Sample Paper 17

Class- X Exam - 2022-23

Mathematics - Basic

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section - A consists of 20 questions of 1 mark each.)

1. **The H.C.F. of the smallest prime number and the smallest composite number is:**

(a) 1 (b) 0

(c) 4 (d) 2 1

2. **A quadratic equation, sum of whose roots is $-3\sqrt{2}$ and their product is 4 is:**

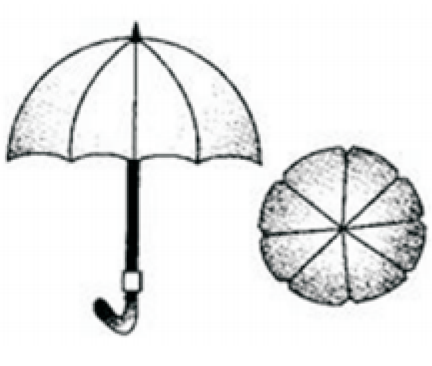
(a) $x^2 + 4x = 0$

(b) $x^2 + 4\sqrt{2}\,x + 3 = 0$

(c) $x^2 + 3\sqrt{2}\,x + 4 = 0$

(d) $x^2 + 3\sqrt{2}\,x - 4 = 0$ 1

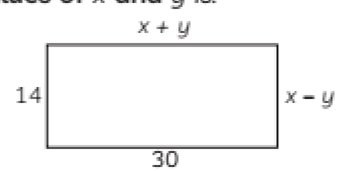
3. **An umbrella has eight evenly spaced ribs (see figure). The space between the umbrella's two consecutive ribs using the assumption that the umbrella is a flat circle with a radius of 45 cm is:**



(a) $\frac{22275}{28}$ cm^2 (b) $\frac{23456}{28}$ cm^2

(c) 5923 cm^2 (d) 2986 cm^2 1

4. **From the adjoining figure of a rectangle, the values of x and y is:**



(a) 22, 8 (b) 30, 8

(c) 20, 6 (d) 25, 4 1

5. **A point which divides the join of A (–3, 4) and B (9, 6) internally in the ratio 3 : 2 is:**

(a) $\frac{15}{2}, \frac{-16}{3}$ (b) $\frac{21}{5}, \frac{26}{5}$

(c) 0, 0 (d) $\frac{-20}{3}, \frac{25}{3}$ 1

6. **If the difference of the roots of the quadratic equation $x^2 - 7x + 2k = 0$ is 1, then the value of k is:**

(a) 2 (b) 4

(c) 6 (d) 8 1

7. If the distance between the points P(2, – 3) and Q(10, y) is 10 units, then the value of 'y' is:

(a) –3, 6 (b) 21, 26
(c) 4, 3 (d) –9, 3 1

8. The value of 'k' for which the linear equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ represent coincident lines is:

(a) 1 (b) 2
(c) 3 (d) 4 1

9. A lighthouse projects a red light over an area of angle 80 at a distance of 16.5 kilometres to alert sailors to the presence of submerged rocks. The area of the sea over which the ships are warned is:

(a) 128.87 km^2 (b) 156.87 km^2
(c) 5923 km (d) 189.97 km^2 1

10. If the curved surface area of a sphere is 4π sq m., then the diameter of the sphere is:

(a) 2 cm (b) 3 cm
(c) 4 cm (d) 6 cm 1

11. A $\triangle$ABC is right angled at C, then the value of cos (A + B) is:

(a) 1 (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) 0 1

12. What is surface area of the resultant cuboid, obtained on joining 2 identical cubes each of edge 2 cm ?

(a) 42 sq. cm (b) 40 sq. cm
(c) 30 sq. cm (d) 35 sq. cm 1

13. A letter is drawn at random from the letters of the word ERROR. What is probability that the drawn letter is R ?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$ 1

14. If cos (A + B) = 0 and sin (A – B) = $\frac{\sqrt{3}}{2}$, then the value of A is:

(a) 30° (b) 60°
(c) 75° (d) 80° 1

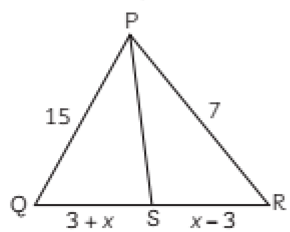
15. In $\triangle$ ABC, right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, then the value of (sin A cos C + cos A sin C) is:

(a) 1 (b) 2
(c) 3 (d) 4 1

16. If a fair dice is thrown once, the probability of getting a number which is even as well as prime is:

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$
(c) $\frac{1}{6}$ (d) $\frac{1}{5}$ 1

17. In the given figure PS is the bisector of $\angle$QPR of $\triangle$PQR. If PQ = 15, PR = 7, QS = $3 + x$ and SR = $x - 3$, the value of x is:



(a) $\frac{3}{2}$ (b) 4
(c) $\frac{5}{2}$ (d) $\frac{33}{4}$ 1

18. If 6^{th} term and 8^{th} term of an A.P. are 12 and 22 respectively, then its 2^{nd} term is:

(a) –8 (b) +8
(c) 0 (d) 1 1

Direction for questions 19 and 20: In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

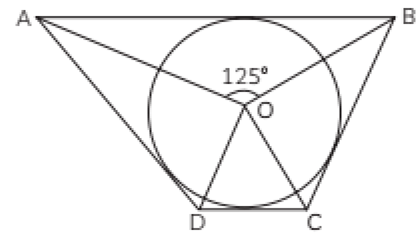
Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : The values of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is ± 2.

