# Application of Integrals
## Short Answer Type Questions

**1. Find the area of the curve $y = sin\ x$ between 0 and $\pi$.**

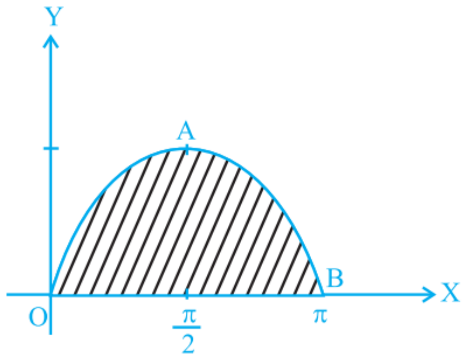


Fig. 8.1

Sol. We have

$$Area\ OAB = \int_o^{\pi} ydx = \int_o^{\pi} \sin x\, dx = |-\cos x|_0^{\pi}$$

$$= \cos 0 - \cos \pi = 2 sq\ units.$$

**2. Find the area of the region bounded by the curve $ay^2 = x^3$, the $y - axis$ and the lines $y = a\ and\ y = 2a.$**

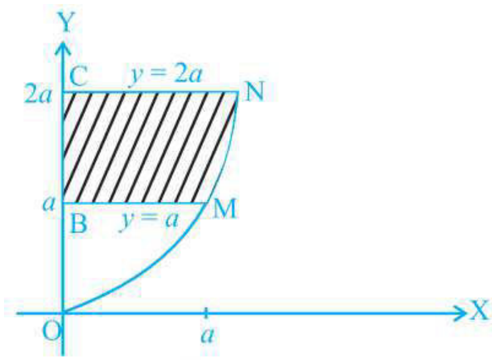


Fig. 8.2

Sol. We have

$$Area\ BMNC = \int_a^{2a} xdy \int_a^{2a} a^{\frac{1}{3}} y^{\frac{2}{3}} dy$$

$$= \frac{3a^{\frac{1}{3}}}{5} \left| y^{\frac{5}{3}} \right|_a^{2a}$$

$$= \frac{3a^{\frac{1}{3}}}{5} \left| (2a)^{\frac{5}{3}} - a^{\frac{5}{3}} \right|$$

$$= \frac{3}{5} a^{\frac{1}{3}} a^{\frac{5}{3}} \left| (2)^{\frac{5}{3}} - 1 \right|$$

$$= \frac{3}{5} a^2 \left| 2.2^{\frac{2}{3}} - 1 \right| sq\, units.$$

**3. Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.**

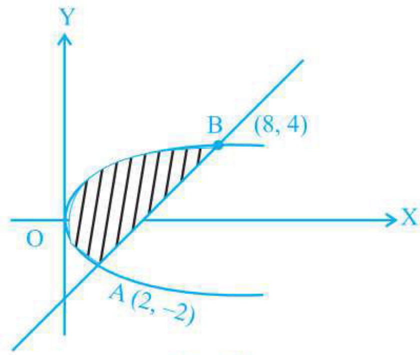



Fig. 8.3

Sol. The intersecting points of the given curves are obtained by solving the equations $x - y = 4$ and $y^2 = 2x$ for $x$ *and* $y$.

We have $y^2 = 8 + 2y$ *i.e.*, $(y-4)(y+2) = 0$ which gives $y = 4, -2$ *and* $x = 8, 2$.

Thus, the points of intersection are $(8,\ 4), (2,\ -2)$. Hence

$$Area = \int_{-2}^{4} \left( 4 + y - \frac{1}{2} y^2 \right) dy$$

$$= \left| 4y + \frac{y^2}{2} - \frac{1}{6} y^3 \right|_{-2}^{4} = 18\, sq\, units.$$

