
CBSE Sample Paper -01 (solved)
SUMMATIVE ASSESSMENT -I
Class - X Mathematics

Time allowed: 3 hours

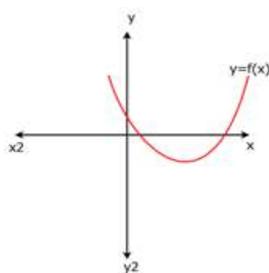
Maximum Marks: 90

General Instructions:

- a) All questions are compulsory.
- b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
- c) Questions 1 to 4 in section A are one mark questions.
- d) Questions 5 to 10 in section B are two marks questions.
- e) Questions 11 to 20 in section C are three marks questions.
- f) Questions 21 to 31 in section D are four marks questions.
- g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

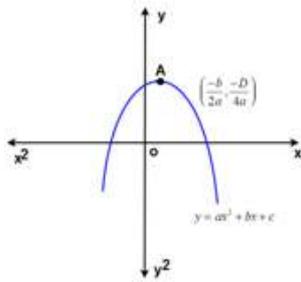
1. Find whether the given system of equations has a unique solution, no solution or infinitely many solutions:
 $x + y = 3,$ $2x + 5y = 12$
2. Without actually performing long division, state whether $\frac{13}{3125}$ will have terminating or non-terminating repeating decimal expansion. Also find the number of decimal places after which the decimal expansion terminates.
3. Identify the given graph corresponds to a linear polynomial or a quadratic polynomial.



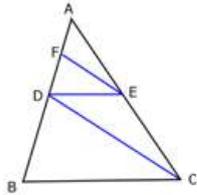
4. Prove that $\cot^2 \theta - \frac{1}{\sin^2 \theta} = -1$

SECTION – B

5. Find the median of the daily wages of ten workers from the following data:
22, 25, 18, 20, 28, 15, 27, 10, 9, 16
 6. The graph of $y = ax^2 + bx + c$ is given in the following figure. Identify the signs of a , b and c .
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7. In the given figure, $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$.



8. If $\sin\theta + \sin^2\theta = 1$, find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$
9. For the following grouped frequency distribution, find the mode.

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

10. ABC is a right triangle, right angled at C. If $A = 30^\circ$ and $AB = 40$ units, find the remaining two sides and $\angle B$ of ΔABC .

SECTION - C

11. Prove that $3\sqrt{2}$ is irrational.
12. Solve: $\frac{x}{a} + \frac{y}{b} = 2$; $ax - by = a^2 - b^2$
13. The mean of the following frequency distribution is 1.46. Find the missing frequencies.

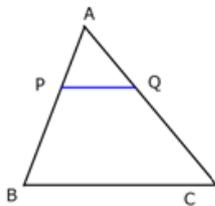
Number of accidents (x)	0	1	2	3	4	5	Total
Frequency (f)	46	f_1	f_2	25	10	5	200

14. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at same point, the ladder is turned to other side of the street to reach a window 12 m high. Find the width of the street.
15. If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0^\circ < A + B \leq 90^\circ$, $A > B$ then find A and B.

16. Prove $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$
17. Find the values of x and y if the total frequency and the median of the following data is 100 and 525, respectively.

Class interval	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	x	12	17	20	y	9	7	4

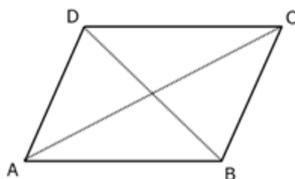
18. P and Q are points on sides AB and AC , respectively of $\triangle ABC$. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3PQ$.



19. If α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find the polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.
20. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

SECTION - D

21. Find all the zeros of the polynomial $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.
22. Show graphically that the system of equations $2x + 4y = 10$; $3x + 6y = 12$ has no solution.
23. Prove that if the corresponding sides of two triangles are proportional, then they are similar.
24. $ABCD$ is a rhombus. Prove that $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$



25. If $\cot B = \frac{12}{5}$, prove that $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.
26. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, prove that each of the side is equal to ± 1 .

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27. Apply step-deviation method to find the arithmetic mean of the following frequency distribution.

Variate (x)	5	10	15	20	25	30	35	40	45	50
Frequency (f)	20	43	75	67	72	45	39	9	8	6

28. If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A}$.

29. Draw a cumulative frequency curve and cumulative frequency polygon for the following frequency distribution by less than method.

