# 3. Statistics

## Exercise 3.1

### 1. Question

Draw a histogram for the following distribution.

| Class Interval | 0-10 | 10-30 | 30-45 | 45-50 | 50-60 |
|---|---|---|---|---|---|
| Frequency | 8 | 28 | 18 | 6 | 10 |

### Answer

Note that, class intervals have varying base widths. This changes how we generally draw a histogram. And there is a need to find length of rectangles for each class interval because length of rectangle will determine y-axis now, while class interval will be represented on the x-axis.

Length of the rectangle is given by,

$$\text{Length of Rectangle} = \frac{\text{Frequency}}{\text{Class width}} \times C$$

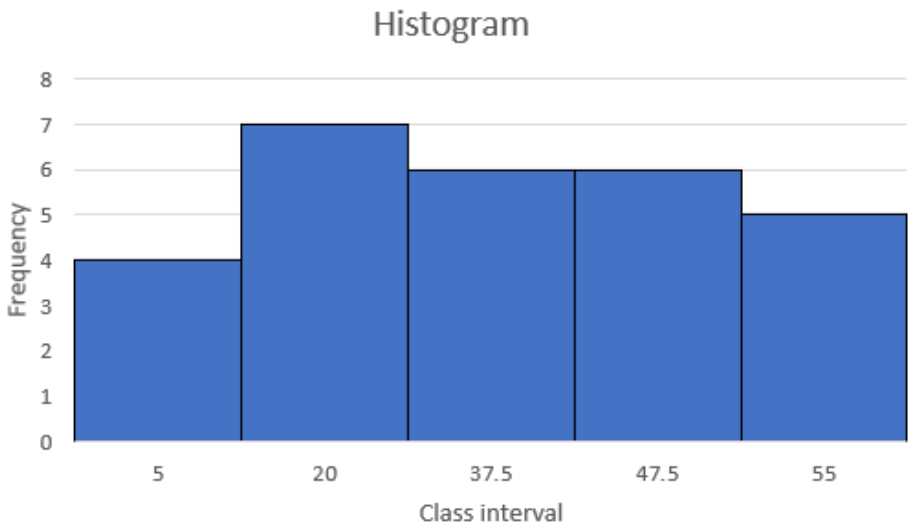
Where C = minimum class width of the data set.

Here, note from the table below that C = 5.

Thus, histogram can be drawn following the given table. Table makes our observation and calculation clearer and ensures accuracy of the final answer.

| Class Interval | Frequency | Class mark | Class width | Length of the Rectangle |
|---|---|---|---|---|
| 0 – 10 | 8 | 5 | 10 | $\frac{8}{10} \times 5 = 4$ |
| 10 – 30 | 28 | 20 | 20 | $\frac{28}{20} \times 5 = 7$ |
| 30 – 45 | 18 | 37.5 | 15 | $\frac{18}{15} \times 5 = 6$ |
| 45 – 50 | 6 | 47.5 | 5 | $\frac{6}{5} \times 5 = 6$ |
| 50 – 60 | 10 | 55 | 10 | $\frac{10}{10} \times 5 = 5$ |

The histogram is as follows:




**2. Question**

Draw a histogram for the monthly wages of the workers in a factory as per data given below.

| Monthly wages (Rs) | 2000-2200 | 2200-2400 | 2400-2800 | 2800-3000 | 3200-3600 | 3200-3600 |
|---|---|---|---|---|---|---|
| No.of workers | 25 | 30 | 50 | 60 | 15 | 10 |

**Answer**

Note that, class intervals have varying base widths. This changes how we generally draw a histogram.

There is a need to find length of rectangles for each class interval because length of rectangle will determine y-axis now, while class interval will be represented on the x-axis.

Length of the rectangle is given by,

$$\text{Length of Rectangle} = \frac{\text{No. of workers}}{\text{Class width}} \times C$$

Where C = minimum class width of the data set.

Here, note from the table below that C = 200.

Thus, histogram can be drawn following the given table. Table makes our observation and calculation clearer and ensures accuracy of the final answer.

| Monthly wages (Rs.) | No. of workers | Class mark | Class width | Length of the Rectangle |
|---|---|---|---|---|
| 2000 – 2200 | 25 | 2100 | 200 | $\frac{25}{200} \times 200 = 25$ |
| 2200 – 2400 | 30 | 2300 | 200 | $\frac{30}{200} \times 200 = 30$ |
| 2400 – 2800 | 50 | 2600 | 400 | $\frac{50}{400} \times 200 = 25$ |
| 2800 – 3000 | 60 | 2900 | 200 | $\frac{60}{200} \times 200 = 60$ |
| 3000 – 3200 | 15 | 3100 | 200 | $\frac{15}{200} \times 200 = 15$ |
| 3200 – 3600 | 10 | 3400 | 400 | $\frac{10}{400} \times 200 = 5$ |

The histogram is as follows:


**3. Question**

The following distribution gives the mass o 48 objects measured to the nearest gram. Draw a histogram to illustrate the data.

| Mass in (gms) | 10-19 | 20-24 | 25-34 | 35-49 | 50-54 |
|---|---|---|---|---|---|
| No. o objects | 6 | 4 | 12 | 18 | 8 |

**Answer**

Note that, class intervals have varying base widths as well as inclusive type of data. This changes how we generally draw a histogram.

We need to convert the data into exclusive type and find length of rectangles for each class interval because length of rectangle will determine y-axis now, while class interval will be represented on the x-axis.

Length of the rectangle is given by,

$$\text{Length of Rectangle} = \frac{\text{No. of objects}}{\text{Class width}} \times C$$

Where C = minimum class width of the data set.

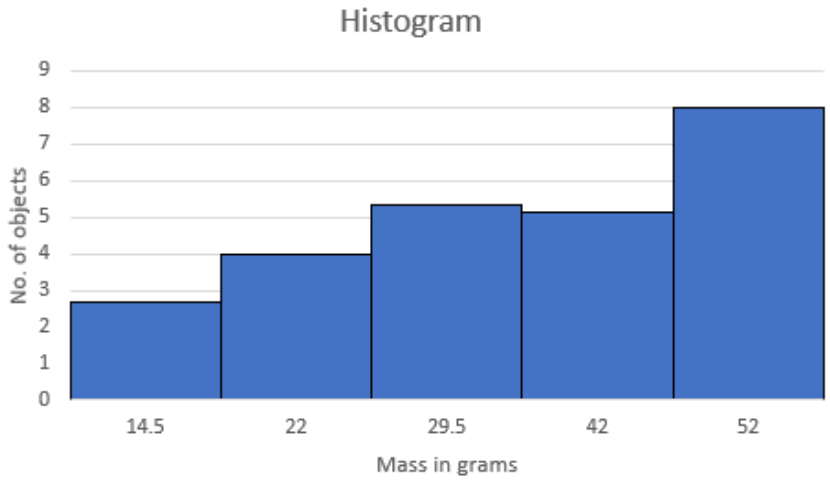
Here, note from the table below that C = 4.

Thus, histogram can be drawn following the given table. Table makes our observation and calculation clearer and ensures accuracy of the final answer.

To convert the inclusive type of data into exclusive type of data, subtract 0.5 from each lower value of the class interval and add 0.5 to each upper value of the class interval.

| Mass (grams) | No. of objects | Class mark | Class width | Length of the Rectangle |
|---|---|---|---|---|
| 9.5 – 19.5 | 6 | 14.5 | 9 | $\frac{6}{9} \times 4 = 2.67$ |
| 19.5 – 24.5 | 4 | 22 | 4 | $\frac{4}{4} \times 4 = 4$ |
| 24.5 – 34.5 | 12 | 29.5 | 9 | $\frac{12}{9} \times 4 = 5.33$ |
| 34.5 – 49.5 | 18 | 42 | 14 | $\frac{18}{14} \times 4 = 5.123$ |
| 49.5 – 54.5 | 8 | 52 | 4 | $\frac{8}{4} \times 4 = 8$ |

The histogram is as follows:




**4. Question**

Draw a histogram to represent the following data.

| Class interval | 10-14 | 14-20 | 20-32 | 32-52 | 52-80 |
|---|---|---|---|---|---|
| Frequency | 5 | 6 | 9 | 25 | 21 |

**Answer**

Note that, class intervals have varying base widths. This changes how we generally draw a histogram.

