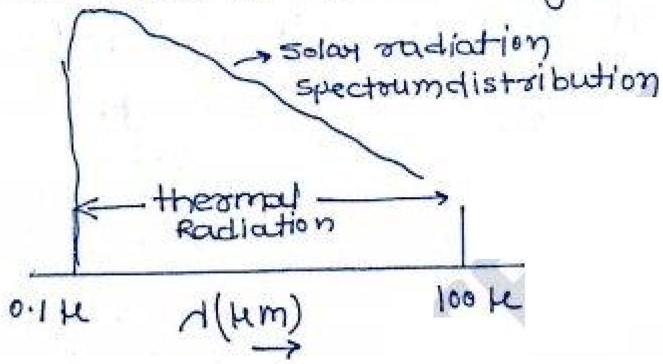
* Radiation

* All bodies at all temprature emit Radiation except the body at 0 K (−273.15 °C).
(In the Form of electromagnetic waves)



λ – wavelength measured in μm

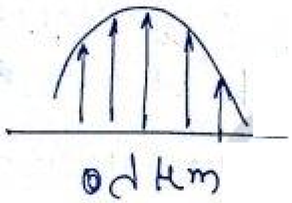
Electromagnetic spectrum

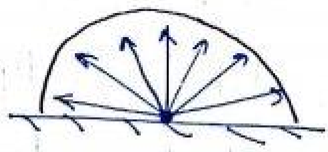
$1\,\mu m = 10^{-6}\,m$

* High temp, large body emit more radiation at shorter wavelength

Thermal Radiation

- wavelength module
- Directional Module





* Any body at any temp(K) shell emit thermal radiation in all possible hemispherical direction at all probable wavelength on E/m spectrum.

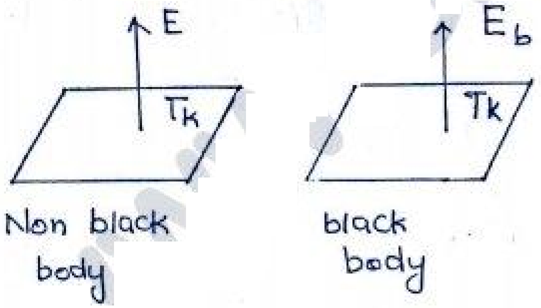
Basic definitions in Radiation Heat transfer

① Total hemispherical emmissive power (E) :-

It is defined as the radiation energy emitted from the surface of a body (By virtue of it's temp) per unit time per unit area in all possible hemi-spherical direction integrated overall the wavelengths

$$E \rightarrow \frac{\text{Joule}}{\text{Sec } M^2} = \left(\text{watt}/m^2\right)$$

② Total emmissivity (ϵ) :-

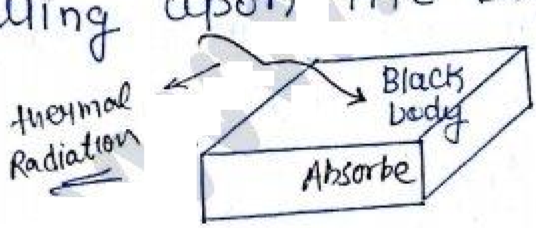


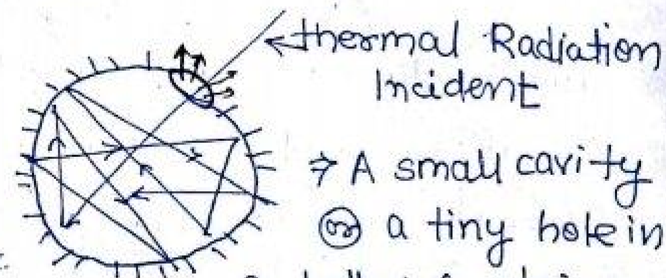
$$\boxed{\epsilon = \frac{E}{E_b}}$$

$\boxed{\epsilon_{\text{any body}} \leq 1}$ $\boxed{\epsilon_b = 1}$ black body

It is defined ratio between total hemispherical power of non black body and total hemispherical power of a black body both being at the same temp

Black body - Black body is the body which Absorve ~~observe~~ all the themal radiation incident or falling upon the body





→ A small cavity or a tiny hole in a hollow container blackbody

Reason:- Since all the thermal Radiation incident upon the cavity will get absorbe after multiple Internal Reflection eq **Example of black body**

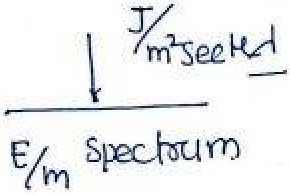
① A small hole in a furnace wall is black body

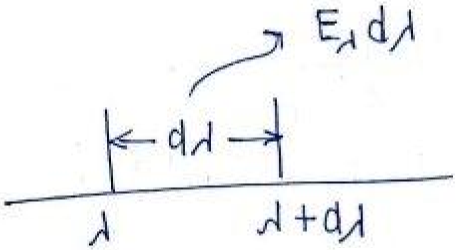
② In Radiation analysis SUN is also treated as blackbody.

→ ice is also close to black body $\epsilon_{ice} = 0.985$

Black body →
- Perfect absorber
- Ideal emitter
- Diffusive

③ Monochromatic (or) Spectral hemispherical emissive power (E_λ)





$d\lambda$ = differentially small Increament in wavelength λ

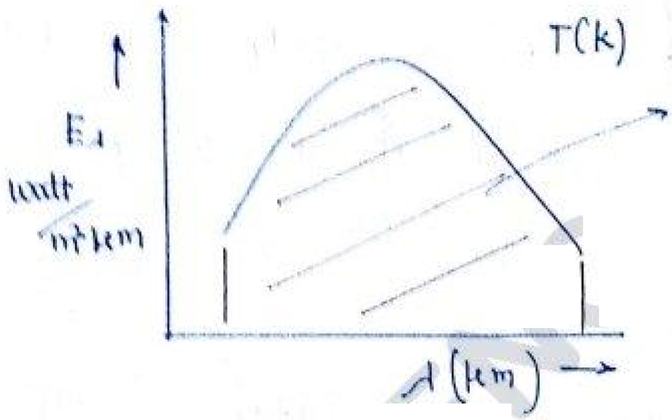
E_λ at a perticular wavelength λ is define as the quantity which one multipled by $d\lambda$ shell give the radiation energy emitted from the surface of a body per unit time per unit area in wavelength region λ to $\lambda + d\lambda$

$$E_\lambda \rightarrow \frac{\text{Joule}}{\text{Sec}\, m^2\, \mu m} = \frac{\text{watt}}{m^2\, \mu m}$$

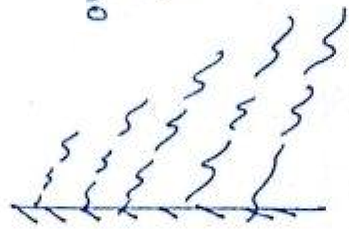
* Diff energies at different λ.

For any given body at a given temp.

$$E_\lambda = f(\lambda)$$



$$\text{Area} = \int_0^\infty E_\lambda \, d\lambda$$



Total hemispherical emissive power

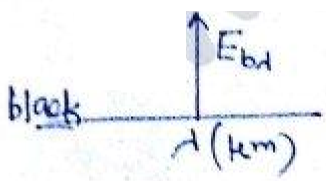
$$\boxed{E = \int_0^\infty E_\lambda \, d\lambda} \quad \text{watt}/m^2$$

$$\boxed{\text{Area} = \int_0^\infty E_\lambda \, d\lambda = \text{total emissive power } (E)}$$

(4) Monochromatic (spectral) emissivity (ϵ_λ): –

It is defined as Ratio b/w monochromatic hemispherical emissive power of a non-black body and monochromatic hemispherical power of a black body, both being at the same temp and wave length.

$$\boxed{\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}} \qquad \boxed{E_\lambda = \epsilon_\lambda E_{b\lambda}}$$



Now $\epsilon = \dfrac{E}{E_b} = \dfrac{\int_0^\infty E_\lambda \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda} = \dfrac{\int_0^\infty \epsilon_\lambda E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda}$

if $\epsilon_\lambda \neq f(\lambda)$ or rather constant

then

$$\epsilon = \frac{\epsilon_\lambda \int_0^\infty E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda}$$

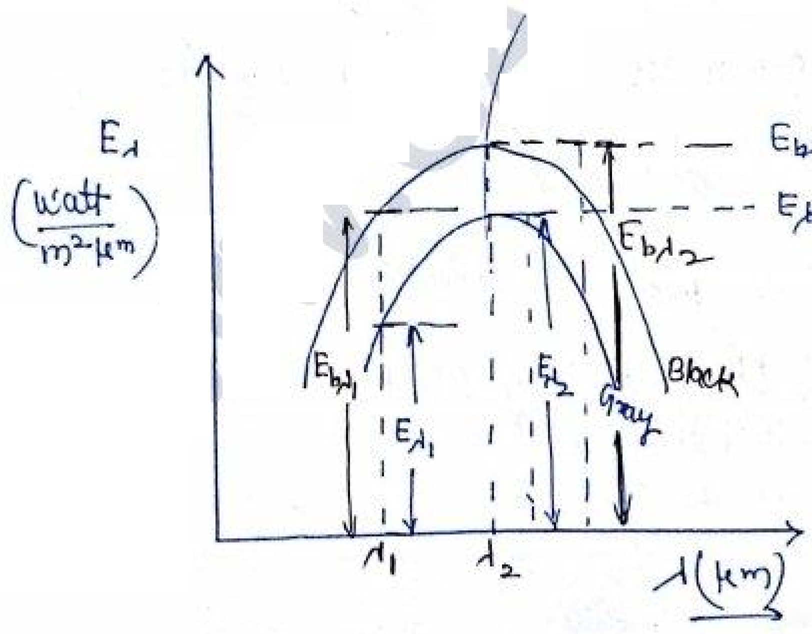
$$\boxed{\epsilon = \epsilon_\lambda}$$

Gray body :-

Such body whose monochromatic emissivity ϵ_λ is independent of wave length λ or rather remaining constant is know Gray body or Gray surface.

