Exercise 2.3

Chapter 2 Derivatives Exercise 2.3 1E

Let $f(x)=2^{40}$

Here 2^{40} is constant, we know that $\frac{d}{dx}c=0$.

$$\boxed{\therefore \frac{d}{dx}2^{40}=0}$$

Chapter 2 Derivatives Exercise 2.3 2E

Let $f(x)=\pi^2$

Here π^2 is constant , we know that $\frac{d}{dx}c=0$

$$\boxed{\therefore \frac{d}{dx}\pi^2=0}$$

Chapter 2 Derivatives Exercise 2.3 3E

The function is $f(t)=2-\frac{2}{3}t$.

The objective is to differentiate the function.

$$\frac{d}{dt}f(t)=\frac{d}{dt}\left(2-\frac{2}{3}t\right)$$

$$=\frac{d}{dt}(2)-\frac{d}{dt}\left(\frac{2}{3}t\right) \text{ Since } \frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

$$=0-\frac{2}{3}\frac{d}{dt}(t) \text{ Since } \frac{d}{dx}c=0, \frac{d}{dx}kf(x)=k\frac{d}{dx}f(x)$$

$$=-\frac{2}{3} \text{ Since } \frac{d}{dx}x^n=nx^{n-1}$$

Therefore, the derivative of the function is $\frac{d}{dt}f(t)=\boxed{-\frac{2}{3}}$.

Chapter 2 Derivatives Exercise 2.3 4E

Differentiate:

$$F(x) = \frac{3}{4}x^8$$

We can use the power rule here which says if n is a positive integer;

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

So using the power rule we simply bring the 8 exponent down and multiply it with 3/4 and then subtract 1 from the exponent, i.e.;

$$8 * \frac{3}{4}x^{8-1}$$

$$8 * \frac{3}{4} = 6, \text{ and } 8 - 1 = 7\ ,$$

So our derivative is;

$$F'(x) = 6x^7$$

Chapter 2 Derivatives Exercise 2.3 5E

Consider the function.

$f(x) = x^3 - 4x + 6$.

Differentiate the function, $f(x) = x^3 - 4x + 6$ with respect to x, to get the following:

$$f'(x) = \frac{d}{dx}\left(x^3 - 4x + 6\right) \quad \text{Differntiate both sides with respect to } x$$

$$= \frac{d}{dx}\left(x^3\right) + \frac{d}{dx}(-4x) + \frac{d}{dx}(6) \quad \text{Use sum rule of differentiation}$$

$$= 3x^2\frac{d}{dx}(x) - 4\frac{d}{dx}(x) + 0 \quad \text{Use power rule of differentiation}$$

$$= 3x^2 \cdot 1 - 4 \cdot 1 \quad \text{Since } \frac{d}{dx}(x) = 1$$

$$= 3x^2 - 4$$

Hence the result is $\boxed{3x^2 - 4}$.

Chapter 2 Derivatives Exercise 2.3 6E

$$f(t) = \frac{1}{2}t^6 - 3t^4 + t$$

$$f'(t) = \frac{1}{2}\frac{d}{dt}\left(t^6\right) - 3\frac{d}{dt}\left(t^4\right) + \frac{d}{dt}(t)$$

$$= \frac{1}{2} \times 6t^5 - 3 \times 4t^3 + 1$$

$$= 3t^5 - 12t^3 + 1$$

Chapter 2 Derivatives Exercise 2.3 7E

Let $g(x) = x^2(1-2x)$

Formula: $\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$

Using the above formula

$$\frac{d}{dx}g(x) = \frac{d}{dx}x^2(1-2x)$$

$$= x^2\frac{d}{dx}(1-2x) + (1-2x)\frac{d}{dx}x^2$$

$$= x^2[-2] + [1-2x](2x)$$

$$= -2x^2 + 2x - 4x^2$$

$$= -6x^2 + 2x$$

$$\boxed{\therefore \frac{d}{dx}g(x) = -6x^2 + 2x}$$

Chapter 2 Derivatives Exercise 2.3 8E

Consider the function

$$h(x) = (x-2)(2x+3)$$

First, to expand $h(x)$:

$$h(x) = (x-2)(2x+3)$$
$$= x(2x+3) - 2(2x+3)$$
$$= 2x^2 + 3x - 4x - 6$$
$$= 2x^2 - x - 6$$

To find differentiate the function $h(x)$ that is $h'(x)$:

Differentiate $h(x) = 2x^2 - x - 6$ with respect to x, to get

$$h'(x) = \frac{d}{dx}[h(x)]$$
$$= \frac{d}{dx}(2x^2 - x - 6)$$

$= \frac{d}{dx}(2x^2) - \frac{d}{dx}(x) - \frac{d}{dx}(6)$ By using the difference rule

$= 2\frac{d}{dx}(x^2) - \frac{d}{dx}(x) - \frac{d}{dx}(6)$ By using the constant multiple rule

$= 2\frac{d}{dx}(x^2) - \frac{d}{dx}(x)$ Since $\frac{d}{dx}(c) = 0, c$ is constant

$= 2(2x) - 1$ By using the power rule: $\frac{d}{dx}(x^n) = nx^{n-1}, n \in \mathbb{R}$

$= 4x - 1$ Simplify.

Therefore, differentiate the function $h(x)$ is

$\boxed{h'(x) = 4x - 1}$.

Chapter 2 Derivatives Exercise 2.3 9E

Let $g(t) = 2t^{\frac{-3}{4}}$

Formula: $\frac{d}{dx}(cf) = c\frac{d}{dx}f$

Using the above formula

$$\frac{d}{dt}g(t) = \frac{d}{dt}\left[2t^{\frac{-3}{4}}\right]$$

$$= 2\frac{d}{dt}t^{\frac{-3}{4}}$$

$$= 2\left(-\frac{3}{4}\right)t^{\frac{-3}{4}-1}$$

$$= \frac{-6}{4}t^{\frac{-7}{4}}$$

$$= \frac{-3}{2}t^{\frac{-7}{4}}$$

$$\boxed{\therefore g'(t) = \frac{-3}{2}t^{\frac{-7}{4}}}$$

Chapter 2 Derivatives Exercise 2.3 10E

$$B(y) = cy^{-6}$$

Differentiating with respect to y by using power rule

$$B'(y) = c(-6)y^{-6-1} = -6cy^{-7}$$

Chapter 2 Derivatives Exercise 2.3 11E

Consider the function,

$$A(s) = -\frac{12}{s^5}.$$

Rewrite the function as,

$A(s) = -12s^{-5}$ Since $\frac{1}{x^n} = x^{-n}$.

Need to find differentiate the given function.

Differentiate $A(s)$ with respect to s.

$$A'(s) = \left(-12s^{-5}\right)'$$

$= -12\left(s^{-5}\right)'$ Since $(cf)' = cf'$.

$= -12\left(-5s^{-5-1}\right)$ Use power rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$.

$$= -12\left(-5s^{-6}\right)$$

$$= 60s^{-6}$$

$= 60\left(\frac{1}{s^6}\right)$ Since $\frac{1}{x^n} = x^{-n}$.

$$= \frac{60}{s^6}$$

Therefore, $A'(s) = \boxed{\frac{60}{s^6}}$.

Chapter 2 Derivatives Exercise 2.3 12E

Let $y = x^{\frac{5}{3}} - x^{\frac{2}{3}}$

Formula: $\frac{d}{dx}(f - g) = \frac{d}{dx}f - \frac{d}{dx}g$

Using the above formula

$$\frac{d}{dx}\left(x^{\frac{5}{3}} - x^{\frac{2}{3}}\right) = \frac{d}{dx}x^{\frac{5}{3}} - \frac{d}{dx}x^{\frac{2}{3}}$$

$$= \frac{5}{3}x^{\frac{5}{3}-1} - \frac{2}{3}x^{\frac{2}{3}-1}$$

$$= \frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{\frac{-1}{3}}$$

$$\boxed{\therefore\ y' = \frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{3}}}$$

Chapter 2 Derivatives Exercise 2.3 13E

Consider the function,

$S(p) = \sqrt{p} - p.$

Rewrite the function as,

$S(p) = p^{1/2} - p.$ Since $\sqrt{a} = a^{1/2}.$

Need to find differentiate the given function.

Differentiate $S(p)$ with respect to $p.$

$$S'(p) = \left(p^{1/2} - p\right)'$$

$= \left(p^{1/2}\right)' - (p)'$ Use difference rule: $(f - g)' = f' - g'.$

$= \frac{1}{2}p^{(1/2)-1} - 1$ Use power rule: $\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(x) = 1.$

$$= \frac{1}{2}p^{-1/2} - 1$$

$= \frac{1}{2p^{1/2}} - 1$ Since $\frac{1}{x^n} = x^{-n}.$

$= \frac{1}{2\sqrt{p}} - 1$ Since $\sqrt{a} = a^{1/2}.$

Therefore, $S'(p) = \boxed{\frac{1}{2\sqrt{p}} - 1}.$

Chapter 2 Derivatives Exercise 2.3 14E

Let $y = \sqrt{x}(x - 1)$

Formula: $\frac{d}{dx}(fg) = fg' + f'g$

Using the above formula

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sqrt{x}(x - 1)\right]$$

$$= \sqrt{x}\frac{d}{dx}(x - 1) + \left(\frac{d}{dx}\sqrt{x}\right)(x - 1)$$

$= \sqrt{x}(1) + \frac{1}{2\sqrt{x}}(x - 1)$ $\left[\because \frac{d}{dx}x^n = nx^{n-1}\right]$

$$= \sqrt{x} + \frac{x - 1}{2\sqrt{x}}$$

$$\boxed{\therefore \frac{dy}{dx} = \sqrt{x} + \frac{x - 1}{2\sqrt{x}}}$$

Chapter 2 Derivatives Exercise 2.3 15E

Consider the following function:

$$R(a)=(3a+1)^2$$

The objective is to find the derivative of the function.

Expand the expression as,

$$(3a+1)^2=(9a^2+6a+1)$$

The Sum Rule states that, if $f(x), g(x)$ are two differentiable functions, then

$$\frac{d}{dx}(f(x)+g(x))=\frac{d}{dx}(f(x))+\frac{d}{dx}(g(x)).$$

The Constant Multiple Rule states that,

$$\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x).$$

And the Power Rule States that,

$$\frac{d}{dx}(x^n)=nx^{n-1}.$$

By using the Sum Rule and the Constant Multiple Rule, we obtain:

$$\begin{aligned}\frac{d}{da}(R(a))&=\frac{d}{da}\left((3a+1)^2\right)\\&=\frac{d}{da}(9a^2+6a+1)\\&=\frac{d}{da}(9a^2)+\frac{d}{da}(6a)+\frac{d}{da}(1)\\&=9\frac{d}{da}(a^2)+6\frac{d}{da}(a)+0\end{aligned}$$

Now by using power rule, we obtain:

$$\begin{aligned}\frac{d}{da}(R(a))&=9(2a^{2-1})+6(1a^{1-1})\\&=18a+6\end{aligned}$$

Therefore, the derivative of the given function is $\boxed{R'(a)=18a+6}$.

Chapter 2 Derivatives Exercise 2.3 16E

Let $S(R)=4\pi R^2$

Formula : $\frac{d}{dx}cf=c\frac{df}{dx}$

Using the above formula

$$\begin{aligned}\frac{d}{dR}S(R)&=\frac{d}{dR}4\pi R^2\\&=4\pi\frac{d}{dR}R^2\\&=4\pi 2R\\&=8\pi R\end{aligned}$$

$$\boxed{\therefore S'(R)=8\pi R}$$

Chapter 2 Derivatives Exercise 2.3 17E

Consider the function:

$$y(x)=\frac{x^2+4x+3}{\sqrt{x}}$$

Use quotient rule of differentiation to find the derivative of the given function.

Quotient rule:

$$\left(\frac{f}{g}\right)'=\frac{gf'-fg'}{g^2}$$

So that,

$$\frac{dy}{dx}=\frac{\sqrt{x}\frac{d}{dx}\left(x^2+4x+3\right)-\left(x^2+4x+3\right)\frac{d}{dx}\left(\sqrt{x}\right)}{\left(\sqrt{x}\right)^2}$$

$$=\frac{\sqrt{x}\left(2x+4+0\right)-\left(x^2+4x+3\right)\left(\frac{1}{2\sqrt{x}}\right)}{x}$$

$$=\frac{\left(2\sqrt{x}\right)\left(\sqrt{x}\right)\left(2x+4\right)-\left(x^2+4x+3\right)}{2x\sqrt{x}}$$

$$=\frac{2x\left(2x+4\right)-x^2-4x-3}{2x\sqrt{x}}$$

Simplify further,

$$=\frac{4x^2+8x-x^2-4x-3}{2x\sqrt{x}}$$

$$=\frac{3x^2+4x-3}{2x\sqrt{x}}$$

$$=\left(\frac{3x^2}{2x\sqrt{x}}+\frac{4x}{2x\sqrt{x}}-\frac{3}{2x\sqrt{x}}\right)$$

$$=\left(\frac{3}{2}\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{3}{2x\sqrt{x}}\right)$$

Therefore,

$$\frac{dy}{dx}=\boxed{\left(\frac{3}{2}\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{3}{2x\sqrt{x}}\right)}$$

Chapter 2 Derivatives Exercise 2.3 18E

Consider the function.

$$y=\frac{\sqrt{x}+x}{x^2}$$

Although it is possible to differentiate the function using the Quotient Rule, it is much easier to perform the **division first.**

$$y=\frac{\sqrt{x}}{x^2}+\frac{x}{x^2}$$

Cancel the terms to further solve the equation.

$$y=\frac{1}{x\sqrt{x}}+\frac{1}{x}$$

$$=\frac{1}{x^{\frac{3}{2}}}+\frac{1}{x}$$

$$=x^{-\frac{3}{2}}+x^{-1}$$

Differentiate both sides with respect to x.

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^{-\frac{3}{2}}+x^{-1}\right)$$

$$=\frac{d}{dx}\left(x^{-\frac{3}{2}}\right)+\frac{d}{dx}\left(x^{-1}\right) \quad \text{By } \frac{d}{dx}(f+g)=\frac{d}{dx}(f)+\frac{d}{dx}(g)$$

$$=-\frac{3}{2}x^{-\frac{3}{2}-1}-1x^{-1-1} \quad \text{By } \frac{d}{dx}\left(x^{n}\right)=nx^{n-1}$$

$$=-\frac{3}{2}x^{-\frac{5}{2}}-1x^{-2}$$

$$=-\frac{3}{2x^{\frac{5}{2}}}-\frac{1}{x^{2}}$$

$$y'=\boxed{-\frac{3}{2x^{\frac{5}{2}}}-\frac{1}{x^{2}}}$$

Chapter 2 Derivatives Exercise 2.3 19E

Let $H(x)=\left(x+x^{-1}\right)^{3}$

$$=x^{3}+x^{-3}+3x^{2}x^{-1}+3x^{-2}x$$

$$=x^{3}+x^{-3}+3x+3x^{-1}$$

Formulas: $\frac{d}{dx}x^{n}=nx^{n-1}$ and

$$\frac{d}{dx}(f+g)=\frac{df}{dx}+\frac{dg}{dx}$$

$\therefore$ Using the above formulas, we get

$$\frac{d}{dx}H(x)=\frac{d}{dx}\left[x^{3}+x^{-3}+3x+3x^{-1}\right]$$

$$=\frac{d}{dx}x^{3}+\frac{d}{dx}x^{-3}+\frac{d}{dx}3x+\frac{d}{dx}3x^{-1}$$

$$=3x^{2}-3x^{-4}+3+3(-1)x^{-2}$$

$$=3x^{2}-3x^{-4}+3-3x^{-2}$$

$$\boxed{\therefore\ H'(x)=3x^{2}-3x^{-4}+3-3x^{-2}}$$

Chapter 2 Derivatives Exercise 2.3 20E

Consider the following function:

$$g(u)=\sqrt{2}u+\sqrt{3u}$$

The objective is to find the derivative of the function.

The Constant Multiple Rule states that,

$$\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x).$$

The Sum Rule states that, if $f(x), g(x)$ are two differentiable functions, then

$$\frac{d}{dx}(f(x)+g(x))=\frac{d}{dx}(f(x))+\frac{d}{dx}(g(x)).$$

And the Power Rule States that,

$$\frac{d}{dx}\left(x^{n}\right)=nx^{n-1}.$$

By using the Sum Rule, we obtain:

$$\frac{dg}{du} = \frac{d}{du}\left(\sqrt{2}u\right) + \frac{d}{du}\left(\sqrt{3u}\right)$$
$$= \sqrt{2}\frac{d}{du}(u) + \sqrt{3}\frac{d}{du}\left(u^{\frac{1}{2}}\right)$$

Now by using power rule, we obtain:

$$\frac{dg}{du} = \sqrt{2}\frac{d}{du}(u) + \sqrt{3}\frac{d}{du}\left(u^{\frac{1}{2}}\right)$$
$$= \sqrt{2}(1) + \sqrt{3}\left(\frac{1}{2}u^{\frac{1}{2}-1}\right)$$
$$= \sqrt{2} + \frac{\sqrt{3}}{2\sqrt{u}}$$

Therefore, the derivative of the given function is $\boxed{\frac{dg}{du} = \sqrt{2} + \frac{\sqrt{3}}{2\sqrt{u}}}$

Chapter 2 Derivatives Exercise 2.3 21E

Consider the function, $u = \sqrt[5]{t} + 4\sqrt{t^5}$.

Find the derivative of u.

Use the following formula:

Suppose f and g are two differentiable functions, then $(f+g)' = f' + g'$.

Differentiate u with respect to t, and then apply the sum rule as shown below:

$$\frac{d}{dt}(u) = \frac{d}{dt}\left(\sqrt[5]{t} + 4\sqrt{t^5}\right)$$
$$u' = \frac{d}{dt}\left(\sqrt[5]{t}\right) + 4\frac{d}{dt}\left(\sqrt{t^5}\right)$$
$$= \frac{d}{dt}\left(t^{\frac{1}{5}}\right) + 4\frac{d}{dt}\left(t^{\frac{5}{2}}\right) \quad \text{Since } \sqrt[m]{a^n} = a^{\frac{n}{m}}$$

Continue further,

$$u' = \frac{d}{dt}\left(t^{\frac{1}{5}}\right) + 4\frac{d}{dt}\left(t^{\frac{5}{2}}\right)$$
$$= \frac{1}{5}t^{\frac{1}{5}-1} + 4\left(\frac{5}{2}t^{\frac{5}{2}-1}\right) \quad \text{Since } \frac{d}{dx}x^n = nx^{n-1}$$
$$= \frac{1}{5}t^{\frac{1-5}{5}} + 4\left(\frac{5}{2}t^{\frac{5-2}{2}}\right)$$
$$= \frac{1}{5}t^{\frac{-4}{5}} + 10t^{\frac{3}{2}}$$
$$= \frac{1}{5t^{\frac{4}{5}}} + 10t^{\frac{3}{2}}$$

Therefore, the derivative of the function $u = \sqrt[5]{t} + 4\sqrt{t^5}$ is $\frac{du}{dt} = \boxed{\frac{1}{5t^{\frac{4}{5}}} + 10t^{\frac{3}{2}}}$.

Chapter 2 Derivatives Exercise 2.3 22E

Given function $v=\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2$

Differentiate v with respect to x, we have

$$v'=\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2$$

$$=2\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^{2-1}\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)$$

$$=2\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)\left(\frac{1}{2\sqrt{x}}+\frac{d}{dx}\left(x^{\frac{-1}{3}}\right)\right)$$

$$=2\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)\left(\frac{1}{2\sqrt{x}}+\left(\frac{-1}{3}\right)\left(x^{\frac{-1}{3}-1}\right)\right)\quad\left(\text{since } \frac{d}{dx}\left(x^n\right)=nx^{n-1}\right)$$

$$=2\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)\left(\frac{1}{2\sqrt{x}}-\frac{1}{3}\left(x^{\frac{-4}{3}}\right)\right)$$

Therefore $v'=2\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)\left(\frac{1}{2\sqrt{x}}-\frac{1}{3x^{\frac{4}{3}}}\right)$

Chapter 2 Derivatives Exercise 2.3 23E

Consider the following function:

$$f(x)=\left(1+2x^2\right)\left(x-x^2\right)$$

The objective is to find the derivative of the function in two ways:

1. Using product rule:

The product rule states that,

If u and v are both differentiable, then

$$\frac{d}{dx}\left[u(x)v(x)\right]=v(x)\frac{d}{dx}\left[u(x)\right]+u(x)\frac{d}{dx}\left[v(x)\right]$$

The Sum Rule states that, if $f(x),g(x)$ are two differentiable functions, then

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left(g(x)\right).$$

And the Power Rule States that,

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}.$$

By using the product rule and power rule,

$$\frac{d}{dx}\left(f(x)\right)=\frac{d}{dx}\left(\left(1+2x^2\right)\left(x-x^2\right)\right)$$

$$=\left(1+2x^2\right)\frac{d}{du}\left(x-x^2\right)+\left(x-x^2\right)\frac{d}{du}\left(1+2x^2\right)$$

$$=\left(1+2x^2\right)\left[1-2x\right]+\left(x-x^2\right)\left(0+4x\right)$$

$$=\left(1+2x^2\right)\left(1-2x\right)+\left(x-x^2\right)\left(4x\right)$$

$$=1-2x+2x^2-4x^3+4x^2-4x^3$$

$$=-8x^3+6x^2-2x+1$$

Therefore, the derivative of the given function is $\boxed{f'(x)=-8x^3+6x^2-2x+1}$.

2. Second Method:

By performing multiplication:

$$f(x) = (1+2x^2)(x-x^2)$$
$$= x + 2x^3 - x^2 - 2x^4$$
$$= -2x^4 + 2x^3 - x^2 + x$$

Now using sum rule and power rule,

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(-2x^4 + 2x^3 - x^2 + x)$$
$$= \frac{d}{dx}(-2x^4) + \frac{d}{dx}(2x^3) - \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$$
$$= -2(4x^3) + 2(3x^2) - (2x) + 1$$
$$= -8x^3 + 6x^2 - 2x + 1$$

Therefore, the derivative of the given function is $\boxed{f'(x) = -8x^3 + 6x^2 - 2x + 1}$.