Age (in years)	0-9	10-19	20-29	30-39	40-49	50-59	60-69
Number of persons	5	15	20	23	17	11	9

30. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.
31. The percentage of salary that 10 households donate to an orphanage is given below:
5, 3, 10, 5, 2, 4, 7, 8, 1, 5
Find the mean, median and mode of the data. Also tell the values depicted by the persons of these households.
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Class - X
Mathematics

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ANSWERS

Maximum Marks: 90

SECTION - A

1. **Solution:**

Here, $a_1 = 1$, $a_2 = 2$, $b_1 = 1$, $b_2 = 5$, $c_1 = -3$ and $c_2 = -12$

We have,

$$\frac{a_1}{a_2} = \frac{1}{2} \quad \text{and} \quad \frac{b_1}{b_2} = \frac{1}{5}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

\therefore The system has a unique **solution**.

2. **Solution:**

We have,

$$\frac{13}{3125} = \frac{13}{2^0 \times 5^5}$$

This shows that the prime factorization of the denominator of $\frac{13}{3125}$ is of the form $2^m \times 5^n$.

Hence, it has terminating decimal expansion which terminates after 5 places of decimals.

3. **Solution:**

We observe that the graph $y = f(x)$ is a parabola opening upwards. Therefore, $f(x)$ is a quadratic polynomial in which coefficient of x^2 is positive.

4. **Solution:**

We have LHS = $\cot^2 \theta - \frac{1}{\sin^2 \theta}$

$$= \cot^2 \theta - \operatorname{cosec}^2 \theta \quad \left[\because \frac{1}{\sin \theta} = \operatorname{cosec} \theta \right]$$

$$= -1 = \text{RHS} \quad \left[\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \Rightarrow \cot^2 \theta - \operatorname{cosec}^2 \theta = -1 \right]$$

SECTION - B

5. **Solution:**

Arranging the wages in ascending order of magnitude, we have

9, 10, 15, 16, 18, 20, 22, 25, 27, 28

Since there are 10 observations, therefore, median is the arithmetic mean of

$\left(\frac{10}{2}\right)^{\text{th}}$ and $\left(\frac{10}{2}+1\right)^{\text{th}}$ observations.

$$\text{Thus, median} = \frac{18+20}{2} = 19$$

6. **Solution:**

We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a < 0$.

We also observe that the vertex of the parabola is in first quadrant.

$$\therefore -\frac{b}{2a} > 0 \Rightarrow -b < 0 \Rightarrow b > 0$$

Parabola $y = ax^2 + bx + c$ cuts Y-axis at P. On Y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of P are $(0, c)$. As P lies on the positive direction of Y-axis, therefore, $c > 0$.

Hence, $a < 0$, $b > 0$ and $c > 0$.

7. **Solution:**

In ΔABC , we have $DE \parallel BC$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad [\text{By basic proportionality theorem}] \quad \dots(i)$$

In ΔADC , we have $FE \parallel DC$

$$\Rightarrow \frac{AD}{AF} = \frac{AC}{AE} \quad [\text{By basic proportionality theorem}] \quad \dots(ii)$$

From (i) and (ii), we get

$$\Rightarrow \frac{AB}{AD} = \frac{AD}{AF} \quad \Rightarrow \quad AD^2 = AB \times AF$$

8. **Solution:**

$$\text{We have } \sin\theta + \sin^2\theta = 1 \quad \Rightarrow \quad \sin\theta = 1 - \sin^2\theta \quad \Rightarrow \quad \sin\theta = \cos^2\theta$$

$$\text{Now, } \cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$$

$$= (\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta) + 2(\cos^4\theta + \cos^2\theta - 1)$$

$$= (\cos^4\theta + \cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$$

$$= (\sin^2\theta + \cos^2\theta)^3 + 2(\sin^2\theta + \cos^2\theta - 1) \quad [\because \cos^2\theta = \sin\theta, \therefore \cos^4\theta = \sin^2\theta]$$

$$= 1 + 2(1 - 1) = 1$$

9. **Solution:**

We observe that the class 12-15 has maximum frequency. Therefore, this is the modal class.