therefor for Gray body ϵ_λ = Constant

Physical Significance of Gray body :-



~~for Gray body~~

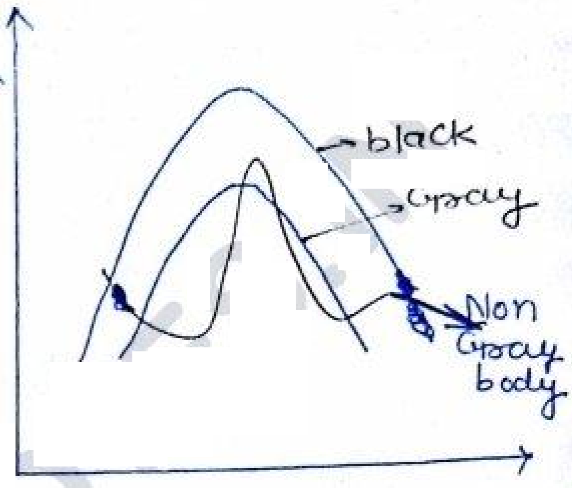
* The ratio b/w the area bottom curve on x-axis and the area under top curve on x-axis shell be equal to total emissivity of Gray body which equal to ϵ_λ

For Gray body

ϵ_λ = constant

i.e. $\epsilon_{\lambda_1} = \epsilon_{\lambda_2}$

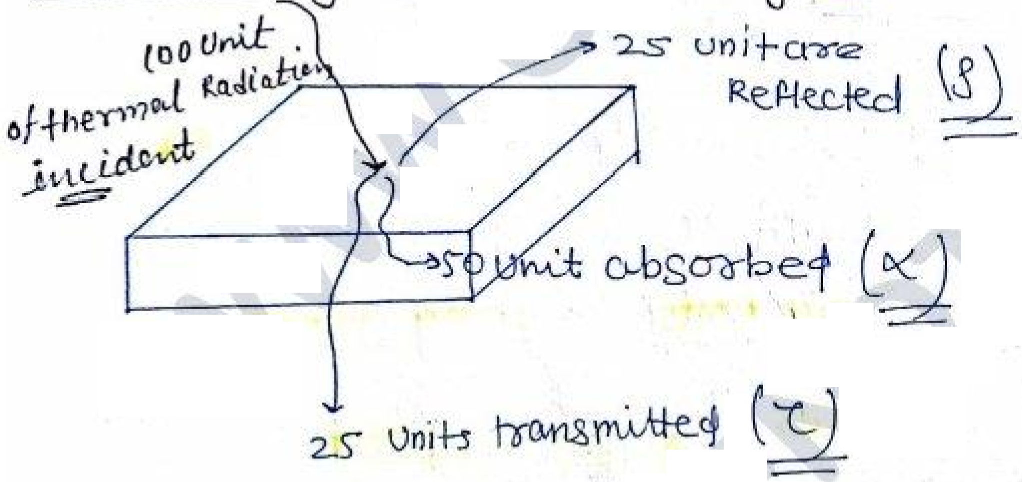
$$\boxed{\frac{E_{\lambda_1}}{E_{b\lambda_1}} = \frac{E_{\lambda_2}}{E_{b\lambda_2}}}$$



Non Gray body

$\epsilon_\lambda = f(\lambda)$

Absorptivity (α), Reflectivity (ρ) & transmissivity (τ):-



Absorptivity (α) = $\frac{50}{100}$ = 0.5

* Fraction of Radiation energy incident upon a surface which is absorbed by body.

Reflectivity (ρ) = $\frac{25}{100}$ = 0.25

* Fraction of Radiation energy incident upon a surface which is Reflected by it

Transmissivity (τ) = $\frac{25}{100} = 0.25$

* Fraction of radiation energy incident upon a surface which is transmitted through it.

∴ For any body.

$$\boxed{\alpha + \rho + \tau = 1}$$

* for opaque body which does not transmit any energy
$\tau = 0$

∴ for opaque body $\boxed{\alpha + \rho = 1}$ eg. concrete wall

* For black body, which absorbe all Radiation energy incident $\boxed{\alpha_b = 1}$

* for Metal Reflectivity is very high as compared to non-metals

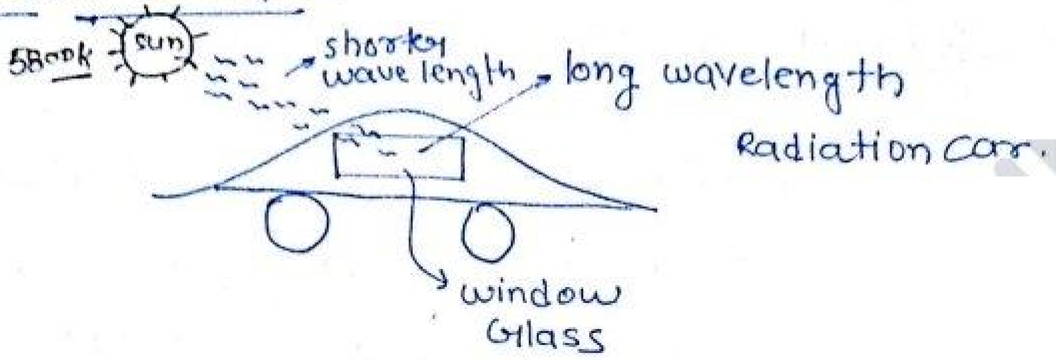
* This is the reason why metallic sheet Generally used in the furnaces to reduce radiation heat exchange

→ * For gases like O_2, N_2 transmissivity (τ) is very high (they are transparent to thermal Radiation)

* The above radiation properties (α, ρ, τ) mentioned change with wavelength of incident thermal radiation, surface roughness of body and also with its temp.

Practical example

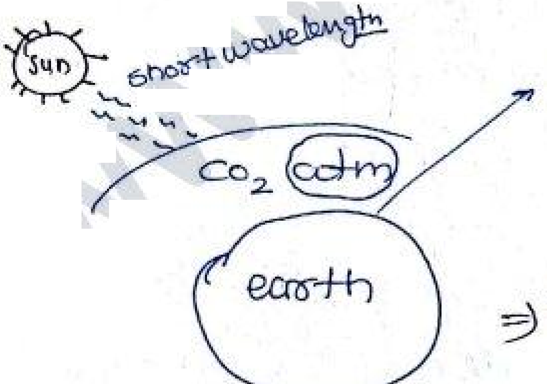
eg.



* So transmissivity change with, λ, time & temp.

The window Glass of a car is very much transparent to the ~~solar~~ short wave length solar radiation, but same window Glass almost becomes opaque to the long wavelength re-radiation given by inside of car thus trapping energy inside the car hence increasing its temp.

eg.



long wavelength
Rediation of eath

Global warming

⇒ CO_2 is called Green house Gases.

⇒ H_2O (water Vapour) is also Greenhouse Gas

As the surface roughness of a body decreases by polishing it, the reflectivity of surface will increase This is the region why highly polished Al, Copper shields (screens) having very good reflectivity are generally used in the furnace to reduce radiation heat exchange.

Laws of Thermal Radiation :-

(1) kirchoff's law of thermal Radiation :-

The law state that whenever a body is in thermal equilibrium with its surrounding, its emissivity is equal to its absorptivity ($\alpha = \epsilon$)

$$\boxed{\alpha = \epsilon}$$

* A good absorber is always a good emitter

Ex for black body $\alpha_b = 1$

$\epsilon_b = 1$

(2) Plank's law :-

$E_{b\lambda} = f(\lambda, T)$ T in kelvin Only

The law state that the monochromatic emissive power of a black body is dependent on both absoulate temp of black body and also on wavelength of radiation energy emitted (λ).

$$\boxed{E_{b\lambda} = \frac{2\pi C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}}$$

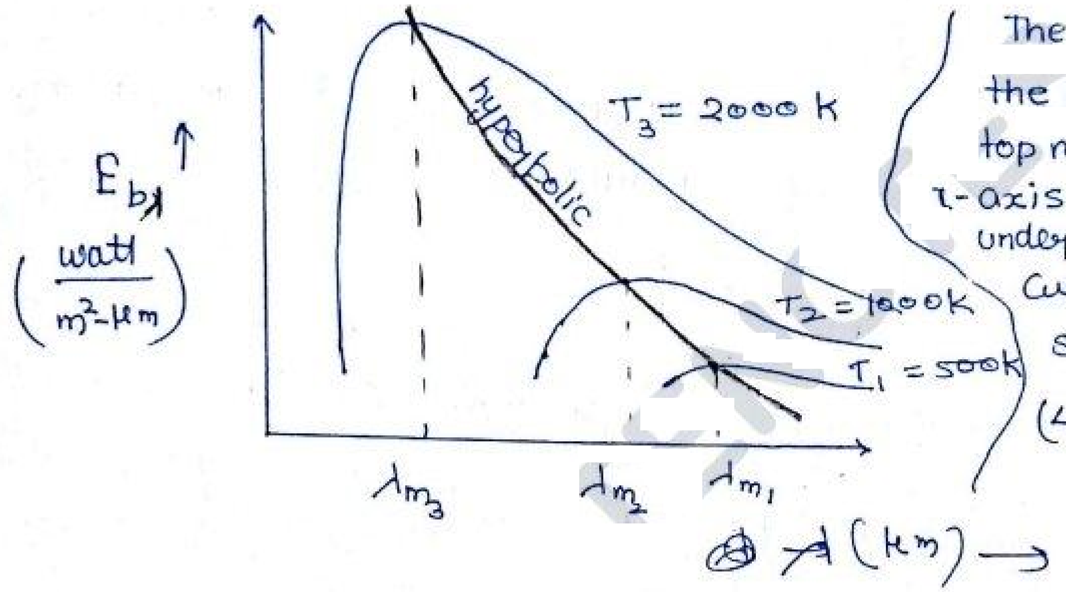
watt/m^2-μm

T - kelvin

λ - μm

C_1 and C_2 are experimental constant.

The functional relationship among three variable can be graphical represented as



The ratio between the area under the top most curve on x-axis and the area under bottom most curve on x-axis shell be equal $(4)^4 = 256$

λ_m = wavelength at which $E_{b\lambda}$ is maximum at a given absolute temp. of blackbody.

At a given absolute temp of black body as wavelength λ increase $E_{b\lambda}$ also increase reaches a. max. and then decrease.

Also as the absolute temp. of black body increase (each times getting doubled) $E_{b\lambda}$ Values anormously at higher temp. will be shifted to smaller wavelength.

As temp increase $\Rightarrow$ λ_m decrease.

$$\lambda_m \propto \frac{1}{T \text{ in } K}$$

$$\lambda_m \propto \frac{1}{T} = \text{Constant}$$

$$\boxed{\lambda_m T = 2898 \ \mu m\text{-}K}$$

For sun (Black body)

$$\lambda_m = \frac{2898}{5800}$$

$$\lambda_m \simeq \frac{1}{2} \ \mu m \quad \text{(In the V.L. Range)}$$

③ Wein's law:- $\boxed{\lambda_m T = C = 2898}$ μm-K

use for measuring high temp. {based on wavelength we measure high temp}

in casting

optical pyrometer very high temp measurement device.