There is a need to find length of rectangles for each class interval because length of rectangle will determine y-axis now, while class interval will be represented on the x-axis.

Length of the rectangle is given by,

$$\text{Length of Rectangle} = \frac{\text{Frequency}}{\text{Class width}} \times C$$

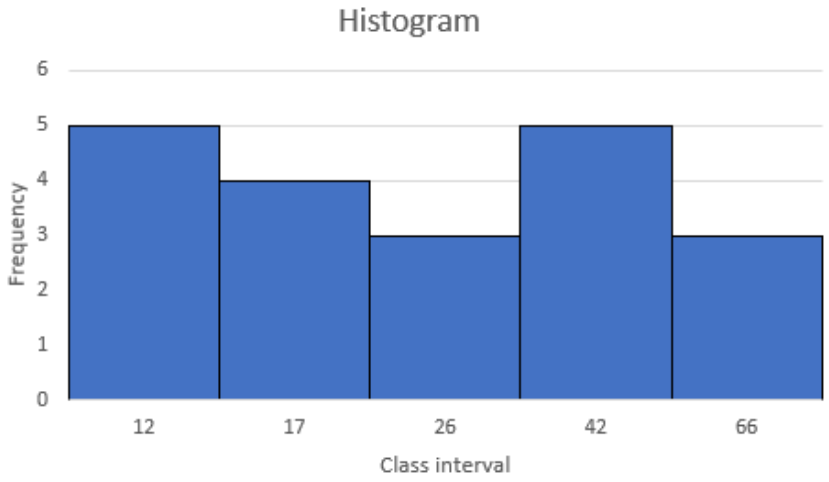
Where C = minimum class width of the data set.

Here, note from the table below that C = 4.

Thus, histogram can be drawn following the given table. Table makes our observation and calculation clearer and ensures accuracy of the final answer.

| Class Interval | Frequency | Class mark | Class width | Length of the Rectangle |
|---|---|---|---|---|
| 10 – 14 | 5 | 12 | 4 | $\frac{5}{4} \times 4 = 5$ |
| 14 – 20 | 6 | 17 | 6 | $\frac{6}{6} \times 4 = 4$ |
| 20 – 32 | 9 | 26 | 12 | $\frac{9}{12} \times 4 = 3$ |
| 32 – 52 | 25 | 42 | 20 | $\frac{25}{20} \times 4 = 5$ |
| 52 – 80 | 21 | 66 | 28 | $\frac{21}{28} \times 4 = 3$ |

The histogram is as follows:




**5. Question**

The age (in years) of360 patients treated in the hospital on a particular day are given below.

| Age in years | 10-20 | 20-30 | 30-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|
| No. of patients | 80 | 50 | 80 | 120 | 30 |

Draw a histogram for the above data.

**Answer**

Note that, class intervals have varying base widths. This changes how we generally draw a histogram.

There is a need to find length of rectangles for each class interval because length of rectangle will determine y-axis now, while class interval will be represented on the x-axis.

Length of the rectangle is given by,

$$\text{Length of Rectangle} = \frac{\text{No. of patients}}{\text{Class width}} \times C$$

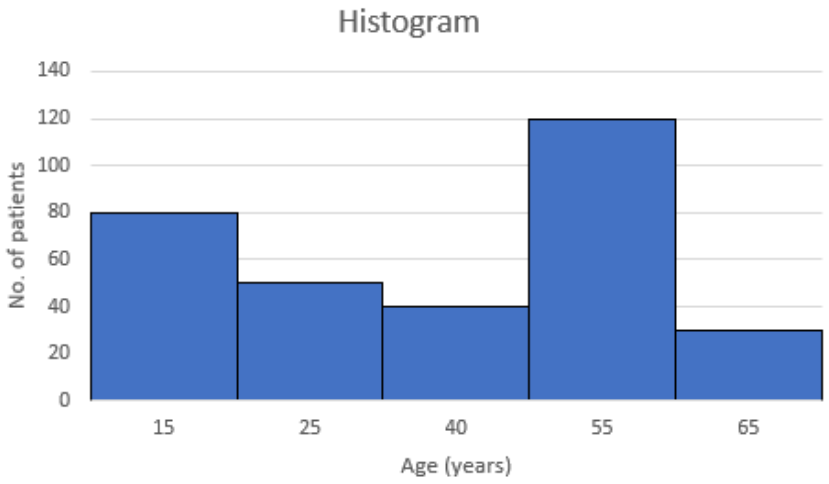
Where C = minimum class width of the data set.

Here, note from the table below that C = 10.

Thus, histogram can be drawn following the given table. Table makes our observation and calculation clearer and ensures accuracy of the final answer.

| Age in years | No. of patients | Class mark | Class width | Length of the Rectangle |
|---|---|---|---|---|
| 10 – 20 | 80 | 15 | 10 | $\frac{80}{10} \times 10 = 80$ |
| 20 – 30 | 50 | 25 | 10 | $\frac{50}{10} \times 10 = 50$ |
| 30 – 50 | 80 | 40 | 20 | $\frac{80}{20} \times 10 = 40$ |
| 50 – 60 | 120 | 55 | 10 | $\frac{120}{10} \times 10 = 120$ |
| 60 – 70 | 30 | 65 | 10 | $\frac{30}{10} \times 10 = 30$ |

The histogram is as follows:




## Exercise 3.2

### 1. Question

Obtain the mean number of bags sold by a shopkeeper on 6 consecutive days from the following table

| Days | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| No.of bags sold | 55 | 32 | 30 | 25 | 10 | 20 |

### Answer

The data given here is raw data of the number of bags sold in their respective days.

Mean of such data is given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where, $\sum x$ = Sum of all observations

& n = Total number of observations

So,

$$\text{Mean} = \frac{55 + 32 + 30 + 25 + 10 + 20}{6}$$

$$\Rightarrow \text{Mean} = \frac{172}{6} = 28.67$$

Thus, mean number of bags sold by the shopkeeper on the given days is 28.67.

**2. Question**

The number of children in 10 families in a locality are 2,4,3,4,1,6,4,5,x,5 find x if the mean number o children in a family is 4

**Answer**

Given is the data of children present in 10 families in a locality.

Mean of number of children in a family = 4

Mean is given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where $\sum x$ = Sum of all observations

& n = Total number of observations.

Here, n = 10

$$\text{So, Mean} = \frac{2+4+3+4+1+6+4+5+x+5}{10}$$

$$\Rightarrow 4 = \frac{34+x}{10}$$

$\Rightarrow 34 + x = 4 \times 10$

$\Rightarrow 34 + x = 40$

$\Rightarrow x = 40 - 34$

$\Rightarrow x = 6$

Thus, x = 6.

**3. Question**

The mean of 20 number is 59. If 3 is added to each number what will be the new mean?

**Answer**

Given that, mean of 20 numbers = 59

Mean can be given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where, $\sum x$ = Sum of all observations

& n = Total number of observations

Here, Mean = 59 and n = 20.

So, $\text{Mean} = \frac{\sum x}{20}$

$\Rightarrow 59 = \frac{\sum x}{20}$

$\Rightarrow \sum x = 59 \times 20$

$\Rightarrow \sum x = 1180$

We have sum of all observations = 1180

If 3 is added to each of the 20 numbers, then new sum of all observations = 1180 + (20 × 3)

⇒ New sum of all observations = 1180 + 60 = 1240

Now, we have fresh data: $\sum x' = 1240$, n = 20

$$\text{New mean} = \frac{1240}{20}$$

⇒ New mean = 62

Thus, new mean of the observation is 62.

**4. Question**

The mean of 15 numbers is 44. If 7 is subtracted from each number what will be the new mean?