From (1) and (2) both answers are same.

Chapter 2 Derivatives Exercise 2.3 24E

Given function $F(x) = \dfrac{x^4 - 5x^3 + \sqrt{x}}{x^2}$

First method:

Quotient Rule: $\left(\dfrac{f}{g}\right)' = \dfrac{gf' - fg'}{g^2}$

$$\therefore \frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{x^4 - 5x^3 + \sqrt{x}}{x^2}\right)$$
$$= \frac{x^2 \frac{d}{dx}(x^4 - 5x^3 + \sqrt{x}) - [x^4 - 5x^3 + \sqrt{x}]\frac{d}{dx}x^2}{x^4}$$
$$= \frac{x^2\left[4x^3 - 15x^2 + \frac{1}{2\sqrt{x}}\right] - [x^4 - 5x^3 + \sqrt{x}]2x}{x^4}$$
$$= \frac{4x^5 - 15x^4 + \frac{1}{2}x^{\frac{3}{2}} - 2x^5 + 10x^4 - 2x^{\frac{3}{2}}}{x^4}$$
$$= \frac{2x^5 - \frac{3}{2}x^{\frac{3}{2}} - 5x^4}{x^4} = 2x - \frac{3}{2}x^{\frac{-5}{2}} - 5$$
$$\boxed{\therefore F'(x) = 2x - \frac{3}{2}x^{\frac{-5}{2}} - 5}$$

Second method: $F(x) = x^2 - 5x + x^{\frac{-3}{2}}$

$$F'(x) = \frac{d}{dx}x^2 - 5x + x^{\frac{-3}{2}}$$
$$= 2x - 5 - \frac{3}{2}x^{\frac{-5}{2}}$$ Both answers are same

$\boxed{\therefore \text{ Second method is simple}}$

Chapter 2 Derivatives Exercise 2.3 25E

$$V(x) = (2x^3+3)(x^4-2x)$$

Then $\dfrac{dV}{dx} = (2x^3+3)\dfrac{d}{dx}(x^4-2x) + (x^4-2x)\dfrac{d}{dx}(2x^3+3)$
$$= (2x^3+3)(4x^3-2) + (x^4-2x)(6x^2)$$
$$= 8x^6 - 4x^3 + 12x^3 - 6 + 6x^6 - 12x^3$$
$$= 14x^6 - 4x^3 - 6$$

Hence $\boxed{\dfrac{dV}{dx} = 14x^6 - 4x^3 - 6}$

Chapter 2 Derivatives Exercise 2.3 26E

Let $L(x)=(1+x+x^2)(2-x^4)$

$= 2+2x+2x^2-x^4-x^5-x^6$

$= -x^6-x^5-x^4+2x^2+2x+2 \left[\because \frac{d}{dx}x^n = x^{n-1}\right]$

$$\therefore L'(x) = -6x^5-5x^4-4x^3+4x+2$$

Chapter 2 Derivatives Exercise 2.3 27E

These type of problems first we can simplify algebraically then differentiating.

$$F(y)=\left(\frac{1}{y^2}-\frac{3}{y^4}\right)(y+5y^3)$$
$$=\frac{1}{y}+5y-\frac{3}{y^3}-\frac{15}{y}$$
$$=5y-\frac{14}{y}-\frac{3}{y^3}$$
$$=5y-14y^{-1}-3y^{-3}$$

Differentiating we get

$$F'(y)=\frac{d}{dy}\left(5y-14y^{-1}-3y^{-3}\right)$$
$$=\left(5-14(-1)y^{-2}-3(-3)y^{-4}\right) \quad \text{(By The Power Rule)}$$
$$=5+\frac{14}{y^2}+\frac{9}{y^4}$$

$$F'(y)=5+\frac{14}{y^2}+\frac{9}{y^4}$$

ALTERNATIVE METHOD:

$$F(y)=\left(\frac{1}{y^2}-\frac{3}{y^4}\right)(y+5y^3)$$

Differentiating we get

$$F'(y)=(y+5y^3)\frac{d}{dy}\left(\frac{1}{y^2}-\frac{3}{y^4}\right)+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\frac{d}{dy}(y+5y^3)$$

(By The Product Rule)

$$=(y+5y^3)\left[\frac{d}{dy}\left(\frac{1}{y^2}\right)-\frac{d}{dy}\left(\frac{3}{y^4}\right)\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[\frac{d}{dy}(y)+\frac{d}{dy}(5y^3)\right]$$

(By Difference Rule and Sum Rule)

$$=(y+5y^3)\left[\frac{d}{dy}\left(\frac{1}{y^2}\right)-3\frac{d}{dy}\left(\frac{1}{y^4}\right)\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[\frac{d}{dy}(y)+5\frac{d}{dy}(y^3)\right]$$

(By The Constant Multiple Rule)

$$=(y+5y^3)\left[\frac{d}{dy}(y^{-2})-3\frac{d}{dy}(y^{-4})\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[\frac{d}{dy}(y)+5\frac{d}{dy}(y^3)\right]$$

$$=(y+5y^3)\left[(-2)y^{-2-1}-3\times(-4)y^{-4-1}\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[1\cdot y^{1-1}+5\times3\cdot y^{3-1}\right]$$

(By The Power Rule)

$$=(y+5y^3)\left[-2y^{-3}+12y^{-5}\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[1\cdot y^0+15\cdot y^2\right]$$

$$=(y+5y^3)\left[-\frac{2}{y^3}+\frac{12}{y^5}\right]+\left(\frac{1}{y^2}-\frac{3}{y^4}\right)\left[1+15y^2\right]$$

Now simplify algebraically

$$F'(y)=\left(y\times-\frac{2}{y^3}+5y^3\times-\frac{2}{y^3}\right)+\left(y\times\frac{12}{y^5}+5y^3\times\frac{12}{y^5}\right)$$
$$+1\times\left(\frac{1}{y^2}-\frac{3}{y^4}\right)+15\cdot y^2\times\left(\frac{1}{y^2}-\frac{3}{y^4}\right)$$

$$= \left(-\frac{2}{y^2} - 10\right) + \left(\frac{12}{y^4} + \frac{60}{y^2}\right) + \left(\frac{1}{y^2} - \frac{3}{y^4}\right) + 15 - \frac{45}{y^2}$$

$$= 5 + \frac{14}{y^2} + \frac{9}{y^4}$$

$$\boxed{F'(y) = 5 + \frac{14}{y^2} + \frac{9}{y^4}}$$

Chapter 2 Derivatives Exercise 2.3 28E

Let $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

$= v^{-1} - 2v^{-3} + v - 2v^{-1}$

$\therefore J'(v) = \frac{d}{dv}(v^{-1} - 2v^{-3} + v - 2v^{-1})$

$= \frac{d}{dv}(-v^{-1} - 2v^{-3} + v) \quad [\because \frac{d}{dx} x^n = x^{n-1}]$

$= -(-1)v^{-2} - 2(-3)v^{-4} + 1$

$= v^{-2} + 6v^{-4} + 1$

$$\boxed{\therefore J'(v) = v^{-2} + 6v^{-4} + 1}$$

Chapter 2 Derivatives Exercise 2.3 29E

Let $g(x) = \frac{1+2x}{3-4x}$

Formula: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Using the above formula

$\frac{d}{dx} g(x) = \frac{d}{dx} \frac{1+2x}{3-4x}$

$$= \frac{(3-4x)\frac{d}{dx}(1+2x) - (1+2x)\frac{d}{dx}(3-4x)}{(3-4x)^2}$$

$$= \frac{(3-4x)(2) - (1+2x)(-4)}{(3-4x)^2}$$

$$= \frac{6-8x+4+8x}{(3-4x)^2} = \frac{10}{(3-4x)^2}$$

$$\boxed{\therefore g'(x) = \frac{10}{(3-4x)^2}}$$

Chapter 2 Derivatives Exercise 2.3 30E

Let $f(x) = \frac{x-3}{x+3}$

Formula: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Using the above formula

$\frac{d}{dx} f(x) = \frac{d}{dx} \frac{x-3}{x+3}$

$$= \frac{(x+3)\frac{d}{dx}(x-3) - (x-3)\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$= \frac{(x+3) - (x-3)}{(x+3)^2}$$

$$= \frac{6}{(x+3)^2}$$

$$\boxed{\therefore f'(x) = \frac{6}{(x+3)^2}}$$

Chapter 2 Derivatives Exercise 2.3 31E

Consider the function,

$$f(x)=\frac{x^3}{1-x^2}.$$

Need to find differentiate the function.

Differentiate $f(x)$ with respect to x.

$$f'(x)=\left(\frac{x^3}{1-x^2}\right)'.$$

Let $u=x^3, v=1-x^2$.

Using the quotient rule, to get

$$f'(x)=\frac{(1-x^2)(x^3)'-x^3(1-x^2)'}{(1-x^2)^2} \qquad \left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}.$$

$$=\frac{(1-x^2)(3x^2)-x^3(0-2x)}{(1-x^2)^2} \qquad \text{since } \frac{d}{dx}(x^n)=nx^{n-1}, \frac{d}{dx}(c)=0.$$

$$=\frac{3x^2-3x^4+2x^4}{(1-x^2)^2}$$

$$=\frac{3x^2-x^4}{(1-x^2)^2}$$

$$=\frac{x^2(3-x^2)}{(1-x^2)^2}$$

Therefore, $f'(x)=\boxed{\frac{x^2(3-x^2)}{(1-x^2)^2}}$.

Chapter 2 Derivatives Exercise 2.3 32E

Quotient rule: If $u(x)$ and $v(x)$ are differentiable, then

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]=\frac{v(x)\frac{d}{dx}[u(x)]-u(x)\frac{d}{dx}[v(x)]}{[v(x)]^2}$$

Consider the function:

$$y=\frac{x+1}{x^3+x-2}$$

Apply the quotient rule to differentiate the function.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{x+1}{x^3+x-2}\right]$$

$$=\frac{\left(x^3+x-2\right)\frac{d}{dx}[x+1]-(x+1)\frac{d}{dx}\left[x^3+x-2\right]}{\left[x^3+x-2\right]^2}$$

$$=\frac{\left(x^3+x-2\right)(1+0)-(x+1)\left(3x^2+1-0\right)}{\left[x^3+x-2\right]^2}$$

$$=\frac{\left(x^3+x-2\right)-\left(3x^3+3x^2+x+1\right)}{\left[x^3+x-2\right]^2}$$

$$=\frac{x^3+x-2-3x^3-3x^2-x-1}{\left[x^3+x-2\right]^2}$$

$$=\frac{-2x^3-3x^2-3}{\left[x^3+x-2\right]^2}$$

Hence,

$$\boxed{\frac{dy}{dx}=\frac{-2x^3-3x^2-3}{\left[x^3+x-2\right]^2}}$$

Chapter 2 Derivatives Exercise 2.3 33E

$$y=\frac{v^3-2v\sqrt{v}}{v}$$

$$=v^2-2\sqrt{v}$$

$$\frac{dy}{dv}=\frac{d}{dv}\left(v^2-2\sqrt{v}\right)=2v-2\times\frac{1}{2\sqrt{v}}$$

$$\frac{dy}{dv}=2v-\frac{1}{\sqrt{v}}$$

Chapter 2 Derivatives Exercise 2.3 34E

Consider the following function:

$$y=\frac{t}{(t-1)^2}$$

The objective is to find the derivative of the function.

The quotient rule:

If u and v are both differentiable, then

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]=\frac{v(x)\frac{d}{dx}[u(x)]-u(x)\frac{d}{dx}[v(x)]}{[v(x)]^2},v(x)\neq 0$$

And the Power Rule States that,

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}.$$

By using the **quotient rule**, we obtain:

$$\frac{dy}{dt} = \frac{d}{dt}\left[\frac{t}{(t-1)^2}\right]$$

$$= \frac{(t-1)^2 \frac{d}{dt}(t) - t\frac{d}{dt}(t-1)^2}{(t-1)^4} \quad \text{Use the quotient rule}$$

$$= \frac{(t-1)^2(1) - t\frac{d}{dt}(t^2-2t+1)}{(t-1)^4} \quad \text{Expand the power}$$

$$= \frac{(t-1)^2 - t(2t-2)}{(t-1)^4} \quad \text{Use the Power rule}$$

$$= \frac{(t-1)^2 - 2t^2 + 2t}{(t-1)^4} \quad \text{Simplify}$$

$$= \frac{t^2 - 2t + 1 - 2t^2 + 2t}{(t-1)^4} \quad \text{Simplify}$$

$$= \frac{-t^2+1}{(t-1)^4}$$

$$= \frac{(1+t)(1-t)}{(t-1)(t-1)^3}$$

$$= \frac{-(1+t)}{(t-1)^3}$$

Therefore, the derivative of the given function is $\boxed{y'(t) = \frac{-(1+t)}{(t-1)^3}}$.

Chapter 2 Derivatives Exercise 2.3 35E

Consider the following function:

$$y = \frac{t^2+2}{t^4-3t^2+1}$$

The objective is to find the derivative of the function.

According to the quotient rule,

If *f* and *g* are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

And the Power Rule States that,

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

By using the **quotient rule**,

$$y'(t)=\frac{d}{dt}\left(\frac{t^2+2}{t^4-3t^2+1}\right)$$

$$=\frac{\left(t^4-3t^2+1\right)\frac{d}{dt}\left(t^2+2\right)-\left(t^2+2\right)\frac{d}{dt}\left(t^4-3t^2+1\right)}{\left(t^4-3t^2+1\right)^2}$$ Quotient rule

$$=\frac{\left(t^4-3t^2+1\right)(2t)-\left(t^2+2\right)\left(4t^3-6t\right)}{\left(t^4-3t^2+1\right)^2}$$ Since $\frac{d}{dx}(c)=0$ and $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$$=\frac{2t^5-6t^3+2t-4t^5+6t^3-8t^3+12t}{\left(t^4-3t^2+1\right)^2}$$

$$=\frac{-2t^5-8t^3+14t}{\left(t^4-3t^2+1\right)^2}$$ Simplify

$$=\boxed{\frac{2t\left(-t^4-4t^2+7\right)}{\left(t^4-3t^2+1\right)^2}}$$

Therefore, the derivative of the given function is $\boxed{y'(t)=\frac{2t\left(-t^4-4t^2+7\right)}{\left(t^4-3t^2+1\right)^2}}$.

Chapter 2 Derivatives Exercise 2.3 36E

Consider the function,

$$g(x)=\frac{t-\sqrt{t}}{t^{\frac{1}{3}}}.$$

Rewrite the function as,

$$g(x)=\frac{t}{t^{\frac{1}{3}}}-\frac{t^{\frac{1}{2}}}{t^{\frac{1}{3}}}$$

$$=t^{1-\frac{1}{3}}-t^{\frac{1}{2}-\frac{1}{3}}$$

$$=t^{\frac{2}{3}}-t^{\frac{1}{6}}$$

The object is to differentiate the above function.

Differentiate on both sides with respect to x.

$$g'(x)=\frac{d}{dx}\left(t^{\frac{2}{3}}-t^{\frac{1}{6}}\right)$$

$$=\frac{d}{dx}\left(t^{\frac{2}{3}}\right)-\frac{d}{dx}\left(t^{\frac{1}{6}}\right)$$

$$=\frac{2}{3}t^{\frac{2}{3}-1}-\frac{1}{6}t^{\frac{1}{6}-1}$$ Use $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$$=\frac{2}{3}t^{\frac{-1}{3}}-\frac{1}{6}t^{\frac{-5}{6}}$$

$$=\frac{2}{3t^{\frac{1}{3}}}-\frac{1}{6t^{\frac{5}{6}}}$$

$$=\frac{4t^{\frac{1}{2}}-1}{6t^{\frac{5}{6}}}$$

$$=\frac{4\sqrt{t}-1}{6t^{\frac{5}{6}}}$$

Therefore, the derivative of the function is,

$$g'(x)=\boxed{\frac{4\sqrt{t}-1}{6t^{\frac{5}{6}}}}.$$

Chapter 2 Derivatives Exercise 2.3 37E

$y = ax^2 + bx + c$

Differentiating using power rule, we get

$\frac{dy}{dx} = 2ax + b$

Chapter 2 Derivatives Exercise 2.3 38E

Given that $y = A + \frac{B}{x} + \frac{C}{x^2}$

Differentiate the above function with respect to x, Then

$\frac{d}{dx}(y) = \frac{d}{dx}\left(A + \frac{B}{x} + \frac{C}{x^2}\right)$

$= \frac{d}{dx}(A) + B\frac{d}{dx}\left(\frac{1}{x}\right) + C\frac{d}{dx}\left(\frac{1}{x^2}\right)$

$= A + Bx^{-1} + Cx^{-2}$

using power rule, we get

$\frac{dy}{dx} = -Bx^{-2} - 2Cx^{-3}$

$\Rightarrow \frac{dy}{dx} = -\frac{B}{x^2} - \frac{2C}{x^3}$

Chapter 2 Derivatives Exercise 2.3 39E

Consider the following function:

$f(t) = \frac{2t}{2+\sqrt{t}}$

The objective is to find the derivative of the function.

According to the quotient rule,

If *f* and *g* are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

And the Power Rule States that,

$\frac{d}{dx}(x^n) = nx^{n-1}$.

By using the **quotient rule**,

$$f'(t) = \frac{d}{dt}\left(\frac{2t}{2+\sqrt{t}}\right)$$

$$= \frac{(2+\sqrt{t})\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(2+\sqrt{t})}{(2+\sqrt{t})^2} \quad \text{Quotient rule}$$

$$= \frac{(2+\sqrt{t})(2) - (2t)\left(0 + \frac{1}{2\sqrt{t}}\right)}{(2+\sqrt{t})^2} \quad \text{Since } \frac{d}{dx}(c) = 0 \text{ and } \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$= \frac{4 + 2\sqrt{t} - 2t\left(\frac{1}{2\sqrt{t}}\right)}{(v)^2}$$

$$= \frac{4 + 2\sqrt{t} - \sqrt{t}}{(2+\sqrt{t})^2} \quad \text{Simplify}$$

$$= \frac{4+\sqrt{t}}{(2+\sqrt{t})^2}$$

Therefore, the derivative of the given function is $\boxed{f'(t) = \frac{4+\sqrt{t}}{(2+\sqrt{t})^2}}$.

Chapter 2 Derivatives Exercise 2.3 40E

$$y = \frac{cx}{1+cx}$$

Using quotient rule, we have

$$\frac{dy}{dx} = \frac{(1+cx)\times\frac{d}{dx}(cx)-(cx)\times\frac{d}{dx}(1+cx)}{(1+cx)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{c(1+cx)-cx\times c}{(1+cx)^2}$$

$$= \frac{c+c^2x-c^2x}{(1+cx)^2} = \frac{c}{(1+cx)^2}$$

Chapter 2 Derivatives Exercise 2.3 41E

Consider the following function:

$$y = \sqrt[3]{t}\left(t^2+t+t^{-1}\right)$$

The objective is to find the derivative of the function.

Expand the expression as,

$$\sqrt[3]{t}\left(t^2+t+t^{-1}\right) = t^{\frac{1}{3}}\left(t^2\right)+t^{\frac{1}{3}}(t)+t^{\frac{1}{3}}\left(t^{-1}\right)$$
$$= t^{\frac{7}{3}}+t^{\frac{4}{3}}+t^{-\frac{2}{3}}$$

The Sum Rule states that, if $f(x), g(x)$ are two differentiable functions, then

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left(g(x)\right).$$

And the Power Rule States that,

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}.$$

By using the Sum Rule, we obtain:

$$\frac{dy}{dt} = \frac{d}{dt}\left[\sqrt[3]{t}\left(t^2+t+t^{-1}\right)\right]$$
$$= \frac{d}{dt}\left[t^{\frac{7}{3}}+t^{\frac{4}{3}}+t^{-\frac{2}{3}}\right]$$
$$= \frac{d}{dt}\left(t^{\frac{7}{3}}\right)+\frac{d}{dt}\left(t^{\frac{4}{3}}\right)+\frac{d}{dt}\left(t^{-\frac{2}{3}}\right)$$

Now by using power rule, we obtain:

$$\frac{dy}{dt} = \frac{d}{dt}\left(t^{\frac{7}{3}}\right)+\frac{d}{dt}\left(t^{\frac{4}{3}}\right)+\frac{d}{dt}\left(t^{-\frac{2}{3}}\right)$$
$$= \frac{7}{3}t^{\frac{7}{3}-1}+\frac{4}{3}t^{\frac{4}{3}-1}-\frac{2}{3}t^{-\frac{2}{3}-1}$$
$$= \frac{7}{3}t^{\frac{4}{3}}+\frac{4}{3}t^{\frac{1}{3}}-\frac{2}{3}t^{-\frac{5}{3}}$$
$$= \boxed{\frac{1}{3}\left(7t^{\frac{4}{3}}+4t^{\frac{1}{3}}-2t^{-\frac{5}{3}}\right)}$$

Therefore, the derivative of the given function is $\boxed{y'(t) = \boxed{\frac{1}{3}\left(7t^{\frac{4}{3}}+4t^{\frac{1}{3}}-2t^{-\frac{5}{3}}\right)}}$.