**4. Find the area of region bounded by the parabolas $y^2 = 6x$ *and* $x^2 = 6y$.**

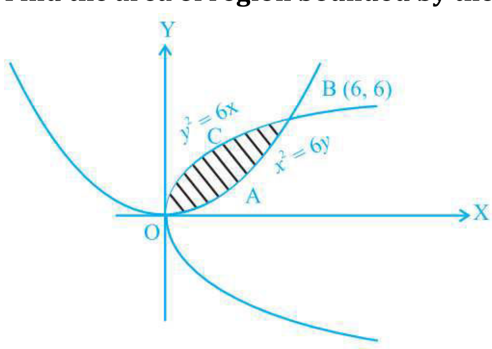



Fig. 8.4

Sol. The intersecting points of the given parabolas are obtained by solving these equations for $x$ *and* $y$, which are $0(0,\ 0)$ *and* $(6,\ 6)$. Hence

$$Area\, OABC = \int_{0}^{6} \left( \sqrt{6x} - \frac{x^2}{6} \right) dx = \left| 2\sqrt{6} \frac{x^{\frac{3}{2}}}{3} - \frac{x^3}{18} \right|_{0}^{6}$$

$$= 2\sqrt{6}\frac{(6)^{\frac{3}{2}}}{3} - \frac{(6)^3}{18} = 12\, sq\, units.$$

**5. Find the area enclosed by the curve $x = 3\cos t,\ y = 2\sin t$.**

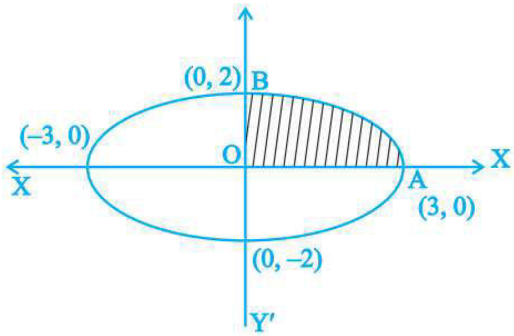


Fig. 8.5

Sol. Eliminating $t$ as follows:

$x = 3\cos t,\ y = 2\sin t \Rightarrow \frac{x}{3} = \cos t, \frac{y}{2} = \sin t,$ we obtain $\frac{x^2}{9} + \frac{y^2}{4} = 1,$ which is the equation of an ellipse.

From Fig. 8.5, we get the required area $= 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\, dx$

$$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 = 6\pi\, sq\, units.$$

## Long Answer Type Questions

**6. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.**

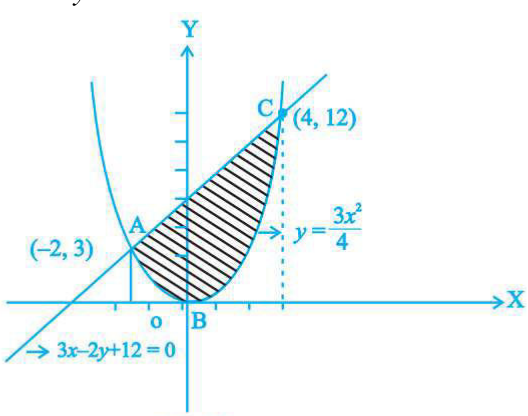


Fig. 8.6

Sol. Solving the equations of the given curves $y = \frac{3x^2}{4}$ and $3x - 2y + 12 = 0,$ we get

$3x^2 - 6x - 24 = 0 \Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow\ x = 4,\ x = -2$ which give $y = 12,\ y = 3$

From Fig. 8.6, the required area = area of ABC

$$= \int_{-2}^{4}\left(\frac{12+3x}{2}\right)dx - \int_{-2}^{4}\frac{3x^2}{4}dx$$

$$= \left(6x+\frac{3x^2}{4}\right)_{-2}^{4} - \left|\frac{3x^3}{12}\right|_{-2}^{4} = 27\, sq\, units.$$

**7.** **Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate coresponding to $t = 1$ and $t = 2$.**

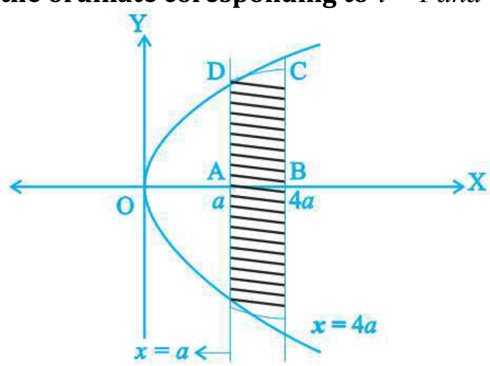



Fig. 8.7

Sol. Given that $x = at^2 \;\ldots(i)$, $y = 2at \;\ldots(ii) \Rightarrow t = \frac{y}{2a}$ putting the value of $t$ in $(i)$, we get