**Answer**

Given that, mean of 15 numbers = 44

Mean can be given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where, $\sum x$ = Sum of all observations

& n = Total number of observations

Here, Mean = 44 and n = 15.

So, $\text{Mean} = \frac{\sum x}{15}$

$\Rightarrow 44 = \frac{\sum x}{15}$

$\Rightarrow \sum x = 44 \times 15$

$\Rightarrow \sum x = 660$

We have sum of all observations = 660

If 7 is subtracted from each of the 15 numbers, then the new sum of all observations = 660 – (15 × 7)

⇒ New sum of all observations = 660 – 105 = 555

Now, we have fresh data: ∑x' = 555, n = 15

$$\text{New mean} = \frac{555}{15}$$

⇒ New mean = 37

Thus, new mean of the observation is 37.

**5. Question**

The mean of 12 numbers is 48. If each numbers is multiplied by 4 what will be the new mean.

**Answer**

Given that, mean of 12 numbers = 48

Mean can be given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where, ∑x = Sum of all observations

& n = Total number of observations

Here, Mean = 48 and n = 12.

$$\text{So, Mean} = \frac{\sum x}{12}$$

$$\Rightarrow 48 = \frac{\sum x}{12}$$

⇒ ∑x = 48 × 12

⇒ ∑x = 576

We have sum of all observations = 576

If 4 is multiplied by each of the 12 numbers, then the new sum of all observations = 576 × 4

⇒ New sum of all observations = 2304

Now, we have fresh data: ∑x' = 2304, n = 12

$$\text{New mean} = \frac{2304}{12}$$

⇒ New mean = 192

Thus, new mean of the observation is 192.

**6. Question**

The mean of 16 numbers is 54. If each number is divided by 9 what will be the new mean.

**Answer**

Given that, mean of 16 numbers = 54

Mean can be given by

$$\text{Mean} = \frac{\sum x}{n}$$

Where, ∑x = Sum of all observations

& n = Total number of observations

Here, Mean = 54 and n = 16.

So, $\text{Mean} = \frac{\sum x}{16}$

$\Rightarrow 54 = \frac{\sum x}{16}$

⇒ ∑x = 54 × 16

⇒ ∑x = 864

We have sum of all observations = 864

If 9 is divided from each of the 16 numbers, then the new sum of all observations = 864 ÷ 9

⇒ New sum of all observations = 96

Now, we have fresh data: ∑x' = 96, n = 16

$$\text{New mean} = \frac{96}{16}$$

⇒ New mean = 6

Thus, new mean of the observation is 6.

**7. Question**

The mean weight of 6 boys in a group is 48kg. The individual weights o 5 of them are 50kg, 45kg, 50kg, 42kg and 40kg. Find the weight of the sixth boy.

**Answer**

Given is that, mean weight of 6 boys = 48 kg

Mean is given by,

$$\text{Mean} = \frac{\sum x}{n}$$

Where, ∑x = sum of all observations

& n = total number of observations

Here, n = 6 and mean = 48.

Let the weight of $6^{th}$ boy be x.

So, $\text{Mean} = \frac{50 + 45 + 50 + 42 + 40 + x}{6}$

$\Rightarrow 48 = \frac{227 + x}{6}$

$\Rightarrow 227 + x = 48 \times 6$

$\Rightarrow 227 + x = 288$

$\Rightarrow x = 288 - 227$

$\Rightarrow x = 61$

Thus, weight of the sixth boy is 61 kg.

**8. Question**

Using assumed mean method find the mean weight of 40 students Using the data given below.

| Weights in kg. | 50 | 52 | 53 | 55 | 57 |
|---|---|---|---|---|---|
| No.of students | 10 | 15 | 5 | 6 | 4 |

**Answer**

In assumed-mean method, we need to assume a mean value (the near to middle or middlemost value) from $x_i$'s.

We have the following table.

| **Weight (kg) ($x_i$)** | **No. of students ($f_i$)** | **$d_i = x_i - A$ ($d_i = x_i - 53$)** | **$f_i d_i$** |
|---|---|---|---|
| 50 | 10 | -3 | 10 × -3 = -30 |
| 52 | 15 | -1 | 15 × -1 = -15 |
| 53 = A | 5 | 0 | 5 × 0 = 0 |
| 55 | 6 | 2 | 6 × 2 = 12 |
| 57 | 4 | 4 | 4 × 4 = 16 |
| Total | $\Sigma f_i = 40$ | | $\Sigma f_i d_i = -17$ |

In assumed mean method, arithmetic mean is given by

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$$

Where, A = assumed mean = 53

$\sum f_i d_i = -17$

& $\sum f_i = 40$

Putting all these value in the formula, we get

$$\bar{x} = 53 + \frac{-17}{40}$$

$$\Rightarrow \bar{x} = \frac{2120-17}{40}$$

$$\Rightarrow \bar{x} = \frac{2103}{40} = 52.575$$

Thus, mean weight of 40 students is 52.575 kg.

**9. Question**

The arithmetic mean of a group of 75 observations was calculated as 27. It was later found that one observation was wrongly read as 43 instead of the correct value 53. Obtain the correct arithmetic mean of the data.

**Answer**

Given that, the total number of observations = 75

Mean of 75 observations = 27

Mean is given by,

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

⇒ Sum of all observation = Mean × Total number of observations

⇒ Sum of all observation = 27 × 75 = 2025

Now, according to the question, one of the value in the observation was wrongly taken as 43 instead of 53.

So, in order to obtain the new mean, we need to subtract 43 from the sum of observations and add the correct value, 53 to it.

$$\text{New mean} = \frac{\text{Sum of all observation} - 43 + 53}{75}$$

$$\Rightarrow \text{New mean} = \frac{2025 + 10}{75}$$

$$\Rightarrow \text{New mean} = \frac{2035}{75} = 27.13$$

Thus, the new mean = 27.13

**10. Question**

Mean of 100 observations is found to be 40. At the time of computation two items were wrongly taken as 30 and 27 instead of 3 and 72. Find the correct mean.

**Answer**

Given that, the total number of observations = 100

Mean of 100 observations = 40

Mean is given by,

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

⇒ Sum of all observation = Mean × Total number of observations

⇒ Sum of all observation = 40 × 100 = 4000

Now, according to the question, two items in the observation were wrongly taken as 30 and 27 instead of 3 and 72.

So, in order to obtain the new mean, we need to subtract 30 and 27 from the sum of observations and add the correct values, 3 and 72 to it.

$$\text{New mean} = \frac{\text{Sum of all observation} - 30 - 27 + 3 + 72}{100}$$

$$\Rightarrow \text{New mean} = \frac{4000 + 18}{100}$$

$$\Rightarrow \text{New mean} = \frac{4018}{100} = 40.18$$

Thus, the new mean = 40.18

**11. Question**

The data on number of patients attending a hospital in a month are given below. Find the average number of patients attending the hospital in a day.

| No.of patients | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| No.of days | 2 | 6 | 9 | 7 | 4 | 2 |

**Answer**

We can make the following table for ease of calculation:

| Number of patients | Midpoint ($x_i$) | No. of days ($f_i$) | $x_if_i$ |
|---|---|---|---|
| 0 – 10 | 5 | 2 | 5 × 2 = 10 |
| 10 – 20 | 15 | 6 | 15 × 6 = 90 |
| 20 – 30 | 25 | 9 | 25 × 9 = 225 |
| 30 – 40 | 35 | 7 | 35 × 7 = 245 |
| 40 – 50 | 45 | 4 | 45 × 4 = 180 |
| 50 – 60 | 55 | 2 | 55 × 2 = 110 |
| Total | | $\Sigma f_i = 30$ | $\Sigma x_if_i = 860$ |

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{860}{30}$$

⇒ Mean = 28.67

Thus, the average number of patients attending the hospital are 28.67.

**12. Question**

Calculate the arithmetic mean for the following data Using step deviation method.

| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| No.of students | 8 | 15 | 22 | 20 | 10 | 5 |

**Answer**

In step deviation method, we divide deviation by the width of the class intervals in order to simplify the calculation.