Chapter 2 Derivatives Exercise 2.3 42E

$$y = \frac{u^6-2u^3+5}{u^2} = u^4-2u+5u^{-2}$$

By power rule, we have

$$\frac{du}{du} = 4u^3-2-10u^{-3} = 4u^3-2-\frac{10}{u^3}$$

Chapter 2 Derivatives Exercise 2.3 43E

$$f(x) = \frac{x}{x + \frac{c}{x}} = \frac{x^2}{x^2 + c}$$

By quotient rule, we have

$$f'(x) = \frac{(x^2 + c) \times \frac{d}{dx}(x^2) - x^2 \times \frac{d}{dx}(x^2 + c)}{(x^2 + c)^2}$$

$$= \frac{2x(x^2 + c) - x^2 \times 2x}{(x^2 + c)^2}$$

$$= \frac{2xc}{(x^2 + c)^2}$$

Chapter 2 Derivatives Exercise 2.3 44E

$$f(x) = \frac{ax + b}{cx + d}$$

Using quotient rule, we get

$$f'(x) = \frac{(cx + d) \times \frac{d}{dx}(ax + b) - (ax + b) \times \frac{d}{dx}(cx + d)}{(cx + d)^2}$$

$$= \frac{a(cx + d) - c(ax + b)}{(cx + d)^2}$$

$$= \frac{ad - bc}{(cx + d)^2}$$

Chapter 2 Derivatives Exercise 2.3 45E

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

Using power rule we get

$$P'(x) = \frac{d}{dx}\left(a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0\right)$$

$$= n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots + 2a_2 x + a_1$$

This is a polynomial of degree $n - 1$

Chapter 2 Derivatives Exercise 2.3 46E

$$f(x) = \frac{x}{x^2 - 1}$$

By the Quotient rule we have $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$

So we have $f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx}\left[\frac{x}{x^2 - 1}\right]$

$$= \frac{(x^2 - 1)\frac{d}{dx}(x) - x.\frac{d}{dx}(x^2 - 1)}{(x^2 - 1)^2}$$

We have $\frac{d}{dx}(x) = 1$ *and* $\frac{d}{dx}(x^2 - 1) = 2x$

Thus $f'(x) = \frac{(x^2 - 1).1 - x.2x}{(x^2 - 1)^2} = \frac{x^2 - 1 - 2x^2}{(x^2 - 1)^2}$

$$\Rightarrow f'(x) = \frac{-(x^2 + 1)}{(x^2 - 1)^2}$$

Here in figure 1 the graph of $f(x)$ and in figure 2 the graph of $f'(x)$ is shown we see that $f(x)$ is not defined at x = 1 and -1, so $f'(x)$ is also not defined. we see that where the graph of $f(x)$ having negative slope so $f'(x)$ is also negative and where f(x) has positive slope so f'(x) is also positive .Both the graphs are shown in figure (1)and (2).

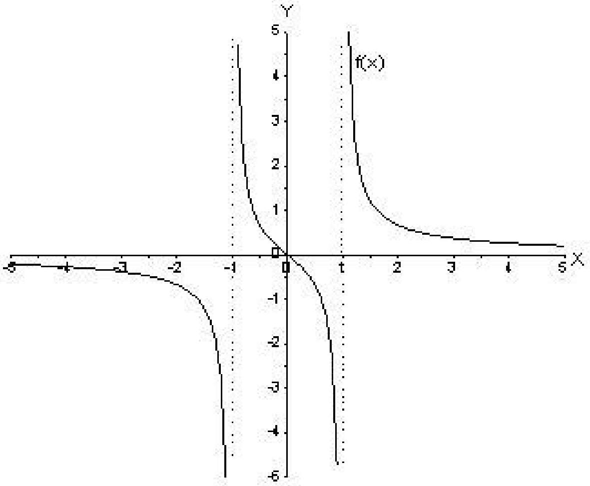


Fig.1

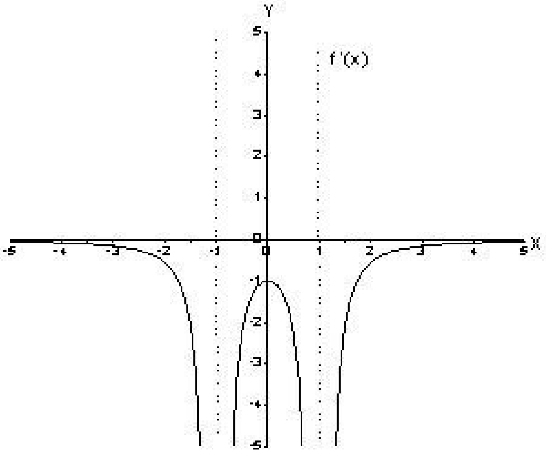


Fig.2

Chapter 2 Derivatives Exercise 2.3 47E

$f(x) = 3x^{15} - 5x^3 + 3$

Then $f'(x) = \frac{d}{dx}\left[3x^{15} - 5x^3 + 3\right]$

$= \frac{d}{dx}\left[3x^{15}\right] - \frac{d}{dx}\left[5x^3\right] + \frac{d}{dx}[3]$ Since $\frac{d}{dx}[f+g] = \frac{d}{dx}f + \frac{d}{dx}g$

$= 3\frac{d}{dx}.x^{15} - 5\frac{d}{dx}x^3 + 0$ Because $\frac{d}{dx}(c) = 0$ where c is any constant

$= 3.15.x^{14} - 5.3x^2$ We have $\left[\frac{d}{dx}x^n = nx^{n-1}\right]$

$\boxed{f'(x) = 45x^{14} - 15x^2}$

In figure 1 the graph of $f(x)$ and in figure 2, the graph of $f'(x)$ are shown

Here we see that at x = 0 the graph of $f(x)$ has horizontal tangent so $f'(0) = 0$

Where $f(x)$ has positive slope, $f'(x)$ is also positive and Where $f(x)$ has negative slope, $f'(x)$ is also negative.

Thus this verify our result of $f'(x)$

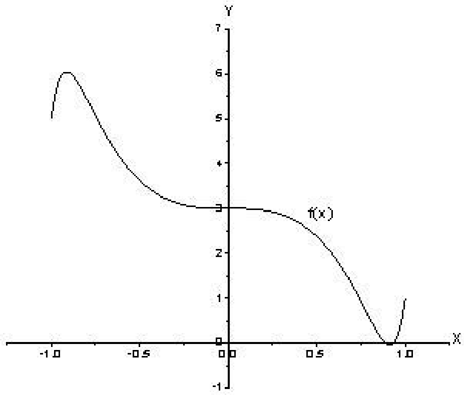


Fig.1

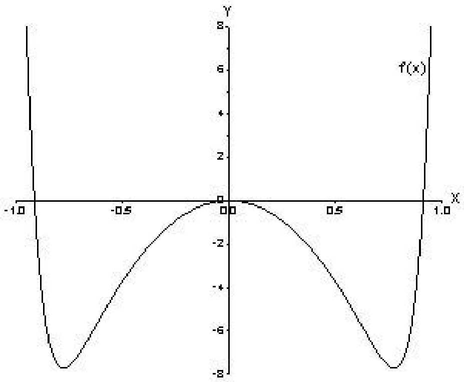


Fig.2

Chapter 2 Derivatives Exercise 2.3 48E

We have $f(x) = x + \frac{1}{x}$

Then $f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right)$

We have $\frac{d}{dx}(f+g) = \frac{d}{dx}f + \frac{d}{dx}g$

Thus $f'(x) = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right)$

$$= 1 + \frac{d}{dx}(x^{-1}) \qquad \left[\frac{d}{dx}(x) = 1\right]$$

$$= 1 + (-1)\frac{1}{x^2} \qquad \left[\frac{d}{dx}x^n = nx^{n-1}\right]$$

$$\boxed{f'(x) = 1 - \frac{1}{x^2}}$$

In the figure 1 the graph of $f(x)$ and in the figure 2, graph of $f'(x)$ is shown.
Here we see that $f(x)$ is not defined at $x = 0$ so $f'(x)$ is also not defined at $x = 0$
Where f(x) has positive slope, the value of $f'(x)$ is also positive and where the slope of $f(x)$ is negative, the value of $f'(x)$ is also negative. This verifies that our answer is reasonable.

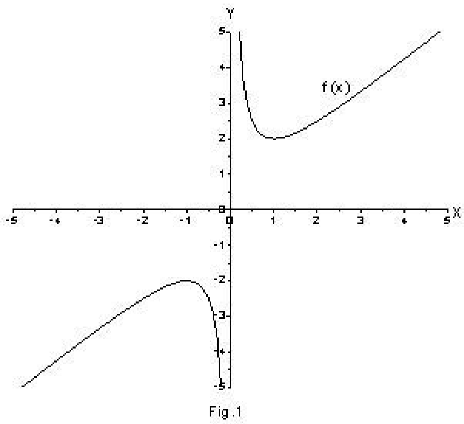


Fig.1

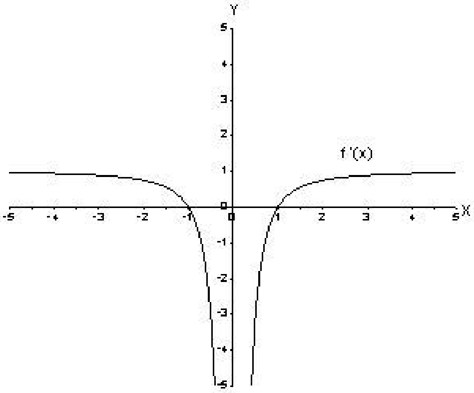


Fig.2

Chapter 2 Derivatives Exercise 2.3 49E

(A) We have $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$

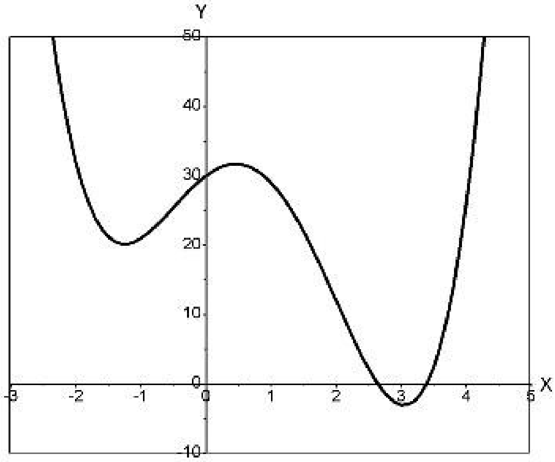


Fig.1

(B) From the graph of $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$ in part (A), we see that the graph has horizontal tangents at $x = -1.25, 0.5$ and 3, so here the derivative will be zero. Since the function $f(x)$ is decreasing on $(-\infty, -1.25)$ and $(0.5, 3)$ so here derivative will be negative and since $f(x)$ is increasing on $(-1.25, 0.5)$ and $(3, \infty)$ so here the derivative will be positive.

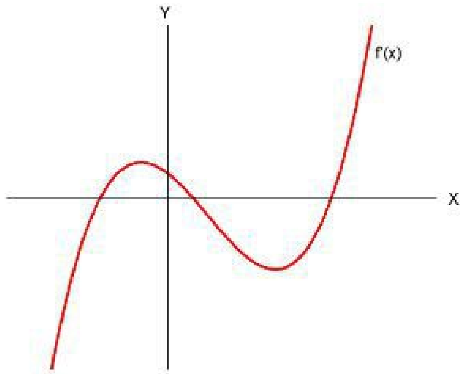


Fig.2

(C) Now we have $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$

Then
$$f'(x) = \frac{d}{dx}\left(x^4 - 3x^3 - 6x^2 + 7x + 30\right)$$
$$= \frac{d}{dx}\left(x^4\right) - 3\frac{d}{dx}\left(x^3\right) - 6\frac{d}{dx}\left(x^2\right) + 7\frac{d}{dx}(x) + \frac{d}{dx}(20)$$
$$= 4x^3 - 3\times 3x^2 - 6\times 2x + 7 + 0$$

Or
$$\boxed{f'(x) = 4x^3 - 9x^2 - 12x + 7}$$

Now we sketch the graph of $f'(x)$ and see that our answer in part (b) is correct

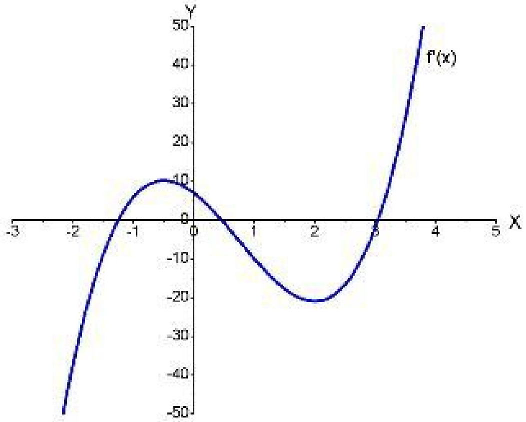


Fig.3

Chapter 2 Derivatives Exercise 2.3 50E

a)

Consider the function $y=\frac{x^2}{x^2+1}$

Sketch the graph of $y=\frac{x^2}{x^2+1}$ in viewing $[-4,4],[-1,1.5]$ is as follows:

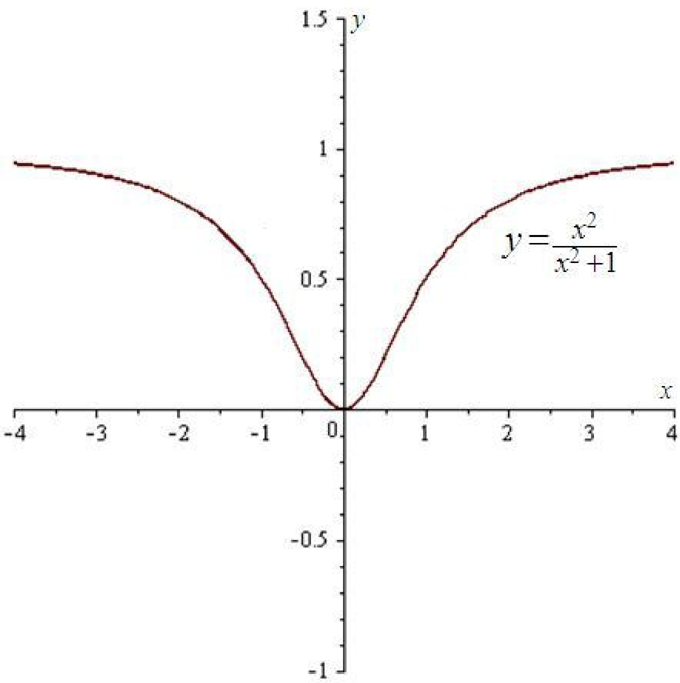
Use Computer Algebra System Maple, to plot the graph:

The input command is

Plot(x^2/x^2+1,x=-4..4,y=-1..5);

The output is

$$> plot\left(\frac{x^2}{x^2+1}, x=-4..4, y=-1..1.5\right);$$



b)

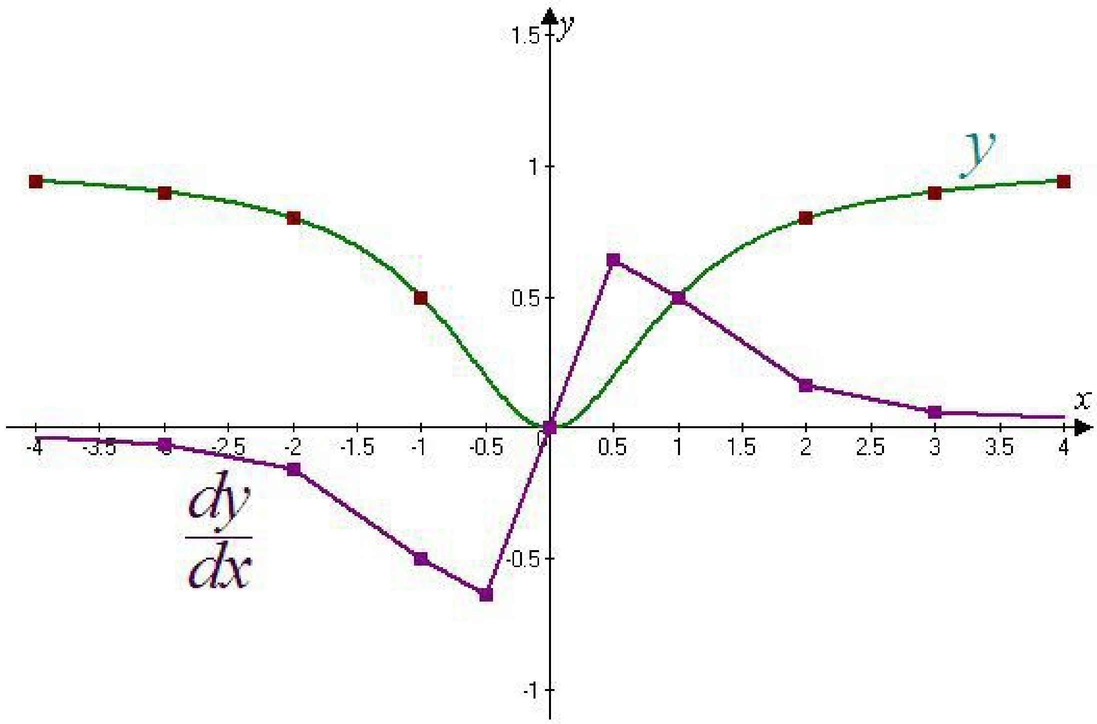
For find graph of y':

For instance, for $x=1$ then its tangent slope is 0.5

For $x=2$ then its tangent slope is 0.16

For $x=3$ then its tangent slope is 0.06

Rough sketch of the graph of y' is as follows:



c)

Find $\frac{dy}{dx}$:

Use Computer Algebra System Maple, to find $\frac{dy}{dx}$:

The input command is

Simplify(diff(x^2/(x^2+1),x));

The output is

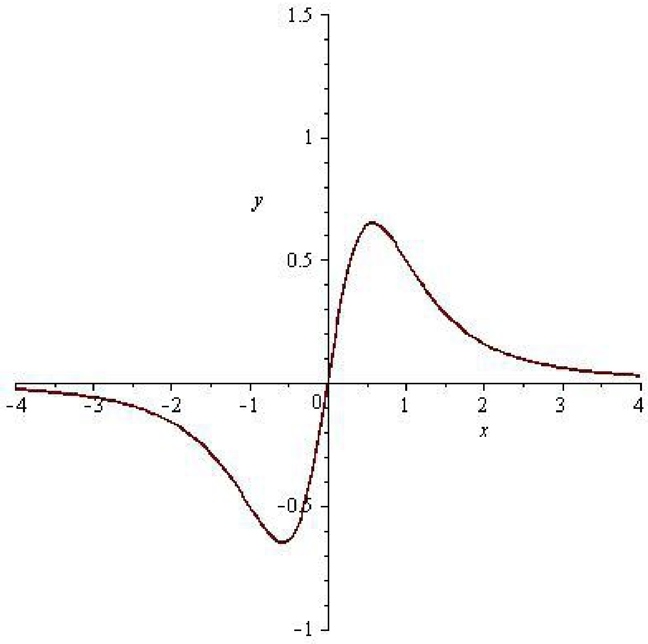
$$> simplify\left(diff\left(\frac{x^2}{(x^2+1)},x\right)\right);$$

$$\frac{2x}{(x^2+1)^2}$$

Therefore, $\frac{dy}{dx} = \boxed{\frac{2x}{(x^2+1)^2}}$

Sketch the graph of $\frac{dy}{dx} = \frac{2x}{(x^2+1)^2}$ is as follows:

$$> plot\left(\frac{2x}{(x^2+1)^2}, x=-4..4, y=-1..1.5\right);$$



Chapter 2 Derivatives Exercise 2.3 51E

We have $y=\frac{2x}{x+1}$

Using quotient rule we get

$$\frac{dy}{dx}=\frac{(x+1)\times\frac{d}{dx}(2x)-2x\times\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$=\frac{2(x+1)-2x}{(x+1)^2}$$

$$=\frac{2}{(x+1)^2}$$

The slope of the tangent at (1, 1) is given by

$$m=\left.\frac{dy}{dx}\right|_{x=1}=\frac{2}{(1+1)^2}=\frac{1}{2}$$

The equation of the tangent at (1, 1) is

$$y-y_1=m(x-x_1)$$

$$\Rightarrow y-1=\frac{1}{2}(x-1)\quad (x_1=1,\ y_1=1)$$

$$\Rightarrow 2y-x=1$$

Or $$\Rightarrow y=\frac{1}{2}x+\frac{1}{2}$$

Chapter 2 Derivatives Exercise 2.3 52E

The slope of the tangent at (1, 1) is given by

$$m=\left.\frac{dy}{dx}\right|_{x=1}=\frac{2}{(1+1)^2}=\frac{1}{2}$$

The equation of the tangent at (1, 1) is

$$y-y_1=m(x-x_1)$$

$$\Rightarrow y-1=\frac{1}{2}(x-1)\quad (x_1=1,\ y_1=1)$$

$$\Rightarrow 2y-x=1$$

Or $$\Rightarrow y=\frac{1}{2}x+\frac{1}{2}$$

Chapter 2 Derivatives Exercise 2.3 53E

(A)

Curve $y = \frac{1}{(1+x^2)}$

Then $y' = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$

We use the Quotient rule $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$

We have $\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(1) - 1.\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

We have $\frac{d}{dx}(C) = 0$ where C is a constant

So $\frac{dy}{dx} = \frac{0-(2x)}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(1+x^2)^2}$

Then slope of tangent at the point $\left(-1, \frac{1}{2}\right)$ is

$$\frac{dy}{dx} = \frac{-2(-1)}{(1+(-1)^2)^2} = \frac{2}{(1+1)^2} = \frac{2}{4}$$

$$\boxed{slope = \frac{1}{2}}$$

Then equation of tangent is $y - y_1 = \frac{dy}{dx}(x - x_1)$

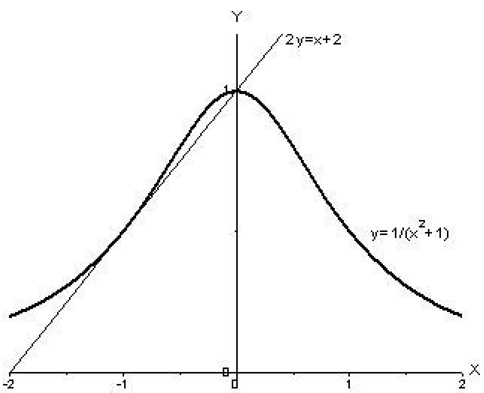
$\Rightarrow \left(y - \frac{1}{2}\right) = \frac{1}{2}(x-(-1))$

$\Rightarrow y - \frac{1}{2} = \frac{1}{2}(x+1)$

$\Rightarrow 2y - 1 = x + 1$

$\Rightarrow \boxed{2y = x + 2}$

(B)



Chapter 2 Derivatives Exercise 2.3 54E

(A) We have $y = \frac{x}{1+x^2}$

After differentiating with respect to x, we get

$\Rightarrow y' = \frac{(1+x^2) - x(2x)}{(1+x^2)^2}$ [Quotient rule]

$\Rightarrow y' = \frac{(1-x^2)}{(1+x^2)^2}$

The slope of the tangent line at (3, 0.3) is

$$\Rightarrow y'(3) = \frac{1-3^2}{\left(1+3^2\right)^2} = -0.08$$

Therefore equation of the tangent at $(3, 0.3)$,

$$(y-0.3) = -0.08(x-3)$$

$$\Rightarrow \boxed{y = -0.08x + 0.54}$$

(B) Now we graph the curve and the tangent line on the same screen.

