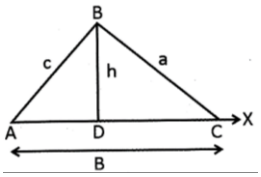
Triangles

Introduction

The word triangle is derived from Greek word, triangle means three and hence it **to** a shape consisting three internal angles. Obviously the shape consists of sides. Hence, a triangle can be defined as a polygon having three sides.

Basic Concepts of Triangles

The general shape of a triangle is shown below:



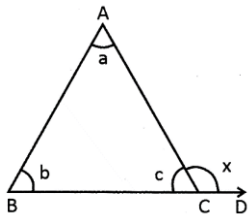
The vertices of a triangle are denoted by the Capital letters of English alphabets.
In the above figure ΔABC, the sides are AB, BC and CA.

Altitude

A Perpendicular drawn from a vertex to the opposite side is called the altitude of the triangle and denoted as h.

Some Basic Facts Related to Triangle

- In any triangle, sum of any two sides is always greater than the 3rd side.
  i.e $b+c>a$ or $a+c>b$ or $c+b>a$.
- The sum of all interior angles of a triangle is $180°$
- The exterior angle at any vertex of the triangle is equal to the sum of other two opposite angles.



Proof: In $\Delta ABC, a+b+c=180°$

but, $c+x=180°$ (Linear pair) $c=180^0-x$

Putting "c" in the above equation, we get

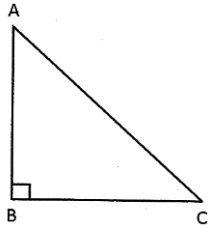
$a+b+180°-x=180°\Rightarrow a+b=x$

There fore, $\Delta ACD=\Delta A+\Delta B$

Types of Triangle

Classification based on angles

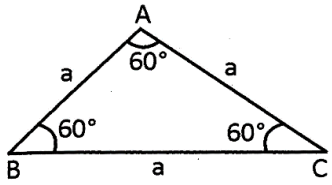
- If one angle of a triangle is right angle then it is called right angled triangle.
  Note that the other two angles are acute.
- From figure, $\angle B=90°$ and $\angle A+\angle C=90°$. So, it is right-angled triangle



- If all the angles of triangle are less than 90° then it is called acute-angled triangle.
  In the figure $\angle A, \angle B$ and $\angle C$ are acute angles.
- If one angle of a triangle is more than 90° then it is called obtuse angled triangle. The other two angles are acute.

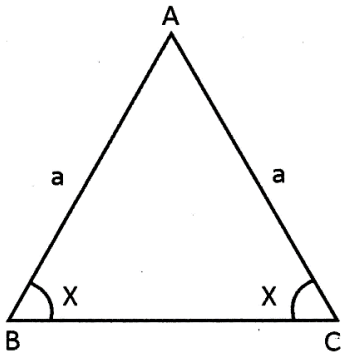
Equilateral Triangle

A triangle in which all sides are equal is known as equilateral triangle.



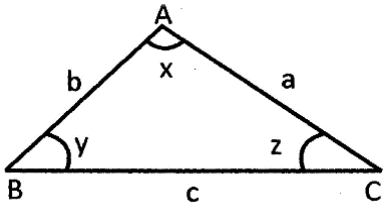
Isosceles Triangle

A triangle in which any two sides are equal is said to be and isosceles triangle.
The angles opposite to the equal sides are equal.
In the triangle given below, sides AB and AC are equal as well as $\angle B$ and $\angle C$ are also equal.



Scalene Triangle

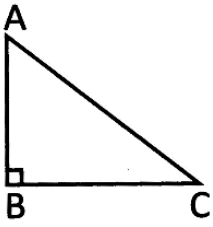
A triangle in which all side are unequal is said to be scalene triangle.
In scalene triangle all angles are different.



In the above given triangle all the sides of triangle denoted by a, b and c are unequal and angles x, y and z are also unequal.