We have,

$$l = 12, h = 3, f = 23, f_1 = 10 \text{ and } f_2 = 21$$

$$\begin{aligned}\therefore \text{ Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3 \\ &= 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6\end{aligned}$$

10. **Solution:**

In ΔABC , we have,

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ \Rightarrow 30^\circ + \angle B + 90^\circ &= 180^\circ \quad [\because \angle A = 30^\circ \text{ and } \angle C = 90^\circ] \\ \Rightarrow \angle B &= 180^\circ - 120^\circ = 60^\circ\end{aligned}$$

$$\text{Now, } \cos A = \frac{AC}{AB}$$

$$\Rightarrow \cos 30^\circ = \frac{AC}{40}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{40}$$

$$\Rightarrow AC = \frac{\sqrt{3}}{2} \times 40 \quad \Rightarrow \quad AC = 20\sqrt{3} \text{ units}$$

$$\text{And, } \sin A = \frac{BC}{AB}$$

$$\Rightarrow \sin 30^\circ = \frac{BC}{40}$$

$$\Rightarrow \frac{1}{2} = \frac{BC}{40}$$

$$\Rightarrow BC = \frac{1}{2} \times 40 \quad \Rightarrow \quad BC = 20 \text{ units}$$

Hence, $AC = 20\sqrt{3}$ units and $BC = 20$ units

SECTION - C

11. **Solution:**

Let us assume, to the contrary, that $3\sqrt{2}$ is rational. Then, there exist co-prime positive integers a and b such that

$$3\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{3b}$$

$$\Rightarrow \sqrt{2} \text{ is rational} \quad [\because 3, a \text{ and } b \text{ are integers, } \therefore \frac{a}{3b} \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is not correct.

Hence, $3\sqrt{2}$ is an irrational number.

12. Solution:

The given system of equations may be written as

$$bx + ay - 2ab = 0$$

$$ax - by - (a^2 - b^2) = 0$$

By cross multiplication, we have

$$\Rightarrow \frac{x}{-a(a^2 - b^2) - (-b)(-2ab)} = \frac{-y}{-b(a^2 - b^2) - a(-2ab)} = \frac{1}{b(-b) - a(a)}$$

$$\Rightarrow \frac{x}{-a(a^2 - b^2) - 2ab^2} = \frac{-y}{-b(a^2 - b^2) + 2a^2b} = \frac{1}{-b^2 - a^2}$$

$$\Rightarrow \frac{x}{-a(a^2 - b^2 + 2b^2)} = \frac{-y}{-b(a^2 - b^2 - 2a^2)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \frac{x}{-a(a^2 + b^2)} = \frac{-y}{-b(-a^2 - b^2)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow x = \frac{-a(a^2 + b^2)}{-(a^2 + b^2)} = a \text{ and } y = \frac{-b(a^2 + b^2)}{-(a^2 + b^2)} = b$$

Hence, solution of the given system of equations is $x = a$ and $y = b$.

13. Solution:

Calculation of mean

x_i	f_i	$f_i x_i$
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$N = 86 + f_1 + f_2$	$\sum f_i x_i = 140 + f_1 + 2f_2$

We have, $N = 200$

$$\Rightarrow 200 = 86 + f_1 + f_2$$

$$\Rightarrow f_1 + f_2 = 114 \quad \dots(i)$$

Also, mean = 1.46

$$\Rightarrow 1.46 = \frac{\sum f_i X_i}{N}$$

$$\Rightarrow 1.46 = \frac{140 + f_1 + 2f_2}{200}$$

$$\Rightarrow 292 = 140 + f_1 + 2f_2$$

$$\Rightarrow f_1 + 2f_2 = 152 \quad \dots(ii)$$

From (i), $f_1 = 114 - f_2$

Putting the value of f_1 in (ii), we have

$$114 - f_2 + 2f_2 = 152$$

$$\Rightarrow f_2 = 152 - 114 = 38$$

Putting the value of f_2 in (i), we have

$$f_1 + 38 = 114$$

$$\Rightarrow f_1 = 114 - 38 = 76$$

Thus, we have $f_1 = 76$ and $f_2 = 38$.

14. **Solution:** Let AB be the width of the street and C be the foot of the ladder. Let D and E be the windows at heights of 9 m and 12 m, respectively from the ground. Then, CD and EF are the two positions of the ladder.

Clearly, AD = 9 m, BE = 12 m, CD = CE = 15 m.