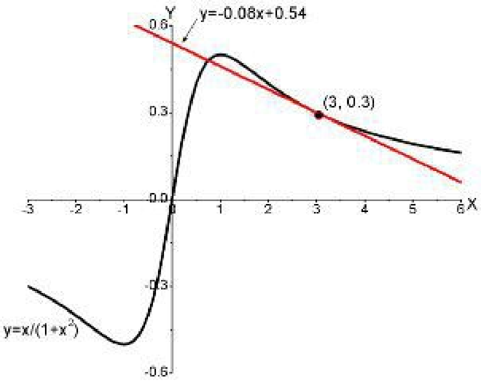


Fig.1

Chapter 2 Derivatives Exercise 2.3 55E

Tangent Line at (1,2)

$y = x + \sqrt{x}$

Differentiate with respect to x

$y' = \frac{d}{dx}[x + \sqrt{x}]$

$= 1 + \frac{1}{2\sqrt{x}}$ Now evaluate at x=1 to find the slope of the tangent at (1,2)

$y'(1) = 1 + \frac{1}{2} = 1.5 = m$

Equation of the tangent line is $\left(y-2\right) = \frac{3}{2}\left(x-1\right)$

Or $y = \frac{3}{2}x + \frac{1}{2}$

Since the normal line is perpendicular to the tangent so its slope m_N will be the negative reciprocal of m, $\frac{-2}{3}$

Then equation of the normal line is

$$\left(y-2\right) = -\frac{2}{3}\left(x-1\right)$$

or $y = -\frac{2}{3}x + \frac{8}{3}$

Chapter 2 Derivatives Exercise 2.3 56E

We are given that the equation of the curve

$y = (1+2x)^2$ and the given point is $(1, 9)$

We first find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2\left(1 + 2x\right)\cdot 2$$

$$= 4(1 + 2x)$$

$$\Rightarrow \frac{dy}{dx} = 4\left(1 + 2x\right)$$

So the slope of the tangent line at $(1, 9)$ is

$$\left[\frac{dy}{dx}\right]_{x=1} = 4\left(1 + 2(1)\right) = 12$$

We use the point -slope form to write an equation of the tangent line at $(1, 9)$

$$y - 9 = 12(x - 1)$$

$$\Rightarrow y - 9 = 12x - 12$$

$$\Rightarrow y = 12x - 3$$

The slope of the normal line at $(1, 9)$ is the negative reciprocal of 12, namely $\frac{-1}{12}$, so an equation is

$$y - 9 = \frac{-1}{12}\left(x - 1\right)$$

$$\Rightarrow y - 9 = \frac{-1}{12}x + \frac{1}{12}$$

$$\Rightarrow y = \frac{-x}{12} + \frac{1}{12} + 9$$

$$\Rightarrow y = \frac{-x}{12} + \frac{109}{12}$$

So, the equation of the tangent line is $y = 12x - 3$ **and the equation of the normal line is** $y = \frac{-x}{12} + \frac{109}{12}$ **at the point** $(1, 9)$

Chapter 2 Derivatives Exercise 2.3 57E

Consider the curve,

$$y=\frac{3x+1}{x^2+1}$$

And the point $(1,2)$

Need to find the equations of the tangent line and normal line to the curve y at $(1,2)$.

Compute $\frac{dy}{dx}$:

According to the quotient rule,

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[\frac{3x+1}{x^2+1}\right]\\&=\frac{\left(x^2+1\right)\frac{d}{dx}(3x+1)-(3x+1)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}\\&=\frac{\left(x^2+1\right)3-(3x+1)(2x)}{\left(x^2+1\right)^2}\\&=\frac{3x^2+3-6x^2-2x}{\left(x^2+1\right)^2}\\&=\frac{-3x^2-2x+3}{\left(x^2+1\right)^2}\end{aligned}$$

Since the slope of the tangent line is the slope of the curve which is the derivative of the function.

Thus, the slope of the tangent line is

$$\frac{dy}{dx}=\frac{-3x^2-2x+3}{\left(x^2+1\right)^2}$$

The slope of the tangent line at $(1,2)$ is

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=1}&=\frac{-3(1)^2-2(1)+3}{\left(1^2+1\right)^2}\\&=\frac{-3-2+3}{4}\\&=\frac{-2}{4}\\&=-\frac{1}{2}\end{aligned}$$

Thus, the slope of the tangent line at $(1,2)$ is $\boxed{-\frac{1}{2}}$

The equation of the tangent line arises from the point-slope equation for a line.

Point-Slope form of the equation of a line:

An equation of line passing through the point (x_1, y_1) and having the slope *m* is

$y - y_1 = m(x - x_1)$

Let $(x_1, y_1) = (1, 2)$

The equation of the tangent line at $(1,2)$ is,

$(y-2) = -\frac{1}{2}(x-1)$

$y - 2 = -\frac{1}{2}x + \frac{1}{2}$ Use distributive property

$y = -\frac{1}{2}x + \frac{5}{2}$ Add 2 to both sides

Therefore, the equation of the tangent line to the curve $y = \frac{3x+1}{x^2+1}$ at the point $(1,2)$ is

$\boxed{y = -\frac{1}{2}x + \frac{5}{2}}$

Recall that, the normal line to a curve C at point *P* is the line through *P* that is perpendicular to the tangent line at *P*.

Two lines are perpendicular; their slopes are negative reciprocals.

So, the slope of the normal line at $(1,2)$ is the negative reciprocal of $-\frac{1}{2}$, namely 2

The equation of the normal line at $(1,2)$ is,

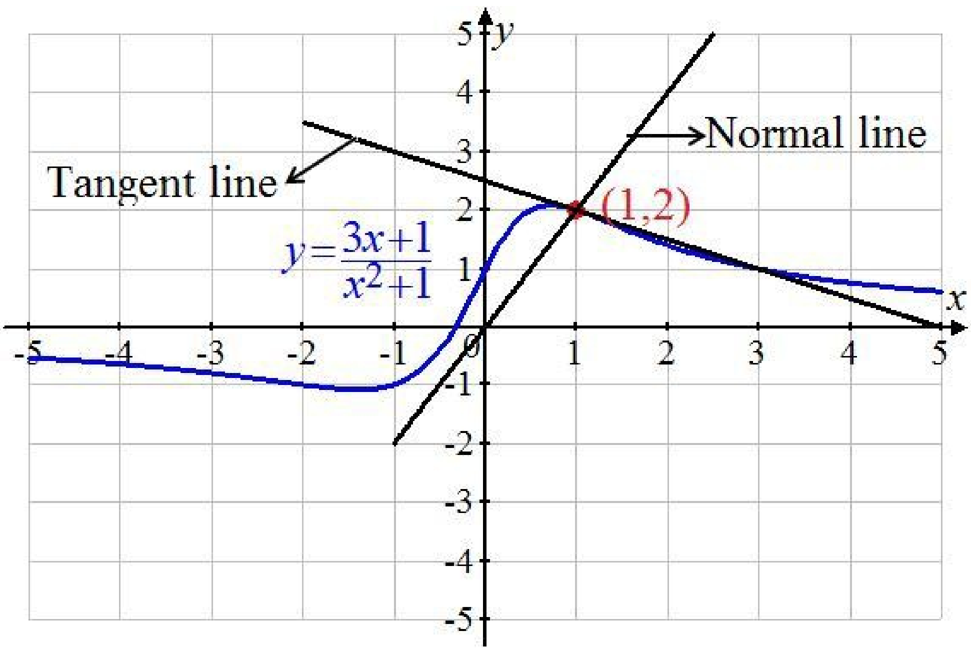
$(y-2) = 2(x-1)$

$y - 2 = 2x - 2$ Use distributive property

$y = 2x$ Add 2 to both sides

Therefore, the equation of the normal line to the curve $y = \frac{3x+1}{x^2+1}$ at the point $(1,2)$ is $\boxed{y = 2x}$

The curve and its tangent and normal lines are graphed in the below figure:



Chapter 2 Derivatives Exercise 2.3 58E

Consider the curve,

$$y=\frac{\sqrt{x}}{x+1}$$

And the point $(4,0.4)$

Need to find the equations of the tangent line and normal line to the curve y at $(4,0.4)$.

Compute $\frac{dy}{dx}$:

According to the quotient rule,

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[\frac{\sqrt{x}}{x+1}\right]\\&=\frac{(x+1)\frac{d}{dx}(\sqrt{x})-(\sqrt{x})\frac{d}{dx}(x+1)}{(x+1)^2}\\&=\frac{(x+1)\frac{1}{2\sqrt{x}}-\sqrt{x}(1)}{(x+1)^2}\\&=\frac{(x+1)-2x}{2\sqrt{x}(x+1)^2}\\&=\frac{1-x}{2\sqrt{x}(x+1)^2}\end{aligned}$$

Since the slope of the tangent line is the slope of the curve which is the derivative of the function.

Thus, the slope of the tangent line is

$$\frac{dy}{dx}=\frac{1-x}{2\sqrt{x}(x+1)^2}$$

The slope of the tangent line at $(4,0.4)$ is

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=4}&=\frac{1-4}{2\cdot\sqrt{4}(4+1)^2}\\&=\frac{-3}{2\cdot 2(4+1)^2}\\&=\frac{-3}{4\cdot 25}\\&=-\frac{3}{100}\end{aligned}$$

Thus, the slope of the tangent line at $(4,0.4)$ is $\boxed{-\frac{3}{100}}$

The equation of the tangent line arises from the point-slope equation for a line.

Point-Slope form of the equation of a line:

An equation of line passing through the point (x_1, y_1) and having the slope *m* is

$$y - y_1 = m(x - x_1)$$

Let $(x_1, y_1) = (4, 0.4)$

The equation of the tangent line at $(4, 0.4)$ is,

$$(y - 0.4) = -\frac{3}{100}(x - 4)$$

$y - 0.4 = -\frac{3}{100}x + \frac{12}{100}$ Use distributive property

$y - \frac{40}{100} = -\frac{3}{100}x + \frac{12}{100}$ Rewrite

$y = -\frac{3}{100}x + \frac{52}{100}$ Add $\frac{40}{100}$ to both sides

$y = -\frac{3}{100}x + \frac{13}{25}$ Simplify

Therefore, the equation of the tangent line to the curve $y = \frac{\sqrt{x}}{x+1}$ at the point $(4, 0.4)$ is

$$\boxed{y = -\frac{3}{100}x + \frac{13}{25}}$$

Recall that, the normal line to a curve C at point *P* is the line through *P* that is perpendicular to the tangent line at *P*.

Two lines are perpendicular; their slopes are negative reciprocals.

So, the slope of the normal line at $(4, 0.4)$ is the negative reciprocal of $-\frac{3}{100}$, namely $\frac{100}{3}$

The equation of the normal line at $(4, 0.4)$ is,

$$(y - 0.4) = \frac{100}{3}(x - 4)$$

$y - 0.4 = \frac{100}{3}x - \frac{400}{3}$ Use distributive property

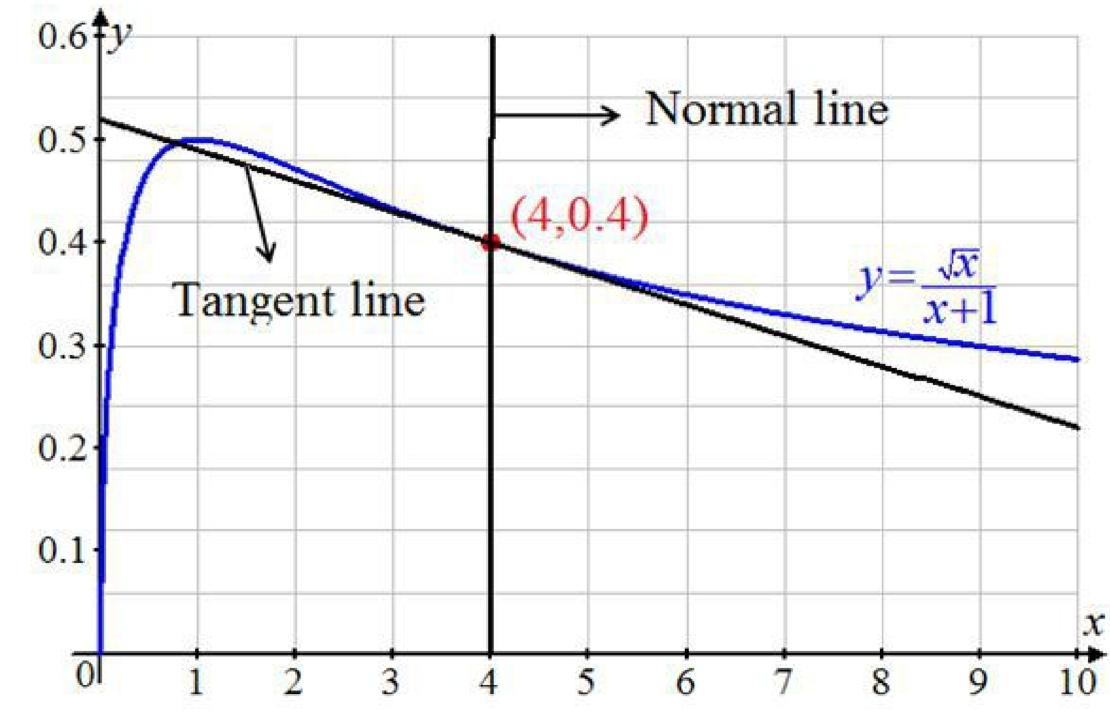
$y - \frac{2}{5} = \frac{100}{3}x - \frac{400}{3}$ Rewrite

$y = \frac{100}{3}x - \frac{1994}{15}$ Add $\frac{2}{5}$ to both sides

Therefore, the equation of the normal line to the curve $y = \frac{\sqrt{x}}{x+1}$ at the point $(4, 0.4)$ is

$$\boxed{y = \frac{100}{3}x - \frac{1994}{15}}$$

The curve and its tangent and normal lines are graphed in the below figure:



Chapter 2 Derivatives Exercise 2.3 59E

Consider the function

$f(x)=x^4-3x^3+16x$

First, to find the first derivative of the function $f(x)$:

Differentiate $f(x)=x^4-3x^3+16x$ with respect to x, to get

$$f'(x)=\frac{d}{dx}[f(x)]$$
$$=\frac{d}{dx}(x^4-3x^3+16x)$$

$=\frac{d}{dx}(x^4)-\frac{d}{dx}(3x^3)+\frac{d}{dx}(16x)$ By using the difference and sum rule

$=\frac{d}{dx}(x^4)-3\frac{d}{dx}(x^3)+16\frac{d}{dx}(x)$ By using the constant multiple rule

$=4x^3-3(3x^2)+16(1)$ By using the power rule: $\frac{d}{dx}(x^n)=nx^{n-1}, n\in\mathbb{R}$

$=4x^3-9x^2+16$ Simplify.

Therefore, the first derivative of the function $f(x)$ is

$\boxed{f'(x)=4x^3-9x^2+16}$.

First, to find the second derivative of the function $f(x)$:

Differentiate $f'(x) = 4x^3 - 9x^2 + 16$ with respect to *x*, to get

$$f''(x) = \frac{d}{dx}\left[f'(x)\right]$$
$$= \frac{d}{dx}\left(4x^3 - 9x^2 + 16\right)$$

$= \frac{d}{dx}\left(4x^3\right) - \frac{d}{dx}\left(9x^2\right) + \frac{d}{dx}(16)$ By using the difference and sum rule

$= 4\frac{d}{dx}\left(x^3\right) - 9\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(16)$ By using the constant multiple rule

$= 4\frac{d}{dx}\left(x^3\right) - 9\frac{d}{dx}\left(x^2\right)$ Since $\frac{d}{dx}(c) = 0, c \text{ is constant}$

$= 4\left(3x^2\right) - 9(2x)$ By using the power rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}, n \in \mathbb{R}$

$= 12x^2 - 18x$ Simplify.

Therefore, the second derivative of the function $f(x)$ is

$$\boxed{f''(x) = 12x^2 - 18x}.$$

Chapter 2 Derivatives Exercise 2.3 60E

Consider the function

$G(r) = \sqrt{r} + \sqrt[3]{r}$.

First, to find the first derivative of the function $G(r)$:

Differentiate $G(r) = \sqrt{r} + \sqrt[3]{r}$ with respect to *r*, to get

$$G'(r) = \frac{d}{dr}\left[G(r)\right]$$
$$= \frac{d}{dr}\left(\sqrt{r} + \sqrt[3]{r}\right)$$

$= \frac{d}{dr}\left(\sqrt{r}\right) + \frac{d}{dr}\left(\sqrt[3]{r}\right)$ By using the sum rule

$= \frac{d}{dr}\left(r^{1/2}\right) + \frac{d}{dr}\left(r^{1/3}\right)$ By using the radical rule: $\sqrt[n]{a} = a^{1/n}$

$= \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3}$ By using the power rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}, n \in \mathbb{R}$

Therefore, the first derivative of the function $G(r)$ is

$$\boxed{G'(r) = \frac{1}{2\sqrt{r}} + \frac{1}{3\sqrt[3]{r^2}}}.$$

Next, to find the second derivative of the function $G(r)$:

Differentiate $G'(r)=\frac{1}{2}r^{-1/2}+\frac{1}{3}r^{-2/3}$ with respect to *r*, to get

$$G''(r)=\frac{d}{dr}\left[G'(r)\right]$$

$$=\frac{d}{dr}\left(\frac{1}{2}r^{-1/2}+\frac{1}{3}r^{-2/3}\right)$$

$$=\frac{d}{dr}\left(\frac{1}{2}r^{-1/2}\right)+\frac{d}{dr}\left(\frac{1}{3}r^{-2/3}\right)$$ By using the sum rule

$$=\frac{1}{2}\frac{d}{dr}\left(r^{-1/2}\right)+\frac{1}{3}\frac{d}{dr}\left(r^{-2/3}\right)$$ By using the constant multiple rule

$$=\frac{1}{2}\left(-\frac{1}{2}r^{-3/2}\right)+\frac{1}{3}\left(-\frac{2}{3}r^{-5/3}\right)$$ By using the power rule: $\frac{d}{dx}\left(x^n\right)=nx^{n-1},n\in\mathbb{R}$

$$=-\frac{1}{4}r^{-3/2}-\frac{2}{9}r^{-5/3}$$ Simplify.

Therefore, the second derivative of the function $G(r)$ is

$$\boxed{G''(r)=-\frac{1}{4\sqrt{r^3}}-\frac{2}{9\sqrt[3]{r^5}}}.$$

Chapter 2 Derivatives Exercise 2.3 61E

Consider the function,

$$f(x)=\frac{x^2}{1+2x}$$

Use the Quotient Rule,

"If *g* and *h* are differentiable, then

$$\frac{d}{dx}\left[\frac{g(x)}{h(x)}\right]=\frac{h(x)\frac{d}{dx}\left[g(x)\right]-g(x)\frac{d}{dx}\left[h(x)\right]}{\left[h(x)\right]^2}."$$

The first derivative of the given function $f(x)=\frac{x^2}{1+2x}$ will be,

$$f'(x)=\frac{(1+2x)\frac{d\left[x^2\right]}{dx}-x^2\frac{d(1+2x)}{dx}}{(1+2x)^2}$$

$$=\frac{(1+2x)\times\left[2x^{2-1}\right]-x^2\times\left[0+2\times1\times x^{1-1}\right]}{(1+2x)^2}$$

$$=\frac{(1+2x)\times2x-x^2\times2}{(1+2x)^2}$$

$$=\frac{2x+4x^2-2x^2}{(1+2x)^2}$$

Simplify further,

$$\boxed{f'(x)=\frac{2x+2x^2}{(1+2x)^2}}$$

Use the Quotient Rule, the second derivative of the given function $f(x)=\frac{x^2}{1+2x}$ will be,

$$f''(x)=\frac{d\left[f'(x)\right]}{dx}$$

$$=\frac{(1+2x)^2\frac{d\left[2x+2x^2\right]}{dx}-\left(2x+2x^2\right)\frac{d(1+2x)^2}{dx}}{\left[(1+2x)^2\right]^2}$$

$$=\frac{(1+2x)^2\times\left[2\times1\times x^{1-1}+2\times2\times x^{2-1}\right]-\left(2x+2x^2\right)\times\frac{d\left(1+4x+4x^2\right)}{dx}}{(1+2x)^4}$$

$$=\frac{(1+2x)^2\times[2+4x]-\left(2x+2x^2\right)\times\left[0+4\times x^{1-1}+4\times2\times x^{2-1}\right]}{(1+2x)^4}$$

Simplify further,

$$f''(x)=\frac{(1+2x)^2\times[2+4x]-\left(2x+2x^2\right)\times[4+8x]}{(1+2x)^4}$$

$$=\frac{[2+4x]\times\left[(1+2x)^2-2\left(2x+2x^2\right)\right]}{(1+2x)^4}$$

$$=\frac{2[1+2x]\times\left[1+4x+4x^2-4x-4x^2\right]}{(1+2x)^4}$$

$$=\frac{2\times[1]}{(1+2x)^{4-1}}$$

Simplification gives,

$$\boxed{f''(x)=\frac{2}{(1+2x)^3}}$$

Chapter 2 Derivatives Exercise 2.3 62E

Consider the function,

$$f(x)=\frac{1}{3-x}$$

Need to find the first and second derivatives of the above function.