Pythagoras Theorem

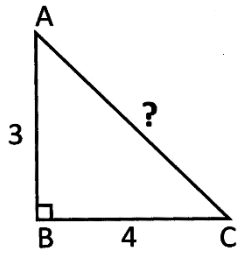
In a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides, mathematically from figure,



$$AC^2 = AB^2 + BC^2$$

Illustrative EXAMPLE

In right angled triangle AB = 3 unit, BC = 4 unit then AC equal to



(a) 3 unit (b) 4 unit
(c) 5 unit (d) 6 unit
(e) None of these

Answer: (c)

Explanation

We know that from Pythagoras theorem

$AB^2 + BC^2 = AC^2 \Rightarrow AC^2 = 3^2 + 4^2 \Rightarrow 9+16 = 25 = 5^2 \Rightarrow AC = 5unit.$

Converse of Pythagoras Theorem

If the square of one side of a triangle is equal to the sum of the square of other two sides then the triangle is right angled triangle, where right angle is opposite to the greatest side.

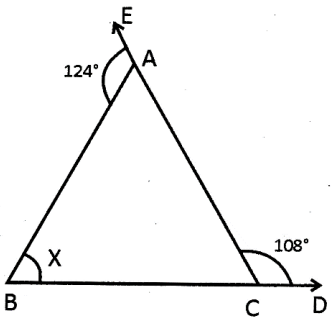
If in a triangle PQR, $PQ^2 = PR^2 + QR^2$ then the angle opposite to PQ is right angle.

Pythagorean Triplets

Three integers p, q and r (such that p > q > r) are said to be a Pythagorean triplet, if $p^2 = q^2 + r^2$

Commonly Asked QUESTIONS

In the adjoining figure, BC is produced to D and CA is produced to E, and $\angle DCA = 108°$ and $\angle EAB = 124°$ then the value of x is:



(a) 48° (b) 52°
(c) 76° (d) 128°
(e) None of these

Answer: (b)

Explanation

$\angle ACB = 180° - \angle ACD$

$\Rightarrow 180^0 - 108^0 = 72^0 \Rightarrow \angle BAC = 180^0 - EAB$

$\Rightarrow 180° - 124° = 56°$. From $\angle ABC$, we have

$\Rightarrow x + (\angle ACB + \angle BAC) = 180^0 \Rightarrow x + (72^0 + 56^0) = 180^0$

$\Rightarrow x = 180° - 128° = 52°$

If the angles of a triangle are in the ratio 1:1:2 then which one of the following statements is incorrect?

(a) Triangle is right angled triangle

(b) The angles of the triangles are $90°,\ 45°$ and $45°$

(c) The angles of the triangles are $90°,\ 45°$ and $45°$. Triangle is right angled isosceles triangle.

(d) The angles of the triangles are $90°,\ 45°$ and $45°$. And it is scalene.

(e) None of these

Answer (d)

Explanation

The angles of triangle are $x,\ x$ and $2x$ therefore, from angle sum property of triangle we get $x+x+2x=180°$ or, $4x=180°$

or $x=45°$, the other angles of the triangle are $90°,\ 45°$ and $45°$.

Here two angles are equal. Therefore, the given triangle is isosceles triangle.

If the bisector of an angle of a triangle is also the median of the triangle then the triangle in which this condition is not possible?

(a) Equilateral

(b) Isosceles

(c) Equiangular

(d) Scalene

(e) None of these

Answer: (c)

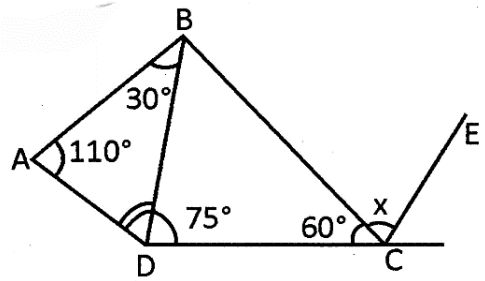
If CE is parallel to DB in the given figure then value of x will be____.

(a) $45°$ (b) $75°$

(c) $30°$ (d) $85°$

(e) None of these

Answer (d)



$\angle ADB=180°-(110°+30°)=40°$

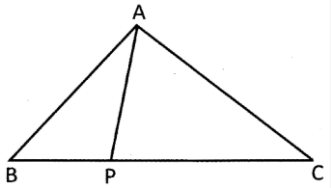
$\angle BDC=75°-40°=35°$

$DB\parallel CE$

$\therefore 35°+(600+x°)=180°$

$\Rightarrow x=85°$

In the given figure P is the point on side BC. Which one of the following is correct?



