S) → (4)
    b) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
    c) (P) → (2), (Q) → (1), (R) → (3), (S) → (5)
    d) (P) → (1), (Q) → (3), (R) → (5), (S) → (4)

15. Consider the ellipse $\frac{x^2}{4} + \frac{y^2}{3}$ = 1. **[3]**
Let H(α, 0), 0 < α < 2, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G Suppose the straight line joining F and the origin makes an angle with the positive x-axis.

List - I	List - II
(I) If $\phi = \frac{\pi}{4}$, then the area of the triangle FGH is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q) 1

List - I	List - II
(III) If $\phi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV) If $\phi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

a) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)

b) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)

c) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

d) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

16. Let p, q, r be nonzero real numbers that are, respectively, the 10th, 100th and 1000th terms of a harmonic progression. Consider the system of linear equations **[3]**

x + y + z = 1
10x + 100y + 1000z = 0
qr x + pr y + pq z = 0.

List-I	List-II
(I) If $\frac{q}{r}$ = 10, then the system of linear equations has	(P) x = 0, y = $\frac{10}{9}$, z = $-\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) x = $\frac{10}{9}$, y = $-\frac{1}{9}$, z = 0 solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q}$ = 10, then the system of linear equations has	(S) no solution
	(T) at least one solution

a) (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

b) (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

c) (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

d) (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

17. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes P$_1$: 7x + y + 2z = 3, P$_2$ = 3x + 5y - 6 z = 4. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L$_1$ and L$_2$, and perpendicular to planes P$_1$ and P$_2$. **[3]**

Match List I with List II and select the correct answer using the code given below the lists :
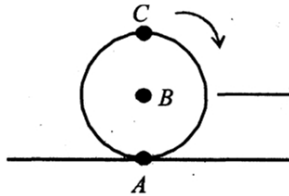
List I	List II
P. $a =$	1. 13
Q. $b =$	2. -3
R. $c =$	3. 1

S. $d =$	4. -2

a) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)

b) (P) → (2), (Q) → (4), (R) → (1), (S) → (3)

c) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)

d) (P) → (3), (Q) → (2), (R) → (4), (S) → (1)

Physics (MRQ)

18. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[4]**

a) $\left|\vec{V}_C - \vec{V}_A\right| = 4\left|\vec{V}_B\right|$

b) $\left|\vec{V}_C - \vec{V}_A\right| = 2\left|\vec{V}_B - \vec{V}_C\right|$

c) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

d) $\vec{V}_C - \vec{V}_A = 2\left(\vec{V}_B - \vec{V}_C\right)$

19. One end of a taut string of length 3 m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) **[4]**

a) y (t) = $A\sin\frac{5\pi x}{2}\cos 250\pi t$

b) y (t) = $A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

c) y (t) = $A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

d) y (t) = $A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

20. A conductor (shown in the figure) carrying constant current I is kept in the x - y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are) **[4]**
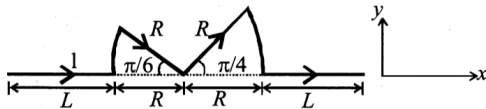
a) If $\vec{B}$ is along $\hat{z}$, F $\propto$ (L + R)

b) If $\vec{B}$ is along $\hat{x}$, F = 0

c) If $\vec{B}$ is along $\hat{z}$, F = 0

d) If $\vec{B}$ is along $\hat{y}$, F $\propto$ (L + R)

Physics (MCQ)

21. A student performs an experiment for determination of $g\left(= \frac{4\pi^2\ell}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for **[3]**

a) Δl = 1 mm, ΔT = 0.1 sec, n = 50

b) Δl = 5 mm, ΔT = 0.2 sec, n = 10

c) Δl = 5 mm, ΔT = 0.1 sec, n = 10   d) Δl = 5 mm, ΔT = 0.2 sec, n = 20

22. A small block is shot into each of the four tracks as shown below. Each of the tracks rises **[3]** to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

a)

b)

c)

d)
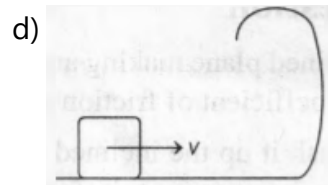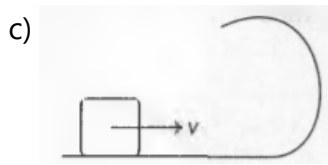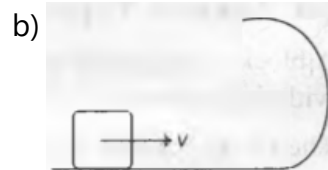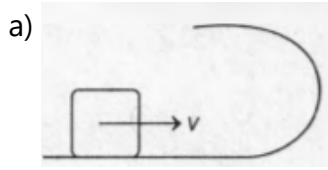
23. A satellite of mass m is orbiting the earth at a height h from its surface. If M is the mass of **[3]** the earth and R its radius, then how much energy must be spent to pull the satellite out of the earth's gravitational field?
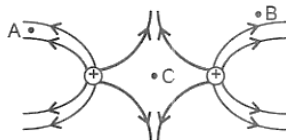
a) $\dfrac{2GmM}{(R+h)}$

b) $\dfrac{2GmM}{(R+h)^2}$

c) $\dfrac{GmM}{2(R+h)}$

d) $\dfrac{GmM}{2(R+h)^2}$

24. The figure below shows the electric field lines due to two positive charges. The **[3]** magnitudes E_A, E_B and E_C of the electric fields at points A, B, and C respectively are related as:

a) $E_B > E_A > E_C$

b) $E_A = E_B > E_C$

c) $E_A > E_B > E_C$

d) $E_A > E_B = E_C$

Physics (NUM)

25. Image of an object approaching a convex mirror of radius of curvature 20 m along its **[4]** optic axis is observed to move from $(\frac{25}{3})$ m to $(\frac{25}{7})$ m in 30 sec. What is the speed (in km/h) of the object?

26. The earth receives at its surface radiation from the sun at the rate of 1400 W/m^{-2}. The **[4]** distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is _____ K.

27. Put a uniform meter scale horizontally on your extended index fingers with the left one at **[4]** 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly

towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is _____.

28. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figure is _____ degrees. **[4]**
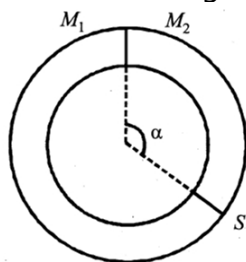
29. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times 10^{-10}$m. The value of the x is _____. **[4]**

(1eV = 1.6×10^{-19} J, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^2/C^2 and electronic charge - 1.6×10^{-19} JC)

30. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in second) does the voltage across them become 4 V? [Take: ln 5 = 1.6, ln 3 = 1.1] **[4]**
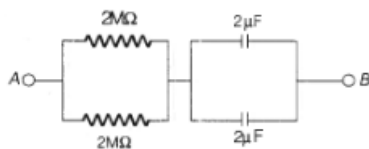
Physics (MATCH)

31. One mole of a monatomic ideal gas is taken along two cyclic processes E → F → G → E and E → F → H → E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[3]**
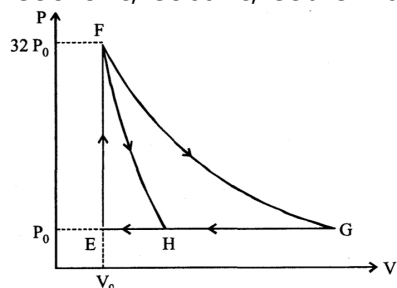
Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.
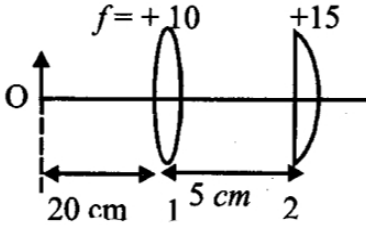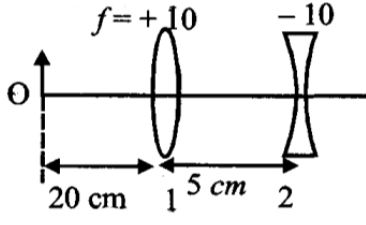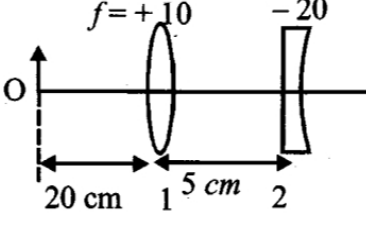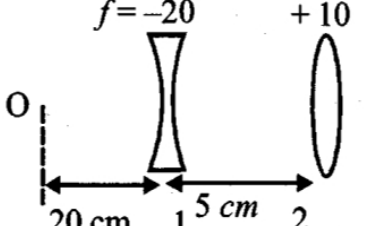
List I	List II
(P) G → E	(1) 160P_0V_0 ln2
(Q) G → H	(2) 36P_0V_0