Reason (R) : If roots are equal, the discriminant is 0. 1

20. Assertion (A) : In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to 55°



Reason (R) : Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angle of the centre of circle. 1

SECTION - B

10 marks

(Section - B consists of 5 questions of 2 marks each.)

21. Using prime factorisation, find the LCM of 90 and 120.

OR

Explain why $3 \times 5 \times 7 \times 11 + 11$ is a composite number. 2

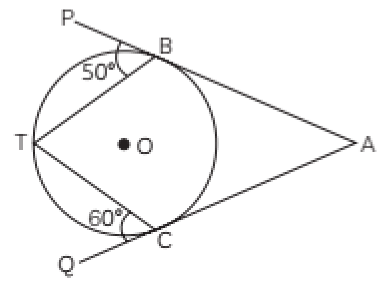
22. Using the quadratic formula, find the roots of the quadratic equation: $x^2 + x - 12 = 0$. 2

23. If $\tan A = \frac{7}{24}$, find the value of sin A cos A.

OR

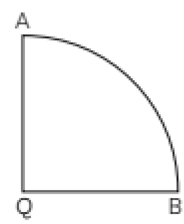
Prove that: $\sin^2 A + \sin^2 A \tan^2 A = \tan^2 A$. 2

24. In the given figure, ABP and ACQ are two tangents to a circle with centre O. If $\angle TBP = 50°$ and $\angle TCQ = 60°$, then find the measure of $\angle BTC$.



2

25. The perimeter of a sheet of paper which is in the shape of a quadrant of a circle, is 75 cm. Find its area.



2

SECTION - C

18 marks

(Section - C consists of 6 questions of 3 marks each.)

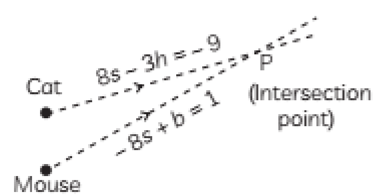
26. If $\sin\theta + \cos\theta = \sqrt{3}$, prove that $\tan\theta + \cot\theta = 1$.

OR

Evaluate: $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

3

27. A computer animation below shows a cat moving in a straight line.



Its height, h metres, above the ground, is given by $3s - 3h = -9$, where s is the time in seconds after it starts moving. In the same animation, a mouse starts to move at the same time as the cat and its movement is given by $-3s + h = 1$.

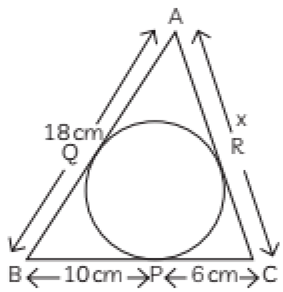
(A) Draw the graph of the two equations on the same sheet of graph paper;

(B) Will the mouse be able to catch the cat?

(C) If yes, after how much time and at what height? 3

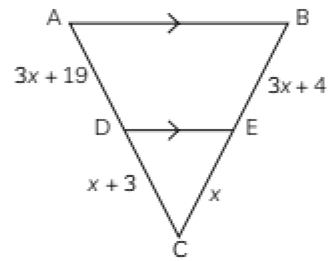
28. If the points A (1, –2), B (2, 3), C (a, 2) and D (–4, –3) from a parallelogram, find the value of a. 3

29. In the figure, all three sides of a triangle ABC touch the circle at points P, Q and R. Find the value of x.



3

30. Find the value of x for which DE || AB in the figure given below:



3

31. The product of the LCM and HCF of two numbers is 24. If the difference of the two number is 2, find the numbers.

OR

How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead having dimensions 66 cm × 42 cm × 21 cm. 3

SECTION - D

20 marks

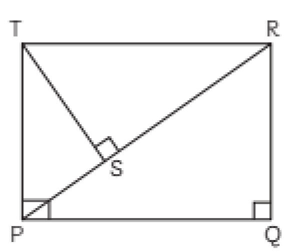
(Section - D consists of 4 questions of 5 marks each.)

32. The first term of an A.P. is 5, the last term is 45 and the sum of its all terms is 400. Find the number of terms of the A.P. and also the common difference. 5

33. A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 metres from its root. Find the whole height of the tree [Use $\sqrt{3}$ = 1.732].

OR

In the figure, RQ ⊥ PQ, PQ ⊥ PT and ST ⊥ PR. Prove that: ST × QR = PS × PQ.