(a) $(AB+BC+CA)<2AP$ (b) $(AB+BC+CA)>2AP$

(c) $(AB+BC+CA)<AP$ (d) $(AB+BC+CA)>AP$

(e) None of these

Answer: (b)

SUMMARY

- A triangle is a polygon with three sides.
- Triangle is classified on the basis of angles and sides.
- According to angle there are three types of triangle (i) acute angled triangle (ii) right angled triangle (iii) obtuse angled triangle.
- According to sides triangles are of also three types (i) equilateral triangle (ii) isosceles triangle (iii) scalene triangle.
- Median is a line segment which joins the midpoint of a side and its opposite vertex.
- Incentre is the point of intersection of angle bisectors of a triangle.
- The point of intersection of altitude of a triangle is known as orthocenter.
- The point of intersection of perpendicular bisectors of the sides of triangle is known as circumventer.
- The point of intersection medians of a triangle is known as centroid.
  Hypotenuse is the longest side of a right angled triangle.

You Must KNOW

- Pascal's triangle is an arithmetical triangle made up of staggered rows of numbers.

Self Evaluation TEST

Duration 10 Minutes

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1. **The triangle formed by joining the mid points of the sides of an equilateral triangle is:**

(a) A right angled triangle (b) An obtuse angled triangle
(c) A scalene triangle (d) An equilateral triangle
(e) None of these

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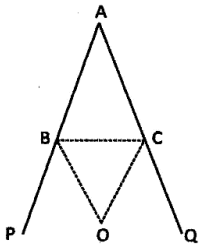
2. **In a triangle ABC, the sides AB and AC are produced to P and Q respectively. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at 0 then ZBOC is equal to:**



(a) $90^0 + \frac{\angle A}{2}$ (b) $90^0 - \frac{\angle A}{2}$
(c) $180^0 + \frac{\angle A}{2}$ (d) $180^0 - \frac{\angle A}{2}$
(e) None of these

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3. **Four triangles are formed by joining the midpoints of the three sides of a triangle then:**

(a) Two triangles are congruent
(b) Three triangles are congruent
(c) All the triangles are congruent
(d) None are congruent
(e) None of these

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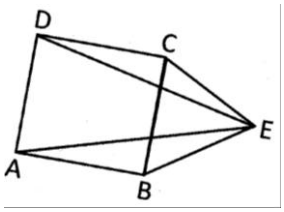
4. **In the adjoining Figure ABCD is a square and EBC is an equilateral triangles. The measure of $\angle EAB$ is equal to:**



(a) $15°$ (b) $30°$
(c) $\left(22\frac{1}{2}\right)^0$ (d) $45°$
(e) None of these

5. **In any triangle the centroid divides the medians in the ratio:**

(a) 1:1 (b) 2:1
(c) 3:2 (d) 5:1
(e) None of these

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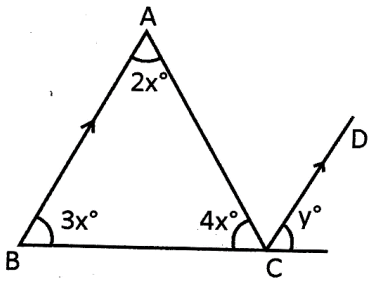
6. **In the figure cd is parallel to AB. The angle Y is equal to:**



(a) $40°$ (b) $60°$
(c) $80°$ (d) $100°$
(e) None of these

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7. **In a triangle ABC, BD and CE are perpendicular on AC and AB respectively. If BD = CE then the triangle ABC is:**

(a) Equilateral (b) Isosceles
(c) Right angled (d) Scalene
(e) None of these

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8. **In the given figure,** $\angle A = 80°,\ \angle B = 60^0.\ \angle C = 2x°$ **and** $\angle BDC = y°$, **BD and CD are bisector of** $\angle B$ **and** $\angle C$ **respectively. The values of** x **and** y respectively are:

(a) 15^0 and 70^0 (b) 10^0 and 160^0
(c) $20°$ and $130°$ (d) 20^0 and 125^0
(e) None of these

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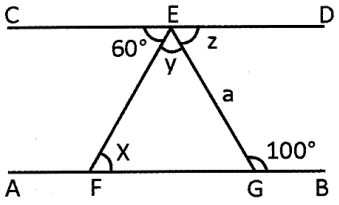
9. **If AB is parallel to CD in the given figure then** $\angle X$ $\angle Y$ **and** $\angle Z$ **respectively are:**



(a) $60°,\ 40°,\ 80°$ (b) $50°,60°,\ 60°$
(c) $60°.\ 60°,\ 60°$ (d) $70°,\ 50°,\ 60°$
(e) None of these