④ Stefan- Boltzman law:-

The Law state that the radition energy or the total hemispherical emissive power of a black body is directly porportional to fourth power of absoulate temp. of the black body.

$$E_b \propto T^4 \quad \text{(T in K only)}$$

$$\boxed{E_b = \sigma T^4} \ J/sec\, m^2 = watt/m^2$$

σ = Stefan Boltzman's Constant

$= 5.67 \times 10^{-8}$ watt$/m^2K^4$

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda$$

$$E_b = \int_0^\infty \frac{2\pi C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}\, d\lambda$$

$$\boxed{E_b = \sigma T^4} \text{ watt/m}^2$$

Plank law

↙ wein's displacement

↘ stefan's boltzman law.

For a non black body whose emissivity is ϵ, the total hemispherical emissive power of non-black body is equal to -

$$E = \epsilon E_b$$

$$\boxed{E = \epsilon \sigma T^4} \text{ watt/m}^2$$

T - kelvin only

If A is the total surface area of non-black body then Radiation energy emitted from entire non-black body = EA watt

$$= \epsilon A \sigma T^4 \text{ watt}$$

Q.30
P.g 80

$E_1 = 500\ W/m^2$ at $T_1 = \cancel{1225}$

$E_2 = 1200\ W/m^2$ at T_2

$E = \epsilon \sigma T^4$ $\quad \epsilon$ - Constant

$$\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4}$$

$$\frac{T_1}{T_2} = \left(\frac{500}{1200}\right)^{1/4} = 0.803$$

Q.32

Sun is a black body

$T_{sun} = 5800\ k$ $\quad T_b = 1000\ k$

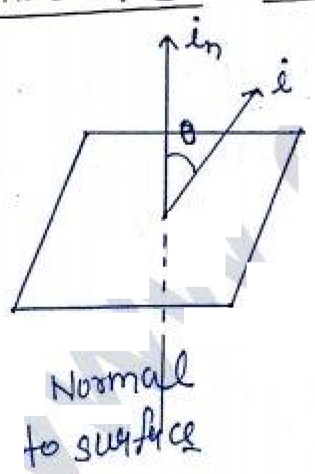
$\lambda = 0.50\ \mu m$ $\quad \lambda = ?$

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$0.50 \times 5800 = 1000 \times \lambda_2$$

$$\lambda_2 = 2.9\ \mu m$$

(5) Lambert's Cosine law :-

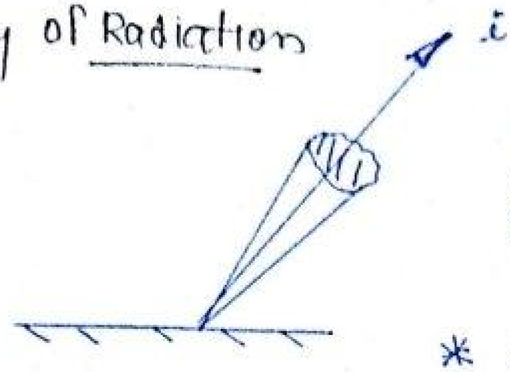


$i = i_n \cos\theta$

i_n = normal intensity of Radiation

i = Intensity of Radiation along any direction making an angle 'θ' w.r.t. normal direction

Intensity of Radiation



$$\frac{\text{Joule}}{\text{Sec. } m^2\text{-steradian}} = \frac{\text{watt}}{m^2\text{-steradian}}$$

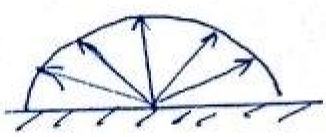
* Steradian is a unit of solid angle.

Intensity of radiation "i" along a given direction is defined as the radiation energy emitted from the surface of body per unit time. per-unit area normal to that direction and per unit solid angle about that direction.

$$i = \frac{dE}{d\omega} \quad \frac{\text{watt}}{m^2\text{-steradian}}$$

total hemispherical emissive power $= E = \int i \, d\omega$ watt/m^2

*



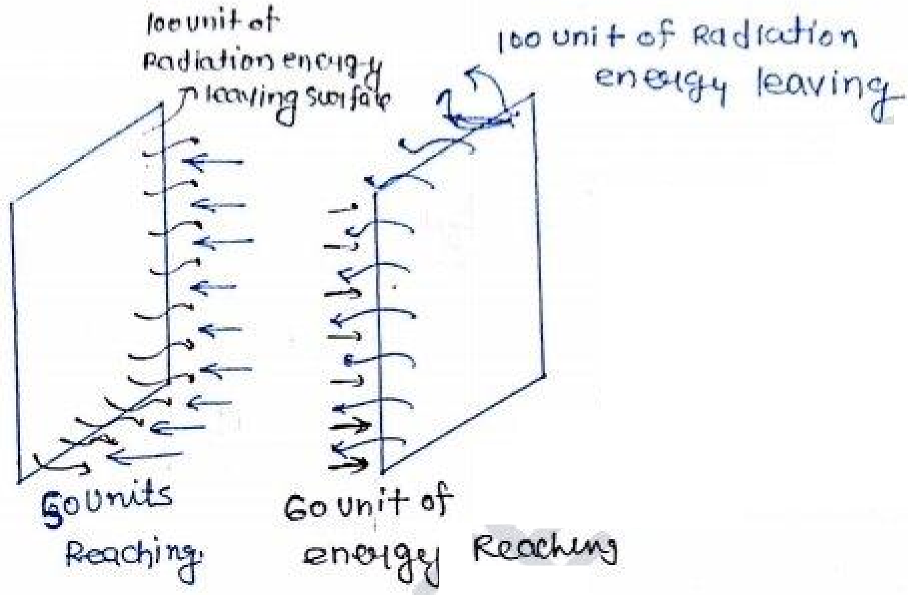
A Diffuse surface has the same intensity of Radiation along all the direction. i.e. for diffuse surface i is uniform in every direction

Eg:- Blackbody is a diffuse surface

$\therefore$ for blackbody $\boxed{E_b = \pi i}$ watt/m^2

V. Imp

Shape factor (OR) View factor (OR) Configuration factor:-



$F_{12} = \frac{60}{100} = 0.6$ = fraction of Radiation energy leaving surface ① that reaching surface ②

$F_{21} = \frac{50}{100} = 0.5$ = fraction of Radiation energy leaving surface ② that reaching surface ①

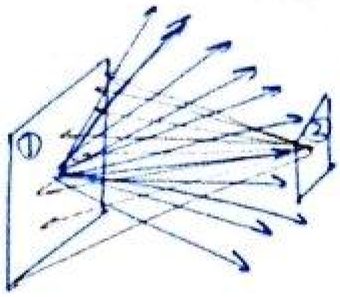
In General

F_{mn} = fraction of Radiation energy surface 'm' that reaches surface 'n'

Imp

* The shape factor between two surface is independent of their temp., their emissivity but depends only one their sizes, their shapes and how closly the two surface are kept w.r.t. to each other,

eg.



F_{21} is more than F_{12}

$$\boxed{0 \le F_{mn} \le 1}$$

eg Plane surface

① Flat

$F_{11} = 0$

eg Concave surface

$F_{11} > 0$

eg Convex surface

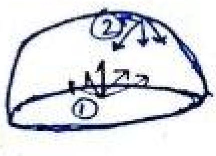
$F_{11} = 0$

while cal. shape factor
Read question carefully
→ Find Area Carefully
→ See Fig Carefully

* When one body or a surface is completaly surrounded by another body or a surface the shape factor of the inner body w.r.t. to outer is equal to 1

$F_{12} = 1$

Ex A hemispherical dom on a plate



$F_{12} = 1$

$F_{21} < 1$

$F_{22} < 1$

$$\boxed{F_{21} + F_{22} = 1}$$

Ex! Two concentric spherical surface

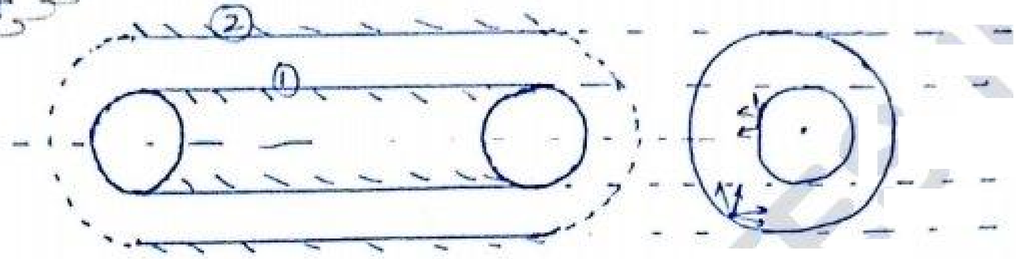
$F_{12} = 1$, $F_{21} < 1$

$F_{22} < 1$

$F_{21} + F_{22} = 1$

Ex Two Infinitely long Concentric Cylinderical Surface.