Here, we shall assume a mean from the given $x_i$'s. Let the assumed mean (A) be 35.

Also, $d_i = \frac{x_i - A}{c}$

Where, A = assumed mean = (here) 35

& c = width of the class intervals = (here) 10

Construct the following table:

| **Marks** | **Midpoint ($x_i$)** | **No. of students ($f_i$)** | $d_i = \frac{x_i - 35}{10}$ | $f_i d_i$ |
|---|---|---|---|---|
| 0 – 10 | 5 | 8 | $\frac{5-35}{10} = -3$ | 8 × -3 = -24 |
| 10 – 20 | 15 | 15 | $\frac{15-35}{10} = -2$ | 15 × -2 = -30 |
| 20 – 30 | 25 | 22 | $\frac{25-35}{10} = -1$ | 22 × -1 = -22 |
| 30 – 40 | 35 = A | 20 | $\frac{35-35}{10} = 0$ | 20 × 0 = 0 |
| 40 – 50 | 45 | 10 | $\frac{45-35}{10} = 1$ | 10 × 1 = 10 |
| 50 – 60 | 55 | 5 | $\frac{55-35}{10} = 2$ | 5 × 2 = 10 |
| Total | | $\Sigma f_i = 80$ | | $\Sigma f_i d_i = -56$ |

Using these values, we can find the arithmetic mean of the following data.

Arithmetic mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} \times c$$

$$\Rightarrow \text{Mean} = 35 + \frac{-56}{80} \times 10$$

⇒ Mean = 35 – 7

⇒ Mean = 28

Thus, arithmetic mean of the following data is 28.

**13. Question**

In a study on patients, the following data were obtained. Find the arithmetic mean.

| Age (In yrs) | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 |
|---|---|---|---|---|---|
| No.of patients | 1 | 0 | 1 | 10 | 13 |

**Answer**

Let's convert the inclusive type of data into exclusive type. We can do this by subtracting 0.5 from each lower limits of the class intervals and by adding 0.5 to each upper limits of the class interval.

Using direct method, we can find arithmetic mean of the data.

Let's make the following table for ease of calculation:

| Age (years) | Midpoint ($x_i$) | No. of patients ($f_i$) | $x_if_i$ |
|---|---|---|---|
| 9.5 – 19.5 | 14.5 | 1 | 14.5 × 1 = 14.5 |
| 19.5 – 29.5 | 24.5 | 0 | 24.5 × 0 = 0 |
| 29.5 – 39.5 | 34.5 | 1 | 34.5 × 1 = 34.5 |
| 39.5 – 49.5 | 44.5 | 10 | 44.5 × 10 = 445 |
| 49.5 – 59.5 | 54.5 | 13 | 54.5 × 13 = 708.5 |
| Total | | $\sum f_i = 25$ | $\sum x_if_i = 1202.5$ |

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{1202.5}{25}$$

⇒ Mean = 48.1

Thus, the arithmetic mean of the data is 48.1.

**14. Question**

The total marks obtained by 40 students in the Annual examination are given below

| Marks | 150-200 | 200-250 | 250-300 | 300-350 | 350-400 | 400-450 | 450-500 |
|---|---|---|---|---|---|---|---|
| Students | 2 | 3 | 12 | 10 | 4 | 6 | 3 |

Using step deviation method to find the mean of the above data.

**Answer**

In step deviation method, we divide deviation by the width of the class intervals in order to simplify the calculation.

Here, we shall assume a mean from the given $x_i$'s. Let the assumed mean (A) be 325.

Also, $d_i = \frac{x_i - A}{c}$

Where, A = assumed mean = (here) 325

& c = width of the class intervals = (here) 50

Construct the following table:

| Marks | Midpoint ($x_i$) | Students ($f_i$) | $d_i = \frac{x_i - 325}{50}$ | $f_id_i$ |
|---|---|---|---|---|
| 150 – 200 | 175 | 2 | $\frac{175-325}{50} = -3$ | 2 × -3 = -6 |
| 200 – 250 | 225 | 3 | $\frac{225-325}{50} = -2$ | 3 × -2 = -6 |
| 250 – 300 | 275 | 12 | $\frac{275-325}{50} = -1$ | 12 × -1 = -12 |
| 300 – 350 | 325 = A | 10 | $\frac{325-325}{50} = 0$ | 10 × 0 = 0 |
| 350 – 400 | 375 | 4 | $\frac{375-325}{50} = 1$ | 4 × 1 = 4 |
| 400 – 450 | 425 | 6 | $\frac{425-325}{50} = 2$ | 6 × 2 = 12 |
| 450 – 500 | 475 | 3 | $\frac{475-325}{50} = 3$ | 3 × 3 = 9 |
| Total | | $\Sigma f_i = 40$ | | $\Sigma f_id_i = 1$ |

Using these values, we can find the arithmetic mean of the following data.

Arithmetic mean is given by

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} \times c$$

$$\Rightarrow \text{Mean} = 325 + \frac{1}{40} \times 50$$

$$\Rightarrow \text{Mean} = (13000 + 50)/40$$

⇒ Mean = 326.25

Thus, arithmetic mean of the following data is 326.25.

**15. Question**

Computer the arithmetic of the following distribution.

| Class Interval | 0-19 | 20-39 | 40-59 | 60-79 | 80-99 |
|---|---|---|---|---|---|
| frequency | 3 | 4 | 15 | 14 | 4 |

**Answer**

Let's convert the inclusive type of data into exclusive type. We can do this by subtracting 0.5 from each lower limits of the class intervals and by adding 0.5 to each upper limits of the class interval.

Using direct method, we can find arithmetic mean of the data.

Let's make the following table for ease of calculation:

| Class interval | Midpoint ($x_i$) | Frequency ($f_i$) | $x_if_i$ |
|---|---|---|---|
| -0.5 – 19.5 | 9.5 | 3 | 9.5 × 3 = 28.5 |
| 19.5 – 39.5 | 29.5 | 4 | 29.5 × 4 = 118 |
| 39.5 – 59.5 | 49.5 | 15 | 49.5 × 15 = 742.5 |
| 59.5 – 79.5 | 69.5 | 14 | 69.5 × 14 = 973 |
| 79.5 – 99.5 | 89.5 | 4 | 89.5 × 4 = 358 |
| Total | | $\Sigma f_i = 40$ | $\Sigma x_if_i = 2220$ |

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{2220}{40}$$

$\Rightarrow$ Mean = 55.5

Thus, the arithmetic mean of the data is 55.5.

## Exercise 3.3

### 1. Question

Find the median of the following data.

(i) 18,12,51,32,106,92,58

(ii) 28,7,15,3,14,18,46,59,1,2,9,21

**Answer**

(i). Let us arrange the numbers in ascending order. (Without arranging the values in ascending/descending order, we will not be able to get the correct median) We have,

12, 18, 32, 51, 58, 92 ,106 ...(1)

Total number of items, n = 7 (odd number).

When n is odd, median is given by

$$\text{Median} = \left(\frac{n + 1}{2}\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \left(\frac{7 + 1}{2}\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \left(\frac{8}{2}\right)^{\text{th}} \text{term}$$

$\Rightarrow$ Median = $4^{th}$ term

Count $4^{th}$ term from the arranged numbers in (1).

The $4^{th}$ term = 51

$\Rightarrow$ Median = 51 [$\because$ Median = $4^{th}$ term]

Thus, median is 51.

(ii). Let us arrange the numbers in ascending order. (Without arranging the values in ascending/descending order, we will not be able to get the correct median) We have,

1, 2, 3, 7, 9, 14, 15, 18, 21, 28, 46, 59 ...(1)

Total number of items, n = 12 (even number).

When n is even, median is given by

$$\text{Median} = \text{mean of } \left(\frac{n}{2}\right)^{th} \text{ and } \left(\frac{n}{2} + 1\right)^{th} \text{ term}$$

$$\Rightarrow \text{Median} = \text{mean of } \left(\frac{12}{2}\right)^{th} \text{ and } \left(\frac{12}{2} + 1\right)^{th} \text{ term}$$

⇒ Median = mean of $6^{th}$ and $7^{th}$ term

Count $6^{th}$ and $7^{th}$ term from the arranged numbers in (1).

