Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

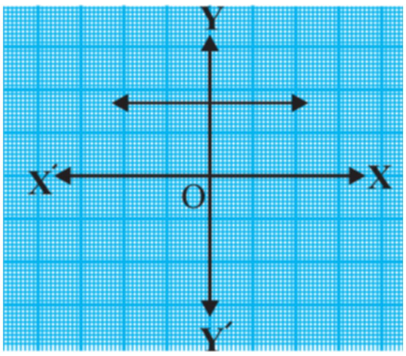
Section A

1. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a = **[1]**

a) 1 b) 2

c) 4 d) 3

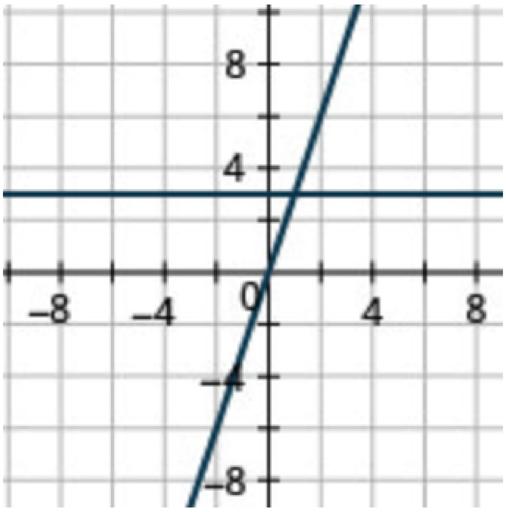
2. The graph of y = p(x) in a figure given below, for some polynomial p(x). Find the number of zeroes of p(x). **[1]**



a) 4 b) 0

c) 1 d) 2

3. The number of solutions of two linear equations representing intersecting lines is/are **[1]**



a) 1

b) 2

c) 0

d) ∞

4. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then **[1]**

a) $-3 < b < 3$

b) $b > 2$

c) $-2 < b < 2$

d) $b < -2$

5. The first term of an A.P., if its $S_n = n^2 + 2n$ is **[1]**

a) 0

b) 1

c) 3

d) 2

6. The distance of the point (4, 7) from the y-axis is **[1]**

a) 11

b) 4

c) $\sqrt{65}$

d) 7

7. The coordinates of the fourth vertex of the rectangle formed by the points (0, 0) (2, 0), (0, 3) are **[1]**

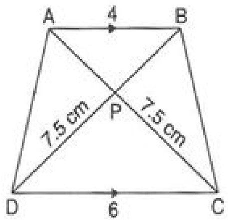
a) (2, 3)

b) (3, 0)

c) (3, 2)

d) (0, 2)

8. In the given figure, AP is equal to **[1]**



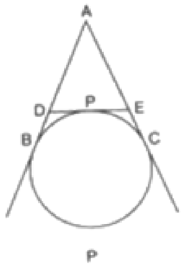
a) 7 cm

b) 6 cm

c) 5 cm

d) 5.5 cm

9. In the given figure, AB = 8 cm. If PE = 3 cm, then the measure of AE is **[1]**



a) 3 cm

b) 11 cm

c) 5 cm

d) 7 cm

10. AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If OA = 15 cm, then AP + AQ = **[1]**

a) 18 cm

b) 36 cm

c) 12 cm

d) 24 cm

11. 9 $\sec^2$A - 9 $\tan^2$A = **[1]**

a) 9

b) 1

c) 0

d) 99

12. If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to **[1]**

a) $\frac{1}{2}$

b) 1

c) $\frac{1}{\sqrt{2}}$

d) $\sqrt{3}$

13. From the top of a building 60m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. The height of the tower is **[1]**

a) 40 m

b) 60 m

c) 45 m

d) 50 m

14. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the sector angle is equal to **[1]**

a) 110°

b) 100°

c) 130°

d) 126°

15. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60^{o} at its centre. The radius of the circle is **[1]**

a) $\frac{20}{6+\pi}$ cm

b) $\frac{30}{6+\pi}$ cm

c) $\frac{60}{\pi}$ cm

d) $\frac{15}{6+\pi}$ cm

16. The probability of an impossible event is **[1]**

a) $\frac{1}{2}$

b) not defined

c) 0

d) 1

17. From a well shuffled pack of 52 cards, one card is drawn at random. The probability of getting a black king is **[1]**

a) None of these

b) $\frac{1}{26}$

c) $\frac{1}{13}$

d) $\frac{2}{39}$

18. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is **[1]**

a) 24.5

b) 24

c) 23.5

d) 22

19. **Assertion (A):** A piece of cloth is required to completely cover a solid object. The solid object is composed of a hemisphere and a cone surmounted on it. If the common radius is 7 m and height of the cone is 1 m, 463.39 cm^2 is the area of cloth required. **[1]**

Reason (R): Surface area of hemisphere = $2\pi r^2$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Common difference of an AP in which $a_{21} - a_7 = 84$ is 14 **[1]**

Reason (R): nth term of AP is given by $a_n = a + (n - 1)d$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

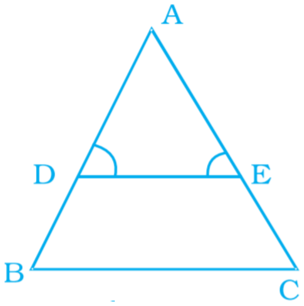
c) A is true but R is false.

d) A is false but R is true.