5

34. Sumit arranges to pay off a debt of ₹ 3,60,000 by 40 installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one – third of the debt unpaid. Determine the first installment's value.

OR

Calculate the median for the following data:

Class	Frequency
More than or equal to 150	0
More than or equal to 140	12
More than or equal to 130	27
More than or equal to 120	60
More than or equal to 110	105
More than or equal to 100	124
More than or equal to 90	141
More than or equal to 80	150

5

35. Compute the mean and mode of the following data:

Marks obtained	25-35	35-45	45-55	55-65	65-75	75-85
Number of students	7	31	33	17	11	1

5

SECTION - E

12 marks

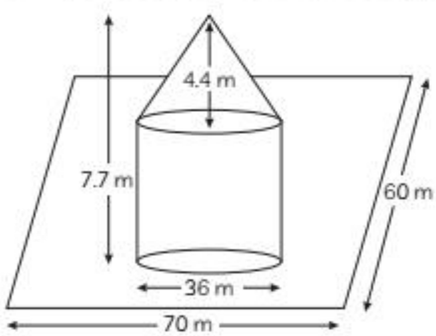
(Case Study Based Questions)

(Section - E consists of 3 questions. All are compulsory.)

36. *Uttar Bantra Sarbojanin Durgotsav Committee had started planning for their Durga puja a year in advance with a mega budget in mind.*

Bholeram Tents is given a contract by the municipal corporation of Budaun (Uttar Pradesh), India to setup a mega function pandal (tent). The architect has designed a tent of height 7.7 m in the form of a right circular cylinder of diameter 36 m and height 4.4 m surmounted by a right circular cone. This tent is setup in a rectangular park of dimensions 70 m × 60 m as shown below.

The tent is made of canvas. (Take $\pi = 3.14$)



On the basis of the above information, answer the following questions:

(A) **For the workers to finalise the purchase of material find the height of the conical part.** 1

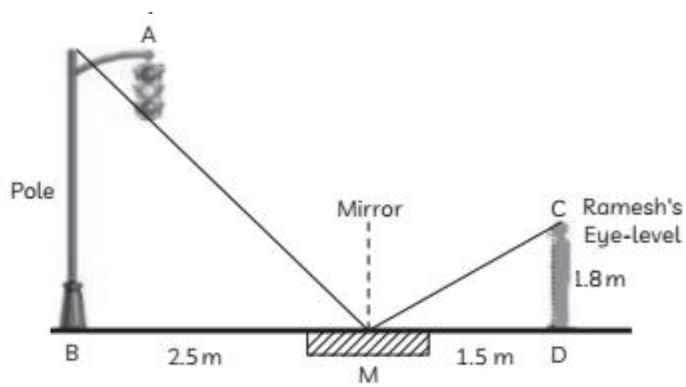
(B) **Find the slant height of the conical part.** 1

(C) **To purchase the canvas, what is the area of the canvas to be used approx in making the tent.**

OR

Find the cost of canvas at ₹ 4.50 sq m and the area of the rectangular park outside the tent? 2

37. *Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it) (see the figure). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.8 m above the ground. The distance of Ramesh and the pole from the mirror are 1.5 m and 2.5 m respectively.*



On the basis of the above information, answer the following questions:

(A) **Which criterion of similarity is applicable to similar triangles?** 1

(B) **Find the height of the pole.** 1

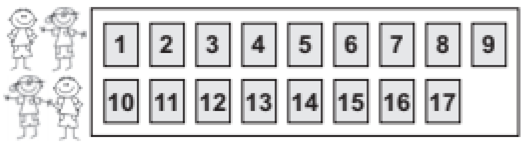
(C) **If Ramesh's eye level is 1.2 m above the ground, then find the height of the pole.**

OR

If the distance of Ramesh and the pole from the mirror are 2.5 m and 1.5 m respectively, then find the height of the pole. 2

38. *4 boys are having a night in and one of the boy's mother decides to play a game. 17 cards numbered 1, 2, 3 ... 17 are put in a box and mixed thoroughly.*

The mother asks each boy to draw a card and after each draw, the card is replaced *back in the box. She shows some magic tricks and at the end, decides to test their mathematical skills.*



On the basis of the above information, answer the following questions:

(A) Find the probability of drawing an odd number card in the first draw by the first boy. 1

(B) Find the probability of drawing a prime number card in the second draw by the second boy. 1

(C) If the card is not replaced after the second draw, find the probability of drawing a card bearing a multiple of 3 greater than 4 in the third draw by the third boy.

OR

If the card is replaced after the third draw, find the probability of drawing a card bearing a number greater than 17 in the fourth draw by the fourth boy and if the card is replaced after the fourth draw, find the probability of drawing a card bearing a multiple of 3 or 7 in the fifth draw by the fourth boy. 2

SOLUTION

SECTION - A

1. *(d) 2*

Explanation: Smallest prime number = 2; smallest composite number = 4

$\therefore$ HCF (2, 4) = 2.