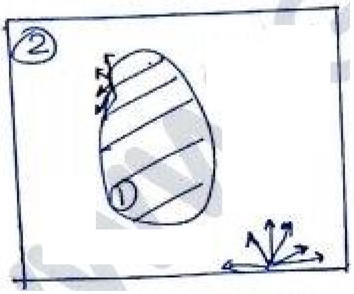
Neglecting end effect



$F_{12} = 1$ (Neglegting end effect

$F_{21} < 1$, $F_{22} < 1$, $F_{21} + F_{22} = 1$

Ex A body kept in an enclose/Room



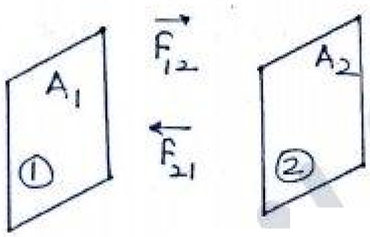
$F_{12} = 1$

$F_{21} < 1$

$F_{22} < 1$

$F_{21} + F_{22} = 1$

Reciprocity Relation between shape factor :-



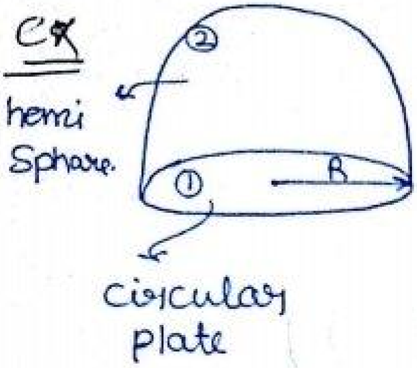
$$\boxed{A_1 F_{12} = A_2 F_{21}}$$

In General

$$\boxed{A_m F_{mn} = A_n F_{nm}}$$

$$\frac{A_1}{A_2} = \frac{F_{21}}{F_{12}}$$

C~~x~~



$F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} = \frac{\pi R^2}{2\pi R^2}$

$F_{21} = 1/2$

$F_{22} = 1 - 1/2 = 1/2$

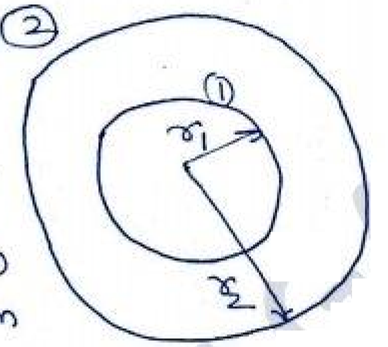
Q.35

P.g 81

~~Soli~~ hollow Sphere

$r_1 = 20\,mm$

$r_2 = 30\,mm$



$A_1 = 4\pi r_1^2$

$A_2 = 4\pi r_2^2$

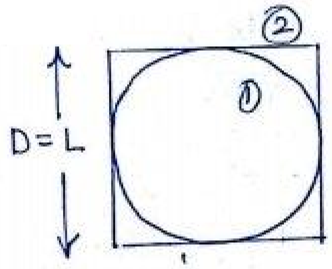
$F_{12} = 1, \quad F_{11} = 0$

$A_1 F_{12} = A_2 F_{21}$

$F_{21} = \frac{A_1}{A_2} = \frac{4\pi(20)^2}{4\pi(30)^2}$

$F_{21} = 4/9$

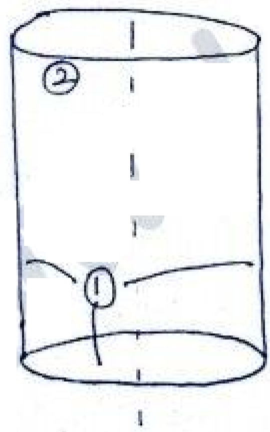
Q.24



$F_{11} = 0, \; F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} = \frac{4\pi(D)^2}{6\times 4\times(D)^2} = \frac{4\times\pi}{6\times 4} = \frac{\pi}{6}$

Q.~~28~~19



① circular base + Cylinder surface

② circular top.

$F_{21} = 1$

$F_{12} = \frac{A_2}{A_1} = \frac{\frac{\pi}{4}d^2}{\frac{\pi}{4}d^2 + \pi d h} = \frac{d}{d+4h}$

Q.31

Closed

$$F_{12} = 1$$

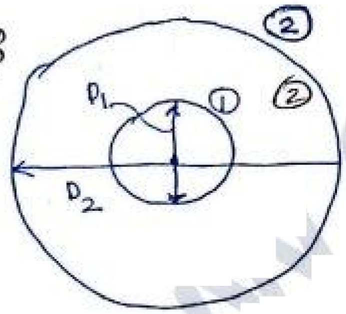
$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} = \frac{4\pi(r)^2}{2\pi(4r)^2 + 2\pi(4r)^2}$$

$$F_{21} = \frac{4\pi r^2}{(32+16)\pi r^2}$$

$$F_{21} = \frac{1}{12}$$

Q.28



if 2 inside

$$F_{12} = 1$$

$$F_{21} = \frac{A_1}{A_2} = \frac{2\pi D_1/2}{2\pi D_2/2} = D_1/D_2$$

$$F_{21} + F_{22} = 1$$

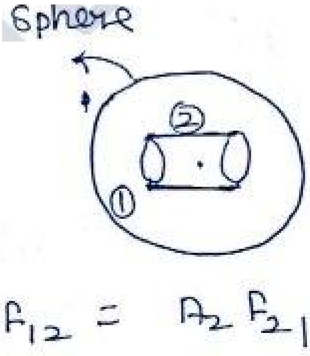
$$F_{22} = 1 - D_1/D_2$$

if 2 outside

$$F_{22} = 0$$

Q.9 Hollow sphere 1

solid ~~sphere~~ Cylinder 2



$D = 1m$

$d = h = 0.5m$

$F_{22} = 0$ $F_{21} = 1$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} = \frac{2\pi(0.25)(0.50) + 2\pi(0.25)^2}{4\pi(0.5)^2}$$

$$F_{12} = 0.375$$

$$F_{11} = 1 - F_{12} = 0.625$$

Q.25

$D = 1m$

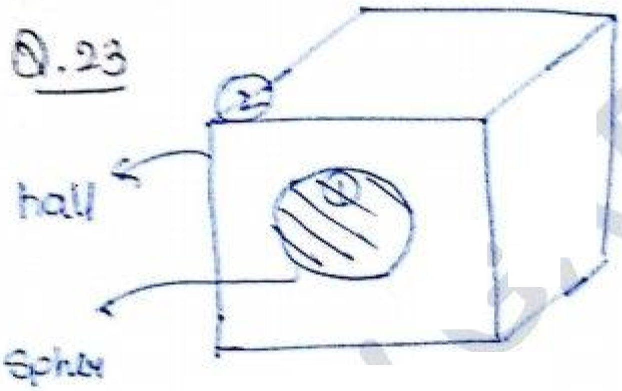
$d = h = 0.5\,m$

$$F_{12} = \frac{2\pi(0.25)(0.5) + 2\pi(0.25)^2}{4\pi(0.5)^2}$$

$F_2 = 0.375$

$F_1 = 0.625$

Q.23



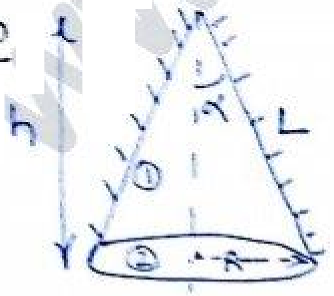
$F_{12} = 1$

$A_1 F_{12} = A_2 F_{21}$

$0.6 \times 1 = 6a^2 \times 0.004$

$a = 5\,m$

Q.20



Surface area of Cone = $\pi R L$

$F_{11} + F_{21} = 1$

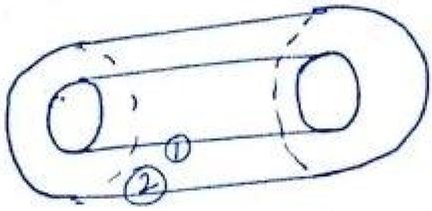
$F_{21} = 1$, $F_{11} = 0$

$A_1 F_{12} = A_2 F_{21}$

$$F_{12} = \frac{A_2}{A_1} = \frac{\pi R^2}{\pi R L}$$

$F_{12} = R/L = \sin\alpha$

Q, 42



outer Cylinder into itself

$$F_{12} = 1$$

$$F_{21} = R_i / R_o$$

$$F_{22} = 1 - R_i / R_o$$

Summation Rule Among ~~sp~~ shape factor:-

If there are 'n' number of surface involved in any radiation heat exchange then

$$F_{11} + F_{12} + F_{13} + \cdots + F_{1n} = 1$$

$$F_{21} + F_{22} + F_{23} + \cdots + F_{2n} = 1$$

$$\vdots \qquad \qquad \vdots$$

$$F_{n1} + F_{n2} + F_{n3} + \cdots + F_{nn} = 1$$

* If any perticalar surface is either flat or Convex then it's self shafe factor become's zero.

* Even now the reciprocity relation between any two surfaces : Ex → $A_1 F_{13} = F_3 F_{31}$

→ $A_2 F_{2n} = F_n A_{n2}$

Q. 21

$F_{11} = 0.1 \qquad A_1 = 4m^2$

$F_{12} = 0.4 \qquad A_4 = 2m^2$

$F_{13} = 0.25$

$F_{11} + F_{12} + F_{13} + F_{14} = 1$

$F_{14} = 1 - 0.1 - 0.4 - 0.25$

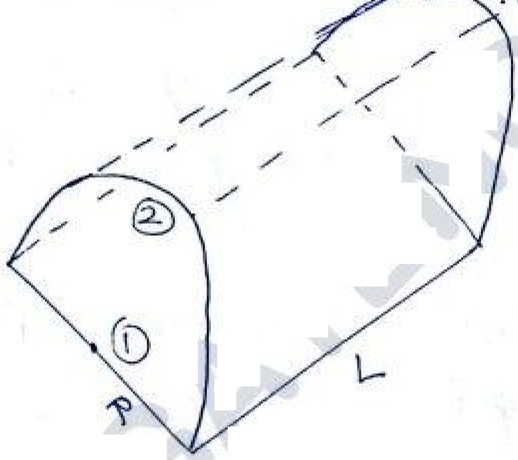
$F_{14} = 0.25$

$A_1 F_{14} = A_4 F_{41}$

$4 \times 0.25 = 2 \times F_{41}$

$F_{41} = 0.25 \times 2 \Rightarrow F_{41} = 0.50$

Quest For a very long semi circular duct as shown in figure calculate F_{11}, F_{12}, F_{21}, F_{22}



open duct (neglecting end effect)

$F_{11} = 0$

$F_{12} = 1$

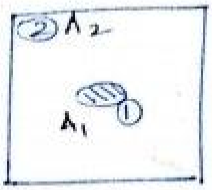
$A_1 F_{12} = A_2 F_{21}$

$$F_{21} = \frac{A_1}{A_2} = \frac{2R \times L}{\pi R \times L} = \frac{2}{\pi}$$

$\cancel{F_{11} + F_{12} = 1}$ $\qquad F_{22} = 1 - \frac{2}{\pi} = \frac{\pi - 2}{\pi}$

Imp

* When a very small body is kept in very large enclosure



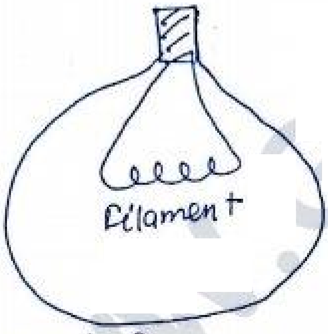
$F_{12} = 1$

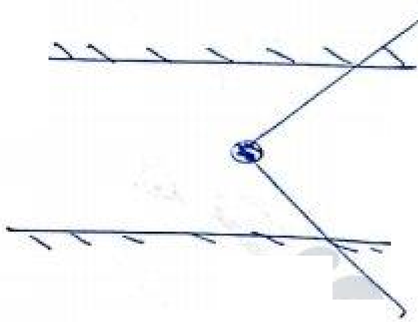
$F_{21} = \frac{A_1}{A_2} \to 0$

But $A_1 <<< A_2$

$A_1/A_2 \to 0$

Ex ① Filament in a bulb ② thermocoule (Bead)