Section B

21. What is the smallest number which when divided by 20, 25, 35 and 40 leaves a remainder of 14, 19, 29 and 34 respectively. **[2]**

22. In given Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that BAC is an isosceles triangle. **[2]**



23. PQ is a tangent drawn from a point P to a circle of centre O and QOR is a diameter of the circle such that $\angle POR = 110°$, Find $\angle OPQ$. **[2]**

24. Prove the trigonometric identity: $\frac{\sec\theta - 1}{\sec\theta + 1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$ **[2]**

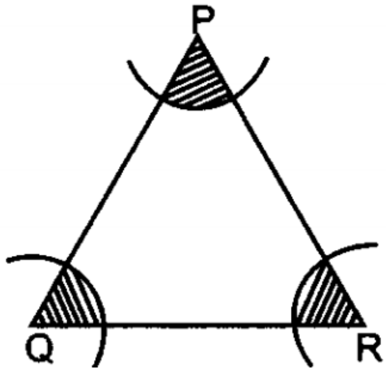
OR

Evaluate: $\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4}\sec^2 45^\circ + \frac{1}{4}cosec^2 30^\circ$

25. A car has two wipers which do not overlap. Each wiper has a blade of length 25cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. **[2]**

OR

In figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Section C

26. In a school there are two sections, namely A and B, of class X. There are 30 students in section A and 28 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B. **[3]**

27. Find a quadratic polynomial whose sum and product of the zeroes are $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$ respectively. Also find the zeroes of the polynomial by factorisation. **[3]**

28. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction? Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

Graphically, solve the following pair of equations:

2x + y=6

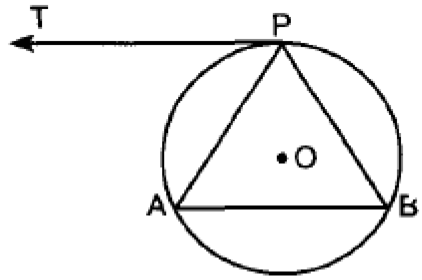
2x - y + 2=0

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

29. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus. **[3]**

OR

A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle.



30. Prove: $\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$ **[3]**

31. Find median for the following data: **[3]**

Class Interval	**Frequency**
10 - 19	2
20 - 29	4
30 - 39	8
40 - 49	9
50 - 59	4

60 - 69	2
70 - 79	1

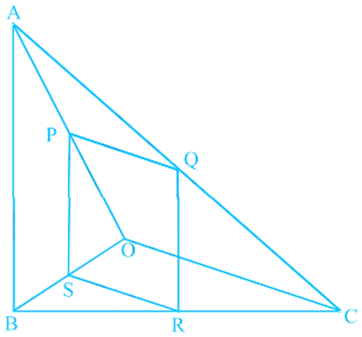
Section D

32. Solve: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x \neq -\frac{1}{2}, 1$ **[5]**

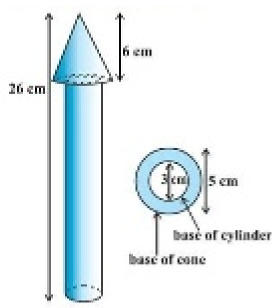
OR

In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight was increased by 30 minutes. Find the duration of flight.

33. In the figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR. **[5]**



34. A wooden toy rocket is in the shape of a cone mounted on a cylinder as shown in given below figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π =3.14) **[5]**



OR

An iron pillar consists of a cylindrical portion 2.8 m high and 20 cm in diameter and a cone 42 cm high is surmounting it. Find the weight of the pillar, given that 1 cm^3 of iron weighs 7.5 g.

35. Find the median from the following data: **[5]**

Class	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45
Frequency	7	10	16	32	24	16	11	5	2

HINT Convert it to exclusive form.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

(i) Find the terms of AP formed in above situation.

(ii) What is the total distance the competitor has to run?

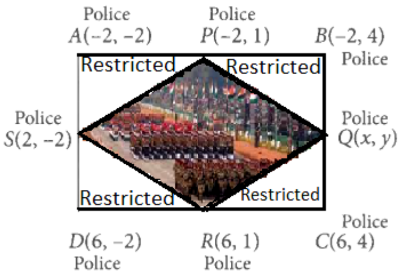
OR

Find the distance covered by competitor in order to put 5^{th} potato in the bucket.

(iii) Find distance cover after 4 potato drop in the bucket?

37. **Read the text carefully and answer the questions:** **[4]**

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux ah rays restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.



(i) If Q is the mid point of BC, then what are the coordinates of Q?

(ii) What is the length of the sides of quadrilateral PQRS?

OR

What is the length of route ABCD?

(iii) What is the length of route PQRS?

38. **Read the text carefully and answer the questions:** **[4]**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^o coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^o after 6 seconds.