$y^2 = 4ax$

Putting $t = 1$ and $t = 2$ in (i), we get $x = a$, and $x = 4a$

Required area = 2 area of ABCD = $2\int_{a}^{4a} y\,dx = 2\times 2\int_{a}^{4a}\sqrt{ax}\,dx$

$$= 8\sqrt{a}\left|\frac{(x)^{\frac{3}{2}}}{3}\right|_{a}^{4a} = \frac{56}{3}a^2\, sq\, units.$$

**8.** **Find the area of the region above the $x-axis$, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.**

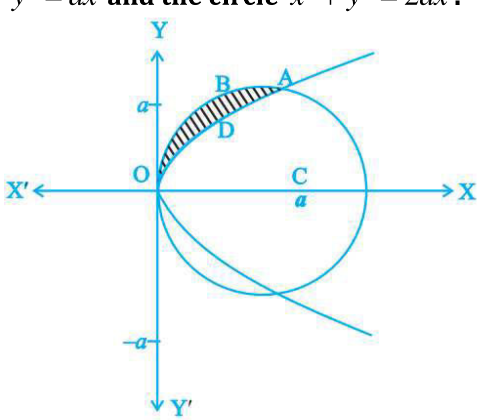



Fig. 8.8

Sol. Solving the given equations of curves, we have

$x^2 + ax = 2ax$

Or $x = 0,\ x = a,$ which give

$y = 0.\quad y = \pm a$

From Fig. 8.8 area ODAB = $\int_0^a \left(\sqrt{2ax - x^2} - \sqrt{ax}\right) dx$

Let $x = 2a\ sin^2\theta$. Then $dx = 4a\ sin\theta\, cos\theta\, d\theta\ and\ x = 0,$

$\Rightarrow \theta = 0,\ x = a\ \Rightarrow \theta = \dfrac{\pi}{4}.$

Again, $\int_0^a \sqrt{2ax - x^2}\, dx$

$$\int_0^{\frac{\pi}{4}} (2a\sin\theta\cos\theta)(4a\sin\theta\cos\theta)\, d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} (1 - \cos 4\theta)\, d\theta = a^2\left(\theta - \frac{\sin 4\theta}{4}\right)_0^{\frac{\pi}{4}} = \frac{\pi}{4}a^2.$$

Further more,

$$\int_0^a \sqrt{ax}\, dx = \sqrt{a}\,\frac{2}{3}\left(x^{\frac{3}{2}}\right)_0^a = \frac{2}{3}a^2$$

Thus the required area $\dfrac{\pi}{4}a^2 - \dfrac{2}{3}a^2 = a^2\left(\dfrac{\pi}{4} - \dfrac{2}{3}\right) sq\, units.$

**9. Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \dfrac{a}{2}$.**

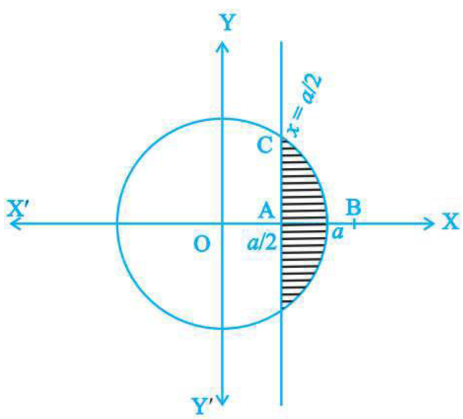



Fig. 8.9

Sol. Solving the equation $x^2 + y^2 = a^2$ and $x = \dfrac{a}{2}$., we obtain their points of intersection which are $\left(\dfrac{a}{2}, \sqrt{3}\dfrac{a}{2}\right)$ and $\left(\dfrac{a}{2}, -\dfrac{\sqrt{3}a}{2}\right)$.