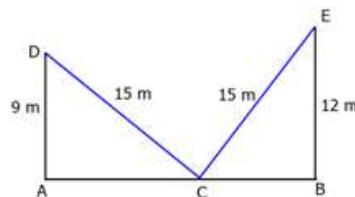
In $\triangle ACD$, we have

$$CD^2 = AC^2 + AD^2$$

$$\Rightarrow 15^2 = AC^2 + 9^2$$

$$\Rightarrow AC^2 = 225 - 81 = 144$$

$$\Rightarrow AC = 12 \text{ m}$$



In $\triangle BCE$, we have

$$CE^2 = BC^2 + BE^2$$

$$\Rightarrow 15^2 = BC^2 + 12^2$$

$$\Rightarrow BC^2 = 225 - 144 = 81$$

$$\Rightarrow BC = 9 \text{ m}$$

Hence, width of the street = AB = AC + CB = (12 + 9) m = 21 m

15. **Solution:**

We have $\sin(A + B) = 1$

$$\Rightarrow \sin(A + B) = \sin 90^\circ$$

$$\Rightarrow A + B = 90^\circ \quad \dots(i)$$

$$\text{And, } \cos(A - B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A - B) = \cos 30^\circ$$

$$\Rightarrow A - B = 30^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

$$(A + B) + (A - B) = 90^\circ + 30^\circ$$

$$\Rightarrow 2A = 120^\circ \quad \Rightarrow \quad A = 60^\circ$$

Putting $A = 60^\circ$ in (i), we get

$$60^\circ + B = 90^\circ \quad \Rightarrow \quad B = 90^\circ - 60^\circ = 30^\circ$$

Thus, $A = 60^\circ$ and $B = 30^\circ$

16. **Solution:**

We have

$$\text{LHS} = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$

$$= \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta \frac{1}{\sin\theta} \right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta \frac{1}{\cos\theta} \right)$$

$$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$$

$$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 4$$

$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4 \quad [\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta]$$

$$= 7 + \cot^2\theta + \tan^2\theta$$

$$= \text{RHS}$$

17. **Solution:**

Calculation of median

Class intervals	Frequency (f)	Cumulative frequency (cf)
0-100	2	2
100-200	5	7
200-300	x	$7 + x$

300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
		Total = 100

We have, $N = \sum fi = 100$

$$\Rightarrow 76 + x + y = 100$$

$$\Rightarrow x + y = 24$$

It is given that the median is 525. Clearly, it lies in the class 500-600.

$$\therefore l = 500, h = 100, f = 20, F = 36 + x \text{ and } N = 100$$

$$\text{Now, median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 525 = 500 + \frac{\frac{100}{2} - (36 + x)}{20} \times 100$$

$$\Rightarrow 525 - 500 = \frac{50 - 36 - x}{20} \times 100$$

$$\Rightarrow 25 = (14 - x) \times 5$$

$$\Rightarrow 25 = 70 - 5x$$

$$\Rightarrow 5x = 45$$

$$\Rightarrow x = \frac{45}{5} \Rightarrow x = 9$$

Putting $x = 9$ in $x + y = 24$, we get

$$9 + y = 24$$

$$\Rightarrow y = 24 - 9 = 15$$

Thus, $x = 9$ and $y = 15$.

18. Solution:

We have,

$$AB = AP + PB = 3 + 6 = 9 \text{ cm}$$

And, $AC = AQ + QC = 5 + 10 = 15$ cm

$$\therefore \frac{AP}{AB} = \frac{3}{9} = \frac{1}{3} \text{ and } \frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$$

Thus, in triangles APQ and ABC, we have

$$\frac{AP}{AB} = \frac{AQ}{AC} \text{ and } \angle A = \angle A$$

Therefore, by SAS criterion of similarity, we have

$$\Delta APQ \sim \Delta ABC$$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow BC = 3PQ$$

19. **Solution:**

Since α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$

$$\therefore \alpha + \beta = -\left(-\frac{5}{2}\right) = \frac{5}{2} \text{ and } \alpha\beta = \frac{7}{2}$$

Let S and P denote respectively the sum and product of zeros of the required polynomial.

$$\text{Then, } S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$$

$$\text{And, } P = (2\alpha + 3\beta)(3\alpha + 2\beta)$$

$$= 6(\alpha^2 + \beta^2) + 13\alpha\beta$$

$$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$$

$$= 6(\alpha + \beta)^2 + \alpha\beta$$

$$= 6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = 6 \times \frac{25}{4} + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$$

Hence, the required polynomial is given by $g(x) = k(x^2 - Sx + P)$

$$= \left(x^2 - \frac{25}{2}x + 41\right), \text{ where } k \text{ is any non-zero real number.}$$