2. *(c)* $x^2 + 3\sqrt{2}x + 4 = 0$

Explanation: A quadratic equation with sum and product of roots as S and P, respectively, is given as $x^2 - Sx + P = 0$ So, a quadratic equation in x whose sum of roots is $-3\sqrt{2}$ and product of roots is 4,

i.e. $x^2 + 3\sqrt{2}x + 4 = 0$

3. *(a)* $\frac{22275}{28}$ cm^2

Explanation: Here, r = 45 cm

and $\theta = \frac{360°}{8} = 45°$

Area between two consecutive ribs of the umbrella

$$= \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{45°}{360°} \times \frac{22}{7} \times 45 \times 45 = \frac{22275}{28} \text{ cm}^2.$$

4. *(a) 22, 8*

Explanation: Since, the figure given is a rectangle,

$\therefore$ $x + y = 30$; $x - y = 14$

Solving the two equations, we get

$x = 22$; $y = 8$.

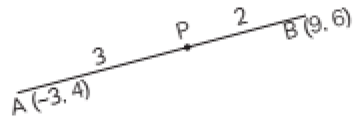
⚠ Caution

➥ *Derive the value of either x or y, but do which is more convenient and don't mess up the process.*

5. *(b)* $\frac{21}{5}, \frac{26}{5}$

Explanation: Let P (x, y) be the required point.

$$\therefore \text{P}(x, y) = \left(\frac{3 \times 9 + 2 \times (-3)}{3+2}, \frac{3 \times 6 + 2 \times 4}{3+2}\right)$$



$$= \left(\frac{21}{5}, \frac{26}{5}\right)$$

6. *(c) 6*

Explanation: Let α and β be the roots of the given quadratic equation.

Then, $\alpha + \beta = -\frac{(-7)}{1} = 7$ ...(i)

and $\alpha\beta = \frac{2k}{1} = 2k$ ...(ii)

It is given that,

$$\alpha - \beta = 1$$

$\Rightarrow (\alpha - \beta)^2 = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow (7)^2 - 4 \times 2k = 1$ [Using (i) and (ii)]

$\Rightarrow 49 - 8k = 1$

$\Rightarrow -8k = -48$

$\Rightarrow k = 6$

7. *(d) –9, 3*

Explanation: Here,

Distance $= \sqrt{(10 - 2)^2 + (y + 3)^2} = 10$

Squaring both sides, we get

$64 + y^2 + 9 + 6y = 100$

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow y + 9 = 0, \quad y - 3 = 0$

$\Rightarrow y = -9, 3.$

8. *(b) 2*

Explanation: The given equations represent coincident lines, when

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

i.e. when $k = 2$.

9. *(d) 189.97 km²*

Explanation: Here, $r = 16.5$ km and $q = 80°$

The area of sea over which the ships are warned

$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{80°}{360°} \times 3.14 \times 16.5 \times 16.5$

$= 189.97 \text{ km}^2$

10. *(a) 2 cm*

Explanation: Let the radius of the sphere be 'r' cm. Then,

CSA $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = 4\pi$

$\Rightarrow r^2 = 1$

$\Rightarrow r = \pm 1$

$\Rightarrow r = 1$ [$\because$ r cannot be negative]

$\Rightarrow$ Diameter of the sphere is 2 cm.

11. *(d) 0*

Explanation: In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B = 180° - \angle C$

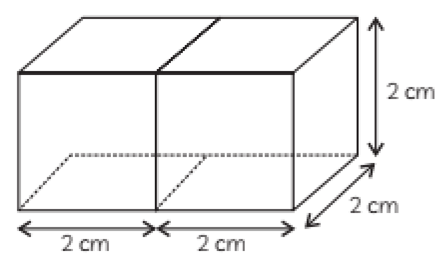
$= 180° - 90°$

$= 90°$

So, $\cos(A + B) = \cos 90° = 0$

12. *(b) 40 sq. cm*

Explanation: The dimensions of the resulting cuboid are 4 cm × 2 cm × 2 cm.



So, its total surface area

$= 2(lb + bh + lh)$

$= 2(8 + 4 + 8)$

$= 40$ sq. cm.

13. *(c) $\frac{3}{5}$*

Explanation: In the given word 'ERROR', R occurs three times.

So, $P(R) = \frac{3}{5}$

14. *(c) 75°*

Explanation:

$\cos(A + B) = 0 \quad \Rightarrow A + B = 90°$

$\sin(A - B) = \frac{\sqrt{3}}{2} \quad \Rightarrow A - B = 60°$

Solving the above equation we get

$A = 75°$

⚠ **Caution**

➥ *You can apply any method for solving the two linear equations to find value of A.*

15. *(a) 1*

Explanation: Given:

$$\tan A = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad A = 30°$

$\Rightarrow \quad C = 60°$

$[\because \angle B = 90°]$

So, $(\sin A \cos C + \cos A \sin C)$

$= \sin 30° \cos 60° + \cos 30° \sin 60°$

$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4}$

$= 1$

⚠ **Caution**

➥ *Learn the table of trigonometric ratios for specific angles properly for solving such types of questions.*

16. *(c)* $\frac{1}{6}$

Explanation: Since 2 is the only number which is even as well as prime,

$\therefore$ Required probability is $\frac{1}{6}$.