Hence, from Fig. 8.9, we get

Required Area $= 2\, Area\, of\ OAB = 2\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}\, dx$

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a^2}{2}.\frac{\pi}{2} - \frac{a}{4}.a\frac{\sqrt{3}}{2} - \frac{a^2}{2}.\frac{\pi}{6}\right]$$

$$= \frac{a^2}{12}\left(6\pi - 3\sqrt{3} - 2\pi\right)$$

$$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right) sq\, units.$$

## Objective Type Questions

**Choose the correct anwer from the given four options in each of the Examples 10 to 12.**

**10. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to**

**(A)** $4\pi\, sq\, units$

**(B)** $2\sqrt{2\pi}\, sq\, units$

**(C)** $4\pi^2\, sq\, units$

**(D)** $2\pi\, sq\, units$

Sol. Correct answer is (D); since Area $= 4\int_{0}^{\sqrt{2}} \sqrt{2 - x^2}$

$$= 4\left(\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}} = 2\pi\, sq\, units.$$

**11. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to**

**(A)** $\pi^2 ab$

**(B)** $\pi ab$

**(C)** $\pi a^2 b$

**(D)** $\pi ab^2$

Sol. Correct answer is (B); since Area $4\int_{0}^{a} \frac{b}{a}\sqrt{a^2 - x^2}\, dx$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a = \pi ab.$$

**12. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$**

**(A)** $\frac{32}{3}$

**(B)** $\frac{256}{3}$

**(C)** $\frac{64}{3}$

**(D)** $\frac{128}{3}$

Sol. Correct answer is (B); since Area $= 2\int_0^{16} \sqrt{y}dy$

**Fill in the blanks in each of the Examples 13 to 14.**

**13. The area of the region bounded by the curve** $x = y^2$, $y - axis$ **and the line** $y = 3$ *and* $y = 4$ **is ________.**

Sol. $\frac{37}{3} sq.units$

**14. The area of the region bounded by the curve** $y = x^2 + x$, $x - axis$ **and the line** $x = 2$ *and* $x = 5$ **is equal to ____________ .**

Sol. $\frac{297}{6} sq.units$

# Application of Integrals

## Objective Type Questions

**Choose the correct answer from the given four options in each of the Exercises 24 to 34.**

**24. The area of the region bounded by the** $y-axis,\ y=cos\ x\ and\ y=sin\,x,\ 0\le x\le \frac{\pi}{2}$ **is**

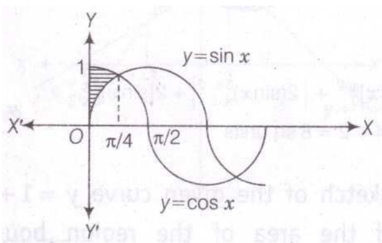
**(A)** $\sqrt{2}\ sq\,units$

**(B)** $(\sqrt{2}+1)\ sq\,units$

**(C)** $(\sqrt{2}-1)\ sq\,units$

**(D)** $(2\sqrt{2}-1)\ sq\,units$

Sol. (C) We have, $y-axis\ i.e.,\ x=0,\ y=cos\ x\ and\ y=sin\,x,$ where $0\le x\le \frac{\pi}{2}$




$\therefore$ Required area $=\int_0^{\pi/4}(\cos x-\sin x)dx$

$=[\sin x]_0^{\pi/4}+[\cos x]_0^{\pi/4}$

$=\left(\sin\frac{\pi}{4}-\sin 0\right)+\left(\cos\frac{\pi}{4}-\cos 0\right)$

$=\left(\frac{1}{\sqrt{2}}-0\right)+\left(\frac{1}{\sqrt{2}}-1\right)$

$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1$

$=-1+\frac{2}{\sqrt{2}}=\frac{-\sqrt{2}+2}{\sqrt{2}}$

$=\frac{-2+2\sqrt{2}}{2}=\left(\sqrt{2}-1\right)sq\,units$

**25. The area of the region bounded by the curve** $x^2=4y$ **and the straight line x= 4y - 2 is**

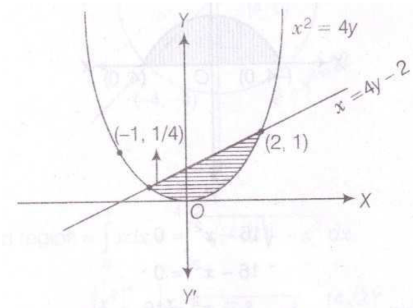
**(A)** $\frac{3}{8}sq\,units$

**(B)** $\frac{5}{8}sq\,units$

**(C)** $\frac{7}{8}sq\,units$

**(D)** $\frac{9}{8}sq\,units$

Sol. (D) Given equation of curve is $x^2 = 4y$ and the straight line $x = 4y - 2$.