Quotient rule:

If *f* and *g* are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Compute the first derivative, $f'(x)$:

$$f'(x)=\frac{d}{dx}\left[\frac{1}{3-x}\right]$$

$$=\frac{(3-x)\frac{d}{dx}(1)-(1)\frac{d}{dx}(3-x)}{(3-x)^2}$$ Quotient rule

$$=\frac{(3-x)(0)-1(-1)}{(3-x)^2}$$ Since: $\frac{d}{dx}(c)=0$ and $\frac{d}{dx}(x)=1$

$$=\frac{1}{(3-x)^2}$$ Simplify

Therefore, the first derivative of $f(x)=\frac{1}{3-x}$ is $\boxed{f'(x)=\frac{1}{(3-x)^2}}$

Compute the first derivative, $f''(x)$:

$$f''(x)=\frac{d}{dx}\left[f'(x)\right]$$

$$=\frac{d}{dx}\left[\frac{1}{(3-x)^2}\right] \quad \text{Since: } f'(x)=\frac{1}{(3-x)^2}$$

$$=\frac{(3-x)^2\frac{d}{dx}(1)-(1)\frac{d}{dx}(3-x)^2}{\left[(3-x)^2\right]^2} \quad \text{Quotient rule}$$

$$=\frac{(3-x)^2(0)-1\left[2(3-x)(-1)\right]}{\left[(3-x)^2\right]^2} \quad \text{Since: } \frac{d}{dx}(c)=0 \text{ and } \frac{d}{dx}(x^n)=nx^{n-1}$$

$$=\frac{-1\left[2(3-x)(-1)\right]}{(3-x)^4} \quad \text{Simplify}$$

$$=\frac{6-2x}{(3-x)^4} \quad \text{Use distributive property}$$

$$=\frac{2(3-x)}{(3-x)^4} \quad \text{Factor out 2 in the numerator}$$

$$=\frac{2}{(3-x)^3} \quad \text{Cancel out the common term, } 3-x$$

Therefore, the second derivative of $f(x)=\frac{1}{3-x}$ is $\boxed{f''(x)=\frac{2}{(3-x)^3}}$

Chapter 2 Derivatives Exercise 2.3 63E

Consider the equation of motion of a particle is

$$s=t^3-3t$$

Where s is in meters and t is in seconds.

(a)

Velocity is the first derivative of the position function $s(t)=t^3-3t$.

Therefore,

$$s'(t)=\frac{d}{dt}(t^3-3t)$$

$$=\frac{d}{dt}(t^3)-\frac{d}{dt}(3t) \quad \left(\frac{d}{dx}\left[f(x)\pm g(x)\right]=\frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)\right)$$

$$=\frac{d}{dt}(t^3)-3\frac{d}{dt}(t) \quad \left(\frac{d}{dx}\left[cf(x)\right]=c\frac{d}{dx}f(x)\right)$$

$$=3t^2-3 \quad \left(\frac{d}{dx}(x^n)=nx^{n-1}\right)$$

Hence, the velocity as function of t is

$\boxed{v(t)=3t^2-3}$.

Acceleration is the first derivative of the velocity function $v(t)=3t^2-3$.

Therefore,

$$\begin{aligned} v'(t) &= \frac{d}{dt}\left(3t^2-3\right) \\ &= \frac{d}{dt}\left(3t^2\right)-\frac{d}{dt}(3) && \left(\frac{d}{dx}\left[f(x)\pm g(x)\right]=\frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)\right) \\ &= 3\frac{d}{dt}\left(t^2\right)-\frac{d}{dt}(3) && \left(\frac{d}{dx}\left[cf(x)\right]=c\frac{d}{dx}f(x)\right) \\ &= 3(2t)-0 && \left(\frac{d}{dx}\left(x^n\right)=nx^{n-1},\frac{d}{dx}(c)=0\right) \end{aligned}$$

$$=6t$$

Hence, the acceleration as function of t is

$\boxed{a(t)=6t}$.

(b)

Evaluate the acceleration after 2 s by substitute 2 into the acceleration function:

$$\begin{aligned} a(t)&=6t \\ a(2)&=6(2) \\ &=12 \end{aligned}$$

Since the units of position are meters with respect to time in seconds, the acceleration after 2 s is $\boxed{12\,\text{m/s}^2}$.

(c)

To find the time when velocity is 0 be setting the velocity function equal to 0:

$$\begin{aligned} v(t)&=3t^2-3 \\ 0&=3t^2-3 \\ 3&=3t^2 \\ 1&=t^2 \end{aligned}$$

$$\pm 1=t$$

Since we will only consider positive times, we recognize only that the velocity is 0 at 1 s.

Evaluate the acceleration after 1 s by substitute 1 into the acceleration function:

$$\begin{aligned} a(t)&=6t \\ a(1)&=6(1) \\ &=6 \end{aligned}$$

Therefore, the acceleration after 1 s, which corresponds to the velocity being 0, is

$\boxed{6\,\text{m/s}^2}$.

Chapter 2 Derivatives Exercise 2.3 64E

The equation of motion of a particle is $s=t^4-2t^3+t^2-t$, where s is in meters and t is in seconds.

(a) Velocity $v=\dfrac{ds}{dt}=\dfrac{d}{dt}\left[t^4-2t^3+t^2-t\right]$

$$=4t^3-6t^2+2t-1$$

$$\boxed{\therefore\ v=4t^3-6t^2+2t-1}$$

Acceleration $a=\dfrac{dv}{dt}=\dfrac{d}{dt}\left(4t^3-6t^2+2t-1\right)$

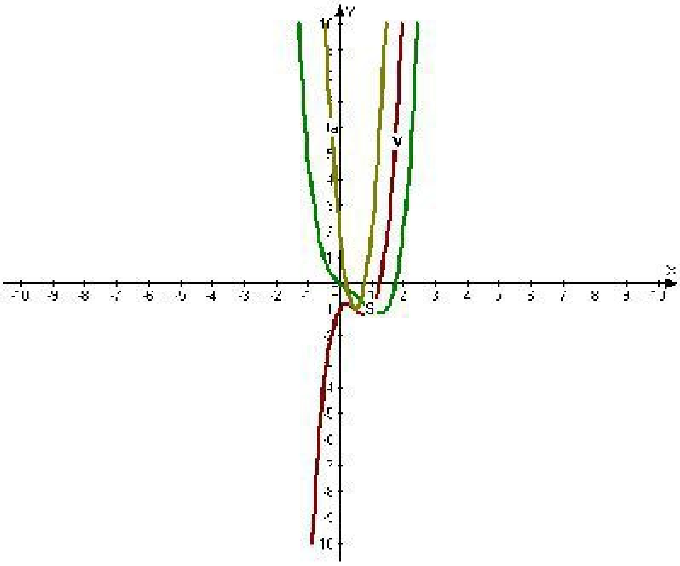
$$=12t^2-12t+2$$

(b) Acceleration after 1s is

$$\left(\frac{dv}{dt}\right)_{t=1}=12\left(1^2\right)-12(1)+2$$

$$=2\ \text{m/s}^2$$

(c)
The graph of S(green curve), v(velocity is red curve) and a(acceleration) are given below



Chapter 2 Derivatives Exercise 2.3 65E

Boyle's law states that when a sample of gas is compressed at a constant pressure, the pressure p of the gas is inversely proportional to the volume v of the gas.

$$\therefore \quad v \propto \frac{1}{p} \quad \Rightarrow \quad v = \frac{k}{p}$$

(a) Suppose that the pressure of a sample of air that occupies $0.106m^3$ at 25^0c is 50

$$\therefore \quad v = (0.106 \times 50)/p$$
$$= 5.3/p$$
$$\boxed{\therefore \ v = 5.3/p}$$

(b) $\dfrac{dv}{dp} = \dfrac{-5.3}{p^2}$

$$\therefore \left(\frac{dv}{dp}\right)_{p=50kpa} = -\frac{5.3}{(50)^2} = -0.00212$$

Derivative is the instantaneous rate of change of the volume with respect to the pressure at 25^0c.

$\boxed{\text{units are } m^3/kpa}$

Chapter 2 Derivatives Exercise 2.3 66E

Given

P	26	25	31	35	38	42	45
L	50	66	78	81	74	70	59

(a) Normal equations to find quadratic expression of the pressure are

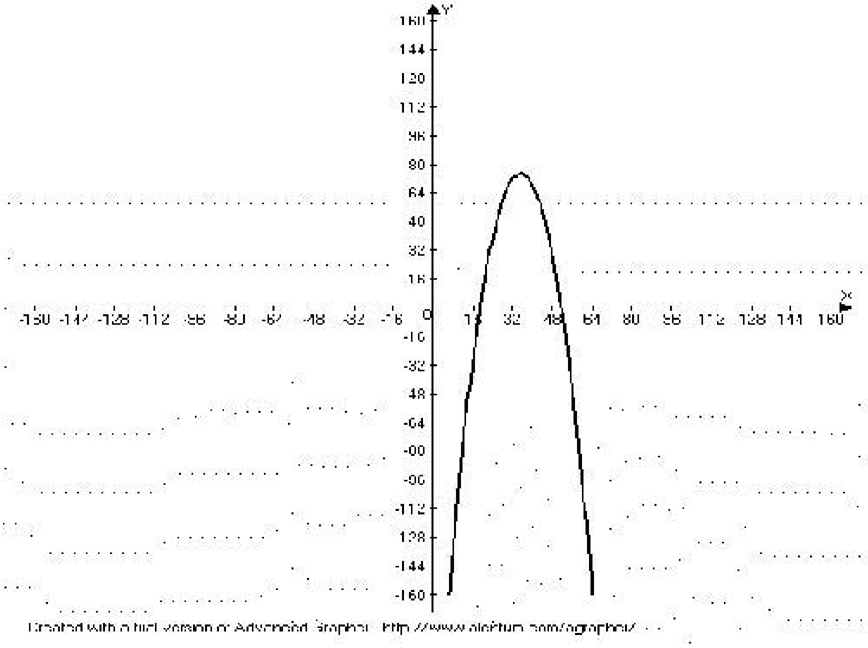
$$na + b\sum P + c\sum P^2 = \sum L$$
$$a\sum P + b\sum P^2 + c\sum P^3 = \sum PL$$
$$a\sum P^2 + b\sum P^3 + c\sum P^4 = \sum P^2 L$$

From the given data the equations be comes

$$7a + 245b + 8879c = 478$$
$$245a + 8879b + 332279c = 16808$$
$$8879a + 332279b + 12793235c = 609538$$

Bu solving these we get a polynomial of second degree as $L = -0.28P^2 + 19.75 - 273.55$

Graph of the quadratic expression given in above is shown in the below graph



(b)

$$\frac{dL}{dP} = -0.56P + 19.75$$

When $P = 30, \frac{dL}{dP} = 2.95$

When $P = 40, \frac{dL}{dP} = -2.65$

The meaning of the derivative is the rate of change in tire life with respect to the pressure.
The units are thousand of miles/lb/in^2

If the value of P increases the $\frac{dL}{dP}$ decreases.

Chapter 2 Derivatives Exercise 2.3 67E

Suppose that $f(5)=1, f'(5)=6, g(5)=-3,$ and $g'(5)=2$.

The product rule:

If *f* and *g* are both differentiable, then

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x)$$

The quotient rule:

If *f* and *g* are both differentiable, then

$$(f/g)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, g(x) \neq 0$$

(a)

By using the product rule, to find the value of $(fg)'(5)$:

$$\begin{aligned}(fg)'(5) &= f(5)g'(5) + g(5)f'(5) \\ &= 1 \cdot 2 + (-3) \cdot 6 \qquad \begin{pmatrix} f(5)=1, f'(5)=6 \\ g(5)=-3, g'(5)=2 \end{pmatrix} \\ &= 2 - 18 \\ &= -16\end{aligned}$$

Therefore $(fg)'(5) = \boxed{-16}$

(b)

By using the quotient rule, to find the value of $(f/g)'(5)$:

$$(f/g)'(5)=\frac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}$$

$$=\frac{(-3)\cdot 6-1\cdot 2}{(-3)^2} \qquad \begin{pmatrix} f(5)=1, f'(5)=6 \\ g(5)=-3,\ g'(5)=2 \end{pmatrix}$$

$$=\frac{-18-2}{9}$$

$$=-\frac{20}{9}$$

Therefore $(f/g)'(5)=\boxed{-\frac{20}{9}}$

(c)

By using the quotient rule, to find the value of $(g/f)'(5)$:

$$(g/f)'(5)=\frac{f(5)g'(5)-g(5)f'(5)}{[f(5)]^2}$$

$$=\frac{1\cdot 2-(-3)\cdot 6}{(1)^2} \qquad \begin{pmatrix} f(5)=1, f'(5)=6 \\ g(5)=-3,\ g'(5)=2 \end{pmatrix}$$

$$=\frac{2-(-18)}{1}$$

$$=20$$

Therefore $(g/f)'(5)=\boxed{20}$

Chapter 2 Derivatives Exercise 2.3 68E

Given $f(2)=-3, g(2), f'(2)=-2, g'(2)=7$

(a) $h(x)=5f(x)-4g(x)$

$\Rightarrow$ $h'(x)=5f'(x)-4g'(x)$

$\Rightarrow$ $h'(2)=5f'(2)-4g'(2)$

$=5(-2)-4(7)$

$=-10-28$

$=\boxed{-38}$

(b) $h(x)=f(x)g(x)$

$h'(x)=(f(x)g(x))'$

$=f(x)g'(x)+g(x)f'(x)$

$h'(2)=f(2)g'(2)+g(2)f'(2)$

$=(-3)(+7)+(4)(-2)$

$=-21-8$

$=\boxed{-29}$

(c) $h(x)=\dfrac{f(x)}{g(x)}$

$\Rightarrow$ $h'(x)=\left(\dfrac{f(x)}{g(x)}\right)'$

$$=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

$$\Rightarrow \quad h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$$

$$= \frac{(4)(-2) - (-3)(7)}{4^2}$$

$$= \frac{-8+21}{16}$$

$$= \boxed{\frac{13}{16}}$$

(d) $$h(x) = \frac{g(x)}{1+f(x)}$$

$$h'(x) = \left(\frac{g(x)}{1+f(x)}\right)'$$

$$= \frac{(1+f(x))g'(x) - g(x)(1+f(x))'}{(1+f(x))^2}$$

$$= \frac{(1+f(x))g'(x) - g(x)f'(x)}{(1+f(x))^2}$$

$$h'(2) = \frac{(1+f(2))g'(2) - g(2)f'(-2)}{(1+f(2))^2}$$

$$= \frac{(1-3)(7) - 4(-2)}{(1-3)^2}$$

$$= \frac{(-2)7+8}{4}$$

$$= \frac{-6}{4}$$

$$= \boxed{-\frac{3}{2}}$$

Chapter 2 Derivatives Exercise 2.3 69E

If f(x) = √(x) * g(x), where g(4) = 8 and g'(4) = 7, find f'(4).

f(x) = √(x) * g(x)

f '(x) = √(x)* g'(x) + g(x) * (1/2)x-1/2 By product rule

f '(4) = √(4)* g'(4) + g(4) * [1/ (2√(4))]

= 2 * 7 + 8 * (1/4)

= 16

Chapter 2 Derivatives Exercise 2.3 70E

Consider, $h(2) = 4$ and $h'(2) = -3$

Need to find the value of $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$.

Quotient rule:

If *f* and *g* are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

According to the quotient rule,

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)=\frac{x\frac{d}{dx}[h(x)]-h(x)\frac{d}{dx}[x]}{[x]^2}$$
$$=\frac{x[h'(x)]-h(x)[1]}{x^2}$$
$$=\frac{xh'(x)-h(x)}{x^2}$$

Thus,

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)=\boxed{\frac{xh'(x)-h(x)}{x^2}}$$

Now,

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}=\frac{x\cdot h'(x)-h(x)}{x^2}\bigg|_{x=2}$$

$=\frac{2\cdot h'(2)-h(2)}{2^2}$ Substitute 2 for x

$=\frac{2\cdot(-3)-4}{4}$ Since: $h(2)=4$ and $h'(2)=-3$

$=\frac{-6-4}{4}$ Multiply

$=\frac{-10}{4}$ Simplify the numerator

$=-\frac{5}{2}$ Divide

Therefore,

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}=\boxed{-\frac{5}{2}}$$

Chapter 2 Derivatives Exercise 2.3 71E

(A)

We have $u(x)=f(x).g(x)$

The product law $u'(x)=f'(x).g(x)+g'(x)f(x)$

Then $x=1$ $u'(1)=f'(1).g(1)+g'(1)f(1)$

Then by the graph we can estimate

$f(1)=2,\ g(1)=1$

$f'(1)=$ Slope of $f(x)$ at $x=1=\frac{|4|}{|2|}=2$

$g'(1)=$ Slope of $g(x)$ at $x=1=-\frac{|2|}{|2|}=-1$

Hence

$u'(1)=+1\times2+(-1)\times2$

$=2-2$

$\boxed{u'(1)=0}$

(B)

We have $V(x)=\frac{f(x)}{g(x)}$

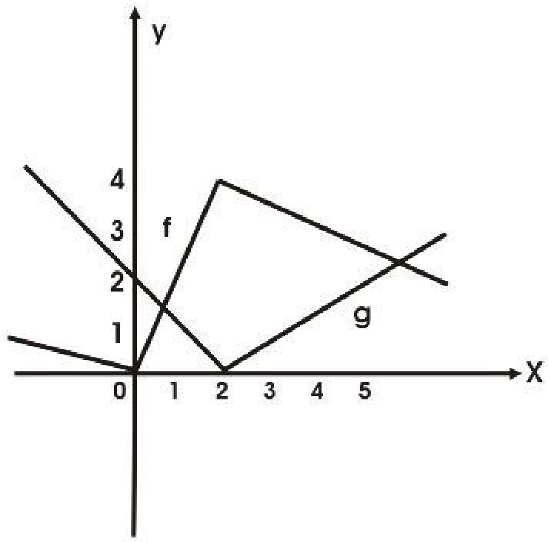
Then by Quotient rule we have $V'(x)=\frac{g(x)f'(x)-g'(x)f(x)}{(g(x))^2}$

Therefore $x=5$ $V'(5)=\frac{g(5)f'(5)-g'(5)f(5)}{(g(5))^2}$

By the graph $f(5)=3, g(5)=2, f'(5)\approx -\frac{1}{3}, g'(5)\approx \frac{2}{3}$

Then $V'(5)=\dfrac{2.\left(-\frac{1}{3}\right)-3.\frac{2}{3}}{(2)^2}=\dfrac{-\frac{2}{3}-\frac{6}{3}}{4}=\dfrac{-\frac{8}{3}}{4}$

$V'(5)=-\frac{8}{12}\Rightarrow \boxed{V'(5)=-\frac{2}{3}}$



Chapter 2 Derivatives Exercise 2.3 72E

(A)

We have $P(x)=F(x).G(x)$

Thus by the product rule we have

$P'(x)=F'(x).G(x)+G'(x).F(x)$

For $x=2$

$P'(2)=F'(2).G(2)+G'(2).F(2)$

By graph we can estimate

$F(2)=3, G(2)=2$

$F'(2)=$ Slope of $F(x)$ at $(x=2)\approx 0$ (here tangent is horizontal)

$G'(2)=$ Slope of $G(x)$ at $(x=2)\approx \frac{1}{2}$

Then $P'(2)=0\times 2+\frac{1}{2}\times 3=\frac{3}{2}$

$\boxed{P'(2)=\frac{3}{2}}$

(B) We have $Q(x)=\dfrac{F(x)}{G(x)}$

Then by the Quotient rule we have $Q'(x)=\dfrac{G(x)F'(x)-F(x)G'(x)}{(G(x))^2}$

For $x=7$ $\quad Q'(7)=\dfrac{G(7)F'(7)-F(7)G'(7)}{(G(7))^2}$

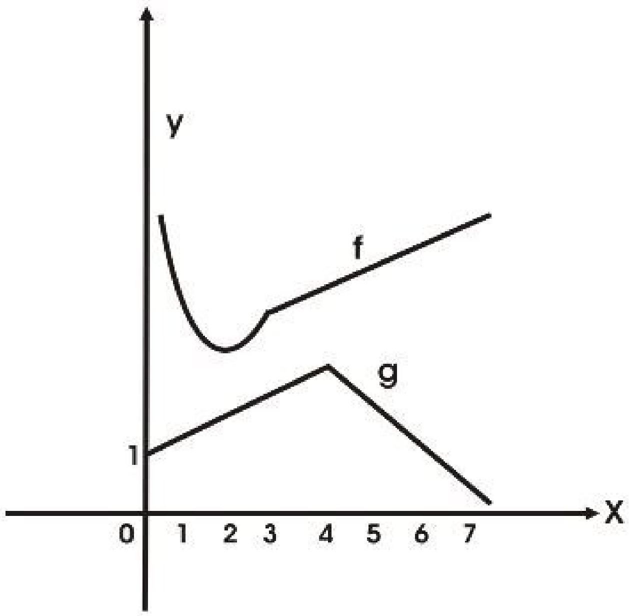
By the graph we can estimate $F(7)=5, G(7)=1$

And $F'(7)=$ Slope of $F(x)$ at $7\approx \frac{1}{4}$

$G'(7)=$ Slope of $G(x)$ at $7\approx -\frac{2}{3}$

Then $Q'(7)=\dfrac{1.\frac{1}{4}-(5).\left(-\frac{2}{3}\right)}{(1)^2}=\dfrac{\frac{1}{4}+\frac{10}{3}}{1}=\dfrac{43}{12}$

$\boxed{Q'(7)=\frac{43}{12}}$



Chapter 2 Derivatives Exercise 2.3 73E

(A)

$$y = xg(x)$$

$$\frac{dy}{dx} = (x)' g(x) + xg'(x) = g(x) + xg'(x)$$

(B)

$$y = \frac{x}{g(x)}$$

$$\frac{dy}{dx} = \frac{g(x) - xg'(x)}{g(x)^2}$$

(C)

$$y = \frac{g(x)}{x}$$

$$\frac{dy}{dx} = \frac{g'(x)x - g(x)}{x^2}$$

Chapter 2 Derivatives Exercise 2.3 74E

(A)

$$y = x^2 f(x)$$

$$\frac{dy}{dx} = 2xf(x) + x^2 f'(x)$$

(B)

$$y = \frac{f(x)}{x^2}$$

$$\frac{dy}{dx} = \frac{f'(x)x^2 - 2xf(x)}{x^4}$$

(C)

$$y = \frac{x^2}{f(x)}$$

$$\frac{dy}{dx} = \frac{2xf(x) - x^2 f'(x)}{f(x)^2}$$

(D)

$$y = \frac{1 + xf(x)}{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{[1 + xf(x)]'\sqrt{x} - [1 + xf(x)](\sqrt{x})'}{x}$$

$$= \frac{[f(x) + xf'(x)]\sqrt{x} - [1 + xf(x)]\frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{\frac{2x[f(x) + xf'(x)] - [1 + xf(x)]}{2\sqrt{x}}}{x}$$

$$= \frac{2x[f(x) + xf'(x)] - [1 + xf(x)]}{2x\sqrt{x}}$$

$$= \frac{2xf(x) + 2x^2 f'(x) - 1 - xf(x)}{2x\sqrt{x}}$$

$$= \frac{xf(x) + 2x^2 f'(x) - 1}{2x\sqrt{x}}$$

Chapter 2 Derivatives Exercise 2.3 75E

Consider the equation of the curve,

$$y = 2x^3 + 3x^2 - 12x + 1 \quad \ldots\ldots (1)$$

The derivative of y at a point $x = a$ gives the slope of the tangent line to the graph of y at the point (a, b).