17. *(d)* $\frac{33}{4}$

Explanation: Since, PS is the bisector of $\angle P$

$\therefore$ By angle-bisector theorem.

$\Rightarrow \quad \frac{PQ}{QS} = \frac{PR}{RS}$

$\Rightarrow \quad \frac{15}{3+x} = \frac{7}{x-3}$

$\Rightarrow \quad 15(x - 3) = 7(3 + x)$

$\Rightarrow \quad 15x - 45 = 21 + 7x$

$\Rightarrow \quad 8x = 66$

$\Rightarrow \quad x = \frac{66}{8} = \frac{33}{4}$

18. *(a) –8*

Explanation: Given, $a_6 = 12$ and $a_8 = 22$

Let, the first term of A.P. be 'a' and common difference be 'd'

Then,

$a + 5d = 12$ ...(i)

$a + 7d = 22$ ...(ii)

On solving equations (i) & (ii), we get

$d = 5$ and $a = -13$

Then, $a_2 = a + d$

$= -13 + 5$

$= -8$

19. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: The equation $2x^2 + kx + 2 = 0$ has equal roots,

When, $D = (k)^2 - 4(2)(2) = 0$

$k^2 = 16$

$k = \pm 4$

20. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: In the given figure, ABCD is a quadrilateral circumscribing a circle.

We know that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

$\therefore \angle AOB + \angle COD = 180°$

$125° + \angle COD = 180°$

$\angle COD = 180° - 125°$

$= 55°$

SECTION - B

21. Here, the prime factorisations of 90 and 120 are:

$90 = 2 \times 3 \times 3 \times 5,$

or $= 2^1 \times 3^2 \times 5^1$

and $120 = 2 \times 2 \times 2 \times 3 \times 5,$

or $= 2^3 \times 3^1 \times 5^1$

So, LCM (90, 120) $= 2^3 \times 3^2 \times 5^1$, *i.e.* 360.

OR

$$(3 \times 5 \times 7 \times 11 + 11) = 11\,(3 \times 5 \times 7 \times 1 + 1)$$
$$= 11\,(105 + 1)$$
$$= 11\,(106)$$
$$= 11 \times 2 \times 53$$

Since $(3 \times 5 \times 7 \times 11 + 11)$ has more than one factor, so the given number is composite.

Caution

➥ *Understand the clear difference between prime and composite number.*

22. Using the quadratic formula to the equation: $x^2 + x - 12 = 0$,

We have, $$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-12)}}{2 \times 1}$$
$$= \frac{-1 \pm \sqrt{49}}{2} = \frac{-1 \pm 7}{2}$$
$$= \frac{-1-7}{2}, \frac{-1+7}{2}$$
$$= -4, 3$$

Thus, $x = -4$ and $x = 3$ are the two roots of $x^2 + x - 12 = 0$.

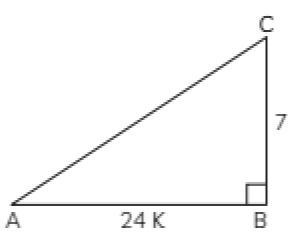
23. Given,
$$\tan A = \frac{7}{24},$$

Let AB = 24 K and BC = 7 K

Using Pythagoras theorem is ΔABC, we get

$\Rightarrow$ AC = 25 K

Thus, $$\sin A = \frac{BC}{AC} = \frac{7}{25}$$



and $$\cos A = \frac{AB}{AC} = \frac{24}{25}$$

Thus, $$\sin A \cos A = \frac{7}{25} \times \frac{24}{25} = \frac{168}{625}$$

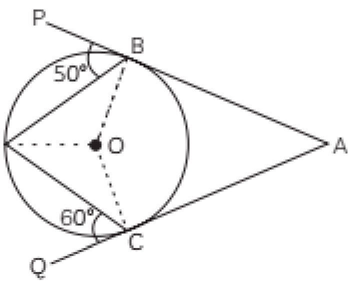
OR

$$\text{L.H.S.} = \sin^2 A + \sin^2 A \tan^2 A$$
$$= \sin^2 A\,(1 + \tan^2 A)$$
$$= \sin^2 A \sec^2 A$$
$$[\because \sec^2 A - \tan^2 A = 1]$$
$$= \sin^2 A \times \frac{1}{\cos^2 A}$$
$$= \tan^2 A = \text{R.H.S.}$$

Caution

➥ *Apply deduction of trigonometric identities, wherever necessary.*

24.



Join OB, OT and OC.

We know, tangent is pependicular to radius at the point of contact.