(i) Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^o.

(ii) Find the distance between two positions of ship after 6 seconds?

OR

Find the distance of ship from the base of the light house when angle of depression is 30^o.

(iii) Find the speed of the ship?

Solution

Section A

1.
   (c) 4
   Explanation: LCM (a, 18) = 36
   HCF (a, 18) = 2
   We know that the product of numbers is equal to the product of their HCF and LCM.
   Therefore,
   18a = 2(36)
   $a = \frac{2(36)}{18}$
   a = 4
2.
   (b) 0
   Explanation: There is no zero as the graph does not intersect the x-axis at any point.
3. **(a)** 1
   Explanation: The number of solutions of two linear equations representing intersecting lines is 1 because two linear equations representing intersecting lines has a unique solution.
4.
   (c) -2 < b < 2
   Explanation: In the equation
   $x^2 - bx + 1 = 0$
   $D = b^2 - 4ac = (-b)^2 - 4 \times 1 \times 1$
   $= b^2 - 4$
   $\because$ it is given that the roots are not real, D < 0
   $\Rightarrow b^2 - 4 < 0$
   $\Rightarrow b^2 < 4 \Rightarrow b^2 < (\pm 2)^2$
   $\therefore$ b < 2 and b > -2 or -2 < b
   $\therefore$ -2 < b < 2
5.
   (c) 3
   Explanation: Given: $S_n = n^2 + 2n$
   Putting n = 1, we get
   $S = a = (1)^2 + 2 \times 1 = 1 + 2 = 3$
6.
   (b) 4
   Explanation: The distance of the point (4, 7) from y-a x is = 4
7. **(a)** (2, 3)
   Explanation: We are given three vertices (0, 0), (2, 0) and (0, 3) of a rectangle.
   We have to find the coordinates of the fourth vertex.
   By plotting the given vertices on an XY plane, C (0, 3) are the consecutive vertices.
   Consider D to represent the fourth vertex.
   Since, AB = 2 units and BC = 3 units.
   Thus, point D is at a horizontal distance of 3 units and a vertical distance of 2 units from the origin.
   Thus, the coordinates of the fourth vertex of the rectangle are (2, 3).

8.

(c) 5 cm

Explanation: In triangles APB and CPD,

$\angle APB = \angle CPD$ [Vertically opposite]

$\angle \mathrm{BAP} = \angle \mathrm{ACD}$ [Alternate angles as AB||CD]

Then $\Delta \mathrm{APB} \sim \Delta \mathrm{CPD}$

Therefore, $\frac{\mathrm{AB}}{\mathrm{CD}} = \frac{\mathrm{CP}}{\mathrm{AP}}$

$\Rightarrow \frac{4}{6} = \frac{AP}{7.5}$ therefore AP = 5cm

9.

(c) 5 cm

Explanation: Since Tangents from an external point to a circle are equal.

$\therefore$ PE = EC = 3 cm and AB = AE = 8 cm

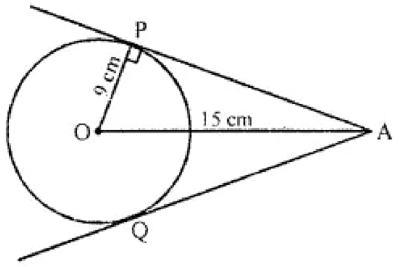
Therefore, AE = AC - EC = 8 - 3 = 5 cm

10.

(d) 24 cm

Explanation: OP is radius, PA is the tangent

$\mathrm{OP} \perp \mathrm{AP}$



Now in right $\Delta \mathrm{OAP}$

$OA^2 = OP^2 + AP^2$

$(15)^2 = (9)^2 + AP^2$

$225 = 81 + AP^2$

$\Rightarrow AP^2 = 225 - 81 = 144 = (12)^2$

AP = 12 cm

But AP = AQ = 12 cm (tangents from A to the circle)

AP + AQ = 12 + 12 = 24 cm

11. **(a)** 9

Explanation: Given: $9 \sec^2 A - 9 \tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$ ... [$\because \sec^2\theta - \tan^2\theta = 1$]

12.

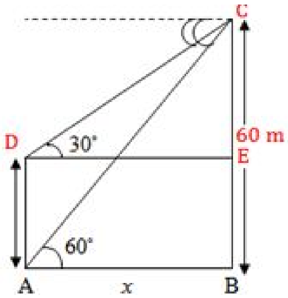
(b) 1

Explanation: We have, x tan 45° cos 60° = sin 60° cot 60°

$\Rightarrow x \times 1 \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{2} = \frac{1}{2}$

$\Rightarrow x = \frac{1}{2} \times 2 = 1$

13. **(a)** 40 m

Explanation: 

In triangle CDE,

$\tan 30^\circ = \frac{60-h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\Rightarrow x = \sqrt{3}(60-h)$ meters…..(i)

Again, in triangle CAB,

$\tan 60^\circ = \frac{60}{x}$

$\Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = \frac{60}{\sqrt{3}}$ meters …..(ii)

From eq. (i), and (ii), we get,

$\sqrt{3}(60-h) = \frac{60}{\sqrt{3}}$

$\Rightarrow 60 - h = 20$

$\Rightarrow h$ = 40 meters

14.