(R) F → H	(3) $24P_0V_0$
(S) F → G	(4) $31P_0V_0$

a) (P) → (4); (Q) → (3); (R) → (1); (S) → (2)

b) (P) → (1); (Q) → (3); (R) → (2); (S) → (4)

c) (P) → (3); (Q) → (1); (R) → (2); (S) → (4)

d) (P) → (4); (Q) → (3); (R) → (2); (S) → (1)

32. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 cm from the first lens on the left, and the distance between the two lenses is 5 cm. List II contains the positions of the final images.   **[3]**

List-I	List-II
(I)	(P) Final image is formed at 7.5 cm on the right side of lens 2.
(II)	(Q) Final image is formed at 60.0 cm on the right side of lens 2.
(III)	(R) Final image is formed at 30.0 cm on the left side of lens 2.
(IV)	(S) Final image is formed at 6.0 cm on the right side of lens 2.
	(T) Final image is formed at 30.0 cm on the right side of lens 2.

a) (I) - (T), (II) - (S), (III) - (Q), (IV) - (R)

b) (I) - (Q), (II) - (P), (III) - (T), (IV) - (S)

c) (I) - (P), (II) - (T), (III) - (R), (IV) - (Q)

d) (I) - (P), (II) - (R), (III) - (Q), (IV) - (T)

33. Four physical quantities are listed in Column I. Their values are listed in Column II in a **[3]** random order:

Column I	Column II
(a) Thermal energy of air molecules at room temp	(e) 0.02 eV
(b) Binding energy of heavy nuclei per nucleon	(f) 2eV
(c) X-ray photon energy	(g) 1 keV
(d) Photon energy of visible light	h) 7 MeV

The correct matching of Columns I and II is given by

a) a - e, b - h, c - g, d - f　　　　　　　b) a - f, b - e, c - g, d - h

c) a - e, b - g, c - f, d - h　　　　　　　d) a - f, b - h, c - e, d - g

34. A musical instrument is made using four different metal strings 1,2,3 and 4 with mass per **[3]** unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List - I gives the above four strings while list - II lists the magnitude of some quantity.

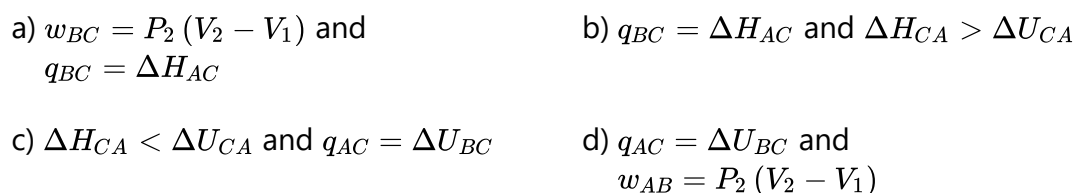| List-I	List-II
(I) String - 1 (μ)	(P) 1
(II) String - 2 (2μ)	(Q) $\frac{1}{2}$
(III) String - 3 (3μ)	(R) $\frac{1}{\sqrt{2}}$
(IV) String - 4 (4μ)	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) $\frac{1}{16}$

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

a) (I) $\rightarrow$ (P), (II)$\rightarrow$ (Q), (III)$\rightarrow$ (T), (IV)$\rightarrow$ (S)

b) (I) $\rightarrow$ (Q), (II)$\rightarrow$ (P), (III)$\rightarrow$ (R), (IV)$\rightarrow$ (T)

c) (I) $\rightarrow$ (Q), (II)$\rightarrow$ (S), (III)$\rightarrow$ (R), (IV)$\rightarrow$ (P)

d) (I) $\rightarrow$ (P), (II)$\rightarrow$ (R), (III)$\rightarrow$ (S), (IV)$\rightarrow$ (Q)

Chemistry (MRQ)

35. A new carbon-carbon bond formation is possible in **[4]**

a) Reimer-Tiemann reaction　　　　　　b) Friedel-Craft alkylation

c) Clemmensen reduction　　　　　　　d) Cannizzaro reaction

36. For a first order reaction, **[4]**

a) the degree of dissociation is equal to $(1 - e^{-kt})$

b) the pre-exponential factor in the Arrhenius equation has the dimension of time, T^{-1}.

c) the time taken for the completion of 75% reaction is thrice the $t_{1/2}$ of the reaction

d) a plot of reciprocal concentration of the reactant vs. time gives a straight line

37. A reversible cyclic process for an ideal gas is shown below. Here, P, V, and T are pressure, volume and temperature, respectively. The thermodynamic parameters q, w, H and U are heat, work, enthalpy and internal energy, respectively. **[4]**
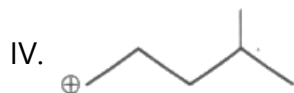

a) $w_{BC} = P_2 (V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$

b) $q_{BC} = \Delta H_{AC}$ and $\Delta H_{CA} > \Delta U_{CA}$

c) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$

d) $q_{AC} = \Delta U_{BC}$ and $w_{AB} = P_2 (V_2 - V_1)$

Chemistry (MCQ)

38. Which of the following compounds are covalent? **[3]**

a) KCl

b) CaO

c) H_2

d) Na_2S

39. Of the given anions, the strongest base is **[3]**

a) ClO^-

b) ClO_3^-

c) ClO_4^-

d) ClO_2^-

40. The correct stability order for the following species is **[3]**

I.

II.

III.

IV.
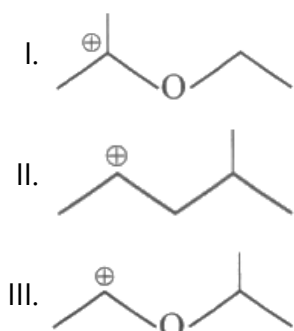
    a) (II) > (I) > (IV) > (III)   b) (II) > (IV) > (I) > (III)

    c) (I) > (III) > (II) > (IV)   d) (I) > (II) > (III) > (IV)

41.   Which of the following will react with water?   **[3]**

    a) Cl_3CCHO b) $ClCH_2CH_2Cl$

    c) $CHCl_3$ d) CCl_4

Chemistry (NUM)

42.   The number of moles of CuO, that will be utilized in Dumas method for estimation nitrogen in a sample of 57.5g of N, A-dimethylaminopentane is _____ $\times 10^{-2}$. (Nearest integer)   **[4]**

43.   29.2% ($\frac{w}{w}$) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 MHCl is: **[4]**

44.   The molar heats of combustion of $C_2H_2(g)$, C(graphite) and $H_2(g)$ are 310.62 kcal, 94.05 kcal and 68.32 kcal, respectively. Calculate the standard heat of formation of $C_2H_2(g)$.   **[4]**

45.   In an atom, the total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -\frac{1}{2}$ is   **[4]**

46. $^{238}_{92}U$ is known to undergo radioactive decay to form $^{206}_{82}Pb$ by emitting alpha and beta particles. A rock initially contained 68×10^{-6} g of $^{238}_{92}U$. If the number of alpha particles that it would emit during its radioactive decay of $^{238}_{92}U$ to $^{206}_{82}Pb$ in three half-lives is Z $\times 10^{18}$, then what is the value of Z?   **[4]**

47.   The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound is   **[4]**
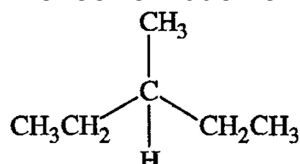
Chemistry (MATCH)

48.   Match List-I with List-II.   **[3]**

List-I (Anion)	List-II (Gas evolved on reaction with dil. H_2SO_4)
(A) CO_3^{2-}	(I) Colourless gas which turns lead acetate paper black
(B) S^{2-}	(II) Colourless gas which turns acidified potassium dichromate solution green.