For intersection point, put $x = 4y - 2$ in equation of curve, we get

$(4y-2)^2 = 4y$

$\Rightarrow 16y^2 + 4 - 16y = 4y$

$\Rightarrow 16y^2 - 20y + 4 = 0$

$\Rightarrow 4y^2 - 5y + 1 = 0$

$\Rightarrow 4y^2 - 4y - y + 1 = 0$

$\Rightarrow 4y(y-1) - 1(y-1) = 0$

$\Rightarrow (4y-1)(y-1) = 0$

$\therefore y = 1, \frac{1}{4}$

For $y = 1,\quad x = \sqrt{4 \cdot 1} = 2$ [since, negative value does not satisfy the equation of line]

For $y = \frac{1}{4},\quad x = \sqrt{4 \cdot \frac{1}{4}} = -1$ [positive value does not satisfy the equation of line]

So, the intersection points are $(2,1)$ *and* $\left(-1, \frac{1}{4}\right)$

$\therefore$ Area of shaded region $= \int_{-1}^{2} \left(\frac{x+2}{4}\right) dx - \int_{-1}^{2} \frac{x^2}{4} dx$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-1}^{2}$

$= -\frac{1}{4}\left[\frac{4}{2} + 4 - \frac{1}{2} + 2\right] - \frac{1}{4}\left[\frac{8}{3} + \frac{1}{3}\right]$

$= \frac{1}{4} \cdot \frac{15}{2} - \frac{1}{4} \cdot \frac{9}{3} = \frac{45-18}{24}$

$= \frac{27}{24} = \frac{9}{8}\, sq\, units$

**26.** **The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is**

**(A)** $8$ *sq units*

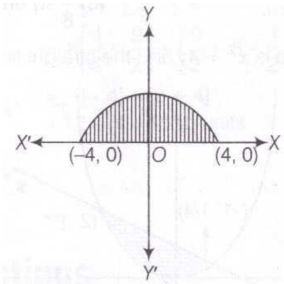
**(B)** $20\pi$ *sq units*

**(C)** $16\pi$ *sq units*

**(D)** $256\pi$ *sq units*

Sol. (A) Given equation of curve is $y=\sqrt{16-x^2}$ and the equation of line is x-axis

$X-axis\ i.e.,\ y=0$




$\therefore \sqrt{16-x^2}=0 \ ...(i)$

$\Rightarrow 16-x^2=0$

$\Rightarrow x^2=16$

$\Rightarrow x=\pm 4$

So, the intersection points are (4, 0) and (-4, 0).

$\therefore$ Area of curve, $A=\int_{-4}^{4}(16-x^2)^{1/2}dx$

$=\int_{-4}^{4}\sqrt{(4^2-x^2)}dx$

$=\left[\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$

$=\left[\frac{4}{2}\sqrt{4^2-4^2}+8\sin^{-1}\frac{4}{4}\right]-\left[-\frac{4}{2}\sqrt{4^2-(-4)^2}+8\sin^{-1}\left(-\frac{4}{4}\right)\right]$

$=\left[2\cdot 0+8\cdot\frac{\pi}{2}-0+8\cdot\frac{\pi}{2}\right]=8\pi\ sq\,units$

**27. Area of the region in the first quadrant enclosed by the $x-axis$, the line $y=x$ and the circle $x^2+y^2=32$ is**

**(A)** $16\pi\,sq\ units$

**(B)** $4\pi\,sq\ units$

**(C)** $32\pi\,sq\ units$

**(D)** $24\pi\ sq\ units$

Sol. (B) We have enclosed by $X-axis\ i.e.,\ y=0,\ y=x$ and the circle $x^2+y^2=32$ in first quadrant.

Since, $x^2+(x)^2=32 \quad [\because y=x]$

$\Rightarrow 2x^2=32$

$\Rightarrow x=\pm 4$

So, the intersection point of circle $x^2+(x)^2=32$ and line $y=x\ are\ (4,\ 4)\ or\ (-4,\ 4)$.