22. **Solution:**

Graph of $2x + 4y = 10$:

We have,

$$2x + 4y = 10 \Rightarrow 4y = 10 - 2x \Rightarrow y = \frac{10 - 2x}{4} \Rightarrow y = \frac{5 - x}{2}$$

When $x = 1$, we have $y = \frac{5 - x}{2} = \frac{5 - 1}{2} = \frac{4}{2} = 2$

When $x = 3$, we have $y = \frac{5 - x}{2} = \frac{5 - 3}{2} = \frac{2}{2} = 1$

Thus, we have the following table:

x	1	3
y	2	1

Graph of $3x + 6y = 12$:

We have,

$$3x + 6y = 12 \Rightarrow 6y = 12 - 3x \Rightarrow y = \frac{12 - 3x}{6} \Rightarrow y = \frac{4 - x}{2}$$

When $x = 2$, we have $y = \frac{4 - x}{2} = \frac{4 - 2}{2} = \frac{2}{2} = 1$

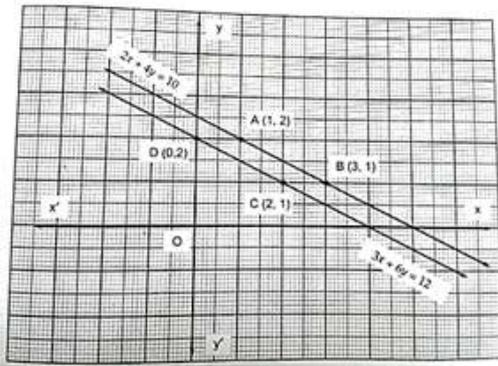
When $x = 0$, we have $y = \frac{4 - x}{2} = \frac{4 - 0}{2} = \frac{4}{2} = 2$

Thus, we have the following table:

x	2	0
y	1	2

Plot the points A (1, 2) and B (3, 1) of $2x + 4y = 10$ on a graph paper. Join A and B and extend it on both sides as shown in the figure.

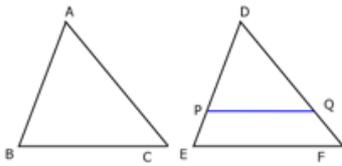
Also, plot the points C (2, 1) and D (0, 2) of $3x + 6y = 12$ on the same graph paper. Join C and D and extend it on both sides as shown in the figure.



We find that the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

23. **Solution:**

Given: Two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



To prove: $\triangle ABC \sim \triangle DEF$

Construction: Let P and Q be points on DE and DF respectively such that $DP = AB$ and $DQ = AC$. Join PQ,

Proof: We have $\frac{AB}{DE} = \frac{AC}{DF}$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} \quad [\because AB = DP \text{ and } AC = DQ]$$

$$\Rightarrow PQ \parallel EF \quad [\text{By the converse of Thale's theorem}]$$

$$\Rightarrow \angle DPQ = \angle E \text{ and } \angle DQP = \angle F \quad [\text{Corresponding angles}]$$

Thus, in triangles DPQ and DEF, we have $\angle DPQ = \angle E$ and $\angle DQP = \angle F$.

Therefore, by AA-criterion of similarity, we have

$$\triangle DPQ \sim \triangle DEF \quad \dots(i)$$

$$\Rightarrow \frac{DP}{DE} = \frac{PQ}{EF} \quad [\text{By definition of similarity}]$$

$$\Rightarrow \frac{AB}{DE} = \frac{PQ}{EF} \quad [\because DP = AB]$$

But, $\frac{AB}{DE} = \frac{BC}{EF}$

$$\therefore \frac{PQ}{EF} = \frac{BC}{EF}$$

$$\Rightarrow PQ = BC$$

Thus, in triangles ABC and DPQ, we have

$$AB = DP, AC = DQ \text{ and } BC = PQ$$

Therefore, by SSS criterion of congruence, we have

$$\triangle ABC \cong \triangle DPQ \quad \dots(\text{ii})$$

From (i) and (ii), we have

$$\triangle ABC \cong \triangle DPQ \text{ and } \triangle DPQ \sim \triangle DEF$$

$$\Rightarrow \triangle ABC \sim \triangle DPQ \text{ and } \triangle DPQ \sim \triangle DEF \quad [\because \triangle ABC \cong \triangle DPQ \Leftrightarrow \triangle ABC \sim \triangle DPQ]$$

$$\Rightarrow \triangle ABC \sim \triangle DEF$$

24. **Solution:**

Let the diagonals AC and BD of rhombus ABCD intersect at O.