(C) SO_3^{2-}	(III) Brown fumes which turns acidified KI solution containing starch blue.
(D) NO_2^-	(IV) Colourless gas evolved with brisk effervescence, which turns lime water milky.

    a) A - IV, B - I, C - II, D - III   b) A - II, B - I, C - IV, D - III

    c) A - III, B - I, C - II, D - IV   d) A - IV, B - I, C - III, D - II

49. LIST-I contains metal species and LIST-II contains their properties.   **[3]**

LIST - I	LIST - II
(I) $[Cr(CN)_6]^{4-}$	(P) t_{2g} orbitals contain 4 electrons
(II) $[RuCl_6]^{2-}$	(Q) μ (spin- only) = 4.9 BM
(III) $[Cr(H_2O)_6]^{2+}$	(R) low spin complex ion
(IV) $[Fe(H_2O)_6]^{2+}$	(S) metal ion in 4+ oxidation state
	(T) d^4 species

[Given: Atomic number of Cr = 24, Ru = 44, Fe = 26]

    a) I → R, S; II → P, T; III → P, Q; IV → Q, T   b) I → R, T; II → P, S; III → Q, T; IV → P, Q

    c) I → P, R; II → R, S; III → R, T; IV → P, T   d) I → Q, T; II → S, T; III → P, T; IV → Q, R

50. The standard reduction potential data at 25^OC is given below: **[3]**

$E^O(Fe^{3+}, Fe^{2+})$ = + 0.77 V; $E^O(Fe^{2+}, Fe)$ = - 0.44 V; $E^O(Cu^{2+}, Cu)$ = + 0.34 V; $E^O(Cu^+, Cu)$ = + 0.52 V

$E^O[O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O]$ = +1.23 V; $E^O[O_2(g) + 2H_2O + 4e^- \rightarrow 4OH^-]$ = + 0.40 V

$E^O(Cr^{3+}, Cr)$ = -0.74 V; $E^O(Cr^{2+}, Cr)$ = -0.91 V

Match E^O of the redox pair in List I with the values given in List II and select the correct answer using the code given below the lists:

List I	List II
(P) $E^O(Fe^{3+}, Fe)$	(1) - 0.18 V
(Q) $E^O(4H_2O \rightleftharpoons 4H^+ + 4OH^-)$	(2) -0.8 V
(R) $E^O(Cu^{2+} + Cu \rightarrow 2Cu^+)$	(3) -0.04 V
(S) $E^O(Cr^{3+}, Cr^{2+})$	(4) -0.83 V

    a) (P) - (3), (Q) - (4), (R) - (1), (S) - (2)   b) (P) - (2), (Q) - (3), (R) - (4), (S) - (1)

    c) (P) - (1), (Q) - (2), (R) - (3), (S) - (4)   d) (P) - (4), (Q) - (1), (R) - (2), (S) - (3)

51. The unbalanced chemical reactions given in List I show missing reagents or conditions (?) which are provided in List II. Match List I with List II and select the correct answer using the   **[3]**

code given below the lists

(List-I)	(List-II)
P. $PbO_2 + H_2SO_4 \xrightarrow{?} PbSO_4 + O_2$ + other product	1. NO
Q. $Na_2S_2O_3 + H_2O \xrightarrow{?} NaHSO_4$ + other product	2. I_2
R. $N_2H_4 \xrightarrow{?} N_2$ + other product	3. Warm
S. $XeF_2 \xrightarrow{?} Xe$ + other product	4. Cl_2

a) P - 1, Q - 4, R - 2, S - 3   b) P - 3, Q - 2, R - 1, S - 4

c) P - 3, Q - 4, R - 2, S - 1   d) P - 4, Q - 2, R - 3, S - 1

JEE Advanced 2024
Sample Paper - 1
Solution
Mathematics (MRQ)

1. **(a)** p = 2, h = 3, k = -4

   Explanation: If (h, k) is the mid point of chord of parabola y^2 = 16x, then equation of chord will be given by

   T = S_1 $\Rightarrow$ yk - 8(x + h) = k^2 - 16h

   $\Rightarrow$ 8x - ky = 8h - k^2 ...(i)

   But given, the equation of chord is

   2x + y = p ...(ii)

   $\therefore$ (i) and (ii) are identical lines

   $\Rightarrow \frac{8}{2} = \frac{-k}{1} = \frac{8h - k^2}{p} \Rightarrow$ k = -4 and p = 2h - 4

   which are satisfied by option (p = 2, h = 3, k = -4).

2. **(a)** f ' is NOT differentiable at x = 1

   (b) f is onto

   (d) f' has a local maximum at x = 1

   Explanation: $f(x) = \begin{cases} \left(x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1\right) - 2x, & x < 0 \\ x^2 - 2 \times \frac{1}{2} \times x + \frac{1}{4} + \frac{3}{4} & , 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & , 1 \le x < 3 \\ (x - 2)\log_e(x - 2) - x + \frac{10}{3}, & , x \ge 3 \end{cases}$

   $= \begin{cases} (x + 1)^5 - 2x & , x < 0 \\ \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} & , 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & , 1 \le x < 3 \\ (x - 2)\log_e(x - 2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$

   For x = 0, f(x) = 1

   For x < 0, f(x) = $(x + 1)^5$ - 2x

   It decreases to -∞.

   $\therefore$ f(x) $\in (-\infty, 1]$ for x $\le$ 0

   For x = 3, f(x) $\frac{1}{3}$

   For x $\ge$ 3, f(x) = increases to ∞

   $\therefore f(x) \in \left[\frac{1}{3}, \infty\right)$ for $x \ge 3$

   On combining the two f(x) $\in R \Rightarrow f$ is onto.

   $f'(x) = \begin{cases} 5(x + 1)^4 - 2 & , x < 0 \\ 2x - 1 & , 0 \le x < 1 \\ 2(x - 2)^2 - 1 & , 1 \le x < 3 \\ \log_e(x - 2) & , x \ge 3 \end{cases}$

   Lf " (1) = 2, Rf " (1) = -4, $\Rightarrow$ f' is not differentiable at x = 1
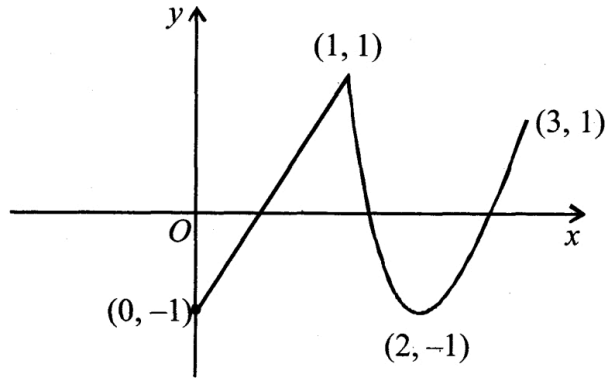
   For x < 0, f'(x) = $5(x + 1)^4$ -1

   Now, f'(x) = 0 $\Rightarrow$ $(x + 1)^4 = \frac{1}{5} \Rightarrow x = -1 \pm \left(\frac{1}{5}\right)^{\frac{1}{4}}$

   $\Rightarrow$ f is changing its nature at x = -1 - $\left(\frac{1}{5}\right)^{\frac{1}{4}}$

   $\therefore$ f is not increasing on (-∞, 0)

$$f'(x) = \begin{cases} 2x - 1 & , 0 \le x < 1 \\ 2(x-2)^2 - 1 & , 1 \le x < 3 \end{cases}$$

From its graph f'(x) has local maxima at x = 1.

3. **(a)** g(x) is continuous but not differentiable at b
   (d) g(x) is continuous but not differentiable at a
   Explanation: Clearly g(x) may or may not be continuous at x = a or x = b.
   But it is continuous at all value of x except x = a, b.
   Let us check the continuity of g(x) at x = a and x = b

   $$\lim_{x \to a^-} g(x) = 0$$

   $$\lim_{x \to a^+} g(x) = \lim_{x \to a^+} \int_a^x f(t)dt = \int_a^a f(t)dt = 0$$

   and g(a) = $\int_a^a$ f(t) dt = 0

   ∴ g(x) is continuous at x = a

   Also $\lim_{x \to b^-} g(x) = \lim_{x \to b^-} \int_a^x f(t)dt = \int_a^b f(t)dt$

   and $\lim_{x \to b^+} g(x) = \int_a^b f(t)dt = \lim_{x \to b^-} g(x) = g(b)$

   ∴ g(x) is continuous at x = b
   Therefore, g(x) is continuous $\forall x \in$ R

   Now $g'(x) = \begin{cases} 0, & x < a \\ f(x), & a \le x \le b \\ 0, & x > b \end{cases}$

   $g'(a^-) = 0$ and $g'(a_1^+) = f(a)$

   g'(b⁻) = f(b) and g'(b⁺) = 0
   Since, $f(a), f(b) \in [1, \infty)$ ∴ $f(a), f(b) \neq 0$
   ∴ $g'(a^-) \neq g'(a^+)$ and $g'(b^-) \neq g'(b^+)$
   ⇒ g is not differentiable at a and b.

   Mathematics (MCQ)

4. **(a)** 7
   Explanation: We know that,
   $(a+b)^5 + (a-b)^5 = {}^5C_0 a^5 + {}^5C_1 a^4 b + {}^5C_2 a^3 b^2 + {}^5C_3 a^2 b^3 + {}^5C_4 ab^4 + {}^5C_5 b^5 + {}^5C_0 a^5 - {}^5C_1 a^4 b$
   $+ {}^5C_2 a^3 b^2 - {}^5C_3 a^2 b^3 + {}^5C_4 ab^4 - {}^5C_5 b^5$
   2[a⁵ + 10a³b² + 5ab⁴]
   ∴ [x + x³ - 1)^(1/2)]⁵ + [x - (x³ - 1)^(1/25)

$$= 2\left[x^5 + 10x^3\left(x^3 - 1\right) + 5x\left(x^3 - 1\right)^2\right]$$

Therefore, the given expression is a polynomial of degree 7.

5.