$\therefore$ $OB \perp AP$ and $OC \perp AQ$

$\therefore$ $\angle OBP = 90°$

$\Rightarrow \angle OBT + \angle TBP = 90°$

$\Rightarrow \angle OBT + 50° = 90°$

$\Rightarrow \angle OBT = 90° - 50° = 40°$

Similarly, $\angle OCQ = 90°$ and $\angle TCQ = 60°$

$\therefore$ $\angle OCT = 30°$

Now, in ΔOBT

OB = OT [Radii]

$\therefore$ $\angle OTB = \angle OBT$

[Equal angles opposite to equal sides]

$\Rightarrow$ $\angle OTB = 40°$

Similarly, in ΔOTC

OT = OC

$\Rightarrow$ $\angle OCT = \angle OTC = 30°$

So, $\angle BTC = \angle OTB + \angle OTC$
$= 40° + 30° = 70°$

25. Here, let OB = OA = r cm. Then,

Perimeter of quadrant = OA + $\widehat{AB}$ + BO
$$= r + \frac{90°}{360°} \times 2\pi r + r$$
$$= 2r + \frac{\pi r}{2} = r\left(2 + \frac{\pi}{2}\right)$$

Equating it to 75 cm, we have
$$r\left(2 + \frac{\pi}{2}\right) = 75$$

$$\Rightarrow \quad r = \frac{2 \times 75}{4 + \pi}$$

$$= \frac{2 \times 75 \times 7}{28 + 22} = \frac{2 \times 75 \times 7}{50}$$

$= 21$ cm.

Thus, area of the quadrant $= \frac{\pi}{4}(r)^2$

$= \frac{\pi}{4}(21)^2 = 346.5$ sq cm.

SECTION - C

26. Given: $\sin\theta + \cos\theta = \sqrt{3}$

squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \quad \sin\theta\cos\theta = 1 \quad$ ....(i)

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{1} \quad \text{[Using (i)]}$$

$$= 1$$

OR

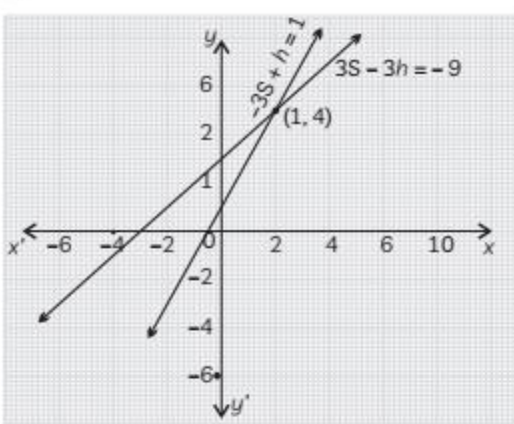
$$\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}}$$

$$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

$$= \frac{24\sqrt{3} - 43}{-11} \text{ or } \frac{43 - 24\sqrt{3}}{11}$$

27. (A)



(B) Yes, if find the several values of the variables s and h for cat as well as mouse, then the same values of s and h show their intersection point. It mean that the cat will definitely catch the mouse.

(C) As mentioned in above statement, the intersection point defines their time and height. Hence, after 1 second at a height of 4 m, the cat will catch the mouse.

$-3s + h = 1$

s	0	1	2	6
h	1	4	7	19

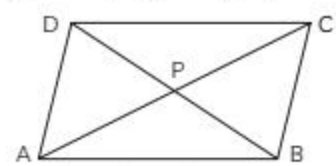
$3s - 3h = -9$

s	0	1	2
h	3	4	5

28. We know that the two diagonals AC and BD of a parallelogram ABCD bisect each other.

So, mid-point of AC = mid-point of BD = P. (say)

$$\Rightarrow P\left(\frac{a+1}{2}, \frac{2-2}{2}\right) = P\left(\frac{2-4}{2}, \frac{3-3}{2}\right)$$



$$\Rightarrow \left(\frac{a+1}{2}, 0\right) = \left(\frac{-2}{2}, 0\right)$$

$$\Rightarrow \quad a = -3.$$

29. From the figure,

AR = AQ, BQ = BP, CP = CR

$\Rightarrow$ BQ = 10 cm and hence

AQ = (18 – 10) cm, *i.e.* 8 cm.

Also, CR = 6 cm

Thus, AC = x = AR + CR = AQ + CR

= (8 cm + 6 cm) = 14 cm.