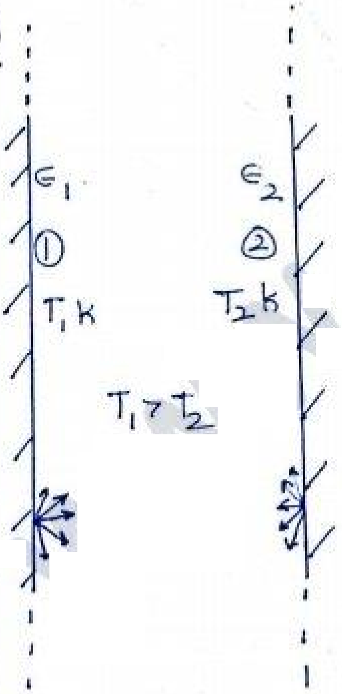




* filament in furnace

Radiation Heat exchange between two Infinitely large parallel planes :-

Case ①



Assume :- ① Steady state H.T. conditions $T \neq f(\text{time})$

② Surface are diffuse and Gray

$\hat{e} \neq f(\text{direction})$ $\quad \epsilon_\lambda = \text{Constant}$

$$\boxed{F_{12} = 1, \quad F_{21} = 1}$$

Net Radiation Heat exchange between ① & ② per unit area

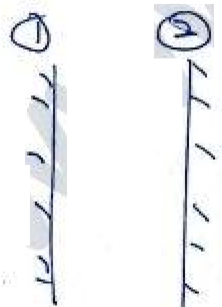
$$\boxed{\left(\frac{q}{A}\right)_{\substack{net\\1-2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}} \quad \text{watt}/m^2$$

if both the plane are black (body)

then $\epsilon_1 = \epsilon_2 = 1$

$$\Rightarrow \quad \boxed{\left(\frac{q}{A}\right)_{\substack{net\\1-2}} = \sigma\left(T_1^4 - T_2^4\right)} \quad \text{watt}/m^2$$

Q 29



400K   300K

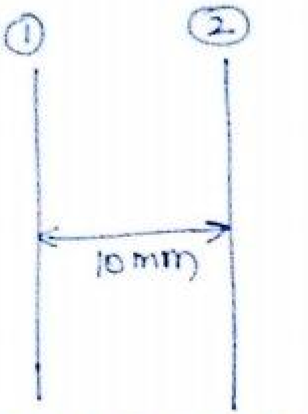
$\epsilon_1 = 0.8$ $\epsilon_2 = 0.8$

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$\left(\frac{q}{A}\right)_{net} = \frac{5.67\times10^{-8}\left((400)^4 - (300)^4\right)}{\frac{1}{0.8} + \frac{1}{0.8} - 1}$$

$$\left(\frac{q}{A}\right)_{net} = 0.66 \ \text{kW}/m^2$$

Q.40



$T_1 = 1000K \quad T_2 = 400K$

$\epsilon_1 = 0.5 \quad \epsilon_2 = 0.25$

$$\left(\frac{q}{A}\right)_{net\,1-2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.67\left(1000^4 - 400^4\right)}{\frac{1}{0.5} + \frac{1}{0.25} - 1}$$

: Gap between plate doesn't matter

$$\left(\frac{q}{A}\right)_{net} = 11049.1 \ W/m^2$$

$$\left(\frac{q}{A}\right)_{net} = 11.0491 \ kW/m^2$$

Q.11

$\epsilon_1 = 0.5 \qquad \epsilon_1 = 0.$

$$\left(\frac{q}{A}\right)_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = 1000 \ W/m^2$$

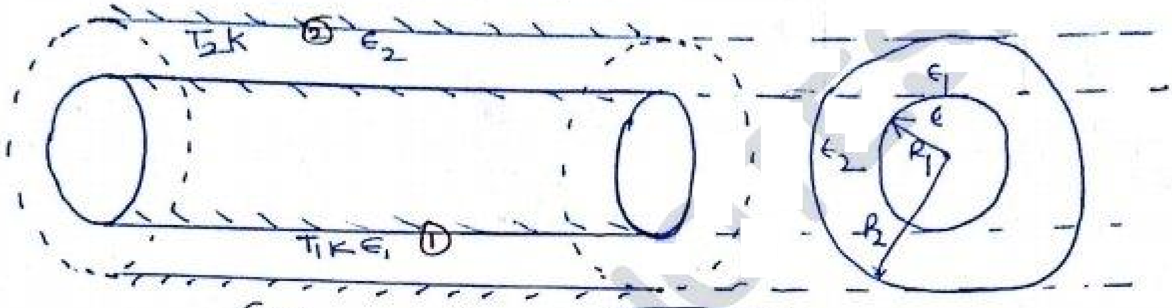
(1) $$\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 = \frac{1}{0.5} + \frac{1}{0.5} - 1$$

$$= 3$$

Now $$\frac{1}{\epsilon_1'} + \frac{1}{\epsilon_2'} - 1 = \frac{1}{0.5} + \frac{1}{0.25} - 1 = 5$$

$$\Rightarrow \frac{1000}{\left(\frac{q}{A}\right)} = \frac{5}{3} \Rightarrow \left(\frac{q}{A}\right)_{net} = 600 \ W/m^2$$

Case ② Radiation Heat Exchange between two Infinitely long Concentric Cylinderical surface:-



$(T_1 > T_2)$ $\boxed{F_{12} = 1}$ $\boxed{F_{21} < 1}$

The outer surface of the inner cylinder at T_1 K having emissivity ϵ_1 is exchanging heat by radiation with the inner surface of outer cylinder at T_2 k having emissivety ϵ_2

Assume:- ① steady state H.T. conditions
i.e. $T \neq f(\text{time})$

② surface are diffuse & Gray ($\epsilon_1 = C$)
$i \neq f(\text{time})$

Then the net Radiation Heat exchange between ① & ②

$$\boxed{(q_{1-2})_{net} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}}$$ watt ~~watt~~.

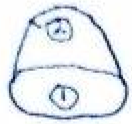
*Also valid when $T_2 > T_1$

Precaution is ① is inner surface ② is outer surface Always

This is Applicable for All radiation Heat exchange cases b/w two surfaces whenever one shape factor b/w them is equal to 1 and other shape factor is less than one.

Example

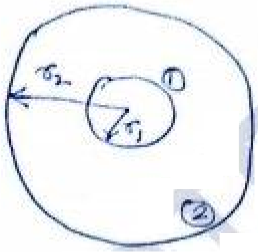
①



$F_{12} = 1$ $\quad \frac{A_1}{A_2} = \frac{1}{2}$

$F_{21} < 1$

② Two concentric sphere

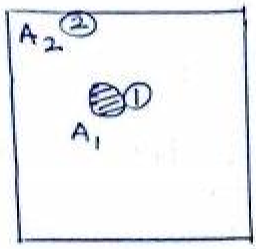


$F_{12} = 1$

$F_{21} < 1$

$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

③ when a very small body is kept in large enclosure



$F_{12} = 1$

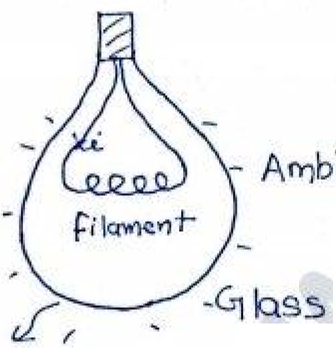
$F_{21} < 1$

$\frac{A_1}{A_2} \to 0$

then

$$\boxed{(q_{net})_{1-2} = \sigma \left(T_1^4 - T_2^4\right) A_1 \epsilon_1 \text{ watt}}$$

Q.8 All the electric power supplied to Fillament will complately converse in heat energy which intern get exchanged by thermal radiation.



For steady state condition of Filament & considering only Radiation Electric power of bulb

P = Net Radiation heat exchange b/w filament & ambient

$$P = \sigma\left(T_{filament}^4 - T_{amb.}^4\right) \epsilon_{filament} \times A_{filament} \text{ watt}$$

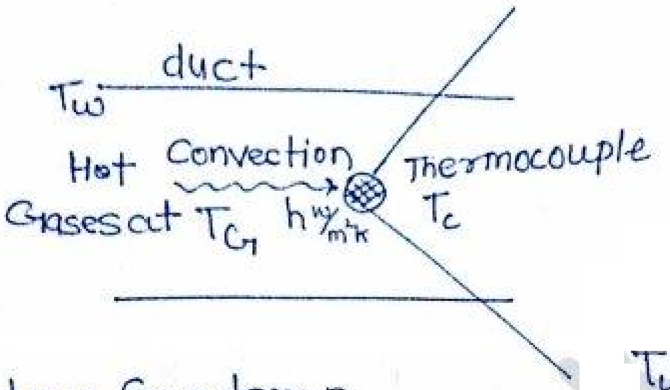
$\frac{A_1}{A_2} \to 0$

$$75 = 5.67 \times 10^{-8}\left(T_{fil}^4 - (343)^4\right) \times 1 \times \pi \times \frac{0.1}{1000} \times \frac{5}{1000}$$

$A_{fila} = \pi d l$

$$T_{fil} = 3029 \text{ K}$$

To Get the error in temprature measurment by using thermocouple.



thermocouple is a very small body in large duct

T_w = wall temp.

T_G = true Gas temp

T_c = temp recorded by thermocouple

The thermocouple bead receive heat by convection from the hot gases with a convective coefficient h, which intern exchange heat by thermal radiation which is duct wall.