The $6^{th}$ term = 14

& $7^{th}$ term = 15

⇒ Median = mean of 14 and 15 ...(2)

Mean of 14 and 15 is calculated as,

$$\text{Mean} = \frac{14 + 15}{2}$$

$$\Rightarrow \text{Mean} = \frac{29}{2} = 14.5$$

Put mean of 14 and 15, 14.5 in equation (2),

Median = 14.5

Thus, median is 14.5.

**2. Question**

Find the median for the following frequency table.

| Value | 12 | 13 | 15 | 19 | 22 | 23 |
|---|---|---|---|---|---|---|
| Frequency | 4 | 2 | 4 | 4 | 1 | 5 |

**Answer**

Since, we have ungrouped data, we need to sort the data in ascending order to find median of the data.

Let us formulate a table for easy and simple calculations.

| Value | Frequency (f) | Cumulative Frequency (cf) |
|---|---|---|
| 12 | 4 | 4 |
| 13 | 2 | 4 + 2 = 6 |
| 15 ← | 4 | 6 + 4 = 10 ← |
| 19 ← | 4 | 10 + 4 = 14 ← |
| 22 | 1 | 14 + 1 = 15 |
| 23 | 5 | 15 + 5 = 20 |
| Total | 20 | |

Total number of items, n = 20 (even)

When n = even, position of median is given by

$$\text{Position of median} = \frac{\left\{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2} + 1\right)^{th}\right\}}{2}$$

$$\Rightarrow \text{Position of median} = \frac{\left\{\left(\frac{20}{2}\right)^{th} + \left(\frac{20}{2} + 1\right)^{th}\right\}}{2}$$

$$\Rightarrow \text{Position of median} = \frac{[10^{th} + 11^{th}]}{2}$$

Note that, from the cumulative frequency column: $10^{th}$ value = 15 & $11^{th}$ value = 19

Then, $\text{Median} = \frac{[15 + 19]}{2}$

$\Rightarrow \text{Median} = \frac{34}{2}$

$\Rightarrow$ Median = 17

Thus, median is 17.

**3. Question**

Find the median for the following data.

| Height (ft) | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 |
|---|---|---|---|---|---|
| No.of trees | 4 | 3 | 10 | 8 | 5 |

**Answer**

This is a grouped frequency distribution, in which median is calculated as follows.

We have the following table:

| Height (ft) | No. of trees (f) | Cumulative frequency (cf) |
|---|---|---|
| 5 – 10 | 4 | 4 |
| 10 – 15 | 3 | 4 + 3 = 7 |
| 15 – 20 ← | 10 ← | 7 + 10 = 17 ← |
| 20 – 25 | 8 | 17 + 8 = 25 |
| 25 – 30 | 5 | 25 + 5 = 30 |
| Total | N = 30 | |

Here, we get total frequency, N = 30. So,

$$\frac{N}{2} = \frac{30}{2} = 15$$

Since, N/2 = 15. Mark a cumulative frequency which is just greater than 15.

Here, it is 17. So, median class would be 15 – 20.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 15

N/2 = 15 (as calculated above)

m = cumulative frequency of the class preceding the median class = 7

f = frequency of the median class = 10

c = width of the median class = 5

Putting all these values in the formula, we get

$$\text{Median} = 15 + \frac{15-7}{10} \times 5$$

$$\Rightarrow \text{Median} = 15 + \frac{8\times5}{10}$$

⇒ Median = 15 + 4 = 19

Thus, median is 19.

**4. Question**

Find the median for the following data.

| Age group | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 |
|---|---|---|---|---|---|---|---|
| No. of persons | 4 | 6 | 10 | 11 | 12 | 6 | 1 |

**Answer**

This is a grouped frequency distribution, in which median is calculated as follows.

First, we need to convert this inclusive type of data values into exclusive type. For this, subtract 0.5 from lower limits of each class and add 0.5 to upper limits of each class interval.

We have the following table:

| Age group | No. of persons (f) | Cumulative frequency (cf) |
|---|---|---|
| 0.5 – 9.5 | 4 | 4 |
| 9.5 – 19.5 | 6 | 4 + 6 = 10 |
| 19.5 – 29.5 | 10 | 10 + 10 = 20 |
| 29.5 – 39.5 ← | 11 ← | 20 + 11 = 31 ← |
| 39.5 – 49.5 | 12 | 31 + 12 = 43 |
| 49.5 – 59.5 | 6 | 43 + 6 = 49 |
| 59.5 – 69.5 | 1 | 49 + 1 = 50 |
| Total | N = 50 | |

Here, we get total frequency, N = 50. So,

$$\frac{N}{2} = \frac{50}{2} = 25$$

Since, N/2 = 25. Mark a cumulative frequency which is just greater than 25.

Here, it is 31. So, median class would be 29.5 – 39.5.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 29.5

N/2 = 25 (as calculated above)

m = cumulative frequency of the class preceding the median class = 20

f = frequency of the median class = 11

c = width of the median class = 10

Putting all these values in the formula, we get

$$\text{Median} = 29.5 + \frac{25 - 20}{11} \times 10$$

$$\Rightarrow \text{Median} = 29.5 + \frac{5 \times 10}{11}$$

⇒ Median = 29.5 + 4.55 = 34.05

Thus, median is 34.05.

**5. Question**

Calculate the median for the following data

| Class interval | 1-5 | 6-10 | 11-15 | 16-20 | 21-25 | 26-30 | 31-35 |
|---|---|---|---|---|---|---|---|
| Frequency | 1 | 18 | 25 | 26 | 7 | 2 | 1 |

**Answer**

This is a grouped frequency distribution, in which median is calculated as follows.

First, we need to convert this inclusive type of data values into exclusive type. For this, subtract 0.5 from lower limits of each class and add 0.5 to upper limits of each class interval.

We have the following table:

| Class Interval | Frequency (f) | Cumulative frequency (cf) |
|---|---|---|
| 0.5 – 5.5 | 1 | 1 |
| 5.5 – 10.5 | 18 | 1 + 18 = 19 |
| 10.5 – 15.5 ← | 25 ← | 19 + 25 = 44 ← |
| 15.5 – 20.5 | 26 | 44 + 26 = 70 |
| 20.5 – 25.5 | 7 | 70 + 7 = 77 |
| 25.5 – 30.5 | 2 | 77 + 2 = 79 |
| 30.5 – 35.5 | 1 | 79 + 1 = 80 |
| Total | N = 80 | |

Here, we get total frequency, N = 80. So,

$$\frac{N}{2} = \frac{80}{2} = 40$$

Since, N/2 = 40. Mark a cumulative frequency which is just greater than 40.

Here, it is 44. So, median class would be 10.5 – 15.5.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 10.5

N/2 = 40 (as calculated above)

m = cumulative frequency of the class preceding the median class = 19

f = frequency of the median class = 25

c = width of the median class = 5

Putting all these values in the formula, we get

$$\text{Median} = 10.5 + \frac{40 - 19}{25} \times 5$$

$$\Rightarrow \text{Median} = 10.5 + \frac{21 \times 5}{25}$$

⇒ Median = 10.5 + 4.2 = 14.7

Thus, median is 14.7.

**6. Question**

The following table gives the distribution of the average weekly wages of 800 workers in a factory. Calculate the median or the data given below.

| Wages(Rs. in hundreds) | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 |
|---|---|---|---|---|---|---|---|---|
| No. o persons | 50 | 70 | 100 | 180 | 150 | 120 | 70 | 60 |

**Answer**

This is a grouped frequency distribution, in which median is calculated as follows.