(d) 126°

Explanation: We have given that area of the sector is $\frac{7}{20}$ of the area of the circle.

Therefore, area of the sector $= \frac{7}{20} \times$ area of the circle

$\therefore \frac{\theta}{360} \times \pi r^2 = \frac{7}{20} \times \pi r^2$

Now we will simplify the equation as below,

$\frac{\theta}{360} = \frac{7}{20}$

Now we will multiply both sides of the equation by 360,

$\therefore \theta = \frac{7}{20} \times 360$

$\therefore \theta = 126$

Therefore, sector angle is 126°.

15.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{\pi r}{3}$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow$ r = $\frac{60}{\pi}$ cm

16.

(c) 0

Explanation: An event which has no chance of occurrence is called an impossible event.

for example: The probability of getting more than 6 when a die is thrown is an impossible event because the highest number in a die is 6

The probability of an impossible event is always 0.

17.

(b) $\frac{1}{26}$

Explanation: black kings = club king + spade king = 2

Number of possible outcomes = 2

Number of Total outcomes = 52

$\therefore$ Required Probability = $\frac{2}{52} = \frac{1}{26}$

18.

(b) 24

Explanation: Mean = 28

Mode = 16

Mode = 3 Median - 2 Mean

Hence, $\text{Median} = \frac{\text{Mode} + 2\text{Mean}}{3}$

$= \frac{16+2(28)}{3}$

$= \frac{16+56}{3}$

$= \frac{72}{3}$

= 24

19.

(c) A is true but R is false.

Explanation: A is true but R is false.

20.

(d) A is false but R is true.

Explanation: We have,

$a_n = a + (n - 1)d$

$a_{21} - a_7 = \{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$

a + 20d - a - 6d = 84

14d = 84

d = $\frac{18}{14}$ = 6

d = 6

So, A is false but R is true.

Section B

21. According to question we have to find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.

Take the LCM of 20, 25, 35 and 40 i.e.,

20 = $2 \times 2 \times 5$

25 = 5×5

35 = $1 \times 5 \times 7$

40 = $2 \times 2 \times 2 \times 5$

Now LCM of 20, 25, 35 & 40 = $2 \times 2 \times 5 \times 5 \times 7 \times 2$ = 1400

If the number 1400 is divided by 20, 25, 35, 40 it leaves a remainder 14, 19, 29, 34.

i.e. 6 less than the divisor in each case

Hence, the required number = 1400 - 6 = 1394.

22. Given that $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$.

Therefore, DE $\|$ BC (Converse of Basic Proportionality Theorem)

So, $\angle$D = $\angle$B and $\angle$E = $\angle$C (Corresponding angles) ....(1)

But $\angle$D = $\angle$E (Given)

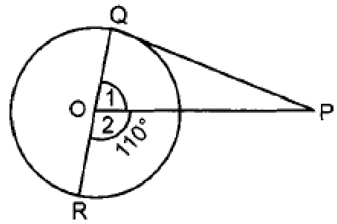
Therefore, $\angle$B = $\angle$C [From (1)]

So, AB = AC (Sides opposite to equal angles)

i.e., BAC is an isosceles triangle.

23. Given PQ is a tangent to the circle with centre O from a point P.

QOR is a diameter of the circle and $\angle$POR =110^o



$\angle$POR = 110°

QR is the diameter of the circle.

$\Rightarrow \angle 1 + \angle 2$ = 180° [Linear pair axiom]

$\Rightarrow \angle 1$ + 110° = 180°

$\Rightarrow \angle 1$ = 70°

$\angle OQP = 90^\circ$

In $\triangle$OPQ

$\angle 1 + \angle OQP + \angle QPO = 180^\circ$

$\Rightarrow 70^\circ + 90^\circ + \angle QPO = 180^\circ$

$\Rightarrow \angle OPQ = 180^\circ - 160^\circ$

$\Rightarrow \angle \mathrm{OPQ} = 20^\circ$

24. LHS $= \frac{\sec\theta - 1}{\sec\theta + 1}$

$= \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$

$= \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

$= \frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos^2\theta}{(1+\cos)^2}\ \left[\because (a-b)(a+b) = a^2b^2\right]$

$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$

$= \left[\frac{\sin\theta}{1+\cos\theta}\right]^2 = RHS$

Hence proved.