Since the diagonals of a rhombus bisect each other at right angles.

$$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ \text{ and } AO = CO, BO = OD.$$

Since $\triangle AOB$ is a right triangle right angled at O,

$$\therefore AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 \quad [\because OA = OC \text{ and } OB = OD]$$

$$\Rightarrow 4AB^2 = AC^2 + BD^2 \quad \dots(\text{i})$$

Similarly, we have

$$4BC^2 = AC^2 + BD^2 \quad \dots(\text{ii})$$

$$4CD^2 = AC^2 + BD^2 \quad \dots(\text{iii})$$

$$4AD^2 = AC^2 + BD^2 \quad \dots(\text{iv})$$

Adding (i), (ii), (iii) and (iv), we get

$$4(AB^2 + BC^2 + CD^2 + AD^2) = 4(AC^2 + BD^2)$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

25. **Solution:**

We have,

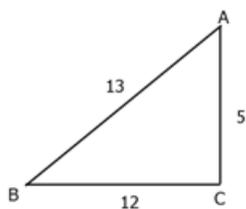
$$\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$$

So, we draw a right triangle ABC, right angled at C such that

Base = BC = 12 units and Perpendicular = AC = 5 units

By Pythagoras theorem, we have

$$\begin{aligned}AB^2 &= BC^2 + AC^2 \\ &= 12^2 + 5^2 = 169\end{aligned}$$



$$\Rightarrow AB = \sqrt{169} = 13$$

$$\therefore \sin B = \frac{AC}{AB} = \frac{5}{13}, \tan B = \frac{AC}{BC} = \frac{5}{12} \text{ and } \sec B = \frac{AB}{BC} = \frac{13}{12}$$

$$\begin{aligned}\text{Now, LHS} &= \tan^2 B - \sin^2 B \\ &= (\tan B)^2 - (\sin B)^2 \\ &= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 \\ &= \frac{25}{144} - \frac{25}{169} \\ &= 25 \left(\frac{1}{144} - \frac{1}{169} \right) \\ &= 25 \left(\frac{169 - 144}{144 \times 169} \right) \\ &= 25 \times \frac{25}{144 \times 169} = \frac{25 \times 25}{144 \times 169} = \frac{5^2 \times 5^2}{12^2 \times 13^2} \quad \dots(i)\end{aligned}$$

$$\begin{aligned}\text{And, RHS} &= \sin^4 B \sec^2 B \\ &= (\sin B)^4 (\sec B)^2 \\ &= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 = \frac{5^4 \times 13^2}{13^4 \times 12^2} \\ &= \frac{5^4}{13^2 \times 12^2} = \frac{5^2 \times 5^2}{13^2 \times 12^2} \quad \dots(ii)\end{aligned}$$

From (i) and (ii), we have $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.

26. **Solution:**

We have,

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$

Multiplying both sides by $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$$\begin{aligned} & (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) \\ & = (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2 \end{aligned}$$

$$\Rightarrow (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$$

$$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$$

$$\Rightarrow 1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$$

$$\Rightarrow (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$$

Similarly, multiplying both sides by $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$, we get

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$$

27. **Solution:**

Let the assumed mean be $A = 25$ and $h = 5$.

Calculation of mean

Variate x_i	Frequency f_i	Deviations $d_i = x_i - 25$	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
5	20	-20	-4	-80
10	43	-15	-3	-129
15	75	-10	-2	-150
20	67	-5	-1	-67
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
$N = \sum f_i = 384$				$\sum f_i u_i = -214$

We have,

$$N = 384, A = 25, h = 5 \text{ and } \sum f_i u_i = -214$$

$$\therefore \text{Mean} = \bar{X} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$= 25 + 5 \times \left(\frac{-214}{384} \right)$$

$$= 25 - 2.786 = 22.214$$

28. **Solution:**

We have,

$$\operatorname{cosec} A = \sqrt{2} \Rightarrow \frac{1}{\sin A} = \sqrt{2} \Rightarrow \sin A = \frac{1}{\sqrt{2}}$$

$$\text{Now, } \cos A = \sqrt{1 - \sin^2 A}$$

$$= \sqrt{1 - \left(\frac{1}{\sqrt{2}} \right)^2} = \frac{1}{\sqrt{2}}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\text{And, } \cot A = \frac{1}{\tan A} = \frac{1}{1} = 1$$

$$\text{Hence, } \frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A} = \frac{2 \times \left(\frac{1}{\sqrt{2}} \right)^2 + 3(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$= \frac{2 \times \frac{1}{2} + 3}{4 - \frac{1}{2}} = \frac{1 + 3}{\frac{7}{2}} = \frac{4}{\frac{7}{2}} = \frac{8}{7}$$