30. Since, DE is parallel to AB.

$\therefore$ By BPT, we have,

$$\frac{CD}{DA} = \frac{CE}{EB}$$

$$\Rightarrow \quad \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$\Rightarrow \quad (x+3)(3x+4) = x(3x+19)$$

$$\Rightarrow \quad 3x^2 + 13x + 12 = 3x^2 + 19x$$

$$\Rightarrow \quad 6x = 12$$

or $\quad x = 2$

31. Let the two numbers be a and b. Then,

$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b) = 24$ ...(i)

Also, $\quad a - b = 2$ ...(ii)

From equation (i) and (ii), we have

$$a - \frac{24}{a} = 2$$

$$\Rightarrow \quad a^2 - 24 = 2a$$

$$\Rightarrow \quad a^2 - 2a - 24 = 0$$

$$\Rightarrow \quad a^2 - (6-4)a - 24 = 0$$

$$\Rightarrow \quad a - 6a + 4a - 24 = 0$$

$$\Rightarrow \quad a(a-6) + 4(a-6) = 0$$

$$\Rightarrow \quad (a-6)(a+4) = 0$$

$$\Rightarrow \quad a = 6, -4$$

But a cannot be negative

$\therefore \quad a = 6$

So, $\quad b = \frac{24}{a} = \frac{24}{6} = 4$

OR

Volume of lead, obtained on melting the rectangular solid = (66 × 42 × 21) cu. cm

Volume of one spherical lead shot

$$= \frac{4}{3}\pi(2.1)^3 \text{ cu. cm}$$

So, Number of spherical lead shots that can be obtained

$$= \frac{\text{Volume of the rectangular solid}}{\text{Volume of 1 spherical lead shot}}$$

$$= \frac{66 \times 42 \times 21}{\frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10}} = 1500$$

SECTION - D

32. Let 'a' be the first term and 'd' be the common difference of AP.

Then, $\quad a = 5$

Here, $\quad$ last term $(l) = 45$

and $\quad$ Sum of all terms = 400

Let the A.P. contains 'n' terms. Then

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad \frac{n}{2}(5 + 45) = 400$$

$$\Rightarrow \quad n = 16$$

As last term is the n^{th} term, we have

$$a + (n-1)d = l$$

$$\Rightarrow \quad 5 + (16-1)d = 45$$

$$\Rightarrow \quad d = \frac{40}{15} \text{ or } \frac{8}{3}$$

Thus, $\quad n = 16$ and $d = \frac{8}{3}$

33. Let the tree AB is broken by the wind at P. Then, PX = PB

Let $\quad \angle PXA = \theta = 30°$

Let $\quad PA = x$ and $PB = h$

From the figure, in ΔAXP,

$$\tan 30° = \frac{AP}{AX}$$

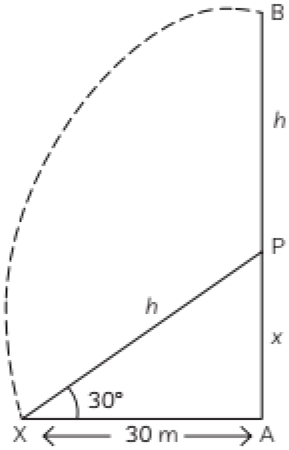
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$\Rightarrow \quad x = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3} \text{ m.}$$

Also, $\quad \cos 30° = \frac{AX}{PX}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{30}{h}$$



$$\Rightarrow \quad h = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$$

Thus, the total height of the tree

$= x + h$

$= (10\sqrt{3} + 20\sqrt{3})$ m

$= (30\sqrt{3})$ m

= 51.96 m.

OR

Consider ∆s PST and RQP, we have

$\angle PST = \angle RQP$ (90° each)

$\angle TPS = \angle PRQ$ (alternate angles)

⇒ By AA similarity criterion,

$\Delta PST \sim \Delta RQP$

Thus, $\frac{PS}{ST} = \frac{QR}{PQ}$

or $ST \times QR = PS \times PQ$

34. Let, the values of first installment be ₹ a.

The monthly installments form an AP, so let us suppose the man increases the value of each installment by ₹ d every month.

∴ The common difference of arithmetic series

= ₹ $(-d)$

Amount paid in 30 installments

$= ₹\ 36{,}000 - \frac{1}{3} \times 36{,}000$

= ₹ 24,000

Let S_n denotes the total amount of money paid in the n installments.

Then, S_{30} = ₹ 24,000

$\Rightarrow \quad \frac{30}{2}[2a + (30-1)d] = 24{,}000$

$\Rightarrow \quad 15[2a + 29d] = 24{,}000$

$\Rightarrow \quad 2a + 29d = 1600$ ...(i)

Also, S_{40} = ₹ 36,000

$\frac{40}{2}[2a + (40-1)d] = 36{,}000$

$\Rightarrow \quad 20[2a + 39d] = 36{,}000$

$\Rightarrow \quad 2a + 39d = 1800$ ...(ii)

Applying (ii) – (i), we get

$(2a + 39d) - (2a + 29d) = 1800 - 1600$

$\Rightarrow \quad 10d = 200$

$\Rightarrow \quad d = 20$

Put $d = 20$, in (i), we get

$2a + 29 \times 20 = 1600$

$\Rightarrow \quad 2a + 580 = 1600$

$\Rightarrow \quad 2a = 1020$

$a = 510$

Hence, the value of first installment is ₹ 510.