(c) $\log_e (1 + x) < x$

Explanation: Let $g(x) = \log_e(1 + x) - x$, $0 < x < 1$

$g'(x) = \frac{1}{1+x} - 1 = -\frac{x}{1+x} < 0$ for $0 < x < 1$

$\Rightarrow g(x)$ decreases for $0 < x < 1$

$\Rightarrow d(x) < g(0)$ for $0 < x < 1$

$\Rightarrow \log_e (1 + x) - x < 0$ for $0 < x < 1$

or $\log_e (1 + x) < x$ for $0 < x < 1$

6. **(a)** a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the X-axis

Explanation: Let $y = \cos x \cos (x + 2) - \cos^2 (x + 1)$

$= \cos (x + 1 - 1) \cos (x + 1 + 1) - \cos^2 (x + 1)$

$= \cos^2 (x + 1) - \sin^2 1 - \cos^2 (x + 1) \Rightarrow y = -\sin^2 1$

This is a straight line which is parallel to X-axis. It passes through $(\pi/2, -\sin^2 1)$

7.

(b) {0, -1}

Explanation: $f(x) = f^{-1}(x) \Rightarrow fof(x) = x$

$\Rightarrow [(x + 1)^2 - 1 + 1]^2 - 1 = x \Rightarrow (x + 1)^4 = x + 1$

$\Rightarrow (x + 1)[(x + 1)^3 - 1] = 0$

$\therefore x = 0$ or -1

$\therefore$ Required set is {0, -1}

Mathematics (NUM)

8. 0

Explanation:

$a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_1, a_2,...., a_{11}$ are in an AP.

$\therefore \frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$

$\Rightarrow 225 + 35d^2 + 150d = 90$

$\Rightarrow 35d^2 + 150d + 135 = 0$

$\Rightarrow d = -3, -\frac{9}{7}$

Given, $a_2 < \frac{27}{2}$

$\therefore d = -3$ and $d \neq -\frac{9}{7}$

$\Rightarrow \frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2}[30 - 10 \times 3] = 0$

9. 3.0

Explanation:

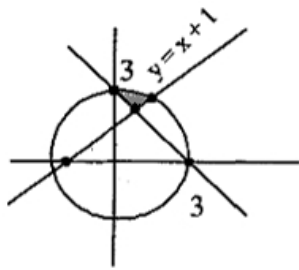$3 - x \leq y \leq y\sqrt{9 - x^2}$

Let $x^2 + y^2 = 9$ ...(i)

$y + x = 3$ ...(ii)

Points $(p, p + 1)$ lies on $y = x + 1$ ...(iii)

So point of intersection of (ii) and (iii) eqn. is $x = 1$, $y = 2$

And point of intersection between (i) and (ii)

$x + 1 = \sqrt{9 - x^2}$ is $x = \frac{-1+\sqrt{17}}{2}$

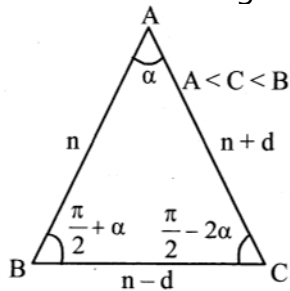Hence $p \in \left(1, \frac{-1+\sqrt{17}}{2}\right)$.

So, $b^2 + b - a^2 = 3$

10. 1008.0

Explanation:

Let sides of triangle are n - d, n, n + d.

$n - d = 2\sin\alpha$ ...(i)

$n + d = 2\sin\left(\frac{\pi}{2} + \alpha\right)$

$\Rightarrow n + d = 2\cos\alpha$ ...(ii)

$n = 2\sin\left(\frac{\pi}{2} - 2\alpha\right)$

$\Rightarrow n = 2\cos 2\alpha$ ...(iii)

Adding (i) and (ii), we get n = sinα + cosα

$\Rightarrow 2\cos 2\alpha = \sin\alpha + \cos\alpha$

$2\left(\cos^2\alpha - \sin^2\alpha\right) - (\cos\alpha + \sin\alpha) = 0$

$(\cos\alpha + \sin\alpha)[2(\cos\alpha - \sin\alpha) - 1] = 0$

$\Rightarrow 2(\cos\alpha - \sin\alpha) = 1 \Rightarrow \sin 2\alpha = \frac{3}{4}$

Then, a = $\frac{1}{2} \cdot n \cdot (n + d) \cdot \sin\alpha = \frac{1}{2} \cdot 2\cos 2\alpha \, 2\cos\alpha \cdot \sin\alpha$

$= \sin 2\alpha \cdot \cos 2a$

$= \frac{3}{4} \times \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{16}$

$(64a)^2 = \left(64 \times \frac{3\sqrt{7}}{16}\right)^2 = 16 \times 9 \times 7 = 1008$

11. 7.0

Explanation:

Given that

$|\vec{u}| = 1; |\vec{v}| = 1; \vec{u}.\vec{v} \neq 0; \vec{u}.\vec{w} = 1; \vec{v}.\vec{w} = 1$

and $\vec{w} \cdot \vec{w} = |\vec{w}|^2 = 4 \Rightarrow |\vec{w}| = 2$

Volume of parallelopiped $= [\vec{u} \quad \vec{v} \quad \vec{w}] = \sqrt{2}$

$\Rightarrow [\vec{u} \quad \vec{v} \quad \vec{w}]^2 = \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{v} & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & \vec{w} \cdot \vec{w} \end{vmatrix} = 2$

$$\Rightarrow \begin{vmatrix} 1 & \vec{u}.\vec{v} & 1 \\ \vec{u}.\vec{v} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2 \Rightarrow \vec{u}.\vec{v} = \tfrac{1}{2}$$

Now, $|3\dot{\vec{u}} + 5\vec{v}|^2 = (3\vec{u} + 5\vec{v})(3\vec{u} + 5\vec{v})$

$= 9|\vec{u}|^2 + 25|\vec{v}|^2 + 30\vec{u} \cdot \vec{v}$

= 9 + 25 + 15 = 49

$\therefore |3\vec{u} + 5\vec{v}| = \sqrt{49} = 7$

12. 0

Explanation:

Given that f(x) = $\int\limits_0^x$ f(t)dt

Clearly f(0) = 0. Also $f'(x)$ = f(x) $\Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating both sides with respect to x, we get
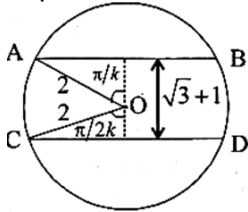
$\int \frac{f'(x)}{f(x)} dx = \int 1 dx$

$\Rightarrow$ In f(x) = x + In C $\Rightarrow$ f(x) = Ce^X

Now f(0) = 0 $\Rightarrow$ Ce^X = 0 $\Rightarrow$ C = 0

$\therefore$ f(x) = 0 $\forall x \Rightarrow$ f(In 5) = 0

13. 3

Explanation:

From the figure

$2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3} + 1$

$\Rightarrow 2 \times 2\cos^2\frac{\pi}{2k} + 2\cos\frac{\pi}{2k} - 2$

$= \sqrt{3} + 1$

$\Rightarrow 4\cos^2\frac{\pi}{2k} + 2\cos\frac{\pi}{2k} - (3 + \sqrt{3}) = 0$

$\Rightarrow \cos\frac{\pi}{2k} = \frac{-2 \pm \sqrt{4 + 16(3+\sqrt{3})}}{8} = \frac{-1 \pm \sqrt{13 + 4\sqrt{3}}}{4}$

$= \frac{-1 \pm (2\sqrt{3}+1)}{4} = \frac{\sqrt{3}}{2}$ or $-\left(\frac{\sqrt{3}+1}{2}\right)$

As $\frac{\pi}{2k}$ is an acute angle, $cos\frac{\pi}{2k} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow$ k = 3

Mathematics (MATCH)

14.

(c) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (3), (S) $\rightarrow$ (5)

Explanation: Given, $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ ...(i)

$|\bar{z}|^3 + 2\bar{z}^2 + 4z - 8 = 0$ [Conjugate both sides]

$2(z^2 - \bar{z}^2) + 4(\bar{z} - z) = 0$

$\Rightarrow 2(z - \bar{z})[z + \bar{z} - 2] = 0$

$\because z = \bar{z}$ (Not possible) or $z + \bar{z} = 2$

$\therefore$ z = 1 + bi(b $\neq$ 0) $\Rightarrow \bar{z}$ = 1 - bi

$\left(1 + b^2\right)^{\frac{3}{2}} + 2(1 - b^2 + 2bi) + 4(1 - bi) - 8 = 0$ [from (i)]

$\left(1+b^2\right)^{\frac{3}{2}}$ - 2(1 + b^2) = 0

$\Rightarrow \left(1+b^2\right)\left(\sqrt{1+b^2}-2\right)$ = 0

$\because$ 1 + b^2 $\neq$ 0 $\Rightarrow$ $\sqrt{1+b^2}$ - 2 = 0 $\Rightarrow$ b^2 = 3

P. |z|^2 = 1 + b^2 = 1 + 3 = 4

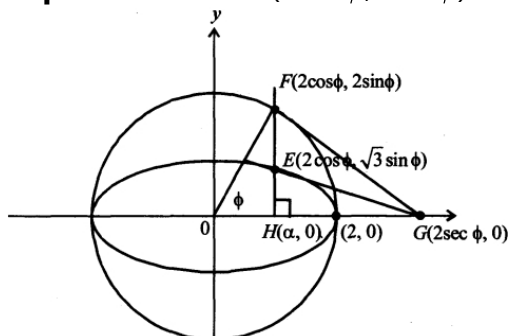Q. |z - z|^2 = |1 + ib -1 + ib|^2 = 4b^2 = 12

R. |z|^2 + |z + $\bar{z}$|^2 = 4 + |1 + ib + 1 - ib|^2 = 4 + 4 = 8

S. |z + 1|^2 = |1 + 1 + ib|^2 = 4 + b^2 = 4 + 3 = 7

15.