Compute the derivative of given curve as follows:

$$\begin{aligned} y' &= 2\frac{d(x^3)}{dx} + 3\frac{d(x^2)}{dx} - 12\frac{d(x)}{dx} + \frac{d(1)}{dx} \\ &= 2\times[3\times x^{3-1}] + 3\times[2\times x^{2-1}] - 12\times[1\times x^{1-1}] + 0 \\ &= 6x^2 + 6x - 12 \\ &= 6(x^2 + x - 2) \end{aligned}$$

For the points, where the tangent to the given curve is horizontal, the slope will be equal to zero. Assume that at point (x_1, y_1) the tangent to the curve is horizontal. Thus,

$$\begin{aligned} (y')_{(x_1, y_1)} &= 0 \\ 6(x_1^2 + x_1 - 2) &= 0 \\ x_1^2 + x_1 - 2 &= 0 \\ x_1^2 + 2x_1 - x_1 - 2 &= 0 \end{aligned}$$

Simplify further,

$$\begin{aligned} x_1(x_1 + 2) - 1(x_1 + 2) &= 0 \\ (x_1 + 2)(x_1 - 1) &= 0 \\ x_1 + 2 = 0 &\Rightarrow x_1 = -2 \\ x_1 - 1 = 0 &\Rightarrow x_1 = 1 \end{aligned}$$

Substitute $x_1 = -2$ in the equation of the curve,

$$\begin{aligned} y_1 &= 2(-2)^3 + 3(-2)^2 - 12(-2) + 1 \\ &= -16 + 12 + 24 + 1 \\ &= 21 \end{aligned}$$

Substitute $x_1 = 1$ in the equation of the curve,

$$\begin{aligned} y_1 &= 2(1)^3 + 3(1)^2 - 12(1) + 1 \\ &= 2 + 3 - 12 + 1 \\ &= -6 \end{aligned}$$

Therefore, at points $\boxed{(-2, 21) \text{ and } (1, -6)}$ the tangent to the curve is horizontal.

Chapter 2 Derivatives Exercise 2.3 76E

The derivative of a function $f(x)$ at any point on the curve gives the slope the tangent at that point and a horizontal tangent has the slope zero.

This follows that the tangents of the function $f(x)$ are horizontal at the points where the derivative of the function $f(x)$ is zero.

So make the derivative of the function $f(x)=x^3+3x^2+x+3$ equal to zero and solve it for x in order to find the points at which the function has horizontal tangents.

Firstly find the derivative of the function.

$$\begin{aligned}\frac{d}{dx}f(x)&=\frac{d}{dx}\left(x^3+3x^2+x+3\right)\\&=\frac{d}{dx}\left(x^3\right)+\frac{d}{dx}\left(3x^2\right)+\frac{d}{dx}(x)+\frac{d}{dx}(3)\\&=3x^2+6x+1+0\\&=3x^2+6x+1\end{aligned}$$

Hence, the derivative of the function $f(x)$ is $3x^2+6x+1$.

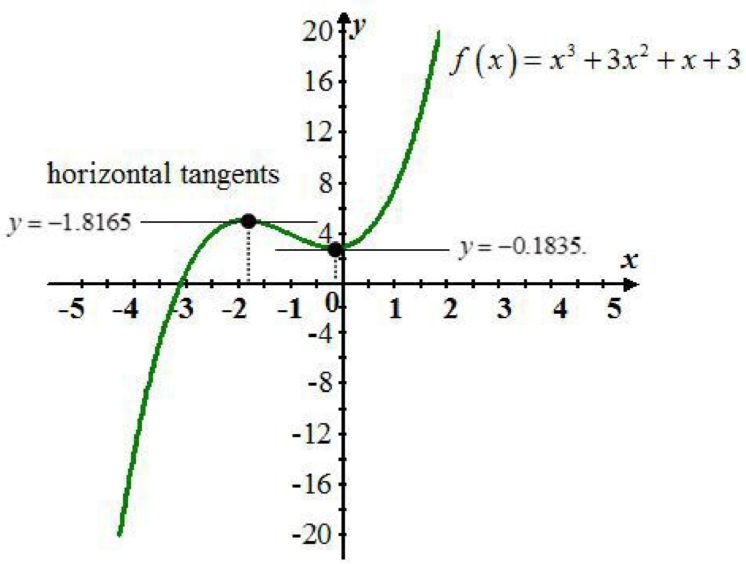
Make the derivative equal to zero and solve it for x.

$$\begin{aligned}f'(x)&=0\\3x^2+6x+1&=0\\3(x+1)^2-2&=0\\3(x+1)^2&=2\\(x+1)^2&=\frac{2}{3}\\x+1&=\pm\sqrt{\frac{2}{3}}\\x&=\pm\sqrt{\frac{2}{3}}-1\\x&=-1-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}-1\\&\approx-1.8165,-0.1835\end{aligned}$$

Hence, the required points are approximately $\boxed{-1.8165}$ and $\boxed{-0.1835}$.

Therefore, the function will have the horizontal tangents at the points $\boxed{-1.8165}$ and $\boxed{-0.1835}$.

The following diagram shows the graph of the function $f(x)=x^3+3x^2+x+3$ and horizontal tangents $y=-1.8165$, $y=-0.1835$ of the function at the points $x=-1.8165$ and $x=-0.1835$.



Chapter 2 Derivatives Exercise 2.3 77E

Curve $y = 6x^3 + 5x - 3$

The slope of the tangent $= \frac{dy}{dx}$

Let slope of the tangent is 4 then we have

$$\frac{dy}{dx} = 4$$

$$\frac{dy}{dx}\left(6x^3 + 5x - 3\right) = 4$$

$$\Rightarrow 6\frac{d}{dx}x^3 + 5\frac{d}{dx}x - \frac{d}{dx}.3 = 4 \qquad \left[\frac{d}{dx}(f+g+h) = f' + g' + h'\right]$$

$$\Rightarrow 18x^2 + 5 - 0 = 4$$

$$\Rightarrow 18x^2 + 5 = 4$$

$$\Rightarrow 18x^2 = 4 - 5$$

$$\Rightarrow 18x^2 = -1$$

$$\Rightarrow x^2 = \frac{-1}{18}$$

$$\Rightarrow x = \sqrt{\frac{-1}{18}}$$

The value of x is not real, so there is no point on the curve where the tangent has slope 4.

Hence, curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

Chapter 2 Derivatives Exercise 2.3 78E

Consider the curve

$$y = x\sqrt{x}$$

Then rewrite the curve $y = x\sqrt{x}$ is

$$y = x\sqrt{x}$$

$$= x\,x^{\frac{1}{2}}$$

$$= x^{1+\frac{1}{2}}$$

$$= x^{\frac{3}{2}}$$

Use the formula $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{3}{2}}\right)$$

$$= \frac{3}{2}x^{\frac{3}{2}-1}$$

$$= \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{3}{2}\sqrt{x}$$

The tangent line is parallel to $y = 1 + 3x$

The slope of the tangent line is same as the slope of this line that is 3.

$$\frac{dy}{dx} = 3$$

$$\frac{3}{2}\sqrt{x} = 3$$

$$\sqrt{x} = 2$$

$$x = 4$$

Substitute 4 for x in the equation $y = x\sqrt{x}$:

$$\begin{aligned} y &= x\sqrt{x} \\ &= 4\sqrt{4} \\ &= 4\cdot 2 \\ &= 8 \end{aligned}$$

Therefore, $(x,y) = (4,8)$

Slope-point form: the equation of the line with slope *m* and point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to $y = 3x+1$ is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= 3(x-4) \\ y - 8 &= 3x - 12 \\ 3x - y - 4 &= 0 \end{aligned}$$

Thus, the equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to $y = 3x+1$ is

$\boxed{3x - y - 4 = 0}$.

Chapter 2 Derivatives Exercise 2.3 79E

Consider the function $y = 1 + x^3$.

Differentiate the given function with respect to x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(1 + x^3\right).$$

The Sum Rule states that, if f and g are both differentiable, then

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right).$$

By using the Sum Rule, we obtain,

$$\frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}\left(x^3\right).$$

The Power Rule states that, If n is real number, then

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}.$$

By using the Power Rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= 0 + 3x^{3-1} \\ &= 3x^2. \end{aligned}$$

The tangent line to $y = f(x)$ at any point has slope $\frac{dy}{dx}$.

Hence the slope of the tangent to the curve $y = 1 + x^3$ is given by

$$\begin{aligned} m &= \frac{dy}{dx} \\ &= 3x^2. \end{aligned}$$

The slope of the line $12x - y = 1$ is given by differentiation with respect x as follows.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(12x - 1) \\ &= \frac{d}{dx}(12x) - \frac{d}{dx}(1) \quad \left[\because \frac{d}{dx}(f-g) = \frac{df}{dx} - \frac{dg}{dx}\right] \\ &= 12\frac{d}{dx}(x) - \frac{d}{dx}(1) \quad \left[\because \frac{d}{dx}(cf) = c\frac{df}{dx}\right] \\ &= 12(1) - 0 \\ &= 12 \end{aligned}$$

If the tangent line to $y=1+x^3$ is parallel to the line $12x-y=1$, then the slopes of these two lines must be equal.

Therefore,

$$m=3x^2=12$$
$$x^2=4$$
$$x=\pm 2$$

Now if $x=2$ then

$$y=1+x^3$$
$$=1+2^3$$
$$=9$$

If $x=-2$ then

$$y=1+x^3$$
$$=1+(-2)^3$$
$$=-7$$

Therefore, the tangent line touches the curve $y=1+x^3$ at the points $(2,9)$ and $(-2,-7)$.

Use the point-slope form of the equation of a line passing through (a,b) with slope m

given as $y-b=m(x-a)$.

Therefore the equation of tangent line at $(2,9)$ with the slope $m=12$ is

$$y-9=12(x-2)$$
$$y-9=12x-24$$
$$\boxed{y=12x-15}.$$

And the equation of tangent line at $(-2,-7)$ with the slope $m=12$ is

$$y+7=12(x+2)$$
$$y+7=12x+24$$
$$\boxed{y=12x+17}.$$

Chapter 2 Derivatives Exercise 2.3 80E

$$x-2y=2$$
$$2y=x-2$$
$$y=\frac{1}{2}x-1$$

The slope of the given line $x-2y=2$ is $\frac{1}{2}$

For the tangent to be parallel to this line, its slope must be $\frac{1}{2}$

i.e. $\frac{dy}{dx}=\frac{1}{2}$

Now

$$y=\frac{x-1}{x+1}$$
$$\frac{dy}{dx}=\frac{(x+1)-(x-1)}{(x+1)^2}$$
$$=\frac{2}{(x+1)^2}$$

Now

$$\frac{2}{(x+1)^2}=\frac{1}{2}$$

$$\Rightarrow \frac{1}{(x+1)^2}=\frac{1}{4}$$

$$\Rightarrow x+1=\pm 2$$

$$\Rightarrow x=\pm 2-1$$

$$\Rightarrow x=-3 \text{ or } x=1$$

If $x=1, y=0$

If $x=-3, y=\frac{-4}{-2}=2$

We use the point-slope form to write an equation of the tangent line at $(1,0)$

Here the required equations is

$$y-y_1=m(x-x_1)$$

$$\Rightarrow y-0=\frac{1}{2}(x-1)\Rightarrow \boxed{2y-x+1=0}$$

And

We use the point-slope form to write an equation of the tangent line at $(-3,2)$

Here the required equations is

$$y-y_1=m(x-x_1)$$

$$\Rightarrow y-2=\frac{1}{2}(x+3)\Rightarrow \boxed{2y-x-7=0}$$

Chapter 2 Derivatives Exercise 2.3 81E

Consider the parabola $y=x^2-5x+4$ that is parallel to the line $x-3y=5$

The normal line needs to have the same slope as the line $x-3y=5$ because they are meant to be parallel to each other. Find the slope of the given line by rewriting the equation in slope-intercept form $y=mx+b$:

$$x-3y=5$$
$$-3y=-x+5$$
$$y=\frac{-x+5}{-3}$$
$$y=\frac{1}{3}x-\frac{5}{3}$$

The slope of the line $x-3y=5$ is $\frac{1}{3}$ and hence the slope of the normal line must be $\frac{1}{3}$.

The normal line to a curve is the line that is perpendicular to the tangent line when both share a point with the curve. The slopes of perpendicular lines are negative reciprocals of each other.

Therefore, since the slope of the normal line is $\frac{1}{3}$, the slope of the tangent line must be -3

By finding the tangent line with slope -3, also find the point shared by the curve, the tangent, and the normal lines.

Therefore, find the derivative of the function $y = x^2 - 5x + 4$ and then set it equal to -3 solve for the *x* values where the tangent line has slope -3:

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 5x + 4)$$

$$y' = \frac{d}{dx}(x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(4) \quad \left(\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)\right)$$

$$= \frac{d}{dx}(x^2) - 5\frac{d}{dx}(x) + \frac{d}{dx}(4) \quad \left(\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)\right)$$

$$= 2x - 5 \quad \left(\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(c) = 0 \text{ where } c \text{ is a constant}\right)$$

Set the derivative equal to -3 solve for the *x* values where the tangent line has slope -3.

$$f'(x) = 0$$

$$2x - 5 = -3 \quad (\text{Add 5 to both sides})$$

$$2x = 2$$

$$x = 1 \quad (\text{Divide both sides by 2})$$

Solving for the *y* value on the curve,

$$y = x^2 - 5x + 4$$

$$= (1)^2 - 5(1) + 4 \quad (\text{Replace } x \text{ with 1})$$

$$= 1 - 5 + 4$$

$$= 0$$

Thus, the point $(1,0)$ on the curve $y = x^2 - 5x + 4$ has a tangent line with a slope of -3

Hence the normal line to $(1,0)$ have a slope of $\frac{1}{3}$.

The equation of a line with slope *m* and which goes through the point (x_1, y_1) in

Point-slope form is: $y - y_1 = m(x - x_1)$

Therefore, the equation of the normal line with slope $\frac{1}{3}$ and through the point $(1,0)$ is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{3}(x - 1)$$

$$y = \frac{1}{3}(x - 1)$$

$$y = \frac{1}{3}x - \frac{1}{3}$$

Hence, the equation of normal line to the curve $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$ is $\boxed{y = \frac{1}{3}x - \frac{1}{3}}$.

Chapter 2 Derivatives Exercise 2.3 82E

The Given equation of the parabola $y = x - x^2$

First we get the equation with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x - x^2\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}\left(-x^2\right)$$

$$\boxed{\frac{dy}{dx} = 1 - 2x = \text{slope of the tangent}}$$

Then the slope of the normal line is $= \frac{-1}{dy/dx} = \frac{-1}{1-2x} = \frac{1}{2x-1}$

At (1, 0), the slope of the normal line is $= \frac{1}{2-1} = 1$

Then equation of the normal is

$$(y - 0) = 1.(x - 1)$$

$$\Rightarrow \boxed{y = x - 1}$$

Put this value of y in the equation of parabola

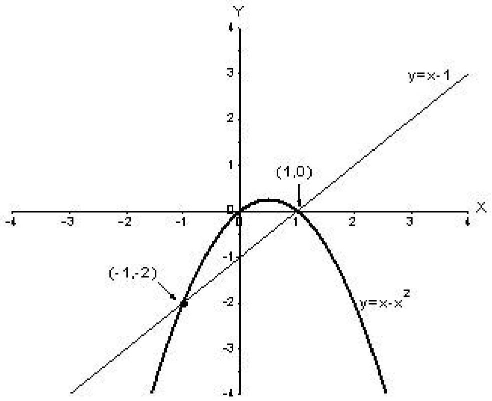
$y = x - x^2 \Rightarrow x - 1 = x - x^2$

$\Rightarrow -1 = -x^2$

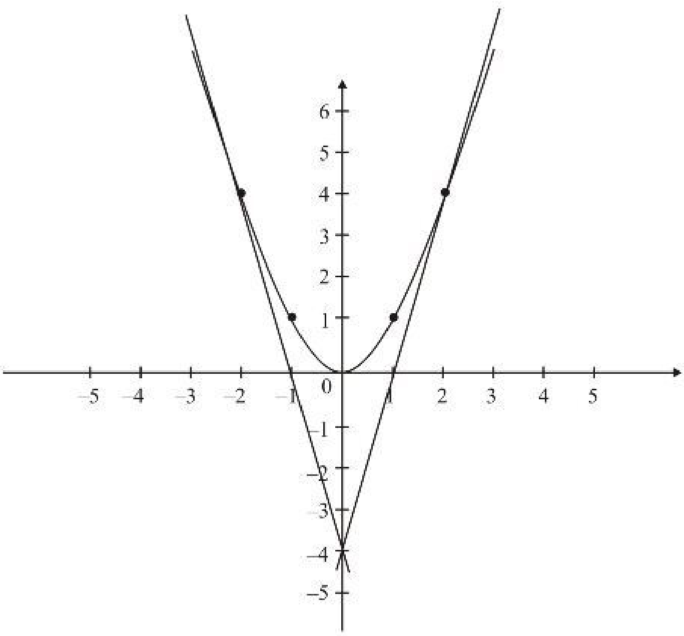
$\Rightarrow x = \sqrt{1} \Rightarrow \boxed{x \pm 1}$

For x = - 1

Y = -1-1 = -2 Hence another point where normal intersect the parabola is (-1, -2)



Chapter 2 Derivatives Exercise 2.3 83E



From the diagram it is clear that the two tangents are passing through $(0, -4)$

And the point if intersection become $(\pm 2, 4)$

Since the parabola and the tangents intersect at $(2,4), (-2,4)$

Chapter 2 Derivatives Exercise 2.3 84E

(a) Let us suppose that a line is tangent to the parabola $y = x + x^2$, touches at the point (a, b) so this point will satisfy the equation of parabola $y = x + x^2$

$$\Rightarrow b = a + a^2$$

Thus we have the point $(a, a + a^2)$

Now slope of the tangent at a

$$y' = \frac{d}{dx}(x + x^2)$$
$$y' = 1 + 2x$$
$$\Rightarrow \boxed{y' = 1 + 2a} \qquad (x = a)$$

Thus the equation of tangent line is

$$(y - (a + a^2)) = (1 + 2a)(x - a)$$

But the tangent goes though (2, -3) then

$$-3 - (a + a^2) = (1 + 2a)(2 - a)$$
$$\Rightarrow -3 - a - a^2 = 2 + 4a - a - 2a^2$$
$$\Rightarrow 2a^2 - a^2 - 3a - a - 3 - 2 = 0$$

$$\Rightarrow a^2 - 4a - 5 = 0$$
$$\Rightarrow a^2 - 5a + a - 5 = 0$$
$$\Rightarrow a(a - 5) + 1(a - 5) = 0$$
$$\Rightarrow (a - 5)(a + 1) = 0$$

Thus we have $\boxed{a = 5, -1}$

Thus we have two points where the tangent touch the parabola

These points are $(5, 5 + 25)\, and\, (-1, -1 + 1)$

Or $(5, 30)\, and\, (-1, 0)$

So equation of tangent at (5, 30) is

$$(y - 30) = y'(x - 5)$$
$$\Rightarrow (y - 30) = (1 + 2 \times 5)(x - 5)$$
$$\Rightarrow y - 30 = 11(x - 5)$$
$$\Rightarrow y - 30 = 11x - 55$$
$$\Rightarrow \boxed{y = 11x - 25}$$

And equation of tangent at (-1, 0) is

$$y - 0 = y'(x - (-1))$$
$$y = (1 + 2(-1))(x + 1)$$
$$\Rightarrow y = (1 - 2)(x + 1)$$
$$\Rightarrow y = -1(x + 1)$$
$$\Rightarrow y = -x - 1$$
$$\Rightarrow \boxed{y + x + 1 = 0}$$

(b) From part (a), the slope of the tangent line to the curve $y = x + x^2$ at the point $(a, a+a^2)$ is $y' = 1+2a$

Then the equation of the tangent line is $(y-(a+a^2)) = (1+2a)(x-a)$

Suppose, the tangent line passes through the point (2, 7), then

$$(7-(a+a^2)) = (1+2a)(2-a)$$

$$\Rightarrow \quad (7-a-a^2) = (2+4a-a-2a^2)$$

$$\Rightarrow \quad 2a^2-a^2-3a-a+7-2=0$$

$$\Rightarrow \quad a^2-4a+5=0$$

$$\Rightarrow \quad a = \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$$

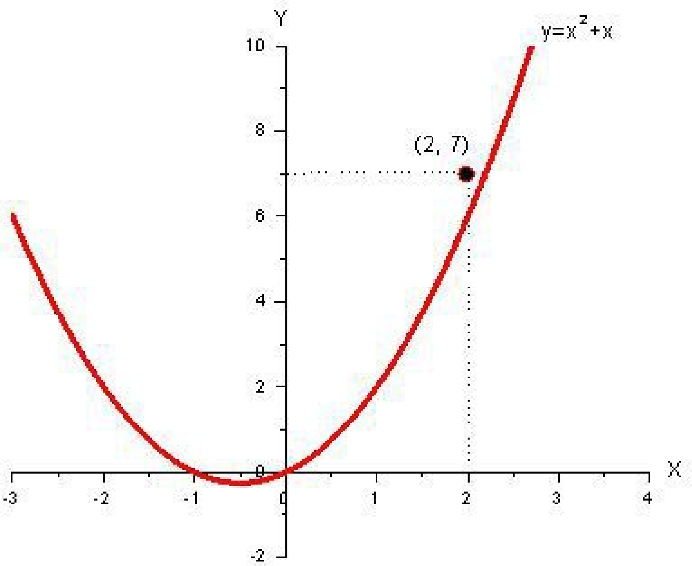
$$\Rightarrow \quad a = \frac{4\pm\sqrt{16-20}}{2}$$

$$\Rightarrow \quad a = \frac{4\pm\sqrt{-4}}{2}$$

Thus for the point (2, 7), 'a' does not have any real value

So no tangent line passes through the point (2, 7)

Now we graph the function and see that in the first quadrant, all tangents must be on right side of the curve (parabola) and the point (2, 7) lies inside the curvature, so no tangent can pass through the point (2, 7)



Chapter 2 Derivatives Exercise 2.3 85E

a)

We have: $f \cdot g \cdot h$ and the fact that f , g and h are differentiable.