For steady state condition of thermocouple bead, writing energy balance

The rate of convection H.T. between Gas & thermocouple is equal (=) Net radiation heat exchange b/w thermocouple and duct wall

$$\Rightarrow \quad hA\left(T_G - T_c\right) = \sigma\left(T_c^4 - T_w^4\right)\epsilon_{T/c} \times A \text{ watt}$$

$\frac{A_{t/c}}{A_{wall}} \rightarrow 0$

$$\boxed{\underbrace{h\left(T_G - T_c\right)}_{\text{Error in measurement}} = \sigma\left(T_c^4 - T_w^4\right) \times \epsilon_{T/c}}$$

Q.39

$T_\omega = 400^\circ C$ $\quad$ $V_{gas} = 4 m/s$

$T_c = 580^\circ C$ $\quad$ $k = 49.72 \times 10^{-3}\ W/m^\circ C$

$\epsilon_c = 0.3$ $\quad$ $\mu = 32.68 \times 10^{-6}\ kg/ms$

$d_c = 1.5\ mm$ $\quad$ $\rho = 0.5224\ kg/m^3$

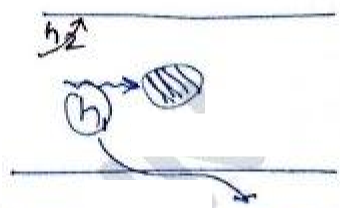
to get convection H.T. coefficient (h)

$$Nu_d = 0.5 (Re_d)^{1/2} \Rightarrow \frac{hd}{k} = 0.5\left(\frac{\rho V d}{\mu}\right)^{1/2}$$

$$Nu_d = \frac{hd}{k} \qquad Re_d = \frac{\rho V d}{\mu}$$

$$Re_d = \frac{0.5224 \times 4 \times 1.5}{32.68 \times 10^{-6} \times 1000}$$

h_1 & h_2 are different



we require this h

so d = dia of bead

$$\frac{h \times 1.5}{49.72 \times 10^{-3} \times 1000} = 0.5 (95.91)^{1/2}$$

$$h = 162.31\ W/m^2 K$$

$$h(T_G - T_c) = \sigma (T_c^4 - T_w^4) \times \epsilon_{T/c}$$

$$162.31 (T_G - 853) = 5.67 \times 10^{-8} \left((853)^4 - (673)^4\right) \times 0.3$$

$$T_G = 886.98\ K$$

$$T_G = 613.98\ ^\circ C$$

Error in measurement $= 613.98 - 580$

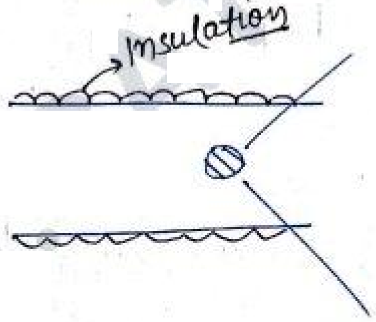
Error $= 33.98\ ^\circ C$

Ans The error in the measurment can be reduced by:-

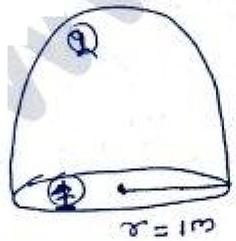
① Increasing the convective H.T. coefficient h i.e. by providing more velocity to Gases.

② By decrease the emissivity of thermocouple bead i.e. by polishing it.

③ By increasing the duct wall temp (T_w) i.e. by insulating the duct from outside.



Q.34



$\epsilon_2 = 1$

$\epsilon_1 = 0.5$

$T_2 = 800K$

$T_1 = 600K$

$\sigma = 5.668 \times 10^{-8}\ W/m^2K^4$

$F_{12} = 1$

$F_{21} < 1$

$$\left(\frac{q}{A}\right)_{net} = \frac{5.668 \times 10^{-8}\left(600^4 - 800^4\right) \times \pi(1)^2}{\frac{1}{0.5} + \frac{1}{2}\left(1 - 1\right)}$$

$$\frac{A_1}{A_2} = \frac{\pi r^2}{2\pi r^2}$$

$$\frac{A_1}{A_2} = \frac{1}{2}$$

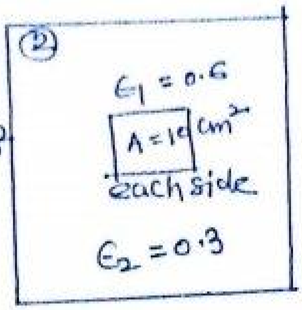
$$\left(\frac{q}{A}\right)_{net} = -24.91\ kW$$

$i-1$

-ve sign shows that net radiation heat exchange is from ② to ①

So $(q/A)_{2-1} = 24.91\ kW$

Q.10



$A_1 = 100\ m^2$

$F_{12} = 1$

$F_{21} < 1$

2nd case

Total Area of plate $= (10+10)\ cm^2$

$A_1 = 20 \times 10^{-4}\ m^2$

$T_1 = 800\ K$

$T_2 = 300\ K$

$$(q_{1-2})_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

$$= \frac{5.67 \times 10^{-8}\,(800^4 - 300^4)}{\frac{1}{0.6} + \frac{20 \times 10^{-4}}{100}\left(\frac{1}{0.3} - 1\right)}$$

$$q_{net} = 27.32 \text{ watt}$$

Q.2

$\cancel{F_{11}}^{\,0} + F_{12} + F_{13} = 1$

flat

$F_{12} = 1 - F_{12}$

$F_{12} = 0.83$

$F_{22} = ?$

From symmetry

$F_{21} = F_{23}$

$F_{21} + F_{22} + F_{23} = 1$

$\therefore F_{22} = 1 - 2(F_{21}) \Rightarrow \underline{F_{22} = 0.585}$

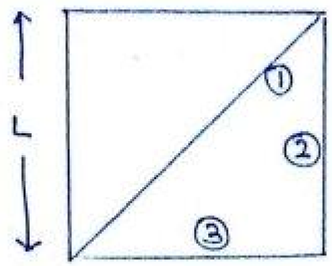
$A_1 F_{12} = A_2 F_{21}$

$F_{21} = \frac{A_1}{A_2} \times F_{12}$

$F_{21} = \frac{\frac{\pi}{4} D^2}{\pi D^2} \times 0.83 \Rightarrow F_{21} = \frac{0.83}{4} \Rightarrow F_{21} = 0.2075$

Q.33 $F_{12} = ?$

$F_{21} = ?$



$F_{12} = F_{13}$

(symetory)

$F_{11} + F_{12} + F_{13} = 1$ ($\because$ Very long duct)

Flat $\cancel{F_{11}}^{0} = 1 - (2F_{12}) \Rightarrow F_{12} = F_{13} = \frac{1}{2}$

$A_1 F_{12} = A_2 F_{21}$

$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2} L \times 1}{L \times 1} \times \frac{1}{2}$

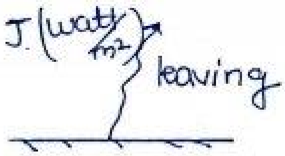
$F_{21} = 0.71$, $F_{12} = 0.50$

Radiation Network

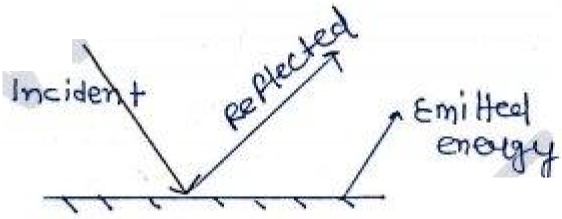
From this radiation network we can calculate the net radiation heat exchange between any two finite or Infinite diffuse gray or black surfaces.

Irradiation (G) and Radiosity (J)

The total thermal Radiation incident upon a surface per unit time per unit area is know as irradiation (G).



The total thermal radiation leaving a surface per unit time per unit area is know as Radiosity (J)



J = Emitted energy + Reflected part of incident energy

$$J = E + \rho G \quad w/m^2$$

$$J = \epsilon E_b + \rho G \quad w/m^2$$

$$\therefore \boxed{J = \epsilon E_b + (1-\epsilon) G} \quad w/m^2$$

for any surface

$$\alpha + \rho + \tau = 1$$

for opaque surface

$$\tau = 0$$

$$\therefore \alpha + \rho = 1$$

$$\Rightarrow \rho = 1 - \alpha$$

But $\alpha = \epsilon$ (Kirchoof)

$$\therefore \boxed{\rho = 1 - \epsilon}$$

$\therefore$ Net Radiation heat exchange between a surface of area (A) and all of its surrounding

$$\underset{\text{surface - surrounding}}{\left(q_{net}\right)} = \left(J - G\right) A \quad \text{watt}$$

$$\left(q_{net}\right) = \left(\epsilon E_b + (1-\epsilon) G - G\right) A \quad \text{watt}$$

$$G = \frac{J - \epsilon E_b}{(1-\epsilon)}$$

Eliminating G, we get

$$\boxed{\underset{\text{surface - surrounding}}{\left(q_{net}\right)} = \frac{E_b - J}{\left(\frac{1-\epsilon}{\epsilon A}\right)}}$$

The equivalent Radiation Circuit is

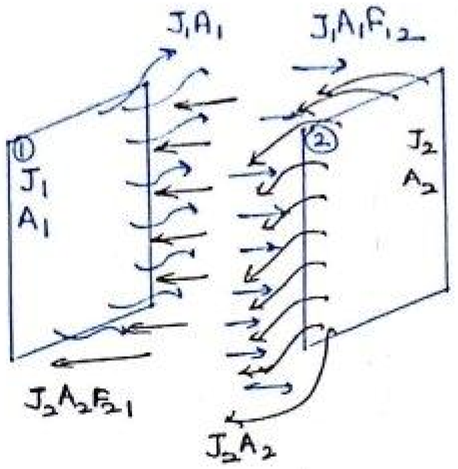
$$E_b \xrightarrow{(q_{1-2})_{net}} J$$

$\left(\frac{1-\epsilon}{\epsilon A}\right)$ = Surface Resistance.

This surface resistance shell exists at every surface which is exchanging heat by radiation with all of it's surrounding.