We have the following table:

| Wages (Rs.) | No. of trees (f) | Cumulative frequency (cf) |
|---|---|---|
| 20 – 25 | 50 | 50 |
| 25 – 30 | 70 | 50 + 70 = 120 |
| 30 – 35 | 100 | 120 + 100 = 220 |
| 35 – 40 | 180 | 220 + 180 = 400 |
| 40 – 45 ← | 150 ← | 400 + 150 = 550 ← |
| 45 – 50 | 120 | 550 + 120 = 670 |
| 50 – 55 | 70 | 670 + 70 = 740 |
| 55 – 60 | 60 | 740 + 60 = 800 |
| Total | N = 800 | |

Here, we get total frequency, N = 800. So,

$$\frac{N}{2} = \frac{800}{2} = 400$$

Since, N/2 = 400. Mark a cumulative frequency which is just greater than 400.

Here, it is 550. So, median class would be 40 – 45.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 40

N/2 = 400 (as calculated above)

m = cumulative frequency of the class preceding the median class = 400

f = frequency of the median class = 150

c = width of the median class = 5

Putting all these values in the formula, we get

$$\text{Median} = 40 + \frac{400 - 400}{150} \times 5$$

$$\Rightarrow \text{Median} = 40 + \frac{0 \times 5}{150}$$

⇒ Median = 40 + 0 = 40

Thus, median is 40.

## Exercise 3.4

### 1. Question

The marks obtained by 15 students of a class are given below. Find the modal marks
42,45,47,49,52,65,65,71,71,72,75,82,72,47 ,72

### Answer

Mode is the number which has highest frequency, that is, largest occurrence.

For ease of observation, we can arrange the data in ascending order. We have,

42, 45, 47, 47, 49, 52, 65, 65, 71, 71, 72, 72, 72, 75, 82

This can be arranged in a tabular form showing frequencies of marks:

| Marks | Frequency |
|---|---|
| 42 | 1 |
| 45 | 1 |
| 47 | 2 |
| 49 | 1 |
| 52 | 1 |
| 65 | 2 |
| 71 | 2 |
| 72 | 3 ← |
| 75 | 1 |
| 82 | 1 |

The highest frequency is 3, and its corresponding mark is 72.

Thus, modal mark is 72.

**2. Question**

Calculate the mode of the following data.

| Size of shoe | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|
| No. of Pairs sold | 15 | 17 | 13 | 21 | 18 | 16 | 11 |

**Answer**

Mode is nothing but a number with highest frequency. Interpret the column 'No. of pairs sold' in the terms of frequency.

| Size of shoe | Frequency |
|---|---|
| 4 | 15 |
| 5 | 17 |
| 6 | 13 |
| 7 | 21 ← |
| 8 | 18 |
| 9 | 16 |
| 10 | 11 |

The highest frequency is 21, and its corresponding size of shoe is 7.

Thus, mode of the following data is 7.

**3. Question**

The age (in years )of 150 patients getting medical treatment in a hospital in a month are given below. Obtain its mode.

| Age (yrs) | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|
| No o patients | 12 | 14 | 36 | 50 | 20 | 18 |

**Answer**

This is a grouped frequency distribution. To find mode of this type of data, we can interpret the column 'No. of patients' as frequency.

| Age (years) | Frequency |
|---|---|
| 10 – 20 | 12 |
| 20 – 30 | 14 |
| 30 – 40 | 36 |
| 40 – 50 | 50 ← |
| 50 – 60 | 20 |
| 60 – 70 | 18 |

Note that, highest frequency = 50.

Corresponding class interval = 40 – 50

⇒ Modal class = 40 – 50

Mode is given by,

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times c$$

Where, l = lower limit of the modal class = 40

f = frequency of the modal class = 50

$f_1$ = frequency of the class preceding the modal class = 36

$f_2$ = frequency of the class succeeding the modal class = 20

c = width of the class interval = 10

Putting all these values in the modal formula,

$$\text{Mode} = 40 + \frac{50 - 36}{2 \times 50 - 36 - 20} \times 10$$

$$\Rightarrow \text{Mode} = 40 + \frac{14}{100 - 56} \times 10$$

$$\Rightarrow \text{Mode} = 40 + \frac{140}{44}$$

$$\Rightarrow \text{Mode} = 40 + 3.18$$

⇒ Mode = 43.18

Thus, the mode is 43.18.

**4. Question**

For the following data obtain the mode.

| Weight (in kg) | 21-25 | 26-30 | 31-35 | 36-40 | 41-45 | 46-50 | 51-55 | 56-60 |
|---|---|---|---|---|---|---|---|---|
| No of students | 5 | 4 | 3 | 18 | 20 | 14 | 8 | 3 |

**Answer**

This is a grouped frequency distribution and an inclusive type of data.

To find mode of this type of data, we must convert this inclusive type of data into exclusive type. For exclusive type data, we need to subtract 0.5 from lower limits of each class interval and add 0.5 to upper limits of each class intervals.

Also, we can interpret the column 'No. of students' as frequency.

| f | Frequency |
|---|---|
| 20.5 – 25.5 | 5 |
| 25.5 – 30.5 | 4 |
| 30.5 – 35.5 | 3 |
| 35.5 – 40.5 | 18 |
| 40.5 – 45.5 | 20 ← |
| 45.5 – 50.5 | 14 |
| 50.5 – 55.5 | 8 |
| 55.5 – 60.5 | 3 |

Note that, highest frequency = 20

Corresponding class interval = 40.5 – 45.5

⇒ Modal class = 40.5 – 45.5

Mode is given by,

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times c$$

Where, l = lower limit of the modal class = 40.5

f = frequency of the modal class = 20

$f_1$ = frequency of the class preceding the modal class = 18

$f_2$ = frequency of the class succeeding the modal class = 14

c = width of the class interval = 5

Putting all these values in the modal formula,

$$\text{Mode} = 40.5 + \frac{20 - 18}{2 \times 20 - 18 - 14} \times 5$$

$$\Rightarrow \text{Mode} = 40.5 + \frac{2}{40-32} \times 5$$

$$\Rightarrow \text{Mode} = 40.5 + \frac{2}{8} \times 5$$

⇒ Mode = 40.5 + 1.25

⇒ Mode = 41.75

Thus, the mode is 41.75.

**5. Question**

The ages of children in a scout camp are 13,13,14,15,13,15,14,15,13,15 years. Find the mean, median and mode of the data.

**Answer**

For Mean:

Since, this is raw data. Mean is given by,

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\Rightarrow \text{Mean} = \frac{13+13+14+15+13+15+14+15+13+15}{10}$$

$$\Rightarrow \text{Mean} = \frac{140}{10}$$

$\Rightarrow$ Mean = 14

Thus, mean of the data is 14.

For Median:

We need to arrange the raw data in ascending order. We have,

13, 13, 13, 13, 14, 14, 15, 15, 15, 15

Total number of items, n = 10 (even)

When n is even, then median is given by

$$\text{Median} = \text{mean of } \left(\frac{n}{2}\right)^{th} \text{ and } \left(\frac{n}{2}+1\right)^{th} \text{ terms} \quad ...(i)$$

Calculate $(n/2)^{th}$ position.

$$\left(\frac{n}{2}\right)^{th} \text{term} = \left(\frac{10}{2}\right)^{th} \text{term}$$

$$\Rightarrow \left(\frac{n}{2}\right)^{th} \text{term} = 5^{th}\text{term}$$

$$\Rightarrow \left(\frac{n}{2}\right)^{th} \text{term} = 14$$

[$\because$ from the arranged data, $5^{th}$ term = 14] ...(ii)

Now, calculate $(n/2 + 1)^{th}$ position.

$$\left(\frac{n}{2}+1\right)^{th} \text{term} = \left(\frac{10}{2}+1\right)^{th} \text{term}$$

$$\Rightarrow \left(\frac{n}{2}+1\right)^{th} \text{term} = 6^{th}\text{term}$$

$$\Rightarrow \left(\frac{n}{2}+1\right)^{th} \text{term} = 14$$

[$\because$ from the arranged data, $6^{th}$ term = 14] ...(iii)

Putting (ii) and (iii) in equation (i), we get

Median = Mean of 14 and 14

$$\Rightarrow \text{Median} = \frac{14+14}{2}$$

$$\Rightarrow \text{Median} = \frac{28}{2}$$

⇒ Median = 14

Thus, median is 14.