OR

cos 30° = $\sqrt{3}$/2 , $\Rightarrow$ cos^2 30° = 3/4

sin 30° = 1/2

$\therefore$ cosec 30° = 1/sin30° = 2 $\Rightarrow$ cosec^2 30° = 4

cot 30° = (cos 30°/sin 30°) = $\sqrt{3}$ $\Rightarrow$ cot^2 30° = 3

cos 45° = 1/$\sqrt{2}$

$\therefore$ sec 45° = 1/cos 45° = $\sqrt{2}$ $\Rightarrow$ sec^2 45° = 2

$\therefore$ cot^{2}30° - 2cos^{2}30° - (3/4)sec^{2}45° + (1/4)cosec^{2}30°

= 3 - 2(3/4) - (3/4) $\times$ 2 + (1/4) $\times$ 4

= 3 - 1.5 - 1.5 + 1

= 1

25. Radius of each wiper = 25cm, Angle = 115°

$\therefore$ θ = 115°

Total area cleaned at each sweep of the blades

$= 2\left[\frac{115}{360} \times \frac{22}{7} \times 25 \times 25\right] \left(\because \text{ Area } = \frac{\theta}{360}\pi r^2\right)$

$= \frac{230 \times 22 \times 5 \times 25}{72 \times 7}$

$= \frac{230 \times 11 \times 125}{36 \times 7}$

$= \frac{115 \times 11 \times 125}{18 \times 7}$

$= \frac{158125}{126}\mathrm{cm}^2$

$= 1254.96\mathrm{cm}^2$

OR

Area of sector on P = $\frac{\angle P}{360^\circ} \times \pi(14)^2$
Area of sector on Q = $\frac{\angle Q}{360^\circ} \times \pi(14)^2$
Area of sector on R = $\frac{\angle R}{360^\circ} \times \pi(14)^2$
Area of shaded region = adding area of all three sectors
$= \frac{\angle \mathbf{P}}{360^\circ} \times \pi(14)^2 + \frac{\angle \mathbf{Q}}{360^\circ} \times \pi(14)^2 + \frac{\angle \mathbf{R}}{360^\circ} \times \pi(14)^2$
$= \frac{\pi(14)^2}{360^\circ}(\angle P + \angle Q + \angle R)$
$= \frac{\pi \times 196}{360^\circ} \times 180^\circ$
$= \frac{22}{7} \times 98 = 308\text{cm}^2$

Section C

26. As per question, the required number of books are to be distributed equally among the students of section A or B.
There are 30 students in section A and 28 students in section B.
So, the number of these books must be a multiple of 30 as well as that of 28.
Consequently, the required number is LCM(30, 28).
Now, 30 = $2 \times 3 \times 5$
and 28 = $2^2 \times 7$.
$\therefore$ LCM(30, 28) = product of prime factors with highest power
= $2^2 \times 3 \times 5 \times 7$
= $4 \times 3 \times 5 \times 7$
= 420
Hence, the required number of books = 420.

27. Here, $\alpha + \beta = \frac{-3}{2\sqrt{5}}$ and $\alpha \cdot \beta = -\frac{1}{2}$ [Given]
f(x) = x^2 - $(\alpha + \beta)$x + $\alpha\beta$ [Formula]
$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$
$\Rightarrow f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$
$\Rightarrow f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$
For zeroes of polynomial f(x), f(x) = 0
$\Rightarrow 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$
$\Rightarrow 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$
$\Rightarrow \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$
$\Rightarrow (2x + \sqrt{5})(\sqrt{5}x - 1) = 0$
$\Rightarrow (2x + \sqrt{5}) = 0$ or $\sqrt{5}x - 1 = 0$
$\Rightarrow x = \frac{-\sqrt{5}}{2}$ or $x = \frac{1}{\sqrt{5}}$
$\therefore\ \alpha = \frac{-\sqrt{5}}{2}$ and $\beta = \frac{1}{\sqrt{5}}$.

28. Let the fraction be $\frac{x}{y}$
Then, according to the question,
$\frac{x+1}{y-1} = 1$ ......(1)
$\frac{x}{y+1} = \frac{1}{2}$ ....(2)
$\Rightarrow$x + 1 = y - 1 ....(3)
2x = y + 1................(4)
$\Rightarrow$x - y = - 2................(5)
2x - y = 1......................(^)
Substituting equation (5) from equation (6), we get x =3
Substituting this value of x in equation (5), we get
3 - y = -2
$\Rightarrow$ y = 3 + 2
$\Rightarrow$ y = 5
Hence, the required fraction is $\frac{3}{5}$
Verification: Substituting the value of x = 3 and y = 5,
we find that both the equations(1) and (2) are satisfied as shown below:

$\frac{x+1}{y-1} = \frac{3+1}{5-1} = \frac{4}{4} = 1$

$\frac{x}{y+1} = \frac{3}{5+1} = \frac{3}{6} = \frac{1}{2}$

Hence, the solution is correct.

OR

Given equation is $2x + y = 6$

⇒ y = 6 - 2x.......(i)

If, $x = 0, y = 6 - 2(0) = 6$

$x = 3, y = 6 - 2(3) = 0$

x	0	3
y	6	0
Points	B	A

Given equation is $2x - y + 2 = 0$

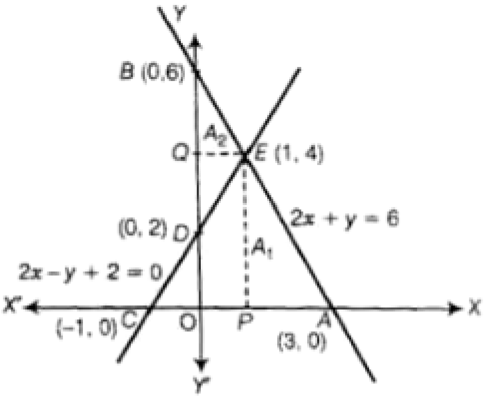
$\Rightarrow y = 2x + 2$ ...(ii)

If, $x = 0, y = 2(0) + 2 = 0 + 2 = 2$

$x = -1, y = 2(-1) + 2 = 0$

x	0	-1
y	2	0
Points	D	C

Plotting 2*x* + *y* = 6 and 2*x* - *y* + 2= 0, as shown below, we obtain two lines AB and CD respectively intersecting at point, *E* (1, 4).