(c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

Explanation: Let F(2cosϕ, 2sinϕ) and E(2cosϕ, $\sqrt{3}$ sin ϕ)

$\alpha \equiv \cos\phi$

Tangent at E(2cosϕ, $\sqrt{3}$ sinϕ) to ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

i.e. $\frac{x\cos\phi}{2} + \frac{y\sin\phi}{\sqrt{3}} = 1$ intersect x-axis at G(2secϕ, 0)

Area of triangle FGH = $\frac{1}{2}$HG $\times$ FT

= $\frac{1}{2}$(2 secϕ - 2cosϕ) 2sinϕ; $\triangle$ = 2sin$^2\phi$ $\cdot$ tan ϕ

$\triangle$ = (1 -cos2ϕ) $\cdot$ tanϕ

I. If $\phi = \frac{\pi}{4}, \triangle = 1 \rightarrow (Q)$

II. If $\phi = \frac{\pi}{3}, \triangle = 2\cdot \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \sqrt{3} = \frac{3\sqrt{3}}{2} \rightarrow (T)$

III. If $\phi = \frac{\pi}{6}, \triangle = 2 \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} \rightarrow (S)$

IV. If $\phi = \frac{\pi}{12}, \triangle = \left(1 - \frac{\sqrt{3}}{2}\right)\cdot(2 - \sqrt{3}) = \frac{(2-\sqrt{3})^2}{2} \rightarrow (P)$

16. **(a)** (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

Explanation: We have system of linear equations

x + y + z = 1 ...(i)

10x + 100y + 1000z = 0

x + 10y + 100z = 0 ...(ii)

qrx + pry + pqz = 0 ...(iii)

$\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r}$ = 0 ($\because$ p, q, r $\neq$ 0)

Let p = $\frac{1}{a+9d}$, q = $\frac{1}{a+99d}$, r = $\frac{1}{a+999d}$

Now, equation (iii) is

(a + 9d)x + (a + 99d)y + (a + 999d)z = 0

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 100 \\ a+9d & a+99d & a+999d \end{vmatrix} = 0$

$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 10 & 100 \\ 0 & a+99d & a+999d \end{vmatrix}$ = 900(d - a)

$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 100 \\ a+9d & 0 & a+999d \end{vmatrix}$ = 990(a - d)

$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 0 \\ a+9d & a+99d & 0 \end{vmatrix}$ = 90(d - a)

Let option I: If $\frac{q}{r}$ = 10 $\Rightarrow$ a = d

$\Delta = \Delta_x = \Delta_y = \Delta_z$ = 0

Since eq. (i) and eq. (ii) represents non-parallel planes and eq. (ii) and eq. (iii) represents same plane

$\Rightarrow$ Infinitely many solutions

So, option I $\to$ P, Q, R, T

Option II: $\frac{p}{r} \neq 100 \Rightarrow a \neq d$

$\Delta = 0, \Delta_x, \Delta_y, \Delta_z \neq 0$

No solution

So, option II $\to$ S

Option III: $\frac{p}{q} \neq 10 \Rightarrow a \neq d$

No solution

So, option III $\to$ S

Option IV: If $\frac{p}{q}$ = 10 $\Rightarrow$ a = d

Infinitely many solution

Hence, IV $\to$ P, Q , R , T

17.

(d) (P) $\to$ (3), (Q) $\to$ (2), (R) $\to$ (4), (S) $\to$ (1)

Explanation: Let any point on L_1 is (2λ + 1, $-\lambda$, λ - 3) and that on L_2 is (μ + 4, μ - 3, 2μ - 3)

For point of intersection of L_1 and L_2

2λ + 1 = μ + 4, $-\lambda$ = μ - 3, λ - 3 = 2μ - 3

$\Rightarrow \lambda$ = 2, μ = 1

$\therefore$ Intersection point of L1 and L_2 is (5, -2, -1)

Equation of plane passing through, (5, -2, -1) and perpendicular to P_1 & P_2 is given by

$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$

$\Rightarrow$ x - 3y - 2z = 13

$\therefore$ a = 1, b = -3, c = -2, d = 13

or (P) $\to$ (3), (Q) $\to$ (2), (R) $\to$ (4), (S) $\to$ (1)

Physics (MRQ)

18. **(b)** $\left| \vec{V}_C - \vec{V}_A \right| = 2 \left| \vec{V}_B - \vec{V}_C \right|$
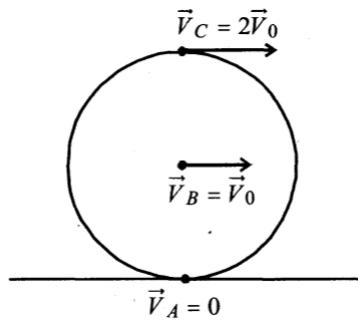
(c) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

Explanation: Here, $\vec{V}_A = 0$

$\vec{V}_B = \vec{V}_0,$

$$\vec{V}_C = 2\vec{V}_0$$
$$\therefore \vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$$
and $\left|\vec{V}_C - \vec{V}_A\right| = \left|\vec{V}_B - \vec{V}_C\right|$

19. **(a)** $y\,(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

    (b) $y\,(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

    (c) $y\,(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

    Explanation: There should be a displacement node at x = 0 and a displacement antinode at x = 3 m. Therefore, y = 0 at x = 0 and y = ± A at x = 3 m

    Speed of wave, $v = \frac{\omega}{k} = 100$ ms^{-1}

20. **(a)** If $\vec{B}$ is along $\hat{z}$, F ∝ (L + R)

    (b) If $\vec{B}$ is along $\hat{x}$, F = 0

    (d) If $\vec{B}$ is along $\hat{y}$, F ∝ (L + R)

    Explanation: Magnetic force acting on a current carrying wire, placed in a uniform magnetic field,

    $\vec{F} = I(\vec{l} \times \vec{B})$

    Here, $\vec{l}$ = displacement of the wire = 2(L + R)$\hat{x}$

    $\therefore \vec{F} = 2I(L + R)(\hat{x} \times \vec{B})$

    If $\vec{B}$ = B$\hat{x}$ then

    $\vec{F} = 2I(L + R)(\hat{x} \times \hat{x})$ B = 0
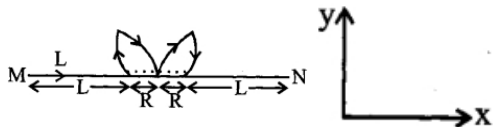
    If $\vec{B}$ = B$\hat{y}$ then

    $\vec{F} = 2I(L + R)(\hat{x} \times \hat{y})$B = 2IB(L + R)$\hat{z}$ or F ∝ (L + R)

    If $\vec{B}$ = B$\hat{z}$ then

    F = 2 I(L + R)($\hat{x} \times \hat{z}$)B = -2IB(L + R)$\hat{y}$
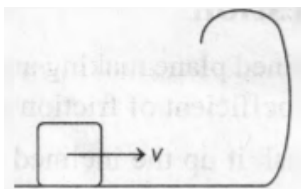
    or, F ∝ (L + R).

Physics (MCQ)

21. **(a)** Δl = 1 mm, ΔT = 0.1 sec, n = 50

    Explanation: The relative error in g, $\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

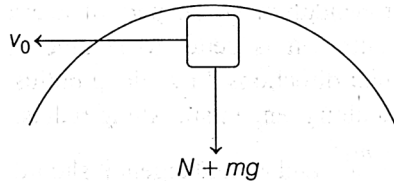 Δl and ΔT are least and a number of readings taken are maximum in option (Δl = 1 mm, ΔT = 0.1 sec, n = 50), therefore the measurement of g is most accurate.

22.

    (d)

Explanation: Since, the block rises to the same heights in all the four cases, from conservation of energy, speed of the block at highest point will be same in all four cases. Say it is v_0. Equation of motion will be

$$N + mg = \frac{mv_0^2}{R}$$

or $N = \frac{mv_0^2}{R} - mg$

R (the radius of curvature) in first case is minimum. Therefore, normal reaction N will be maximum in first case.