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10. **If one angle of a triangle is greater than other two angles by 30° then the angles of the triangle are:**

(a) $40°, 40°100°$ (b) $50°, 50°, 80°$

(c) $30°, 30°, 120°$ (d) $35°, 35°, 110°$

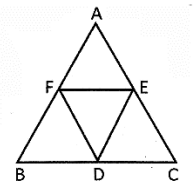
(e) None of these

Answers – Self Evaluation Test																			
1.	D	**2.**	B	**3.**	C	**4.**	A	**5.**	B	**6.**	B	**7.**	B	**8.**	C	**9.**	A	**10.**	B

Self Evaluation Test

SOLUTIONS

1.



ΔABC is an equilateral triangle. D, E and F are the mid – points of its sides

$\therefore \ \angle A = \angle B = \angle C = 60^O$

AB=BC=CA

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

$\Rightarrow BD = DC = CE = EA = AF = FB$

In $\Delta BDF, \angle \mathrm{BDF}, \angle \mathrm{B} = 60^O$ and BD=DF,

$\angle BDF = \angle BFD$ ($\therefore$ Angles opposite the equal sides are also equal)

So, $\angle B + \angle BDF + \angle BFD$

$= 180^O \Rightarrow 60^O + 2\angle BDF = 180^O$

$\Rightarrow \angle BDF = 60^O = \angle BFD$

Thus each of the angles may be shown equilateral triangle.

Similarly other triangles may be shown equilateral.

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2. From the given figure

$\angle A + \angle B + \angle C = 180^O \Rightarrow \angle B + \angle C = 180^O - \angle A$

$\frac{1}{2}\angle PBC + \frac{1}{2}\angle QCB + \angle BOC = 180^O$ (Form ΔBOC)

$= \frac{1}{2}(180^O - \angle \mathrm{B}) + \frac{1}{2}(180^O - \angle \mathrm{C}) + \angle \mathrm{BOC} = 180^O \Rightarrow 180^O - \frac{1}{2}(\angle \mathrm{B} + \angle \mathrm{C}) + \angle \mathrm{BOC} = 180^O$

$\Rightarrow 180^O - \frac{1}{2}(180^O - \angle \mathrm{A}) + \angle \mathrm{BOC} = 180^O$

$\Rightarrow \angle BOC = 90^O - \frac{\angle A}{2}$

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4. $\angle BOC = 90^O - \frac{\angle A}{2}$

From the figure in triangle ABE, AB = BE

$\Rightarrow \angle BAE = \angle EAB = 15^O$

6. $2x^o + 3x^o + 4x^o = 180^o \Rightarrow 9x = 180^o \Rightarrow x^o = 20^o$

$y^o = 3x^o$ (corresponding angles)

$y^o = 3 \times 20^o = 60^o$

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7. in $\Delta BCD \& \Delta BEC$

$BC = BC$ (Common)

$\angle BDC = \angle BEC$ (each equal 90^o)

$BD = CE$ (given)

$\therefore ABDC$ and $\therefore ABEC$ are congruent

$\therefore \angle BCD = \angle CEB$

$\Rightarrow AB = AC$

Hence, the ΔABC is an isosceles triangle.

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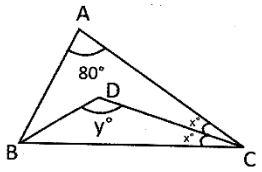
8. in $\Delta ABC, 80^O + 60^O + 2x^O = 180^O$

$\Rightarrow 2x^O = 40^O$



$\therefore x^o = 20^o$

$\therefore BD$ and CD are bisectors of $\angle B$ and $\angle C$

$Y^o = 90^o + \frac{\angle A}{2} = 90^o + \frac{80^o}{2} = 130^o$

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9. $x = 60^o$ (x and $\angle CEF$ are alternate angles)

From $\Delta EFGY + 60^o = 100^o$ (100^o is exterior to the A)

$\therefore y = 40^o$

$\therefore CED$ is a straight line

$60^o + 40^o + Z = 180^o \Rightarrow Z = 80^o$

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10. Let, $\angle A = \angle B = X^o$

Then $\angle C = x + 30^o$

$\therefore x^o + x^o + x^o + 30^o$

$= 180^o \Rightarrow 3x^o = 150^o$

$\therefore x^o = 50^o$

Hence, the angles of a triangle are $50^o, 50^o$ and 80^o