Now, A_1 = Area of ACE = $\frac{1}{2} \times AC \times PE$

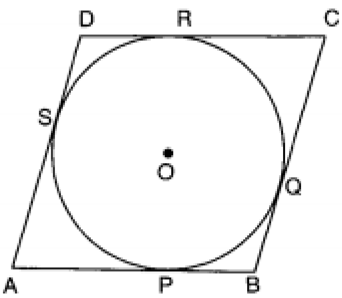
$= \frac{1}{2} \times 4 \times 4 = 8$

And A_2 = Area of BDE = $\frac{1}{2} \times BD \times QE$

$= \frac{1}{2} \times 4 \times 1 = 2$

∴ $A_1 : A_2$ = 8:2 = 4: 1

∴ Ratio of areas of two $\triangle s \quad = \frac{\text{Area }\Delta\text{ACE}}{\text{Area }\Delta\text{BDE}} = \frac{8}{2} = \frac{4}{1} = 4:1$

29. 

Let ABCD be a parallelogram such that its sides touch a circle with centre O.

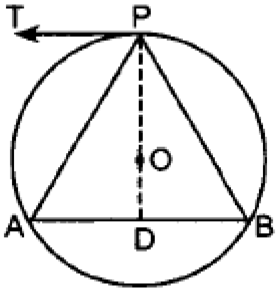
We know that the tangents to a circle from an exterior point are equal in length.

Therefore,AP = AS [From A] ...(i)

BP = BQ [FromB] ...(ii)
CR = CQ [From C] ...(iii)
and , DR = DS [From D] ...(iv)
Adding (i), (ii), (iii) and (iv), we get,
AP + BP + CR + DR = AS + BQ +CQ + DS
⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
⇒ AB + CD = AD + BC
⇒ 2 AB = 2 BC
⇒ AB = BC
Therefore, AB = BC = CD = AD
Thus, ABCD is a rhombus.

OR

Given,



Construction: Join PO and produce it to D.
Proof: Here, $OP \perp TP$
$\angle OPT = 90^\circ$
Also, $TP || AB$
$\therefore \angle \text{TPD} + \angle \text{ADP} = 180^\circ$
$\Rightarrow \angle \text{ADP} = 90^\circ$
OD bisects AB [Perpendicular from the centre bisects the chord]
In $\triangle$ADP and $\triangle$BDP
AD = BD
$\angle ADP = \angle BDP$ [Each 90°]
$PD = PD$
$\therefore \triangle ADP \cong \triangle BDP$ [SAS]
$\angle PAB = \angle PBA$ [C.P.C.T.]
$\therefore \triangle$PAB is isosceles triangle.

30. Given- $\sin^6A + 3\sin^2A \cos^2A = 1 - \cos^6A$
Now, taking
$\sin^6A + \cos^6A = 1 - 3\sin^2A \cos^2A$
Taking LHS
$= \sin^6A + \cos^6A = (\sin^2A)^3 + (\cos^2A)^3$
$= (\sin^2A + \cos^2A)^3 - 3\sin^2A \cos^2A(\sin^2A + \cos^2A)$ $\{\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)\}$
$= (1)^3 - 3\sin^2A \cos^2A(1)$
$= 1 - 3\sin^2A \cos^2A$ = RHS

31. Calculation of Median:

Class Interval	**Frequency**	**c.f.**
9.5 - 19.5	2	2
19.5 - 29.5	4	6
29.5 - 39.5	8	14
39.5 - 49.5	9	23
49.5 - 59.5	4	27

59.5 - 69.5	2	29
69.5 - 79.5	1	30

n = 30, $\frac{n}{2}$ = 15, Median class = 39.5 - 49.5

$l = 39.5, c.f. = 14, f = 9, h = 10$

Median = 39.5 + $\left(\frac{15-14}{9}\right) \times 10$

= 39.5 + $\frac{1}{9} \times 10$

$= 39.5 + 1.11 = 40.61$

Section D

32. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y-1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2

OR

Let the original speed of the aircraft be x km/hr.

Then, new speed = (x- 200) km/hr.