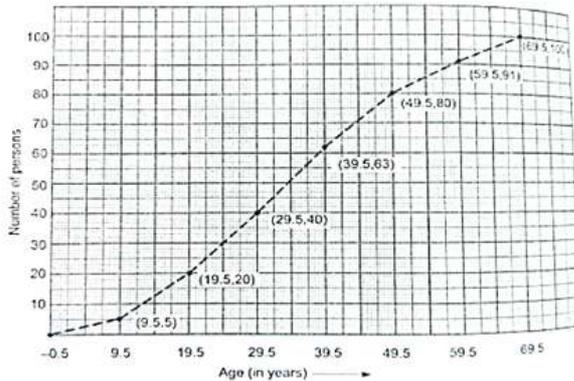
29. **Solution:**

The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under.

Age (in years)	Frequency	Age less than	Cumulative frequency
-0.5-9.5	5	9.5	5
9.5-19.5	15	19.5	20
19.5-29.5	20	29.5	40
29.5-39.5	23	39.5	63
39.5-49.5	17	49.5	80

49.5-59.5	11	59.5	91
59.5-69.5	9	69.5	100

Now, we plot points (9.5, 5), (19.5, 20), (29.5, 40), (39.5, 63), (49.5, 80), (59.5, 91) and (69.5, 100) and join them by a free hand smooth curve to obtain the required ogive as shown in the figure.



The cumulative frequency polygon is obtained by joining these points by line segments as shown below.

30. **Solution:**

Let the actual speed of the train be x km/hr and the actual time taken by y hours. Then,

$$\text{Distance covered} = (xy) \text{ km} \quad \dots\text{(i)} \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$$

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours, i.e., when speed is $(x + 6)$ km/hr, time of journey is $(y - 4)$ hours.

$$\therefore \text{Distance covered} = (x + 6)(y - 4)$$

$$\Rightarrow xy = (x + 6)(y - 4) \quad \text{[using (i)]}$$

$$\Rightarrow -4x + 6y - 24 = 0$$

$$\Rightarrow -2x + 3y - 12 = 0 \quad \dots\text{(ii)}$$

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours, i.e., when speed is $(x - 6)$ km/hr, time of journey is $(y + 6)$ hours.

$$\therefore \text{Distance covered} = (x - 6)(y + 6)$$

$$\Rightarrow xy = (x - 6)(y + 6) \quad \text{[using (i)]}$$

$$\Rightarrow 6x - 6y - 36 = 0$$

$$\Rightarrow x - y - 6 = 0 \quad \dots\text{(iii)}$$

Thus, we obtain the following system of equations:

$$-2x + 3y - 12 = 0$$

$$x - y - 6 = 0$$

By using cross-multiplication, we have,

$$\frac{x}{3 \times -6 - (-1) \times 12} = \frac{-y}{-2 \times -6 - 1 \times -12} = \frac{1}{-2 \times -1 - 1 \times 3}$$

$$\Rightarrow \frac{x}{-30} = \frac{-y}{24} = \frac{1}{-1}$$

$$\Rightarrow x = 30 \text{ and } y = 24.$$

Putting the values of x and y in equation (i), we get

$$\text{Distance} = 30 \times 24 = 720 \text{ km}$$

Hence, the length of the journey is 720 km.

31. **Solution:**

$$\text{Mean of the given data} = \frac{5+3+10+5+2+4+7+8+1+5}{10} = \frac{50}{10} = 5$$

Arranging the given data in ascending order, we get

1, 2, 3, 4, 5, 5, 5, 7, 8, 10

We observe that maximum occurring observation is 5. Thus, mode of the given data is 5.

$$\begin{aligned} \text{Median} &= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2} = \frac{5+5}{2} = \frac{10}{2} = 5 \end{aligned}$$

The values depicted by the persons of these households are social service and caring.