We can rewrite as:

$$(f \cdot g \cdot h) = (f \cdot g) \cdot h$$

Using the Product Rule we have:

$$(f \cdot g \cdot h)' = (f \cdot g) \cdot h' + h \cdot (f \cdot g)'$$

Using the Product Rule:

$$(f \cdot g \cdot h)' = (f \cdot g) \cdot h' + h \cdot (f \cdot g' + g \cdot f')$$

Then:

$(f \cdot g \cdot h)' =$ f g h'+h f g' +h g f '

We can rewrite as:

$(f \cdot g \cdot h)' =$ f 'g h +f g' h +f g h'

b) Taking f =g=h and substituting in $(f\cdot g\cdot h)' = f'gh + fg'h + fgh'$:

$$(f\cdot f\cdot f)' = [f'\cdot f\cdot f] + [f\cdot f'\cdot f] + [f\cdot f\cdot f']$$

Factorize f ':

$$(f\cdot f\cdot f)' = [(f\cdot f) + (f\cdot f) + (f\cdot f)]\cdot f'$$

$$(f\cdot f\cdot f)' = [3\cdot f\cdot f]\cdot f'$$
$$(f\cdot f\cdot f)' = \left[3f^2\right]\cdot f'$$
$$\left(f^3\right)' = \left[3f^2\right]\cdot f'$$

and therefore:

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2\cdot f'\left(x\right)$$

c)

Let $[f(x)]^3 = \left(x^4+3x^3+17x+82\right)^3$

$$f\left(x\right) = x^4+3x^3+17x+82$$
$$f'\left(x\right) = 4x^3+9x^2+17$$

Substituting in $\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2\cdot f'\left(x\right)$:

$$\frac{d}{dx}\left[f(x)\right]^3 = 3\left(x^4+3x^3+17x+82\right)^2\cdot\left(4x^3+9x^2+17\right)$$
$$y' = 3\left(x^4+3x^3+17x+82\right)^2\cdot\left(4x^3+9x^2+17\right)$$

Chapter 2 Derivatives Exercise 2.3 86E

The notation for the *n*th derivative of a function *f* with respect to the variable *x* is mathematically represented as follows:

$$\frac{d^n f}{dx^n}$$

Find the first several derivatives of the function and then make an educated guess as to the *n*th derivative by recognizing a pattern in the derivatives.

(a)

Each successive derivative is the first derivative of the previous result.

The first several derivatives of $f\left(x\right)=x^n$ are as follows:

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d^2}{dx^2}\left(x^n\right)=\frac{d}{dx}\left(nx^{n-1}\right)$$
$$=n\frac{d}{dx}\left(x^{n-1}\right)$$ By using the power rule: $\frac{d}{dx}\left(x^n\right)=nx^{n-1}, n\in\mathbb{R}$
$$=n\left(n-1\right)x^{n-2}$$

$$\frac{d^3}{dx^3}\left(x^n\right)=\frac{d}{dx}\left(n\left(n-1\right)x^{n-2}\right)$$
$$=n\left(n-1\right)\frac{d}{dx}\left(x^{n-2}\right)$$ By again using the power rule
$$=n\left(n-1\right)\left(n-2\right)x^{n-3}$$

Follow this same pattern until the *n*th derivative, to guess the following:

$$\frac{d^n}{dx^n}(x^n) = n(n-1)(n-2)\cdots(n-(n-2))(n-(n-1))x^{n-n}$$
$$= n(n-1)(n-2)\cdots(2)(1)x^0$$
$$= n(n-1)(n-2)\cdots(2)(1)$$
$$= 1\cdot 2\cdot 3\cdot \ldots \cdot n$$

The resulting expression is called the *n* factorial and is expressed with the notation $n!$.
Therefore, it is concluded as follows:

$$\frac{d^n}{dx^n}(x^n) = \boxed{n!}.$$

(b)

The first several derivatives of $f(x) = 1/x$ are as follows:

$$\frac{d}{dx}(1/x) = \frac{d}{dx}(x^{-1})$$
$$= -x^{-2} \quad \text{By using the power rule: } \frac{d}{dx}(x^n) = nx^{n-1}, n \in \mathbb{R}$$
$$= -1/x^2$$

$$\frac{d^2}{dx^2}(1/x) = \frac{d}{dx}\left[\frac{d}{dx}(1/x)\right]$$
$$= \frac{d}{dx}(-x^{-2}) \quad \text{By again using the power rule}$$
$$= -1(-2)x^{-3}$$
$$= 2/x^3$$

$$\frac{d^3}{dx^3}(1/x) = \frac{d}{dx}\left[\frac{d^2}{dx^2}(1/x)\right]$$
$$= \frac{d}{dx}(2x^{-3}) \quad \text{By again using the power rule}$$
$$= 2(-3)x^{-4}$$
$$= -6/x^4$$

The pattern that is recognized involves a few characteristics: the signs alternate, the coefficient is the product *n* factorial, and the exponent is 1 less than negative *n*. By following this same pattern until the *n*th derivative, this implies the following:

$$\frac{d^n}{dx^n}(1/x) = (-1)^n n(n-1)(n-2)\cdots(n-(n-2))(n-(n-1))x^{-n-1}$$
$$= (-1)^n n(n-1)(n-2)\cdots(2)(1)x^{-n-1}$$
$$= (-1)^n n!x^{-(n+1)}$$
$$= \boxed{(-1)^n n!/x^{n+1}}$$

The alternating coefficient factor $(-1)^n$ ensures that when *n* is odd, the coefficient will be negative; hence every odd derivative is negative and every even derivative is positive.

Chapter 2 Derivatives Exercise 2.3 87E

To find a second degree polynomial P such that $P(2) = 5, P'(2) = 3,$ and $P''(2) = 2$;

Suppose that the second degree polynomial is,

$$P(x) = a_0 + a_1x + a_2x^2$$

Since $P(2) = 5$, then

$$P(2) = a_0 + a_1(2) + a_2(2)^2$$

This implies that,

$$5 = a_0 + 2a_1 + 4a_2$$

This implies that,

$$a_0 + 2a_1 + 4a_2 = 5$$

Differentiate $P(x)$ with respect to x, to get

$$P'(x) = a_1 + 2a_2 x$$

Since $P'(2) = 3$, then

$$P'(2) = a_1 + 2a_2(2)$$

This implies that,

$$3 = a_1 + 4a_2$$

This implies that,

$$a_1 + 4a_2 = 3$$

Differentiate $P'(x)$ with respect to x, to get

$$P''(x) = 2a_2$$

Since $P''(2) = 2$, then

$$P''(2) = 2a_2$$

This implies that,

$$2 = 2a_2$$

This implies that,

$$a_2 = 1$$

Substitute $a_2 = 1$ into $a_1 + 4a_2 = 3$, to get

$$\begin{aligned} a_1 + 4a_2 &= 3 \\ a_1 + 4(1) &= 3 \\ a_1 &= 3 - 4 \end{aligned}$$

This implies that,

$$a_1 = -1$$

Substitute $a_1 = -1$ and $a_2 = 1$ into $a_0 + 2a_1 + 4a_2 = 5$, to get

$$\begin{aligned} a_0 + 2(-1) + 4(1) &= 5 \\ a_0 - 2 + 4 &= 5 \\ a_0 + 2 &= 5 \\ a_0 &= 5 - 2 \end{aligned}$$

This implies that,

$$a_0 = 3$$

Substitute $a_0 = 3$, $a_1 = -1$, and $a_2 = 1$ into $P(x)$, to get

$$\begin{aligned} P(x) &= 3 + (-1)x + (1)^2 x^2 \\ &= 3 - x + x^2 \\ &= x^2 - x + 3 \end{aligned}$$

Hence, a second degree polynomial $P(x)$ is $\boxed{x^2 - x + 3}$.

Chapter 2 Derivatives Exercise 2.3 88E

Given differential equation $y'' + y' - 2y = \sin x$ ....(1)

Given function $y = A\sin x + B\cos x$

Differentiating y with respect to x

$$\begin{aligned} y' &= \frac{dy}{dx} \\ &= \frac{d}{dx}(A\sin x + B\cos x) \\ &= A\frac{d}{dx}\sin x + B\frac{d}{dx}\cos x \\ &= A\cos x + B(-\sin x) \\ y' &= A\cos x - B\sin x \end{aligned}$$

Again differentiating y' with respect to x gives

$$y'' = \frac{dy'}{dx}$$
$$= \frac{d}{dx}(A\cos x - B\sin x)$$
$$= A\frac{d}{dx}\cos x - B\frac{d}{dx}\sin x$$
$$= -A\sin x - B\cos x$$
$$y'' = -A\sin x - B\cos x$$

Since $\frac{d}{dx}\cos x = -\sin x$ and $\frac{d}{dx}\sin x = \cos x$

Substituting y', y'', y values in (1) gives

$$(-A\sin x - B\cos x) + (A\cos x - B\sin x) - 2(A\sin x + B\cos x) = \sin x$$
$$-A\sin x - B\cos x + A\cos x - B\sin x - 2A\sin x - 2B\cos x = \sin x$$
$$\sin x(-A - 2A - B) + \cos x(-B + A - 2B) = \sin x$$
$$\sin x(-3A - B) + \cos x(A - 3B) = \sin x$$

Since by taking $\sin x$ and $\cos x$ as common factors

Now comparing $\sin x$, $\cos x$ coefficients both sides gives

$$-3A - B = 1 \quad ....(2)$$
$$A - 3B = 0 \quad ....(3)$$

From (3) $A = 3B$

Substituting A value in (2)

$$-3(3B) - B = 1$$
$$-9B - B = 1$$
$$-10B = 1$$
$$B = \frac{-1}{10}$$

Substituting B values in A gives

$$A = 3\times\frac{-1}{10}$$
$$= -\frac{3}{10}$$
$$\therefore \quad A = -\frac{3}{10} \text{ and } B = -\frac{1}{10}$$

Chapter 2 Derivatives Exercise 2.3 89E

The slope of the given curve is given by its derivative so

$$m = \frac{dy}{dx}$$
$$= \frac{d}{dx}(ax^3 + bx^2 + cx + d)$$
$$= a\frac{d}{dx}x^3 + b\frac{d}{dx}x^2 + c\frac{d}{dx}x + \frac{d}{dx}(d)$$
$$= a\cdot 3x^2 + b\cdot 2x + c\cdot 1 + 0$$
$$= 3ax^2 + 2bx + c$$

Since by derivative rules and derivative principle

The curve has horizontal tangents at $(-2,6)$ and $(2,0)$

So at these points the slope of the curve is "0"

$$3a(-2)^2 + 2b(-2) + c = 0$$
$$\Rightarrow \quad 12a - 4b + c = 0 \quad ....(1)$$

When at $(2,0)$ it becomes

$$3a(2)^2 + 2b(2) + c = 0$$
$$\Rightarrow \quad 12a + 4b + c = 0 \quad ....(2)$$

Subtracting (1) and (2) gives

$$-8b = 0 \quad \Rightarrow \quad b = 0$$

Substitute $b = 0$ in (1) and (2) gives

$$12a + c = 0$$
$$\Rightarrow \quad c = -12a \quad ....(3)$$

The curve passing through $(-2,6)$ since it has horizontal tangent at that point

So $6=a(-2)^3+6(-2)^2+c(-2)+d$

Substituting $b=0$ gives

$6=-8a-2c+d \qquad ....(4)$

Substitute $(2,0)$ and $b=0$ with curve gives

$0=a(2)^3+c(2)+d$

$\Rightarrow 8a+2c+d=0 \qquad ....(5)$

Adding (4) and (5) gives

$2d=6$

$d=3$

Now substitute $d=3$ and $c=-12a$ in (5) gives

$8a+2(-12a)+3=0$

$8a-24a+3=0$

$-16a+3=0$

$16a=3$

$a=\frac{3}{16}$

Now $c=-12a$

$=-12\times\frac{3}{16}$

$=\frac{-9}{4}$

$\therefore$ Equation of the cubic function

$y=\frac{3}{16}x^3-\frac{9}{4}x+3$

Chapter 2 Derivatives Exercise 2.3 90E

The slope of the given parabola is given by its derivative so

$m=\frac{dy}{dx}$

$=\frac{d}{dx}(ax^2+bx+c)$

$=a\frac{d}{dx}x^2+b\frac{d}{dx}x+\frac{d}{dx}(c)$

$=a\cdot 2x+b\cdot 1+0$

$m=2ax+b$ Since by derivative rules

Given that the slope is 4 when $x=1$

$2a\cdot 1+b=4$

$\Rightarrow 2a+b=4 \qquad ....(1)$

Again slope is -8 at $x=-1$

$2a\times -1+b=-8$

$-2a+b=-8 \qquad ....(2)$

Adding (1) and (2) gives

$2a+b-2a+b=4-8$

$2b=-4$

$b=-2$

Substituting b value in (1) gives

$2a-2=4$

$2a=4+2$

$2a=6$

$a=3$

Given that the curve is passing through $(2,15)$ so

$15=a\cdot 2^2+b\cdot 2+c$

$15=4a+2b+c$

Substituting $a=3$, $b=-2$ in the above gives

$15=4\times 3+2\times -2+c$

$15=12-4+c$

$15=8+c$

$c=15-8$

$c=7$

$\therefore$ The equation of parabola is $y=3x^2-2x+7$

Chapter 2 Derivatives Exercise 2.3 91E

Let
Total personal income = TPI = x
Average annual income = AAI = y
Total population at time t = z
Then $x = y.z$

$$\Rightarrow \frac{dx}{dt} = y\frac{dz}{dt} + z\frac{dy}{dt}$$

$$\Rightarrow x'(t) = yz'(t) + zy'(t) \text{ (Rate of change in total personal income)}$$

Here $z'(t) =$ Rate of increase in population at time t
$x'(t) =$ Rate of increase in total personal income at time t
And $y'(t) =$ Rate of increase in average annual income at time t
We have been given that

$$z(1999) = 961{,}400 \quad \text{and} \quad z'(1999) = 9200$$

$$y(1999) = \$30{,}593 \quad \text{and} \quad y'(1999) = \$1400$$

Putting values, we have, the rate at which total personal income was rising in 1999, is

$$x'(1999) = y(1999)z'(1999) + z(1999)y'(1999)$$

$$= (30593)(9200) + (961400)(1400)$$

$$= \boxed{\$1{,}627{,}415{,}600 \text{ Per year}}$$

Chapter 2 Derivatives Exercise 2.3 92E

(A)

$f(20) = 10000$ means that 10000 yards fabric sold with the price of \$20 per yards.
And $f'(20) = -350$ means rate of change in quantity of fabric, in other words
We can say that the rate is decreased by 350

(B)

We have $R(p) = pf(p)$
Then differentiate with respect to p

$$R'(p) = \frac{dR(p)}{dp} = \frac{d}{dp}(pf(p))$$

$$= [pf'(p) - f(p).1]$$

$$R'(p) = pf'(p) - f(p)$$

Then put $p = 20$

$$R'(20) = 20.f'(20) - f(20)$$

$$= 20.(-350) - 10000$$

$$\boxed{R'(20) = -17000}$$

This means the graph of $R(p)$ is having negative slope at p = 20. It means the rate of total revenue earned with selling price \$20 is decreasing.

Chapter 2 Derivatives Exercise 2.3 93E

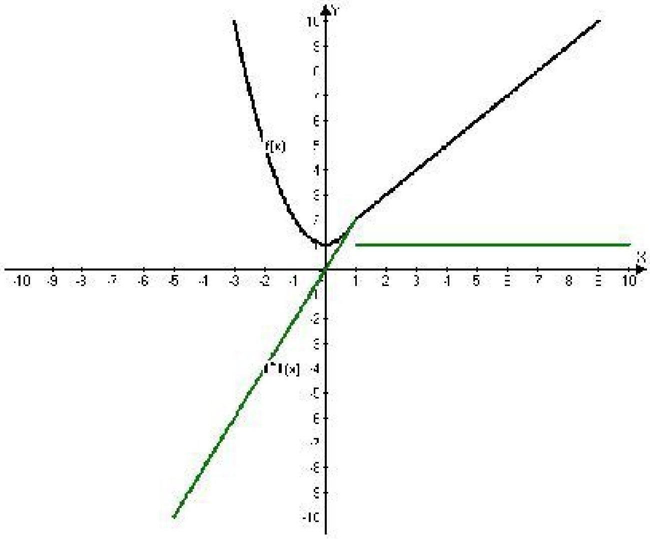
Let $f(x) = \begin{cases} x^2+1 & \text{if} \quad x<1 \\ x+1 & \text{if} \quad x \ge 1 \end{cases}$

Then $f'(x) = \begin{cases} 2x & \text{if} \quad x<1 \\ 1 & \text{if} \quad x \ge 1 \end{cases}$

$f(x)$ is not differentiable at $x = 1$, because $f'(1^-) = 2(1) = 2 \ne f'(1^+) = 1$

$\boxed{\therefore f(x) \text{ is not diffentiable at } x=1 \text{ and it is differentiable remaining all points}}$

The graph f and f' is



Chapter 2 Derivatives Exercise 2.3 94E

Let $g(x)=\begin{cases} 2x & \text{if} & x\le 0 \\ 2x-x^2 & \text{if} & 0<x<2 \\ 2-x & \text{if} & x\ge 2 \end{cases}$

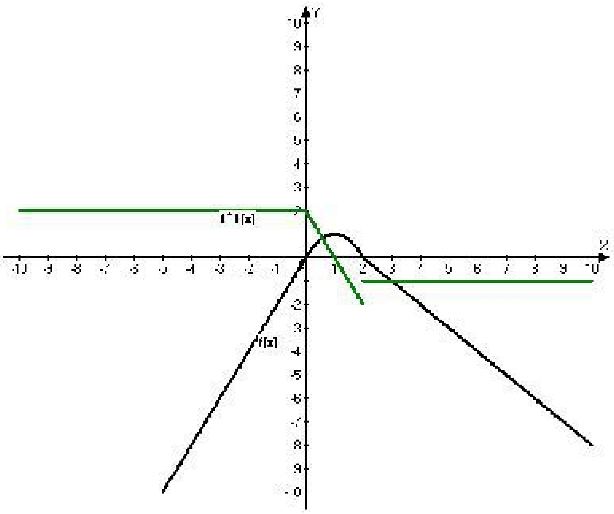
Then $g'(x)=\begin{cases} 2 & \text{if} & x\le 0 \\ 2-2x & \text{if} & 0<x<2 \\ -1 & \text{if} & x\ge 2 \end{cases}$

g is not differentiable at $x=2$, because $g'(2^-)=2-2(2)=-2$ and $g'(2^+)=-1$

$\therefore\ g'(2^-)\ne g'(2^+)$, g is not differentiable at $x=2$

Hence g is differentiable at every point except $x=2$

The graph of g and g' is



Chapter 2 Derivatives Exercise 2.3 95E

(A) We have $f(x)=|x^2-9|=\begin{cases} x^2-9, & \text{if } x<-3 \\ 9-x^2, & \text{if } x\in[-3,3] \\ x^2-9, & \text{if } x>3 \end{cases}$

For a modulus function, it is always not differentiable at the breaking point.
Therefore f(x) is not differentiable at $x=3$ *and* $x=-3$

$$f'(x)=\begin{cases} 2x, & \text{if } x<-3 \\ -2x, & \text{if } x\in(-3,3) \\ 2x, & \text{if } x>3 \end{cases} = \begin{cases} 2x & \text{if } |x|>3 \\ -2x & \text{if } |x|<3 \end{cases}$$

(B) Graph of $f(x)$:

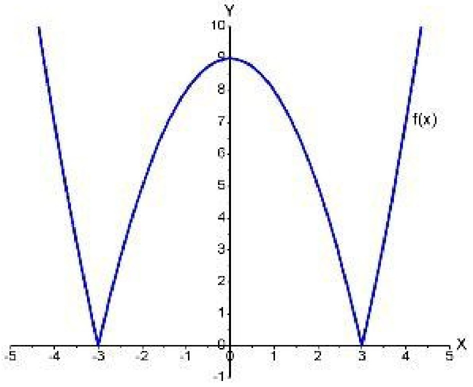


Fig.1

Graph of $f'(x)$:

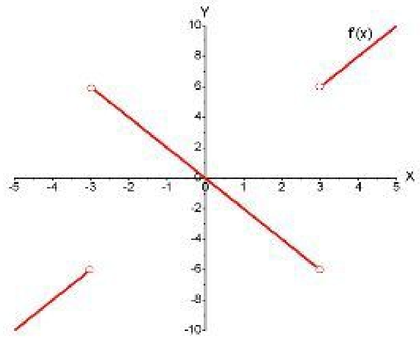


Fig.2

Chapter 2 Derivatives Exercise 2.3 96E

$h(x) = |x-1| + |x+2| = -2x-1 \ (x \le -2)$

$= 3 \quad (-2 < x < 1)$

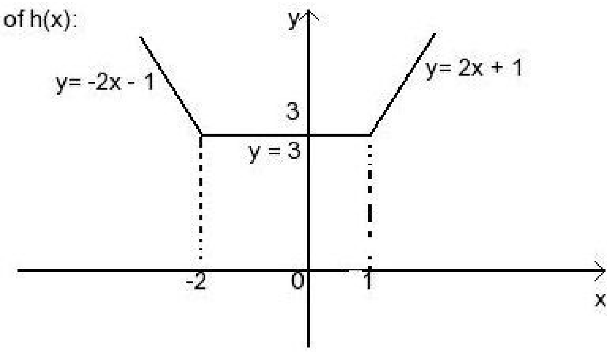
$= 2x+1 \quad (x \ge 1)$, so

$h'(x) = -2 \quad (x \le -2)$

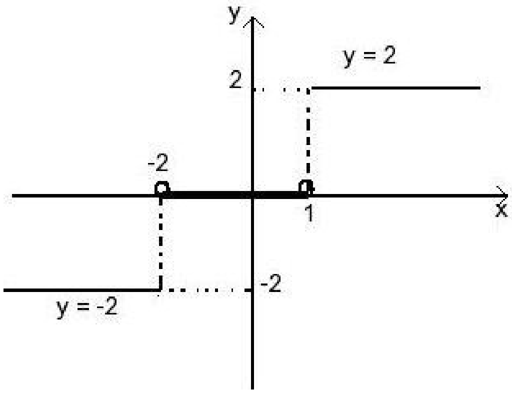
$= 0 \quad (-2 < x < 1)$

$= 2 \quad (x \ge 1)$

Graph of h(x):



Graph of h'(x):



Therefore, h(x) is differentiable at x $\in$ (-∞, -2) $\cup$ (-2, 1) $\cup$ (1, ∞).