For Mode:

We need to arrange the raw data in ascending order for ease of observation. We have,

13, 13, 13, 13, 14, 14, 15, 15, 15, 15

From observing from the arranged data, note that

13 → 4 times

14 → 2 times

15 → 4 times

Representing it in tabular form,

| Data | Frequency |
|---|---|
| 13 | 4 ← |
| 14 | 2 |
| 15 | 4 ← |

Highest frequency = 4

And the corresponding data are 13 and 15.

This is a bimodal distribution, because it has two modes.

Thus, modes are 13 and 15.

**6. Question**

The following table gives the numbers of branches and number plants in a garden of a school.

| No.of branches | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| No of plants | 14 | 21 | 28 | 20 | 17 |

Calculate the mean, median and mode of the above data.

**Answer**

We have

| No. of branches ($x_i$) | No. of plants ($f_i$) | $x_if_i$ | Cumulative frequency (cf) |
|---|---|---|---|
| 2 | 14 | 2 × 14 = 28 | 14 |
| 3 | 21 | 3 × 21 = 63 | 14 + 21 = 35 |
| 4 | 28 | 4 × 28 = 112 | 35 + 28 = 63 |
| 5 | 20 | 5 × 20 = 100 | 63 + 20 = 83 |
| 6 | 17 | 6 × 17 = 102 | 83 + 17 = 100 |
| Total | $\sum f_i = 100$ | $\sum x_if_i = 405$ | |

For Mean:

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$\Rightarrow \text{Mean} = \frac{405}{100}$ [from the table]

$\Rightarrow$ Mean = 4.05

For Median:

Total number of items, n = 5 (odd number).

When n is odd, median is given by

$$\text{Median} = \left(\frac{n + 1}{2}\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \left(\frac{5 + 1}{2}\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \left(\frac{6}{2}\right)^{\text{th}} \text{term}$$

$\Rightarrow$ Median = $3^{rd}$ term

Count $3^{rd}$ term from the arranged numbers in the table.

The $3^{rd}$ term = 4

$\Rightarrow$ Median = 4 [$\because$ Median = $3^{rd}$ term]

For Mode:

Mode is nothing but a number with highest frequency. Interpret the column 'No. of plants' in the terms of frequency.

| No. of branches | Frequency |
|---|---|
| 2 | 14 |
| 3 | 21 |
| 4 | 28 ← |
| 5 | 20 |
| 6 | 17 |

Here, highest frequency is 28. Then, corresponding no. of branches are 4.

⇒ Mode = 4

Thus, mean is 4.05, median is 4 and mode is 4.

**7. Question**

The following table shows the age distribution of cases of certain disease reported during a year in a particular city.

| Age in year | 5-14 | 15-24 | 25-34 | 35-44 | 45-54 | 55-64 |
|---|---|---|---|---|---|---|
| No. of cases | 6 | 11 | 12 | 10 | 7 | 4 |

Obtain the mean, median and mode of the above data.

**Answer**

We have

| **Age (year)** | **Midpoint ($x_i$)** | **No. of cases ($f_i$)** | **$x_if_i$** | **Cumulative frequency (cf)** |
|---|---|---|---|---|
| 4.5 – 14.5 | 9.5 | 6 | 9.5 × 6 = 57 | 6 |
| 14.5 – 24.5 | 19.5 | 11 | 19.5 × 11 = 214.5 | 6 + 11 = 17 |
| 24.5 – 34.5 ← | 29.5 | 12 ← | 29.5 × 12 = 354 | 17 + 12 = 29 ← |
| 34.5 – 44.5 | 39.5 | 10 | 39.5 × 10 = 395 | 29 + 10 = 39 |
| 44.5 – 54.5 | 49.5 | 7 | 49.5 × 7 = 346.5 | 39 + 7 = 46 |
| 54.5 – 64.5 | 59.5 | 4 | 59.5 × 4 = 238 | 46 + 4 = 50 |
| Total | | $\Sigma f_i = 50$ | $\Sigma x_if_i = 1605$ | |

For Mean:

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{1605}{50} \text{ [from the table]}$$

⇒ Mean = 32.1

For Median:

Here, we get total frequency, $\sum f_i = N = 50$. So,

$$\frac{N}{2} = \frac{50}{2} = 25$$

Since, N/2 = 25. Mark a cumulative frequency which is just greater than 25.

Here, it is 29. So, median class would be 24.5 – 34.5.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 24.5

N/2 = 25 (as calculated above)

m = cumulative frequency of the class preceding the median class = 17

f = frequency of the median class = 12

c = width of the median class = 10

Putting all these values in the formula, we get

$$\text{Median} = 24.5 + \frac{25 - 17}{12} \times 10$$

$$\Rightarrow \text{Median} = 24.5 + \frac{8 \times 10}{12}$$

⇒ Median = 24.5 + 6.67 = 31.167

For Mode:

We have the following data in exclusive type in tabular form. We can interpret the column 'No. of cases' as frequency.

| Age (year) | Frequency |
|---|---|
| 4.5 – 14.5 | 6 |
| 14.5 – 24.5 | 11 |
| 24.5 – 34.5 | 12 ← |
| 34.5 – 44.5 | 10 |
| 44.5 – 54.5 | 7 |
| 54.5 – 64.5 | 4 |

Note that, highest frequency = 12

Corresponding class interval = 24.5 – 34.5

⇒ Modal class = 24.5 – 34.5

Mode is given by,

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times c$$

Where, l = lower limit of the modal class = 24.5

f = frequency of the modal class = 12

$f_1$ = frequency of the class preceding the modal class = 11

$f_2$ = frequency of the class succeeding the modal class = 10

c = width of the class interval = 10

Putting all these values in the modal formula,

$$\text{Mode} = 24.5 + \frac{12-11}{2\times 12-11-10}\times 10$$

$$\Rightarrow \text{Mode} = 24.5 + \frac{1}{24-21}\times 10$$

$$\Rightarrow \text{Mode} = 24.5 + \frac{1}{3}\times 10$$

⇒ Mode = 24.5 + 3.33

⇒ Mode = 27.83

Thus, mean is 32.1, median is 32.167 and mode is 27.83.

**8. Question**

Find the mean, mode and median of marks obtained by 20 students in an examination.

The marks are given below.

| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| No. of students | 1 | 4 | 5 | 8 | 2 |

**Answer**

We have

| **Marks** | **Midpoint ($x_i$)** | **No. of students ($f_i$)** | **$x_if_i$** | **Cumulative frequency (cf)** |
|---|---|---|---|---|
| 0 – 10 | 5 | 1 | 5 × 1 = 5 | 1 |
| 10 – 20 | 15 | 4 | 15 × 4 = 60 | 1 + 4 = 5 |
| 20 – 30 | 25 | 5 | 25 × 5 = 125 | 5 + 5 = 10 |
| 30 – 40 | 35 ← | 8 ← | 35 × 8 = 280 | 10 + 8 = 18 ← |
| 40 – 50 | 45 | 2 | 45 × 2 = 90 | 18 + 2 = 20 |
| Total | | $\sum f_i$ = 20 | $\sum x_if_i$ = 560 | |

For Mean:

Mean is given by

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{560}{20} \text{ [from the table]}$$

⇒ Mean = 28

For Median:

Here, we get total frequency, $\sum f_i = N = 20$. So,

$$\frac{N}{2} = \frac{20}{2} = 10$$

Since, N/2 = 10. Mark a cumulative frequency which is just greater than 10.

Here, it is 18. So, median class would be 30 – 40.