Duration of flight at original speed = $\frac{600}{x} hrs$

Duration of flight at reduced speed = $\frac{600}{x-200} hrs$

According to the question

$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$

$\implies \frac{600x-600x+120000}{x(x-200)} = \frac{1}{2}$

$\implies \frac{120000}{x^2-200x} = \frac{1}{2}$

$\implies$ x^2 - 200x -240000 = 0

$\implies$ x^2 - 600 x + 400 x - 240000 = 0

$\implies$ x (x - 600) + 400 (x - 600) = 0

$\implies$ (x- 600) (x + 400) = 0

Either x - 600 =0 or x + 400 = 0

$\implies$ x = 600, -400

since Speed cannot be negative. So x = 600

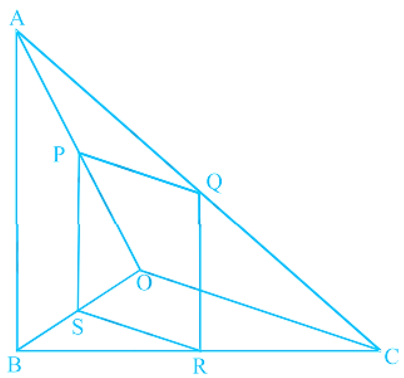
So, original speed of the aircraft was 600 km /hr.

Hence, duration of flight = $\frac{600}{x} hrs = \frac{600}{600} hrs = 1 hr$

33. It is given that PQRS is a parallelogram,

So, PQ || SR and PS || QR.

Also, AB || PS.



To prove OC || SR

In $\triangle$OPS and OAB,

PS || AB

$\angle$POS = $\angle$AOB [common angle]

$\angle$OSP = $\angle$OBA [corresponding angles]

$\therefore$OPS $\sim$ $\triangle$OAB [by AAA similarity criteria]

Then,

$\frac{PS}{AB} = \frac{OS}{OB}$ …(i) [by basic proportionality theorem]

In $\triangle$CQR and ΔCAB,

QR || PS || AB

$\angle$QCR = $\angle$ACB [common angle]

$\angle$CRQ = $\angle$CBA [corresponding angles]

$\therefore \triangle$CQR $\sim \triangle$CAB

Then, by basic proportionality theorem

$= \frac{QR}{AB} = \frac{CR}{CB}$

$\Rightarrow \frac{PC}{AB} = \frac{CR}{CB}$ .....(ii)

[PS ≅ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$\frac{\mathrm{OS}}{\mathrm{OB}} = \frac{\mathrm{CR}}{\mathrm{CB}}$

or $\frac{\mathrm{OB}}{\mathrm{OS}} = \frac{\mathrm{CB}}{\mathrm{CR}}$

On subtracting from both sides, we get,

$\frac{\mathrm{OB}}{\mathrm{OS}} - 1 = \frac{\mathrm{CB}}{\mathrm{CR}} - 1$

$\Rightarrow \frac{\mathrm{OB}-\mathrm{OS}}{\mathrm{OS}} = \frac{(\mathrm{CB}-\mathrm{CR})}{\mathrm{CR}}$

$\Rightarrow \frac{\mathrm{BS}}{\mathrm{OS}} = \frac{\mathrm{BR}}{\mathrm{CR}}$

By converse of basic proportionality theorem, SR || OC

Hence proved.

34. Let radius, slant height and height of cone be r, *l* and h respectively and radius and height of cylinder be r_1 and h_1 respectively.

r = 2.5 cm, h = 6 cm, r_1 = 1.5 cm and h_1 = 26 - 6 = 20 cm

$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{(2.5)^2 + 6^2}$

$= \sqrt{6.25 + 36} = \sqrt{42.25}$

$= 6.5\mathrm{cm}$

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange = Curved surface area of the cone + Base area of the cone - Base area of the cylinder

$= \pi rl + \pi r^2 - \pi r_1^2$

$= \pi\left\{rl + r^2 - r_1^2\right\}$

$= \pi\left\{2.5 \times 6.5 + (2.5)^2 - (1.5)^2\right\}$

$= 3.14(16.25 + 6.25 - 2.25) = 3.14 \times 20.25 = 63.585\mathrm{cm}^2$

Now, the area to be painted yellow = Curved surface area of the cylinder + Area of the base of the cylinder

$= 2\pi r_1 h_1 + \pi r_1^2$

$= \pi r_1\left(2h_1 + r_1\right)$

$= 3.14 \times 1.5(2 \times 20 + 1.5)$

$= 3.14 \times 1.5 \times 41.5 = 4.71 \times 41.5 = 195.465\mathrm{cm}^2$

OR

We are Given that,

An iron pillar consists of a cylindrical portion and a cone mounted on it.

The height of the cylindrical portion of the pillar, H = 2.8 m = 280 cm.

The height of the conical portion of the pillar, h = 42 cm..

The diameter of the cylindrical portion of the pillar = diameter of the circular base of cone = D = 20 cm.

The radius of the circular base of cylinder/ cone r = $\frac{D}{2}$ = 10 cm.

Now, we have,

Volume of the pillar, (V) = Volume of the cylindrical portion of pillar + volume of the conical portion of the pillar.

