

Quadratic Equations

21. Solve the given equation for :

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$$

2011/2012/2014 [3 marks]

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{a+b+x} - \frac{1}{x}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{x-(a+b+x)}{x(a+b+x)} = \frac{-(a-b)}{x(a+b+x)}$$

$$\Rightarrow \frac{1}{ab} = -\frac{1}{x(a+b+x)}$$

$$\Rightarrow x(a+b+x) = -ab$$

$$\Rightarrow x^2 + ax + bx + ab = 0$$

$$\Rightarrow x(x+a) + b(x+a) = 0$$

$$\Rightarrow (x+a)(x+b) = 0$$

$$\Rightarrow (x+a) = 0 \text{ or } (x+b) = 0$$

$$\text{So, } x = -a \text{ or } x = -b$$

22) A two digit number is such that the product of the digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

2011/2015/2015 [3 Marks]

Let units digit be x. then tens digits = $\frac{18}{x}$

$$\text{Number} = \frac{180}{x} + x.$$

On interchanging the digits, the number becomes $10x + \frac{18}{x}$

$$\text{Therefore, } \frac{180}{x} + x - 63 = 10x + \frac{18}{x}$$

$$\Rightarrow 9x^2 + 63x - 162 = 0$$

$$\Rightarrow x^2 + 7x - 18 = 0$$

$$\Rightarrow x^2 + 9x - 2x - 18 = 0$$

$$\Rightarrow x(x+9) - 2(x+9) = 0$$

$$\Rightarrow (x+9)(x-2) = 0$$

$$x = -9 \text{ or } x = 2$$

∴ neglecting the negative value of x as the digits is positive, $x = 2$

$$\text{Hence, the number} = \frac{180}{2} + 2 = 90 + 2 = 92$$

23) A certain group of students uses the internet services for a monthly charge of Rs. 4800. If 4 more students join the group, each person would pay Rs.200 less. Find the number of students in the group in the beginning.

2014/2015 (4 Marks)

Let the number of students in the group in the beginning be x .

So, share of each student for using internet services in a month = Rs. $\frac{4800}{x}$

As per given condition,

$$(x + 4) \left(\frac{4800}{x} - 200 \right) = 4800$$

$$\text{Or } 4800 - 200x + \frac{19200}{x} - 800 = 4800$$

$$\text{Or } \frac{-200x^2 + 19200}{x} = 800$$

$$\text{Or } -200x^2 + 19200 = 800x$$

$$\text{Or } x^2 - 96 = -4x$$

$$\text{Or } x^2 + 4x - 96 = 0$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times (-96)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 4 \times 96}}{2}$$

$$\Rightarrow x = \frac{-4 \pm 4\sqrt{1+24}}{2} = \frac{-4 \pm 20}{2}$$

$$\Rightarrow x = \frac{-24}{2} = -12$$

$$\Rightarrow x = \frac{16}{2} = 8$$

i.e. $x = 8$ or $x = -12$.

Since number of students cannot be negative,

Therefore number of students in the beginning = 8.

24) If the equation $(1 + m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ of x has equal roots, prove that

$$c^2 = a^2(1 + m^2).$$

2012/2014 (3 marks)

$(1 + m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$, It has equal roots.

$$\therefore D = 0 \text{ or } b^2 = 4ac \Rightarrow (2mnc)^2 = 4\{(1 + m^2)n^2\}\{c^2 - a^2\}$$

$$4m^2n^2c^2 = 4(n^2)(1 + m^2)(c^2 - a^2)$$

$$\Rightarrow m^2c^2 = c^2 - a^2 + m^2c^2 - m^2a^2 \Rightarrow 0 = c^2 - a^2 - m^2a^2$$

$$\Rightarrow c^2 = a^2(1 + m^2).$$

25) -4 is a root of the quadratic $x^2 + kx - 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots. Find the values of p and k.

2012/2015 (3 marks)

$$k = 3, p = \pm 2\sqrt{3}.$$