23.

(c) $\frac{GmM}{2(R+h)}$

Explanation: The total energy of the orbiting satellite at a height h is

$E = -\frac{GM_E m}{2(R_E+h)}$

The total energy of the satellite at infinity is zero.

∴ Energy expended to rocket the satellite out of the earth's gravitational field is

$\Delta E = E_\infty - E$

$= 0 - \left(-\frac{GM_E m}{2(R_E+h)}\right) = \frac{GM_E m}{2(R_E+h)}$

24.

(b) $E_A = E_B > E_C$

Explanation: $E_A = E_B > E_C$

Physics (NUM)

25. 3

Explanation:

For v = $\frac{25}{3}$m, let object distance be u_1 and for v = $\frac{50}{7}$m.

Let object distance be u_2, then from mirror's formula,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{\frac{25}{3}} + \frac{1}{u_1} = \frac{1}{\frac{20}{2}}$ or u_1 = -50 m
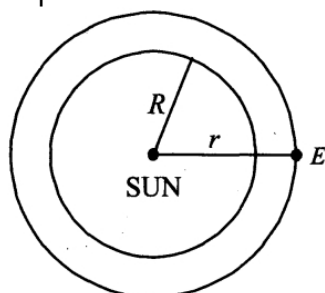
Also $\frac{1}{\frac{50}{7}} + \frac{1}{u_2} = \frac{1}{\frac{20}{2}}$ or u_2 = -25 m

So, it means object moves from 50 m from mirror to 25 m from mirror in 30 sec. Assuming that the object moves with constant speed, the velocity of object is given by :

$v_0 = \frac{25}{30}$m/s = $\frac{5}{6}$m/s = $\frac{5}{6} \times \frac{18}{5}$ = 3 km/h.

26. 5803

Explanation:

According to Stefan's law, Energy radiated per unit time per unit area = σT^4.

∴ $\frac{\text{Energy}}{\text{time}} = (\sigma T^4) \times (4\pi R^2)$

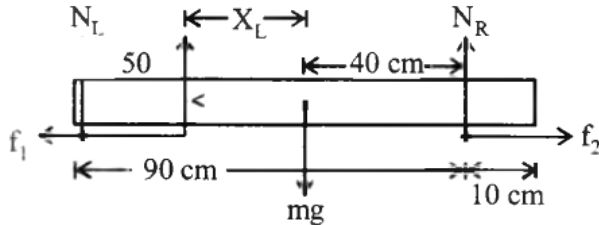$\therefore$ Energy received on earth per unit time = $1400 \times 4\pi r^2$

$\therefore \sigma T^4 \times 4\pi R^2 = 1400 \times 4\pi r^2$ or $T^4 = \frac{1400 \times r^2}{\sigma R^2}$

or $T^4 = \frac{1400 \times (1.5 \times 10^{11})^2}{(5.67 \times 10^{-8})(7 \times 10^8)^2}$

or $T^4 = \frac{14 \times 2.25 \times 10^{24}}{5.67 \times 49 \times 10^8}$ or T = 5803 K.

27. 25.60

Explanation:

Initially,

$N_L + N_R = Mg$; $N_L = \frac{40}{90} Mg = \frac{4Mg}{9}$

Torque about centre $\tau_{\text{center}} = 0$

$\therefore N_1(50) = N_2(40)$; $N_R = \frac{50}{90} Mg = \frac{5Mg}{9}$

$5N_L = 4N_R$

$f_{1_K} = \mu_K N_L$; $f_{1_L} = \mu_S N_L$

$f_{1\,K} = 0.32\,N_L$; $f_{1_L} = 0.4\,N_L$

$f_{2\,K} = 0.32\,N_{LR}$; $f_{2_L} = 0.4\,NR$

If X_L = distance of left finger from centre when right finger starts moving

$(\tau_n = 0)$about centre $\Rightarrow N_{LX_L} = N_R(40)$

$f_{K_1} = f_{L_2} \Rightarrow 0.32\,N_L = 0.40\,N_R$

$4N_L = 5N_R$

$N_{LX_L} = \frac{4\,N_L}{5}(40) \Rightarrow X_L = 32$

Now x_R = distance when right finger stops and left finger starts moving

$N_{LX_L} = N_R(X_R)$ [Torque about centre, $T_{\text{centre}} = 0$]

$f_{L_1} = f_{K_2} \Rightarrow 0.4\,N_L = 0.32\,N_R$

$4N_L = 4N_R$

$\frac{4\,N_2}{5}(32) = N_R X_R$

$\therefore X_R = \frac{128}{5} = 25.6$ cm

28. 192

Explanation:

The movable stopper will adjust to a position with equal pressure on either sides. Therefore,

$P_1 = P_2$

$P_1 = \frac{n_1 RI}{V_1} = \frac{m}{M_1} RT$, $P_2 = \frac{n_2 RT}{V_2} = \frac{m}{M_2} RT$

$\Rightarrow \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$ $\therefore \alpha = \frac{360°}{(8+7)} \times 8 = 192°$

29. 16.0

Explanation:

Binding energy of system is given by

$\Rightarrow \frac{Ke^2}{2r} = 12.8$ eV

$\Rightarrow \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2r}$ = 12.8 ×1.6 × 10^{-19}

($\because$ K = $\frac{1}{4\pi \in_0}$ = 9 × 10^9 Nm^2/C^2 and electronic charge = 1.6 × 10^{-19} C)

$\Rightarrow$ r = $\frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{12.8 \times 2}$ $\Rightarrow$ r = $\frac{9 \times 10^{-10}}{16}$

The value of x is 16.

30. 2

Explanation:

Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time twill be given by

V = 10 $\left(1 - e^{-t/\tau}C\right)$

Here, τ_c = Cnet Rnet

= $\left(1 \times 10^6\right) \left(4 \times 10^{-6}\right)$ = 4s

V = 10(1 - e$^{-t/4}$)

or e$^{-t/4}$ = 0.6 = $\frac{3}{5}$

Taking log on both sides we have,

$-\frac{t}{4}$ = ln 3 - ln 5

or t = 4 (ln 5 - ln 3) = 2 s

Hence, the answer is 2

Physics (MATCH)

31.

(d) (P) → (4); (Q) → (3); (R) → (2); (S) → (1)

Explanation: $W_{GE} = P_0 (V_0 - 32 V_0) = -31P_0 V_0$

$W_{GH} = P_0 (8 V_0 - 32 V_0) = -24P_0 V_0$

$\left(W_{FH}\right)_{adiabatic} = \frac{P_f V_f - P_i V_i}{1 - \gamma} = \frac{P_0 (8 V_0) - 32P_0 (V_0)}{1 - \frac{5}{3}}$ = 36P$_0$V$_0$

$\left(W_{FG}\right)_{isothermal} = nRT \ln\left(\frac{V_f}{V_i}\right) = P_0 V_0 \ln\left(\frac{V_f}{V_i}\right)$

$= 1 \left(32P_0 V_0\right) \log_e \frac{32 V_0}{V_0}$

$= 32P_0 V_0 \log_e 2^5 = 160P_0 V_0 \log_e 2$

32.

(d) (I) - (P), (II) - (R), (III) - (Q), (IV) - (T)

Explanation:

i. u_1 = -20 cm

f_1 = +10 cm

$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u} \Rightarrow$ v = $\frac{uf}{u+f}$

So, v_1 = $\frac{-20 \times 10}{-20+10}$ = 20 cm

Now, u_2 = +15 cm, f_2 = +15 cm

So, v_2 = $\frac{15 \times 15}{15+15}$ = 7.5 cm (from lens 2). So (I) → (P)

ii. u_1 = -20 cm

f_1 = +10 cm

So, v_1 = $\frac{-20 \times 10}{-20+10}$ = 20 cm

Now, u_2 = +15 cm, f_2 = -10 cm

So, v_2 = $\frac{15 \times -10}{15-10}$ = -30 cm. So (II) → (R)

iii. Preceeding as above

$u_2 = +15$ cm, $f_2 = -20$ cm

So, $v_2 = \frac{15 \times -20}{15 - 20} = 60$ cm. So (III) $\to$ (Q)

iv. $u_1 = -20$ cm, $f_1 = -20$ cm

$v_1 = \frac{-20 \times -20}{-20 - 20} = -10$ cm

So, $u_2 = -15$ cm and $f_2 = +10$ cm

Then, $v_2 = \frac{-15 \times 10}{-15 + 10} = 30$ cm. So (IV) $\to$ (T)

33. **(a)** a - e, b - h, c - g, d - f

Explanation: At room temperature, thermal energy of air molecule $= 0.02\text{eV}$

photon energy of visible light $(\lambda = 4000\overset{o}{A}$ to $700\overset{o}{A}) = 2\text{eV}$.