$\Rightarrow$ V = $\pi r^2 H + \frac{1}{3}\pi r^2 h$

$\Rightarrow$ V = $\left(\frac{22}{7} \times 10^2 \times 280 + \frac{1}{3} \times \frac{22}{7} \times 10^2 \times 42\right)$ cm^3

$\Rightarrow$ V = (22 $\times$ 100 $\times$ 40 + 22 $\times$ 100 $\times$ 2) cm^3

$\Rightarrow$ V = (88000 + 4400) cm^3

$\Rightarrow$ V = 92400 cm^3

Hence, volume of iron pillar is 92400 cm^3

Given,

Weight of 1 cm^3 iron = 7.5 gm.

Hence, weight of 92400 cm^3 iron = 7.5 $\times$ 92400 gm.

= 693000 gm.

= 693 Kg.

Since, 1Kg = 1000 gm.

Hence, the weight of iron piller is 693 Kg.

35.

Class Interval	Frequency f_i	Cumulative frequency
0.5 - 5.5	7	7
5.5 - 10.5	10	17
10.5 - 15.5	16	33
15.5 - 20.5	32	65
20.5 - 25.5	24	89
25.5 - 30.5	16	105
30.5 - 35.5	11	116
35.5 - 40.5	5	121
40.5 - 45.5	2	123

Here, N = 123$\Rightarrow \frac{N}{2} = 61.5$

Median class is 15.5 - 20.5

$\therefore l = 15.5, h = 5, f = 32, c.f. = 33$

Now, Median = $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$= 15.5 + \left[5 \times \frac{(61.5-33)}{32}\right]$

$= 15.5 + 4.45 = 19.95$

Section E

36. **Read the text carefully and answer the questions:**

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

(i) Distance travel by the competitor to pick up each potato form an AP

10, 16, 22 ...

(ii) $S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_{10} = \frac{10}{2}$ {2 $\times$ 10 + 9 $\times$ 6}

S_{10} = 5{20 + 54}

$S_{10} = 5 \times 74$

$S_{10} = 370$ m

i.e., The competitor has to run 370 m.

OR

$t_n = a + (n - 1)d$

$t_5 = 10 + (5 - 1)6$

$t_5 = 10 + 24$

$t_5 = 34$ m

(iii) $S_4 = \frac{4}{2}\{2 \times 10 + (4 - 1)6\}$

$= 2\{20 + 18\}$

$= 2 \times 38$

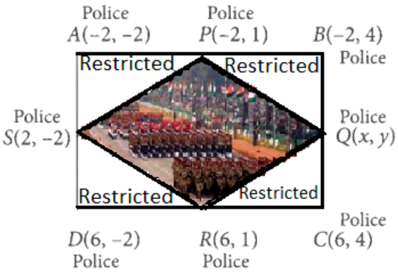
$S_4 = 76$

$\therefore$ Required distance = 370 - 76

= 294

37. **Read the text carefully and answer the questions:**

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux ah rays restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.



(i) Q(x, y) is mid-point of B(-2, 4) and C(6, 4)

$\therefore (x, y) = \left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right) = (2, 4)$

(ii) Since PQRS is a rhombus, therefore, PQ = QR = RS = PS.

$\therefore PQ = \sqrt{(-2-2)^2 + (1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

Thus, length of each side of PQRS is 5 units.

OR

Length of CD = 4 + 2 = 6 units and length of AD = 6 + 2 = 8 units

$\therefore$ Length of route ABCD - 2(6 + 8) = 28 units

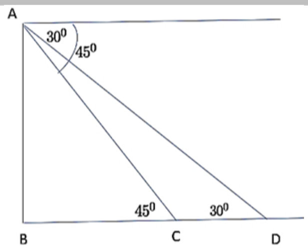
(iii) Length of route PQRS = 4 PQ

$= 4 \times 5 = 20$ units

38. **Read the text carefully and answer the questions:**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^o coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^o after 6 seconds.

(i)



The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^o.

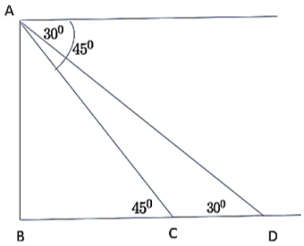
In $\triangle$ABC

$\tan 45^o = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{40}{BC}$

$\Rightarrow$ BC = 40 m

(ii)



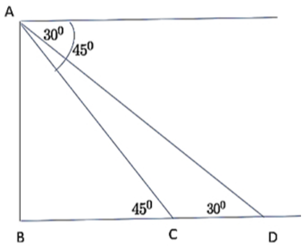
The distance between two positions of ship after 6 seconds

CD = BD - BC

$\Rightarrow$ CD = $40\sqrt{3} - 40 = 40(\sqrt{3} - 1)$

$\Rightarrow$ CD = 29.28 m

OR



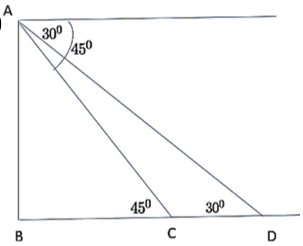
The distance of ship from the base of the light house when angle of depression is 30^o.

In $\triangle$ABD

$\tan 30^o = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$

$\Rightarrow$ BD = $40\sqrt{3}$ m

(iii)


Speed of ship = $\frac{Distance}{Time} = \frac{29.28}{6}$ = 4.88 m/sec