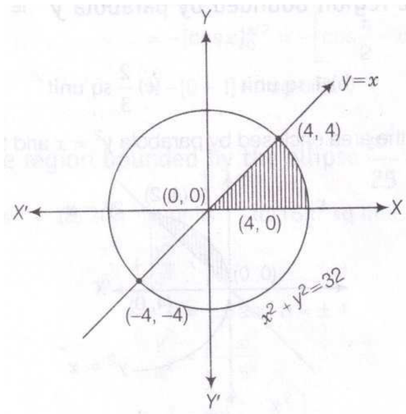
And $x^2+y^2=(4\sqrt{2})^2$

Since, $y=0$

$\therefore x^2+(0)^2=32$

$\Rightarrow x=\pm 4\sqrt{2}$

So, the circle intersects the X-axis at $\left(\pm 4\sqrt{2},\, 0\right)$.




Area of shaded region= $\int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\, dx$

$= \left|\frac{x^2}{2}\right|_0^4 + \left|\frac{x}{2}\sqrt{\left(4\sqrt{2}\right)^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right|_4^{4\sqrt{2}}$

$= \frac{16}{2} + \left[\frac{4\sqrt{2}}{2}\cdot 0 + 16\sin^{-1}\frac{\left(4\sqrt{2}\right)}{\left(4\sqrt{2}\right)} - \frac{4}{2}\sqrt{\left(4\sqrt{2}\right)^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$

$= 8 + \left[16\cdot\frac{\pi}{2} - 2\cdot\sqrt{16} - 16\cdot\frac{\pi}{2}\right]$

$= 8 + [8\pi - 8 - 4\pi] = 4\pi\, sq\, units$

**28.** **Area of the region bounded by the curve** $y = cos\, x$ **between** $x = 0$ *and* $x = \pi$ **is**

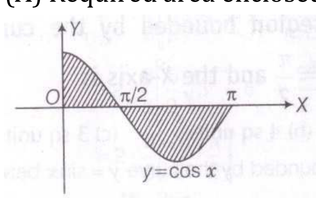
**(A)** $2$ *sq units*

**(B)** $4$ *sq units*

**(C)** $3$ *sq units*

**(D)** $1$ *sq units*

Sol. (A) Required area enclosed by the curve $y = cos\, x,\;\; x = 0$ *and* $x = \pi$




$A = \int_0^{\pi/2} \cos x\, dx + \left|\int_{\pi/2}^{\pi} \cos x\, dx\right|$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\sin\frac{\pi}{2} - \sin \pi\right|$

$= 1 + 1 = 2\, sq\, units$

**29.** **The area of the region bounded by parabola** $y^2 = x$ **and the straight line** $2y = x$ **is**

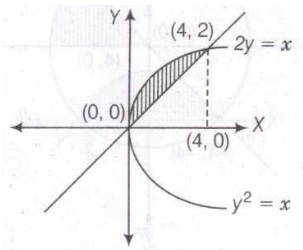
**(A)** $\frac{4}{3}\, sq\, units$

**(B)** $1\, sq\, units$

**(C)** $\frac{2}{3} sq\, units$

**(D)** $\frac{1}{3} sq\, units$

Sol. (A) We have to find the area enclosed by parabola $y^2 = x$ and the straight line $2y = x$.




$$\therefore \left(\frac{x}{2}\right)^2 = x$$

$$\Rightarrow x^2 = 4x \Rightarrow x(x-4) = 0$$

$$\Rightarrow x = 4 \Rightarrow y = 2\, and\, x = 0 \Rightarrow y = 0$$

So, the intersection points are $(0, 0)$ *and* $(4, 2)$.

Area enclosed by shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2} \cdot \frac{x^2}{2}\right]_0^4 = \left[2 . \frac{x^{\frac{3}{2}}}{3} - \frac{x^2}{4}\right]_0^4$$

$$= \frac{2}{3} 4^{3/2} - \frac{16}{4} - \frac{2}{3} \cdot 0 + \frac{1}{4} \cdot 0$$

$$= \frac{16}{3} - \frac{16}{4} = \frac{64-48}{12} = \frac{16}{12} = \frac{4}{3} sq\, units$$

**30. The area of the region bounded by the curve $y = sin\, x$ between the ordinates x = 0, $x = \frac{\pi}{2}$ and the x-axis is**