$\rightarrow$

⇒ Consider two finite surface of Area's A_1 & A_2 having Radiosity J_1 & J_2, exchange heat by thermal radiation as shown in fig



out of the total radiation energy leaving surface one ① J_1A_1 the energy reaches upon ② is $J_1A_1F_{12}$ similarly out of the total radiation energy leaving surface ② J_2A_2 the energy reaches upon ① is $J_2A_2F_{21}$

∴ Net Radiation Heat exchange between ① & ②

$$(q_{1-2})_{net} = (J_1A_1F_{12} - J_2A_2F_{21}) \quad \text{watt}$$

But $A_1F_{12} = A_2F_{21}$ (Reciprocity Relation)

$$\therefore (q_{1-2})_{net} = (J_1 - J_2)A_1F_{12} \quad \text{watt}$$

$$(q_{net})_{1-2} = \frac{(J_1 - J_2)}{\left(\frac{1}{A_1F_{12}}\right)}$$

$$\therefore (q_{1-2})_{net} = \frac{J_1 - J_2}{\left(\frac{1}{A_1 F_{12}}\right)} \text{ watt}$$

$\left(\frac{1}{A_1 F_{12}}\right) \rightarrow$ Space Resistance

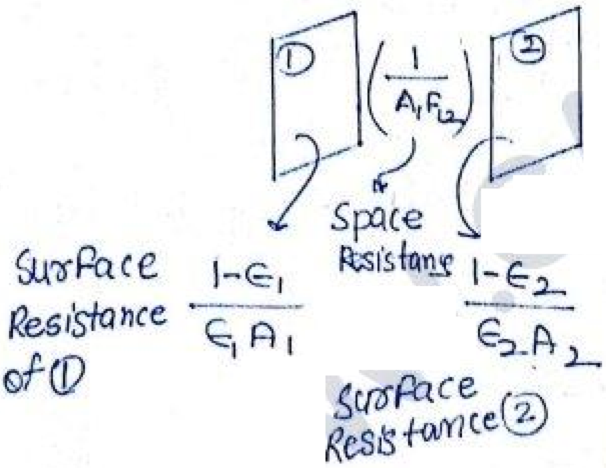
The equivalent Radiation Circuit is

$$J_1 \xrightarrow{(q_{1-2})_{net}} \underset{\left(\frac{1}{A_1 F_{12}}\right)}{\text{www}} \longrightarrow J_2$$

$\left(\frac{1}{A_1 F_{12}}\right) \rightarrow$ Space Resistance

This space resistance shell exists in the space or the Gap ~~prevelli~~ prevailing between 2 surface exchanging heat by radiation.

⇒ Hence the Complate Radiation Network for radiation heat ~~each~~ exchange between two finite Gray surface is Given by.



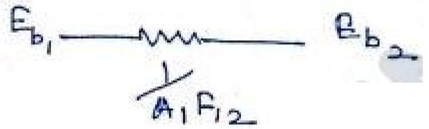
$$E_{b_1} \underset{\left(\frac{1-\epsilon_1}{\epsilon_1 A}\right)}{\text{www}} J_1 \underset{\left(\frac{1}{A_1 F_{12}}\right)}{\text{www}} J_2 \underset{\left(\frac{1-\epsilon_2}{\epsilon_2 A}\right)}{\text{www}} E_{b_2}$$

$$(q_{1-2})_{net} = \frac{E_{b_1} - E_{b_2}}{\sum R}$$

$$\boxed{\left(q_{1-2}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_1 A_2}\right)}} \text{ watt}$$

i.e. $\epsilon_1 = \epsilon_2 = 1$

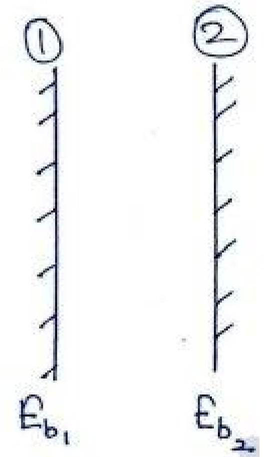
then Network is :-



$$\boxed{\left(q_{1-2}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(1/A_1 F_{12}\right)}} \text{ Watt}$$

Appication of Network Resistance :-

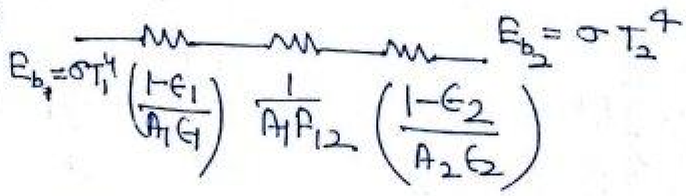
Case ① Infinitely large Parallel plates :-



Since large planes we always calculate Radiation flux i.e. $\left(q/A\right)$

Put $\Rightarrow A_1 = A_2 = 1$

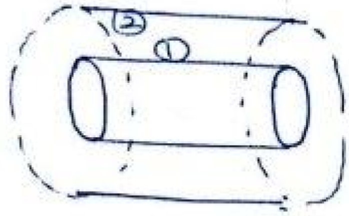
Radiation Network



$$\left(q/A\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1 \times 1} - \frac{1}{1} + \frac{1}{1 \times 1} + \frac{1}{\epsilon_2 \times 1} - 1}$$

$$\boxed{\left(q/A\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}}$$

Case - 2 similarly Case ② formula can be obtain by substituting $F_{12} = 1$ in Radiation Network.



$F_{12} = 1$

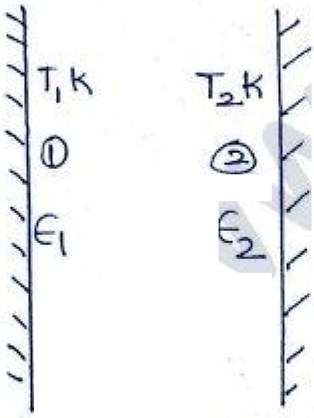
$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 A_2} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{A_1\epsilon_1} - \cancel{\frac{1}{A_1}} + \cancel{\frac{1}{A_1}} + \frac{1}{A_2\epsilon_2} - \frac{1}{A_2}}$$

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right)A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

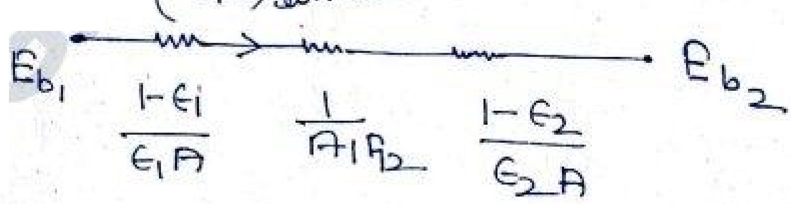
Case ③

Radiation shield :-



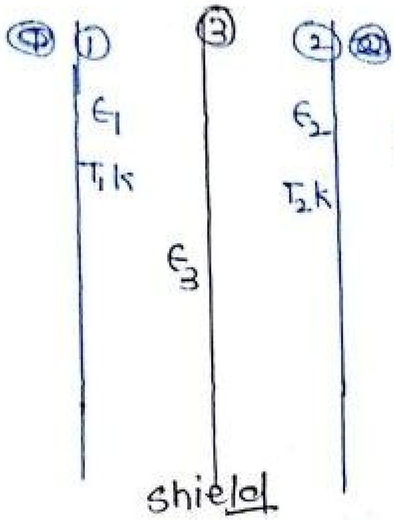
Infinitely large Plate $F_{12} = F_{21} = 1$

(q_{1-2}) without shield



$$\left(\frac{q}{A}\right)_{net\ \text{without shield}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Placing a shield between ① & ②



total 6 Resistance

$$E_{b_1} \quad E_{b_3} = T_3 \quad E_{b_2}$$

$$\left(\frac{1-\epsilon_1}{\epsilon_1 A_1}\right)\left(\frac{1}{A_1 F_{13}}\right)\left(\frac{1-\epsilon_3}{\epsilon_3 A_3}\right)\left(\frac{1-\epsilon_3}{\epsilon_3 A_3}\right)\left(\frac{1}{A_3 F_{32}}\right)\left(\frac{1-\epsilon_2}{\epsilon_2 A_2}\right)$$

Put $A_1 = A_2 = A_3 = 1$ ($\because$ flux being calculated)

$F_{13} = F_{32} = 1$ (large plate.

$$\boxed{\left(\frac{q}{A}\right)_{1-2\,net} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{2}{\epsilon_3} + \frac{1}{\epsilon_2} - 2\right)}} \quad \text{Watt}/m^2$$

with one shield

∴ To Reduce the radiation heat exchange by a very good percentage by keeping the shield, the shield kept between the plane must have very low emissity or very high reflectivity for which highly polished Al, Cu Shield Generaly used

Polishing ↑ Reflectivity (ρ)

$$\left(\boxed{\underset{low}{\epsilon} = 1 - \underset{high}{\rho}}\right)$$

* Each sheild kept between the plate shell bring in three additional resistance extra into the network out of which 2 are Surface resistance and the remaining one is space resistance.

* Hence if there are 'n' no. of shield kept between the plates there would be total $(2n+2)$ no. of surface resistance and $(n+1)$ no. of space resistance in the entire ~~net~~ radiation network drawn with 'n' shield.

Special case :-

In case if all the surface have the same emissivity i.e. $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$

without any shield

$$\boxed{\left(\frac{q}{A}\right)_{1\text{-}2\,net} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{2}{\epsilon} - 1\right)}}$$

with one shield

$$\boxed{\left(\frac{q}{A}\right)_{1\text{-}2\,net} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{4}{\epsilon} - 2\right)}} = \left(\frac{q}{A}\right)_{\text{without shield}} \times \frac{1}{2}$$

50% drop

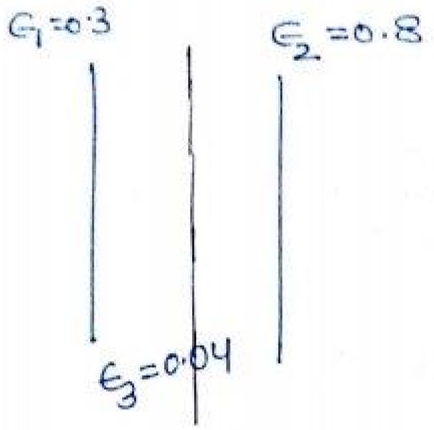
If there are 'n' no. of shield being used then

$$\boxed{\left(\frac{q}{A}\right)_{\substack{net\\1\text{-}2\\\text{with 'n' shields}}} = \frac{1}{(n+1)}\left(\frac{q}{A}\right)_{\substack{net\\1\text{-}2\\\text{without shield}}}}$$

No. of shield	% Reduction
1	50%
2	66.66%
3	75%
4	80%

Provided {all ϵ same}

Q. 37



without shield

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{0.3} + \frac{1}{0.8} - 1}$$

$$\left(\frac{q}{A}\right)_{net} = \sigma(T_1^4 - T_2^4) \times 0.2790$$

with shield

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{2}{\epsilon_3} + \frac{1}{\epsilon_2} - 2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{0.3} + \frac{2}{0.04} + \frac{1}{0.8} - 2} = 0.0190\,\sigma(T_1^4 - T_2^4)$$

$$\%\ \text{Reduction} = \frac{0.2790 - 0.0190}{0.0190} \times 100 = 93.189\ \%$$