Chapter 2 Derivatives Exercise 2.3 97E

We have the parabola $y = ax^2$ and tangent line $y + 2x = b$

We know that at the given point the slope of the tangent is equal to the slope of the curve.

$$\frac{dy}{dx} = \frac{d}{dx}(ax^2)$$

$$= a\frac{d}{dx}(x^2)$$

$$\boxed{\frac{dy}{dx} = 2ax}$$

Now if x = 2

Then $y = a(2)^2 = 4a$ and $\frac{dy}{dx} = 2.2a = 4a$

We use the point-slope form to write an equation of the tangent line at $(2, 4a)$

Here the required equation of tangent is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4a = 4a(x - 2)$$

$$\Rightarrow \boxed{y - 4ax + 4a = 0}$$

Compare this equation with given equation $y + 2x = b$

Thus we have $-4a = 2$ and $b = -4a$

$$\Rightarrow a = -\frac{2}{4}$$

$$\Rightarrow \boxed{a = -\frac{1}{2}} \text{ So } b = -4 \times -\frac{1}{2}$$

$$\Rightarrow \boxed{b = 2}$$

Chapter 2 Derivatives Exercise 2.3 98E

The product rule:

If *f* and *g* are both differentiable, then

$$(fg)' = fg' + gf'$$

The sum rule:

If *f* and *g* are both differentiable, then

$$(f + g)' = f' + g'$$

(a)

Consider the function

$F(x) = f(x)g(x)$

where *f* and *g* have derivatives of all orders

To show that $F'' = f''g + 2f'g' + fg''$:

The first derivative is

$$F' = (fg)' \quad \text{By using the product rule}$$
$$= fg' + gf'$$

Therefore $F' = fg' + gf'$

Now, find the second derivative of *F* by finding the derivative of *F'* and using the product rule again:

$$F'' = (F')'$$
$$= (fg' + gf')' \quad \text{By using the sum rule}$$
$$= (fg')' + (gf')'$$
$$= f(g')' + g'f' + g(f')' + f'g' \quad \text{By using the product rule}$$
$$= fg'' + 2f'g' + f''g$$

Therefore, $\boxed{F'' = f''g + 2f'g' + fg''}$

(b)

To find formula for F''':

Similarly, find the third derivative of *F* by finding the derivative of F'' and using the product rule again:

$$F''' = (F'')'$$
$$= (f''g + 2f'g' + fg'')' \quad \text{By using the sum rule}$$
$$= (f''g)' + (2f'g')' + (fg'')'$$
$$= f''g' + g(f'')' + 2f'(g')' + 2g'(f')' + f(g'')' + g''f' \quad \text{By using the product rule}$$
$$= f''g' + gf''' + 2f'g'' + 2g'f'' + fg''' + g''f'$$

Therefore, $\boxed{F''' = f'''g + 3f''g' + 3f'g'' + fg'''}$

To find formula for $F^{(4)}$:

Now, find the fourth derivative of *F* by finding the derivative of F''' and using the product rule again:

$$F^{(4)} = (F''')'$$
$$= (f'''g + 3f''g' + 3f'g'' + fg''')' \quad \text{By using the sum rule}$$
$$= (f'''g)' + (3f''g')' + (3f'g'')' + (fg''')'$$
$$= f'''g' + g(f''')' + 3f''(g')' + 3g'(f'')' + 3f'(g'')' + 3g''(f')' + f(g''')' + g'''f'$$

By using the product rule

$$= f'''g' + gf^{(4)} + 3f''g'' + 3g'f''' + 3f'g''' + 3g''f'' + fg^{(4)} + g'''f'$$

Therefore, $\boxed{F^{(4)} = f^{(4)}g + 4f'''g' + 6f''g'' + 4f'g''' + fg^{(4)}}$

(c)

To find formula for $F^{(n)}$:

From the previous formulas, we see that the first term has the nth derivative of *f* with no derivative of *g*. Each successive term has the one lower derivative of *f* and one derivative higher of *g*.

The coefficients follow a pattern displayed in the Binomial Theorem. The Binomial Theorem is used to find the nth power of a binomial. The formula is

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \binom{n}{k}x^{n-k}y^k + \ldots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{1\cdot 2\cdot 3\cdot\ldots\cdot k}$

The coefficients for the nth derivative of a product are the same as in the Binomial Theorem. Therefore, the formula for the nth derivative of a product is

$$F^{(n)} = \boxed{f^{(n)}g + nf^{(n-1)}g' + \frac{n(n-1)}{2}f^{(n-2)}g'' + \ldots + \binom{n}{k}f^{(n-k)}g^{(k)} + \ldots + nf'g^{(n-1)} + fg^{(n)}}$$

Chapter 2 Derivatives Exercise 2.3 99E

The line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$ when

$$\frac{3}{2}x + 6 = c\sqrt{x}$$

Comparing $y = \frac{3}{2}x + 6$ with y = mx + b we see that the slope of the tangent line is m = $\frac{3}{2}$

Using the Power Rule

$$y = c\sqrt{x}$$
$$y' = \frac{c}{2\sqrt{x}}$$

The slope of the tangent line at x =a is given by f '(a)

$$\frac{3}{2} = \frac{c}{2\sqrt{x}}$$
$$c = 3\sqrt{x}$$

We can use $\frac{3}{2}x + 6 = c\sqrt{x}$ with $c = 3\sqrt{x}$ to find c

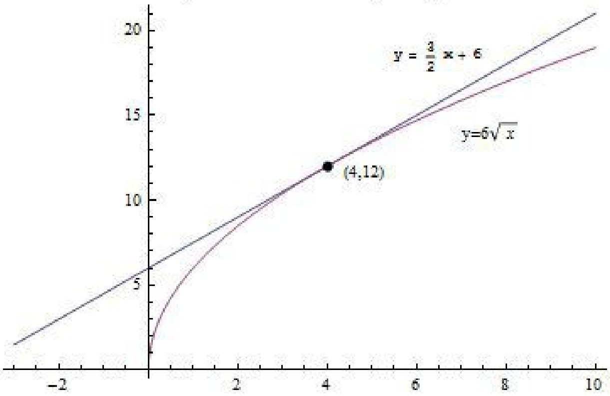
$$\frac{3}{2}x + 6 = 3\cdot\sqrt{x}\sqrt{x}$$
$$\frac{3}{2}x + 6 = 3|x|$$
$$\frac{1}{2}x + 2 = |x|$$

For x>0

$$\frac{1}{2}x + 2 = x$$
$$-\frac{1}{2}x + 2 = 0$$
$$-\frac{x-4}{2} = 0$$
$$x - 4 = 0$$
$$x = 4$$
$$c = 3\sqrt{4}$$
$$c = 6$$

For x<0 $y = c\sqrt{x}$ is not defined

Therefore the only real solution is $c = 6$



Chapter 2 Derivatives Exercise 2.3 100E

Consider the function,

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

The function $f(x)$ is differentiable everywhere.

Need to find the values of *m*, *b*.

The derivative of a function at *a* :

The derivative of a function *f* at a number *a*, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

It can be written as,

$$\lim_{h^- \to 0} \frac{f(a-h) - f(a)}{h} = \lim_{h^+ \to 0} \frac{f(a+h) - f(a)}{h}$$

The function $f(x)$ is differentiable everywhere.

Assume the function $f(x)$ is differentiable at 2.

The left derivative of $f(2)$ is equal to right derisive of $f(2)$.

That is,

$$\lim_{h \to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \quad \ldots\ldots (1)$$

Find $\lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$

$$\lim_{h \to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \to 0} \frac{(2-h)^2 - (2)^2}{-h}$$

$$= \lim_{h \to 0} \frac{4 - 4h + h^2 - 4}{-h}$$

$$= \lim_{h \to 0} \frac{h^2 - 4h}{-h}$$

$$= \lim_{h \to 0} \frac{-h(4-h)}{-h}$$

$$= \lim_{h \to 0} (4-h)$$

$$= 4$$

Thus,

$$\lim_{h \to 0} \frac{f(2-h) - f(2)}{-h} = 4 \quad \ldots\ldots (2)$$

Find $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$.

$$\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h \to 0} \frac{(m(2+h)+b)-(m \cdot 2+b)}{h}$$

$$= \lim_{h \to 0} \frac{2m+mh+b-2m-b}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= \lim_{h \to 0} m$$

$$= m$$

Thus,

$$\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = m \quad \ldots\ldots (3)$$

From equation (1)

$$\lim_{h \to 0} \frac{f(2-h)-f(2)}{-h} = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$$

From equations (2) and (3)

$$4 = m$$

Recollect,

If a function *f* is differentiable at *a*, then *f* is continuous at *a*.

A function *f* is continuous at a number *a* if

$$\lim_{x \to a} f(x) = f(a)$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$
$$= f(a)$$

The function $f(x)$ is differentiable at 2so, the function $f(x)$ continuous at 2.

That is,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$
$$= f(2)$$

Need to find the left and right hand limits:

The left hand side limit is,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 \quad f(x) = x^2 \quad \text{if } x \le 2$$

$$= \lim_{h \to 0} (2-h)^2$$

$$= 4$$

The right hand limit is,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} mx+b$$

$$= \lim_{h \to 0} m(2+h)+b$$

$$= 2m+b$$

Since,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$4 = 2m+b$

$4 = 2(4)+b$ Substitute $m = 4$

$4 = 8+b$ Simplify

$b = -4$ Add 8 on both sides

Hence, the values are $\boxed{m = 4, b = -4}$

Chapter 2 Derivatives Exercise 2.3 101E

We write $f = Fg$ where, it is assumed that $F'(x)$ exists

By the product rule

$$f' = (Fg)'$$
$$= Fg' + F'g$$

$$\Rightarrow F'g = f' - Fg'$$

$$\Rightarrow F'g = f' - \frac{f}{g}.g' \qquad \Rightarrow \text{where } F = \frac{f}{g}$$

$$\Rightarrow F'g = \frac{gf' - fg'}{g}$$

$$\boxed{\Rightarrow F' = \frac{gf' - fg'}{g^2}}$$

This the resulting equation for F'

This result is same as would have been obtained by differentiating $F = \frac{f}{g}$ using Quotient rule

Chapter 2 Derivatives Exercise 2.3 102E

(A) The given equation of the hyperbola is $xy = c$

Then $y' = -\frac{c}{x^2} = \frac{-y}{x}$ $\left(as\ \frac{c}{x} = y\right)$

Let the co-ordinates of the point P be $(x_0, c/x_0)$.

Then the slope of the tangent line at P is $y'(x_0) = -\frac{c}{x_0^2}$.

The equation of the tangent line at P is

$$(y - c/x_0) = -\frac{c}{x_0^2}(x - x_0)$$

Or $$y = -\frac{c}{x_0^2}x + \frac{2c}{x_0}$$

Then we find y –intercept by putting $x = 0$ in the equation of the tangent line.

$$y = \frac{2c}{x_0}$$

We find x-intercepts by putting $y = 0$ in the equation of the tangent line.

$$-\frac{c}{x_0^2}x + \frac{2c}{x_0} = 0$$

Or $$\frac{c}{x_0^2}x = \frac{2c}{x_0}$$

Or $$x = 2x_0$$

Then mid-point of the line joining the points $(0,\ 2c/x_0)$ and $(2x_0,\ 0)$ is

$$\left(\frac{0 + 2x_0}{2},\ \frac{2c/x_0 + 0}{2}\right) = (x_0,\ c/x_0) = P$$

Therefore mid-point of the line segment cut from this tangent line by the coordinate axes is P.

(B) Since from part (A) the tangent line intersects x-axis at $(2x_0,\ 0)$ and y-axis at $(0,\ 2c/x_0)$.

Then the base of triangle formed by the tangent line and the coordinate axes is

$$\text{base} = 2x_0$$

And the height of the triangle is $= 2c/x_0$

Then the area of the triangle is

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (2x_0) \times (2c/x_0) = 2c = \text{ constant}$$

Therefore, the area of the triangle formed by the tangent line and the coordinate axes does not depend on the position of the point P.

Chapter 2 Derivatives Exercise 2.3 103E

Let $f(x) = x^{1000}$

Then by the definition of derivative at a is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$
$$= \lim_{x \to a} \frac{x^{1000} - a^{1000}}{x - a}$$

Here we have to get $\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}$

Or we can write as $\lim_{x \to 1} \frac{x^{1000} - 1^{1000}}{x - 1}$

Then by comparing, we have $\Rightarrow a = 1$

Now we differentiate the function $f(x) = x^{1000}$ by the formula $\frac{d}{dx}(x^n) = nx^{n-1}$

$$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx}(x^{1000})$$
$$= 1000(x^{1000-1})$$
$$f'(x) = 1000(x^{999})$$

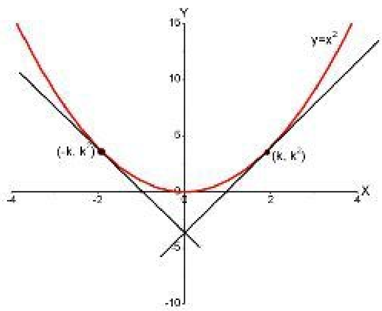
For $x = 1$ $\quad f'(1) = 1000(1^{999})$

$$\Rightarrow f'(1) = 1000$$

Hence $\lim_{x \to 1} \frac{x^{1000} - 1^{1000}}{x - 1} = 1000$

$$\boxed{\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1} = 1000}$$

Chapter 2 Derivatives Exercise 2.3 104E



Let the points of tangency be $(-k, k^2)$ and (k, k^2)

Equation of the curve is $y = x^2$

Then $\quad \frac{dy}{dx} = 2x$

Slope of the tangent line at $(-k, k^2)$ is

$$m_1 = \left(\frac{dy}{dx}\right)_{x=-k} = -2k \qquad \text{....(1)}$$

So the equation of the tangent line at $(-k, k^2)$ is

$$(y - k^2) = -2k(x + k)$$

Or $\quad y = -2kx - k^2 \qquad$ ....(2)

Slope of the tangent line at (k, k^2) is

$$m_2 = \left(\frac{dy}{dx}\right)_{x=k} = 2k \qquad \text{....(3)}$$

So the equation of the tangent line at (k, k^2) is

$$(y - k^2) = 2k(x - k)$$

Or $\quad y = 2kx - k^2 \qquad$ ....(4)

Since y –intercepts of these two tangent lines are $(-k^2)$

So these tangent line intersect each other at $(0, -k^2)$

Chapter 2 Derivatives Exercise 2.3 105E

Consider the following parabola:

$$y = x^2$$

A normal line at a point (x, y) is perpendicular to the tangent line at (x, y). Perpendicular lines have slopes, which are negative reciprocals of each other.

The slope of any tangent line to $y = x^2$ is found by the derivative:

$$\begin{aligned} y &= x^2 \\ y' &= \frac{d}{dx}(x^2) \\ &= 2x \end{aligned}$$

Choose $x = a$, then since $y = x^2$, the point (a, a^2) is where the tangent line has slope $2a$.

Then the slope of a normal line at $x = a$ is $-\frac{1}{2a}$ when $a \neq 0$.

Also, find the slopes of the normal lines by using the formula for slope between two points.

The slope, m, between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Therefore, the slope of the normal line between points $(0, c)$ and (a, a^2) is:

$$\begin{aligned} m &= \frac{a^2 - c}{a - 0} \\ &= \frac{a^2 - c}{a} \end{aligned}$$

As the slope of the normal line at the point (a, a^2) is equal to $\frac{a^2 - c}{a}$ and $-\frac{1}{2a}$, so equate these quantities:

$$\begin{aligned} \frac{a^2 - c}{a} &= -\frac{1}{2a} \\ a^2 - c &= -\frac{1}{2} \\ a^2 &= c - \frac{1}{2} \\ a &= \pm\sqrt{c - \frac{1}{2}} \end{aligned}$$

As the square root of non-negative values are defined, a exists only if $c \geq \frac{1}{2}$.

If $c > \frac{1}{2}$, and it has the two solutions $a = \pm\sqrt{c - \frac{1}{2}}$ from the previous formula. Whereas if $c < \frac{1}{2}$ the formula $a = \pm\sqrt{c - \frac{1}{2}}$ has no solutions.

Consider the case where $a = 0$.

For this case, the slope $-\frac{1}{2a}$ is not defined. Instead, if $a = 0$, then slope of the tangent line at $(0, 0)$ is:

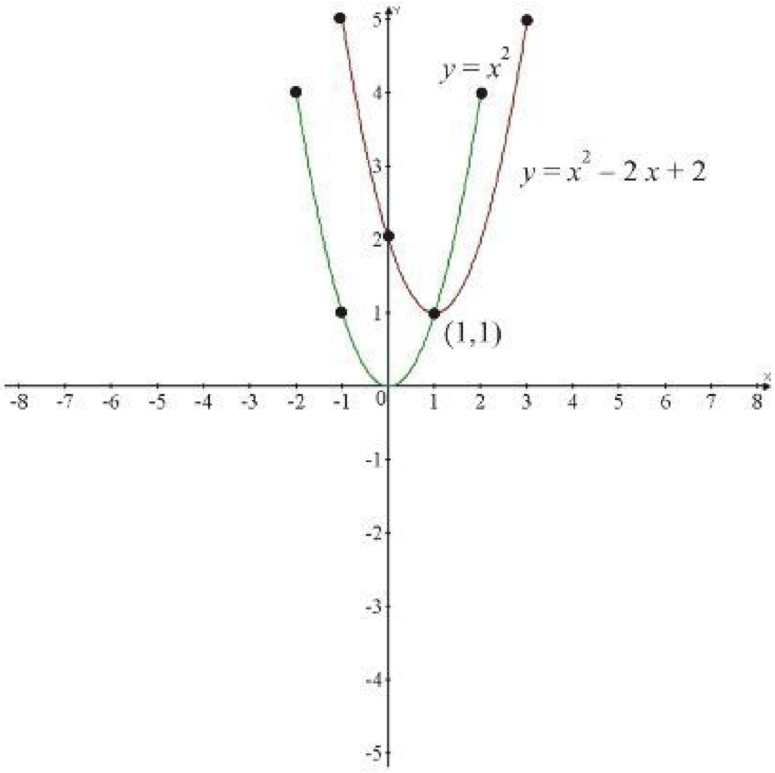
$$\begin{aligned} 2a &= 2(0) \\ &= 0 \end{aligned}$$

The line perpendicular to the tangent line through $(0, 0)$ with slope 0 is the *y*-axis. The *y*-axis is perpendicular to the parabola and always goes through the point $(0, c)$.

Consider the y-axis as always being a normal line to the parabola, when $c > \frac{1}{2}$ a total of $\boxed{3 \text{ normal lines}}$ through the point $(0, c)$.

However, when $c \le \frac{1}{2}$, a total of $\boxed{1 \text{ normal line}}$ through the point $(0, c)$ since it become very nearer to the origin, so only one normal can be drawn.

Chapter 2 Derivatives Exercise 2.3 106E



The two curves are intersecting at $(1,1)$ so they have a common tangent at that point

The slope of the curve is given by their derivatives at that point so

$$\frac{dy}{dx} = \frac{d}{dx}x^2$$
$$= 2x$$

At $(1,1)$ slope of the tangent $m = 2 \times 1$
$$= 2$$

Equation of the Tangent at $(1,1)$ is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x - 1)$$
$$y - 1 = 2x - 2$$
$$y = 2x - 2 + 1$$
$$y = 2x - 1$$

Hence the equation of the line that is tangent to both the given curves is $y = 2x - 1$