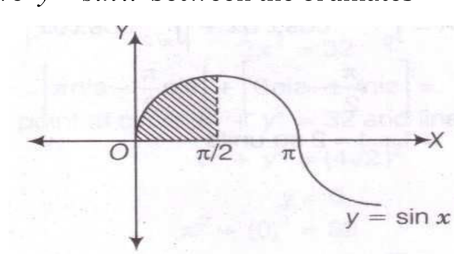
**(A)** $2$ *sq units*

**(B)** $4$ *sq units*

**(C)** $3$ *sq units*

**(D)** $1$ *sq units*

Sol. (D) Area of the region bounded by the curve $y = sin\, x$ between the ordinates $x = 0, x = \frac{\pi}{2}$ and the X-axis is




$$A = \int_0^{\pi/2} \sin x\, dx$$

$$= -[\cos x]_0^{\pi/2} = -\left[\cos\frac{\pi}{2} - \cos 0\right]$$

$= -[0-1] = 1\, sq\, unit$

**31.** **The area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is**

**(A)** $20\pi\, sq\ units$

**(B)** $20\pi^2 sq\ units$

**(C)** $16\pi^2 sq\ units$

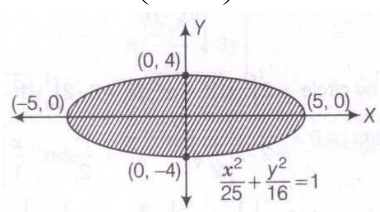
**(D)** $25\pi\, sq\ units$

Sol. (A) We have, $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$

Here, $a = \pm 5\, and\, b = \pm 4$

And $\frac{y^2}{4^2} = 1-\frac{x^2}{5^2}$

$\Rightarrow y^2 = 16\left(1-\frac{x^2}{25}\right)$




$\Rightarrow y = \sqrt{\frac{16}{25}(25-x^2)}$

$\Rightarrow y = \frac{4}{5}\sqrt{(5^2-x^2)}$

$\therefore$ Area enclosed by ellipse, $A = 2\cdot\frac{4}{5}\int_{-5}^{5}\sqrt{5^2-x^2}\,dx$

$= 2\cdot\frac{8}{5}\int_{0}^{5}\sqrt{5^2-x^2}\,dx$

$= 2\cdot\frac{8}{5}\left[\frac{x}{2}\sqrt{5^2-x^2}+\frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$

$= 2\cdot\frac{8}{5}\left[\frac{5}{2}\sqrt{5^2-5^2}+\frac{5^2}{2}\sin^{-1}\frac{5}{5}-0-\frac{25}{2}\cdot 0\right]$

$= 2\cdot\frac{8}{5}\left[\frac{25}{2}\cdot\frac{\pi}{2}\right]$

$= \frac{16}{5}.\frac{25\pi}{4}$

$= 20\pi\, sq\, units$

**32.** **The area of the region bounded by the circle $x^2 + y^2 = 1$ is**

**(A)** $2\pi\, sq\ units$

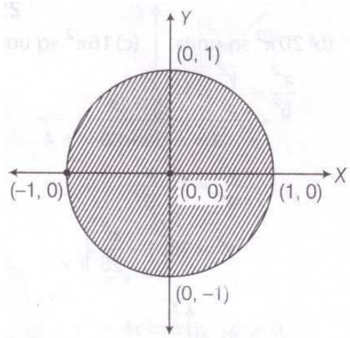
**(B)** $\pi\, sq\ units$

**(C)** $3\pi\, sq\ units$

**(D)** $4\pi\, sq\ units$

Sol. (A) We have, $x^2 + y^2 = 1^2$ $\quad [\because r = \pm 1]$

$\Rightarrow y^2 = 1 - x^2 \Rightarrow y = \sqrt{1-x^2}$




$\therefore$ Area enclosed by circle$= 2\int_{-1}^{1}\sqrt{1^2 - x^2}\,dx = 2\cdot 2\int_0^1 \sqrt{1^2 - x^2}\,dx$

$$= 2.2\left[\frac{x}{2}\sqrt{1^2 - x^2} + \frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot\frac{\pi}{2} - 0 - \frac{1}{2}\cdot 0\right]$$