OR

Converting the given distribution into continuous distribution

CI	CF	Frequency	*c.f. (Less than type)*	
80-90	150	9	9	
90-100	141	17	26	
100-110	124	19	45	Median
110-120	105	45	90	class
120-130	60	33	123	
130-140	27	15	138	
140-150	12	12	150	
150 or more	9	9	150	
Total		150		

Here, $N = 150, \frac{N}{2} = 75$

∴ Median class is 110 – 120

Here, $l = 110, cf = 45, f = 45, h = 10$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

$$= 110 + \frac{(75-45)}{45} \times 10$$

$$= 110 + \frac{300}{45}$$

$$= 110 + 6.67$$

$$= 116.67 \text{ (approx)}$$

35. Calculation of Mode:

Here, the modal class is 45-55

For this class,

$l = 45, h = 10, f = 33, f_0 = 31, f_2 = 17, h = 10$

$$\therefore \quad \text{Mode} = \frac{f - f_0}{2f - f_0 - f_2} \times h$$

$$\Rightarrow \quad \text{Mode} = 45 + \frac{33-31}{66-31-17} \times 10$$

$$= 45 + \frac{20}{18} = 46.1 \text{ (approx.)}$$

Calculation of Mean:

Marks obtained	Class marks	Frequency	$d_i = x_i - A$ where A = 50	$f_i d_i$
25-35	30	7	–20	– 140
35-45	40	31	–10	– 310
45-55	50	33	0	0
55-65	60	17	10	170
65-75	70	11	20	220
75-85	80	1	30	30
		$\Sigma f_i = 100$		$\Sigma f_i d_i = -30$

So, $\quad \text{Mean} = \frac{\Sigma f_i d_i}{\Sigma f_i}$

$$= 50 + \frac{(-30)}{100}$$

$$= 49.7$$

SECTION - E

36. (A) Height of conical part,

$$h = 7.7 - 4.4$$

$$= 3.3 \text{ m}$$

(B) Slant height of conical part,

$$l = \sqrt{r^2 + h^2} = \sqrt{18^2 + 3.3^2}$$

$$= \sqrt{324 + 10.89}$$

$$= \sqrt{334.89}$$

$$= 18.3 \text{ m}$$

(C) Area of canvas used in making tent

$$= \pi r l + 2\pi r H$$

$$= \pi r(l + 2H)$$

$$= 3.14 \times 18 \, (18.3 + 2 \times 4.4)$$

$$= 3.14 \times 18 \, (18.3 + 8.8)$$

$$= 1531.692$$

$$\approx 1533 \text{ sq m}$$

OR

Cost of canvas = ₹ 1533 × 4.50

= ₹ 6898.5

Area of rectangular park

= Area of park – Area of circular base

$= 60 \times 70 - 3.14 \times 18^2$

= 3182.64 sq m

37. (A) Since, angle of incidence and angle of reflection are the same, $\angle AMB = \angle CMD$

Also, $\angle ABM = \angle CDM = 90°$

So, by AA similarity criterion

$\Delta ABM \sim \Delta CDM$

⚠ **Caution**

➥ *While expressing similarity of two triangles, the corresponding vertices must be written in the same order.*

(B) As $\Delta ABM \sim \Delta CDM$,

$$\frac{AB}{CD} = \frac{BM}{DM}$$

$$\frac{AB}{1.8} = \frac{2.5}{1.5}$$

$\Rightarrow \quad AB = \frac{5}{3} \times 1.8$

$\Rightarrow \quad AB = 3$

Thus, the height of the pole is 3 metres.

(C) As $\Delta ABM \sim \Delta CDM$,

$$\frac{AB}{CD} = \frac{BM}{DM}$$

$$\frac{AB}{1.2} = \frac{2.5}{1.5}$$

$\Rightarrow \quad AB = \frac{5}{3} \times 1.2$

$= 2$

OR

As $\Delta ABM \sim CDM$

Then, $\frac{AB}{CD} = \frac{BM}{DM}$

$\Rightarrow \quad \frac{AB}{1.8} = \frac{1.5}{2.5}$

$\Rightarrow \quad AB = \frac{1.5 \times 1.8}{2.5}$

$= \frac{5.4}{5} = 1.08$

38. (A) Number of possible outcomes = 17

Number of favourable outcomes = 9{1, 3, 5, 7, 9, 11, 13, 15, 17}

$\therefore$ P(getting on odd number on card) = $\frac{9}{17}$

(B) Number of possible outcomes = 17

Number of favourable outcomes = 7{2, 3, 5, 7, 11, 13, 17}

$\therefore$ P(getting a prime number) = $\frac{7}{17}$

(C) If the card drawn is not replaced, then total number of cards remaining is 16.

Now, total number of outcomes = 16

Favourable outcomes = 4{6, 9, 12, 15}

$\therefore$ P{getting a multiple of 3} = $\frac{4}{16} = \frac{1}{4}$

OR

Since, there is no card with a number greater than 17.

Total number of outcomes = 17

Favourable outcomes = 7{3, 6, 7, 9, 12, 14, 15}

$\therefore$ P{getting a multiple of 3 or 7} = $\frac{7}{17}$