34.

(d) (I) $\to$ (P), (II)$\to$ (R), (III)$\to$ (S), (IV)$\to$ (Q)

Explanation: Frequency, $v = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ for first mode of vibration

For 'v' to be maximum, 'l' should be minimum.

String - 1 $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

String - 2 $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

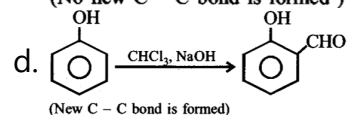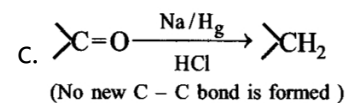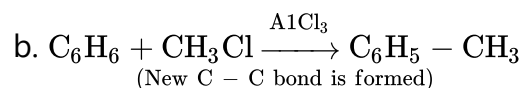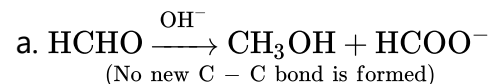**String - 3** $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

String - 4 $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

Chemistry (MRQ)

35. **(a)** Reimer-Tiemann reaction

(b) Friedel-Craft alkylation

Explanation:

a. $\text{HCHO} \xrightarrow{\text{OH}^-} \text{CH}_3\text{OH} + \text{HCOO}^-$
(No new C – C bond is formed)

b. $\text{C}_6\text{H}_6 + \text{CH}_3\text{Cl} \xrightarrow{\text{A1Cl}_3} \text{C}_6\text{H}_5 - \text{CH}_3$
(New C – C bond is formed)

c. $\text{\Large$>$}\text{C=O} \xrightarrow[\text{HCl}]{\text{Na/Hg}} \text{\Large$>$}\text{CH}_2$
(No new C – C bond is formed)

d. $\xrightarrow{\text{CHCl}_3,\ \text{NaOH}}$
(New C – C bond is formed)

36. **(a)** the degree of dissociation is equal to $(1 - e^{-kt})$

(b) the pre-exponential factor in the Arrhenius equation has the dimension of time, T^{-1}.

Explanation: In first order reaction, if a is the degree of dissociation then

$kt = \log_e \frac{1}{(1-\alpha)} = -\log_e(1-\alpha)$ or $e^{-kt} = 1 - \alpha$

$\therefore \alpha = 1 - e^{-kt}$

The Arrhenius equation is, $k = Ae^{-E_a/RT}$

Plot of reciprocal concentration of the reactant vs time is linear. Dimensions of pre-

exponential factor 'A' are equivalent to dimensions of k, which is T^{-1} for a first order reaction.

37. **(a)** $w_{BC} = P_2(V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$

(c) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$

Explanation: A - C $\Rightarrow$ isochoric process

A - B $\Rightarrow$ isothermal process

B - C $\Rightarrow$ isobaric process

a. $q_{AC} = \Delta U_{AC} = nC_{V,m}(T_2 - T_1) = \Delta U_{BC}$

$W_{AB} = -nRT_1 \ln\left(\frac{V_2}{V_1}\right)$ (pressure is not constant)

b. $W_{BC} = -P_2(V_1 - V_2) = P_2(V_2 - V_1)$

$q_{BC} = \Delta H_{BC} = nC_{P,m}(T_2 - T_1) = \Delta H_{AC}$

c. $\Delta H_{CA} = nC_{P,m}(T_1 - T_2)$

d. $\Delta U_{CA} = nC_{V,m}(T_1 - T_2)$

$\Delta H_{CA} < \Delta U_{CA}$ since both are negative $(T_1 < T_2)$ and $C_{p,m} > C_{V,m}$

Chemistry (MCQ)

38.

(c) H$_2$

Explanation: H$_2$

= H - H

39. **(a)** ClO⁻

Explanation: The order of acidic strength of conjugate acids is

HOCl < HClO$_2$ < HClO$_3$ < HClO$_4$

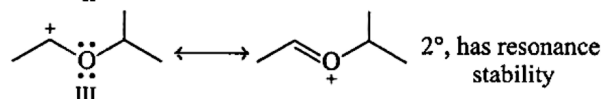Reverse is the order of basic strength of their conjugate base, i.e.

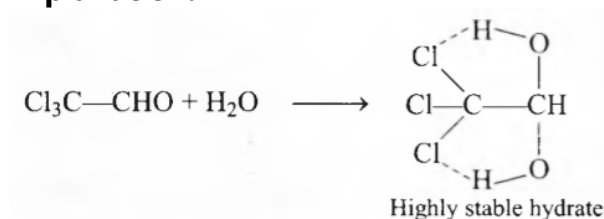ClO⁻ is the strongest base.

40.

(c) (I) > (III) > (II) > (IV)
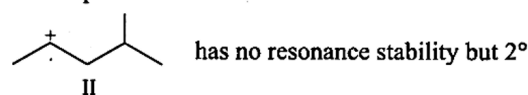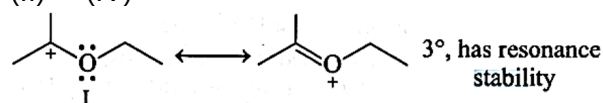
Explanation:
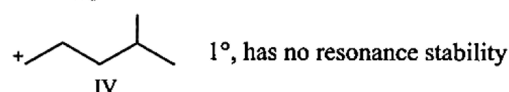
Therefore, overall stability order is : I > III > II > IV
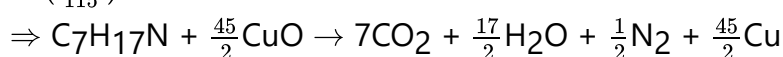
41. **(a)** Cl$_3$CCHO

Explanation:

Highly stable hydrate

Chemistry (NUM)

42. 1125.0

Explanation:

Moles of N in N, N-dimethylaminopentane

$= \left(\frac{57.5}{115}\right) = 0.5$ mol

$\Rightarrow$ C$_7$H$_{17}$N + $\frac{45}{2}$CuO $\rightarrow$ 7CO$_2$ + $\frac{17}{2}$H$_2$O + $\frac{1}{2}$N$_2$ + $\frac{45}{2}$Cu

For 0.5 moles of $C_7H_{17}N$ number of moles of CuO required

$= \frac{1}{2}\left(14 + \frac{17}{2}\right) = \frac{45}{4} = 1125 \times 10^{-2}$

43. 8

Explanation:

Molarity of stock solution of HCl

$= \frac{29.2 \times 1000 \times 1.25}{100 \times 36.5}$

Let the volume of stock solution required = V mL

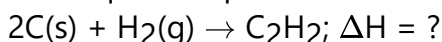Thus, $V \times \frac{29.2 \times 1000 \times 1.25}{100 \times 36.5} = 200 \times 0.4$

$\Rightarrow V = 8$ mL

44. 54.20

Explanation:
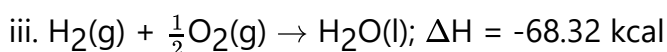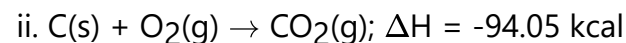
The required equation is:

$2C(s) + H_2(g) \rightarrow C_2H_2; \Delta H = ?$

Write the thermochemical equations for the given data

  i. $C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(l); \Delta H = -310.62$ kcal

  ii. $C(s) + O_2(g) \rightarrow CO_2(g); \Delta H = -94.05$ kcal

  iii. $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l); \Delta H = -68.32$ kcal
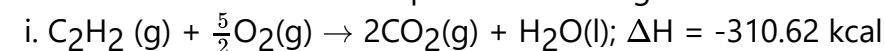
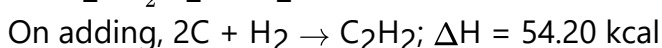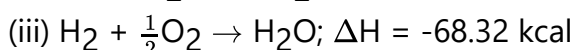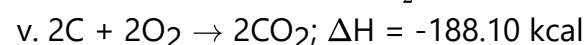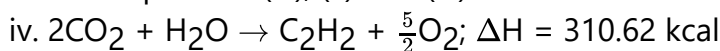For getting the above-required reaction, we will have to

NOTE:

  a. Bring C_2H_2 in the product that can be done by reversing the equation (i) to give equation (iv).

  b. Multiply equation (ii) by 2 to get 2C atoms in the reactants and thus equation (v) is obtained.

  c. Keep equation (iii) as such.

  d. Add equations (iv), (v) and (iii).

  iv. $2CO_2 + H_2O \rightarrow C_2H_2 + \frac{5}{2}O_2; \Delta H = 310.62$ kcal

  v. $2C + 2O_2 \rightarrow 2CO_2; \Delta H = -188.10$ kcal

  (iii) $H_2 + \frac{1}{2}O_2 \rightarrow H_2O; \Delta H = -68.32$ kcal

On adding, $2C + H_2 \rightarrow C_2H_2; \Delta H = 54.20$ kcal

Hence, the standard heat of formation of $C_2H_2(g)$ = 54.20 kcal

45. 6

Explanation:

Then count all possible electrons having a given set of quantum numbers.