Q. 36 All emissivities are same

the Ratio $= \frac{1}{2}$

Q. 25 & 26   large Plate

① $\epsilon_1 = 1, T_1 = 1000K$

② $\epsilon_2 = 0.7, T_2 = 500K$

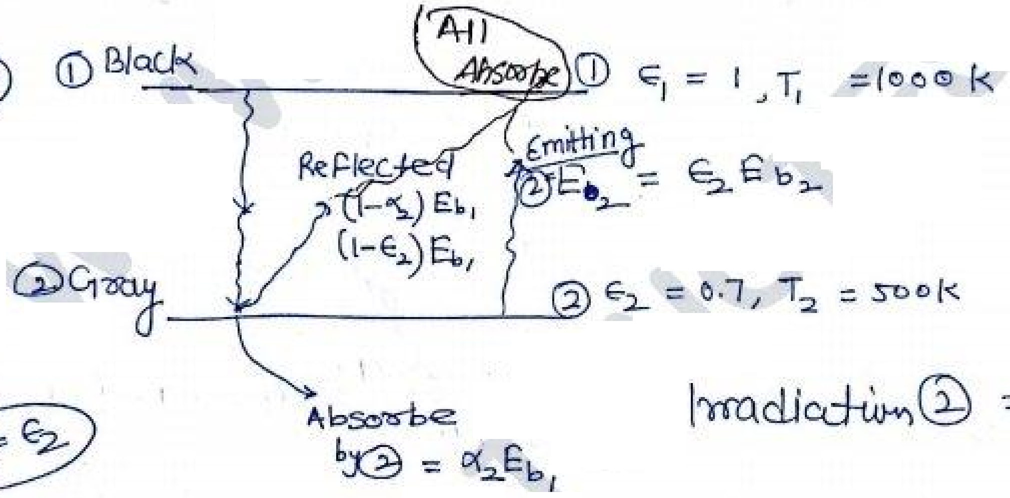
(26) If $\epsilon_1 = 0.8$
$\epsilon_2 = 0.7$

$$\left(\frac{q}{A}\right)_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{5.67\times10^{-8}(1000^4 - 500^4)}{\frac{1}{0.8} + \frac{1}{0.7} - 1}$$

$$\left(\frac{q}{A}\right)_{net} = 31667.55\ W/m^2$$

$$= 31.7\ KW/m^2$$

(25)



$F_{12} = 1$

$\alpha_2 = \epsilon_2$

Irradiation ② $= \underline{E_{b_1}}$

Irradiation of ① = total thermal Radiation incident upon ①

$$= (1-\epsilon_2)E_{b_1} + \epsilon_2 E_{b_2}$$

$$= (1-\epsilon_2)\sigma T_1^4 + \epsilon_2 \sigma T_2^4$$

$$= (1-0.7)\times5.67\times10^{-8}\times(1000)^4 + (0.7)(5.67\times10^{-8})(500)^4$$

Irradiation $= 19490.7\ W/m^2$

Irradiation ① $= 19.490\ kW/m^2$

Q.15

$J = E + \rho G$

$J = 32 + (0.6)(93)$

$J = 87.8 \ W/m^2$

Q.16

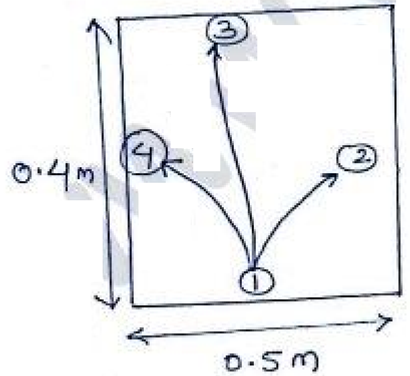
$J = E + \rho G$

$\Rightarrow J = E + (1-\epsilon) G$

$12 = 10 + (1-\epsilon) 20$

$\epsilon = 0.9$

Q.41



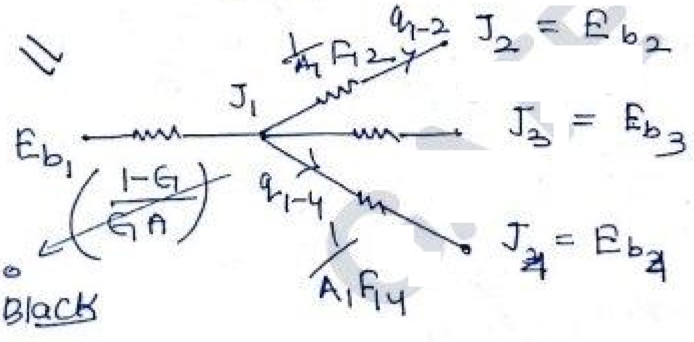
$(\epsilon = 1)$ Black

$F_{12} = 0.26$

$F_{11} = 0$

$F_{12} = F_{14} = 0.26$

$F_{13} =$



$E_{b_1} = E_{b_3}$

$(\because T_1 = T_3)$

total Radiation Heat loss $= q_{1-2} + q_{1-4}$

$= 2(q_{1-2}) = \frac{\cancel{2} \times 5.67 \times 10^{-8} (1200^4 - 800^4)}{\cancel{2 \times 10} \ 0.5 \times 1 \times 0.26}$

$(q_1)_{net} = \cancel{98122.75}$ $24530.688 \ W/m^2$

Q.38

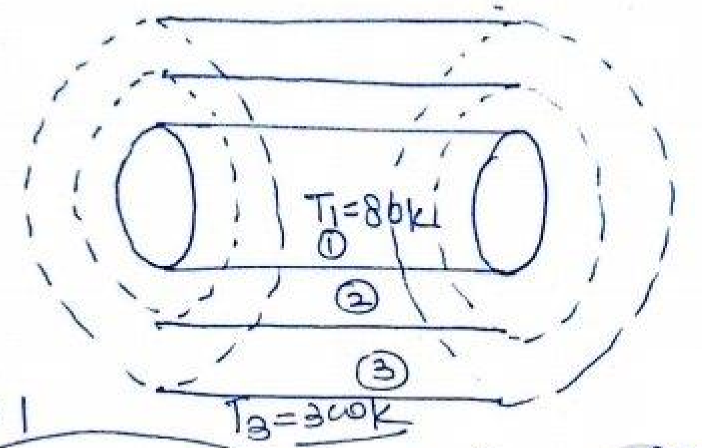
$\epsilon_1 = 0.05$

$\epsilon_2 = 0.1$

$\epsilon_3 = 0.2$

$F_{12} = 1$

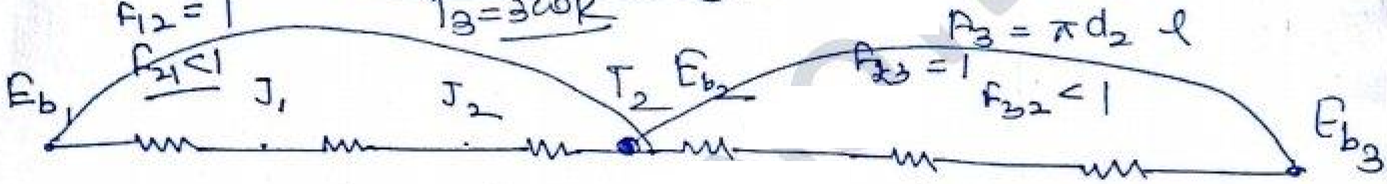
$F_{23} = 1$



$A_1 = \pi d_1 l$

$A_2 = \pi d_2 l$

$A_3 = \pi d_2 l$



$$\frac{1-\epsilon_1}{A_1\epsilon_1} \quad \frac{1}{A_1F_{12}} \quad \frac{1-\epsilon_2}{A_2\epsilon_2} \quad \frac{1-\epsilon_2}{A_2\epsilon_2} \quad \frac{1}{A_2F_{23}} \quad \frac{1-\epsilon_3}{A_3\epsilon_3}$$

$$\frac{A_1}{A_2} = \frac{R_1}{R_2}$$

$$\frac{A_2}{A_3} = \frac{R_2}{R_3}$$

$$\frac{E_{b_1} - E_{b_2}}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}} = \frac{E_{b_2} - E_{b_3}}{\frac{1-\epsilon_2}{A_2\epsilon_2} + \frac{1}{A_2F_{23}} + \frac{1-\epsilon_3}{A_3\epsilon_3}}$$

$$\frac{\sigma T_1^4 - \sigma T_2^4}{\frac{1-0.05}{0.150\times0.05} + \frac{1}{0.150\times1} + \frac{1-0.1}{0.250\times0.1}} = \frac{\sigma T_2^4 - \sigma T_3^4}{\frac{1-0.1}{0.1\times0.250} + \frac{1}{0.25\times1} + \frac{1-0.2}{0.35\times0.02}}$$

$$\underline{T_2 = 280.73\ K}$$

or

$$\frac{\sigma\left(T_1^4 - T_2^4\right)A_1}{1/\epsilon_1 + A_1/A_2\left(1/\epsilon_2 - 1\right)} = \frac{\sigma\left(T_2^4 - T_4^4\right)A_2}{1/\epsilon_2 + A_2/A_3\left(1/\epsilon_3 - 1\right)}$$

$$A_1/A_2 = \frac{d_1}{d_2} \quad \& \quad \frac{A_2}{A_3} = \frac{d_2}{d_3}$$

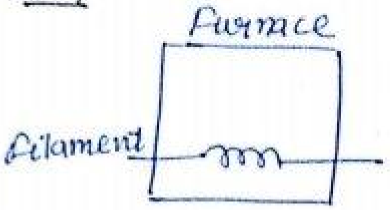
$$T_2 = 280.6\ \underline{K}$$

(6)

$$E = \int_0^\infty E_\lambda d\lambda = 150(12-3) + 300(25-12)$$

$$E = 5250 \ w/m^2$$

Q.9



element is very small in large furnace

$$q_{1-2} = \sigma(1073^4 - 573^4) \times \epsilon_{ele} \times 1 = 8 \ kw/m^2 \quad -(1)$$

$$q'_{1-2} = (1273^4 - 573^4) \times \epsilon_{ele} \times 1 = 16.5 \ kw/m^2 \quad -(2)$$

Q.18

$$E_b = \pi i_n = \sigma T^4$$

$$i_n = \frac{\sigma T^4}{\pi} = \left(\frac{\sigma \times 800^4}{\pi}\right) \ w/m^2\text{-steradian.}$$

$$i = i_n \cos 30^\circ = 4750 \ watt/m^2\text{-steradian}$$