Median is given by

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where, l = lower limit of the median class = 30

N/2 = 10 (as calculated above)

m = cumulative frequency of the class preceding the median class = 10

f = frequency of the median class = 8

c = width of the median class = 10

Putting all these values in the formula, we get

$$\text{Median} = 30 + \frac{10 - 10}{8} \times 10$$

$$\Rightarrow \text{Median} = 30 + \frac{0 \times 10}{8}$$

⇒ Median = 30 + 0 = 30

For Mode:

We have the following data in exclusive type in tabular form. We can interpret the column 'No. of students' as frequency.

| Marks | Frequency |
|---|---|
| 0 – 10 | 1 |
| 10 – 20 | 4 |
| 20 – 30 | 5 |
| 30 – 40 | 8 ← |
| 40 – 50 | 2 |

Note that, highest frequency = 8

Corresponding class interval = 30 – 40

⇒ Modal class = 30 – 40

Mode is given by,

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times c$$

Where, l = lower limit of the modal class = 30

f = frequency of the modal class = 8

$f_1$ = frequency of the class preceding the modal class = 5

$f_2$ = frequency of the class succeeding the modal class = 2

c = width of the class interval = 10

Putting all these values in the modal formula,

$$\text{Mode} = 30 + \frac{8-5}{2\times 8-5-2}\times 10$$

$$\Rightarrow \text{Mode} = 30 + \frac{3}{16-7}\times 10$$

$$\Rightarrow \text{Mode} = 30 + \frac{3}{9}\times 10$$

⇒ Mode = 30 + 3.33

⇒ Mode = 33.3

Thus, mean is 28, median is 30 and mode is 33.3.

## Exercise 3.5

### 1. Question

| The mean of the first 10 natural numbers is

A. 25

B. 55

C. 5.5

D. 2.5

**Answer**

Let us list out first 10 natural numbers. They are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Total number of observations = 10

Mean is given by

$$\text{Mean} = \frac{\text{Sum of all observation}}{\text{Total number of observations}}$$

$$\Rightarrow \text{Mean} = \frac{1+2+3+4+5+6+7+8+9+10}{10}$$

$$\Rightarrow \text{Mean} = \frac{55}{10}$$

⇒ Mean = 5.5

Thus, option (C) is correct.

### 2. Question

The Arithmetic mean of integers from-5 to 5 is

A. 3

B. 0

C. 25

D. 10

**Answer**

The integers from -5 and 5 are:

-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

Total number of observations = 11

Mean is given by

$$\text{Mean} = \frac{\text{Sum of all observation}}{\text{Total number of observations}}$$

$$\Rightarrow \text{Mean} = \frac{-5-4-3-2-1+0+1+2+3+4+5}{11}$$

⇒ Mean = 0

Thus, option (B) is correct.

**3. Question**

If the mean of x,x + 2,x + 4,x + 6,x + 8 is 20 then x is

A. 32

B. 16

C. 8

D. 4

**Answer**

Here, total number of observations = 5

And given that, mean = 20

Mean is given by

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\Rightarrow 20 = \frac{x+x+2+x+4+x+6+x+8}{5}$$

⇒ 5x + 20 = 5 × 20

⇒ 5x + 20 = 100

⇒ 5x = 100 – 20

⇒ 5x = 80

$$\Rightarrow x = \frac{80}{5}$$

⇒ x = 16

Thus, option (B) is correct.

**4. Question**

The mode of the data 5,5,5,5,5,1,2,2,3,3,3,4,4,4,4 is

A. 2

B. 3

C. 4

D. 5

**Answer**

We can arrange the data in ascending form as:

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5

This gives us an ease of observation.

Here, 5 is coming 5 times and it is the most occurring value in the data.

Since, mode is a number having the highest frequency of all numbers in the data.

⇒ Mode = 5

Thus, option (D) is correct.

**5. Question**

The median of 14, 12, 10,9,11 is

A. 11

B. 10

C. 9.5

D. 10.5

**Answer**

To find median, we need to arrange data in ascending order.

We have

9, 10, 11 ,12, 14

Total number of observations, n = 5 (odd).

When n is odd, median is found as follows.

$$\text{Median} = \left(\frac{n + 1}{2}\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \left(\frac{5 + 1}{2}\right)^{\text{th}} \text{term}$$

$\Rightarrow \text{Median} = \left(\frac{6}{2}\right)^{\text{th}} \text{term}$

$\Rightarrow$ Median = $3^{rd}$ term

$\Rightarrow$ Median = 11 [$\because$ the $3^{rd}$ term in the arranged data = 11]

Thus, option (A) is correct.

**6. Question**

The median of 2,7,4,8,9,1 is

A. 4

B. 6

C. 5.5

D. 7

**Answer**

To find median, we need to arrange data in ascending order.

We have

1, 2, 4, 7, 8, 9

Total number of observations, n = 6 (even).

When n is even, median is found as follows.

$$\text{Median} = \text{Mean of } \left(\frac{n}{2}\right)^{\text{th}} \text{term and } \left(\frac{n}{2}+1\right)^{\text{th}} \text{term}$$

$$\Rightarrow \text{Median} = \text{Mean of } \left(\frac{6}{2}\right)^{\text{th}} \text{term and } \left(\frac{6}{2}+1\right)^{\text{th}} \text{term}$$

$\Rightarrow$ Median = Mean of $3^{rd}$ term and $4^{th}$ term

$\Rightarrow$ Median = Mean of 4 and 7 [$\because$ from the arranged data, $3^{rd}$ term = 4 and $4^{th}$ term = 7]

$$\Rightarrow \text{Median} = \frac{4+7}{2}$$

$$\Rightarrow \text{Median} = \frac{11}{2}$$

$\Rightarrow$ Median = 5.5

Thus, option (C) is correct.

**7. Question**

The mean of first 5 whole number is

A. 2

B. 2.5

C. 3

D. 0

**Answer**

The first whole numbers are: 0, 1, 2, 3, 4

Total number of observations = 5

Mean is given by

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\Rightarrow \text{Mean} = \frac{0+1+2+3+4}{5}$$

$$\Rightarrow \text{Mean} = \frac{10}{5}$$

⇒ Mean = 2

Thus, option (A) is correct.

**8. Question**

The arithmetic mean of 10 number is-7. If 5 is added to every number, then the new Arithmetic mean is

A. -2

B. 12

C. -7

D. 17

**Answer**

Given that, arithmetic mean of 10 numbers = -7

We know, total number of observations, n = 10

Then, mean is given by

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

$$\Rightarrow -7 = \frac{\text{Sum of all observations}}{10}$$

⇒ Sum of all observations = 10 × -7 = -70

According to question, if 5 is added to every 10 number then,

New sum of all observations = Old sum + (5 × 10)

⇒ New sum = -70 + 50 = -20

$$\text{New mean} = \frac{\text{New sum}}{10}$$

$$\Rightarrow \text{New mean} = -\frac{20}{10}$$

⇒ New mean = -2

Thus, option (A) is correct.

**9. Question**

The Arithmetic mean of all the factors of 24 is

A. 8.5

B. 5.67

C. 7

D. 7.5

**Answer**

All the factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. [These numbers divide 24 completely]

Total number of observations = 8

Mean is given by

$$\text{Mean} = \frac{1 + 2 + 3 + 4 + 6 + 8 + 12 + 24}{8}$$

$$\Rightarrow \text{Mean} = \frac{60}{8}$$

⇒ Mean = 7.5

Thus, option (D) is correct.

**10. Question**

The mean of 5 numbers is 20. If one number is excluded their mean is 15. Then the excluded number is

A. 5

B. 40

C. 20

D. 10

**Answer**

Given that, Total number of observations = 5

Mean = 20

So, Sum of observations can be given as

Sum of observations = Mean × Total number of observations [$\because$ $\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$]

$\Rightarrow$ Sum of observations = 20 × 5

$\Rightarrow$ Sum of observations = 100

Let the excluded number be x.

New Sum of observations = 100 – x

New Mean = 15

New total number of observations = 4

Mean is given by

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$\Rightarrow 15 = \frac{100-x}{4}$

$\Rightarrow$ 100 – x = 15 × 4

$\Rightarrow$ 100 – x = 60

$\Rightarrow$ x = 100 – 60 = 40

$\Rightarrow$ The excluded number is 40.

Thus, option (B) is correct.