For n = 4, the total number of possible orbitals are

According to question $|m_l| = 1$, i.e. there are two possible values of m_l i.e. +1 and -1 and one orbital can contain a maximum of two electrons one having s = $+\frac{1}{2}$ and the other having s = $-\frac{1}{2}$
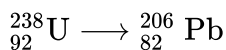
So, the total number of orbitals having $\{|m_l| = 1\}$ = 6

Total number of electrons having

$\{|m_l| = 1$ and $m_s = -\frac{1}{2}\}$ = 6

46. 1.20

Explanation:

$$_{92}^{238}U \longrightarrow _{82}^{206}Pb$$

Initial moles of U^{238} = $\frac{68 \times 10^{-6}}{238}$ = x

Number of α-particles emitted per nuclei of

$_{92}^{238}U = \frac{238-206}{4}$ = 8
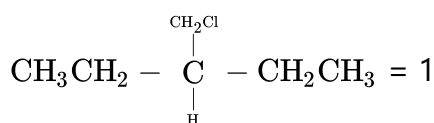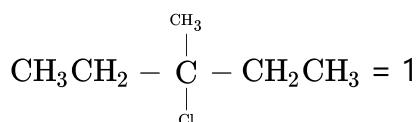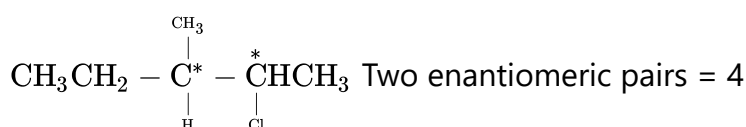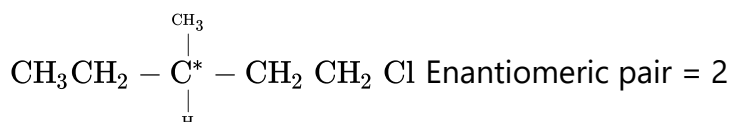
Moles of U^{238} decayed in three half-lives = $\frac{7}{8}x$

No. of α-particles emitted = $\left(\frac{7}{8}x\right) \times 8 \times N_A$

= $7 \times \frac{68 \times 10^{-6}}{238} \times 6.022 \times 10^{23}$ = 1.204×10^{18}

$\therefore$ Z = 1.20

47. 8

Explanation:

$$CH_3CH_2 - \overset{\overset{CH_3}{|}}{\underset{|}{C^*}} - CH_2\ CH_2\ Cl \quad \text{Enantiomeric pair = 2}$$
$$_{H}$$

$$CH_3CH_2 - \overset{\overset{CH_3}{|}}{\underset{|}{C^*}} - \overset{*}{\underset{|}{C}}HCH_3 \quad \text{Two enantiomeric pairs = 4}$$
$$_{H \quad Cl}$$

$$CH_3CH_2 - \overset{\overset{CH_3}{|}}{\underset{\underset{Cl}{|}}{C}} - CH_2CH_3 \quad \text{= 1}$$

$$CH_3CH_2 - \overset{\overset{CH_2Cl}{|}}{\underset{\underset{H}{|}}{C}} - CH_2CH_3 \quad \text{= 1}$$

Total = 2 + 4 + 1 + 1 = 8

Chemistry (MATCH)

48. **(a)** A - IV, B - I, C - II, D - III

Explanation: Each anions have particular confirmatory test.

- CO_3^{2-} ion will evolved CO_2(g) which will turns lime water milky.

- S^{2-} ion will give H_2S(g), will turns lead acetate paper black.

- SO_3^{2-} ion will give SO_2(g), which will turns acidified potassium dichromate solution green.

- NO_2^- ion will give brown fumes of NO_2(g), which turns KI solution blue.

49.

(b) I $\rightarrow$ R, T; II $\rightarrow$ P, S; III $\rightarrow$ Q, T; IV $\rightarrow$ P, Q

Explanation:

I. $[Cr(CN)_6]^4 \Rightarrow Cr^{2+} (d^4) \Rightarrow$ low spin Oh complex as CN^- is strong field ligand.

Electronic configuration = $t_{2g}^4 e_g^0; \Delta_0 > P$;

$\mu_{S.o.} = \sqrt{2(2+2)}$ = 2.82BM

(P), (R), (T)

II. $[RuCl_6]^{2-} \Rightarrow Ru^{4+} (d^4) \Rightarrow$ Low spin Oh complex as **Ru** is of large size.

Electronic configuration = $t_{2g}^4 e_g^0; \Delta_0 > P$;

$\mu_{S.o.} = \sqrt{2(2+2)}$ = 2.82 BM

(P), (R), (S), (T)

III. $[Cr(H_2O)_6]^{2+} \Rightarrow Cr^{2+} (d^4) \Rightarrow$ high spin Oh complex as H_2O is weak field ligand.

Electronic configuration $= t_{2g}^3 e_g^1; \Delta_0 < P;$

$\mu_{S.O.} = \sqrt{4(4+2)}$ = 4.89 Bm

(Q), (T)

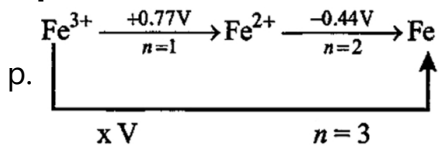IV. $[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} (d^6) \Rightarrow$ high spin Oh complex

Electronic configuration $= t_{2g}^4 e_g^2; \Delta_0 < P;$

$\mu_{S.o.} = \sqrt{4(4+2)}$ = 4.89 Bm

(P), (Q)

50. **(a)** (P) - (3), (Q) - (4), (R) - (1), (S) - (2)

Explanation:

p.

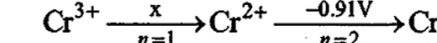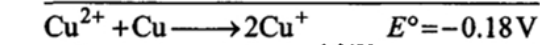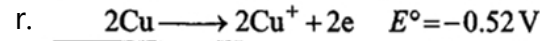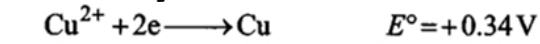$Fe^{3+} \xrightarrow[n=1]{+0.77V} Fe^{2+} \xrightarrow[n=2]{-0.44V} Fe$

$\underset{x\,V}{\underline{\qquad}} \quad \underset{n=3}{\longleftarrow}$

$\Delta G^o_{Fe^{3+}/Fe} = \Delta G^o_{Fe^{3+}/Fe^{2+}} + \Delta G^0_{Fe^{2+}/Fe}$

$\Rightarrow -3 \times FE^o_{(Fe^{+3}/Fe)} = -1 \times FE^o_{(Fe^{+3}/Fe^{+2})} + \left(-2 \times FE^o_{Fe^{+2}/Fe}\right)$

$\Rightarrow 3 \times x = 1 \times 0.77 + 2 \times (-0.44)$

$\Rightarrow x = -\frac{0.11}{3} V \simeq$ - 0.04 V.

q.

$\begin{array}{ll} 2H_2O \longrightarrow O_2 + 4H^+ + 4e^- & E^\circ = -1.23\,V \\ 4e + O_2 + 2H_2O \longrightarrow 4OH^- & E^\circ = +0.40\,V \\ \hline 4H_2O \longrightarrow 4H^+ + 4OH^- & E^\circ = -0.83\,V \end{array}$

r.

$\begin{array}{ll} Cu^{2+} + 2e \longrightarrow Cu & E^\circ = +0.34\,V \\ 2Cu \longrightarrow 2Cu^+ + 2e & E^\circ = -0.52\,V \\ \hline Cu^{2+} + Cu \longrightarrow 2Cu^+ & E^\circ = -0.18\,V \end{array}$

s.

$Cr^{3+} \xrightarrow[n=1]{x} Cr^{2+} \xrightarrow[n=2]{-0.91V} Cr$
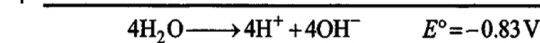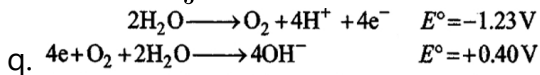
$\underset{-0.74V, \quad n=3}{\underline{\qquad}}$

$x \times 1 + 2 \times (-0.91) = 3 \times (-0.74)$

$x - 1.82 = -2.22 \Rightarrow x =$ -0.4 V

51.

(c) P - 3, Q - 4, R - 2, S - 1

Explanation:

P. $2PbO_2 + 2H_2SO_4 \xrightarrow{warm} 2PbSO_4 + 2H_2O + O_2$

Q. $Na_2S_2O_3 + 5H_2O + 4Cl_2 \longrightarrow 2NaHSO_4 + 8HCl$

R. $N_2H_4 + 2I_2 \longrightarrow N_2 + 4HI$

S. $XeF_2 + 2NO \longrightarrow Xe + 2NOF$