$$= 4\cdot\frac{\pi}{4} = \pi\, sq\, units$$

**33. The area of the region bounded by the curve y = x+1 and the lines x = 2 and x =3 is**

**(A)** $\frac{7}{2} sq\, units$

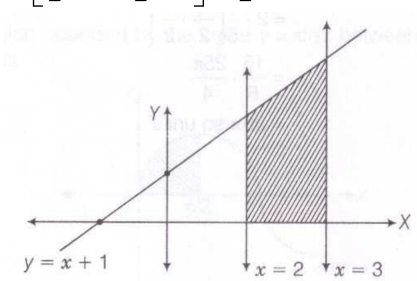
**(B)** $\frac{9}{2} sq\, units$

**(C)** $\frac{11}{2} sq\, units$

**(D)** $\frac{13}{2} sq\, units$

Sol. (A) Required area, $A = \int_2^3 (x+1)dx = \left[\frac{x^2}{2} + x\right]_2^3$

$$= \left[\frac{9}{2} + 3 - \frac{4}{2} - 2\right] = \frac{7}{2} sq\, units$$




**34. The area of the region bounded by the curve** $x = 2y + 3$ **and the** $y$ **lines.**
$y = 1$ *and* $y = -1$ **is**

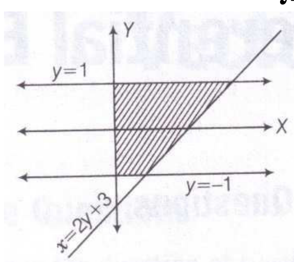
**(A)** 4 *sq units*

**(B)** $\frac{3}{2}$ *sq units*

**(C)** 6 *sq units*

**(D)** 8 *sq units*

Sol. (C) Required area, $A=\int_{-1}^{1}(2y+3)dy$




$=\left[\frac{2y^2}{2}+3y\right]_{-1}^{1}$

$=\left[y^2+3y\right]_{-1}^{1}$

$=[1+3-1+3]$

$=6\,sq\,units$

## Application of Integrals

## Short Answer Type Questions

**1. Find the area of the region bounded by the curves $y^2 = 9x,\ y = 3x$.**

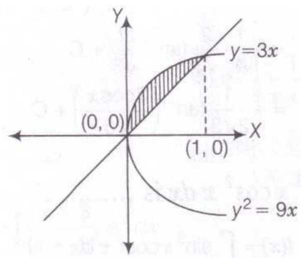
Sol. We have $y^2 = 9x$ *and* $y = 3x$

$\Rightarrow \quad (3x)^2 = 9x$

$\Rightarrow \quad 9x^2 - 9x = 0$

$\Rightarrow \quad 9x(x-1) = 0$

$\Rightarrow \quad x = 1,\ 0$




$\therefore$ Required area, A= $\int_0^1 \sqrt{9x}dx - \int_0^1 3xdx$

$= 3\int_0^1 x^{1/2}dx - 3\int_0^1 xdx$

$= 3\left[\frac{x^{3/2}}{3/2}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$

$= 3\left(\frac{2}{3} - 0\right) - 3\left(\frac{1}{2} - 0\right)$

$= 2 - \frac{3}{2} = \frac{1}{2} sq\ units$

**2. Find the area of the region bounded by the parabola $y^2 = 2px,\ x^2 = 2py$.**

Sol. We have, $y^2 = 2px$ *and* $x^2 - 2py$

$\therefore \quad y = \sqrt{2px}$

$\Rightarrow \quad x^2 = 2p.\sqrt{2px}$

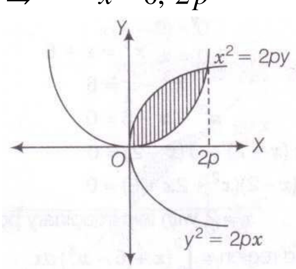
$\Rightarrow \quad x^4 = 4p^2.(2px)$

$\Rightarrow \quad x^4 = 8p^3x$

$\Rightarrow \quad x^4 = 8p^3x = 0$

$\Rightarrow \quad x^3(x - 8p^3) = 0$

$\Rightarrow \quad x = 0,\ 2p$



$\therefore$ Required area $= \therefore \int_0^{2p} \sqrt{2px}\, dx - \int_0^{2p} \frac{x^2}{2p} dx$

$= \sqrt{2p} \int_0^{2p} x^{1/2} dx - \frac{1}{2p} \int_0^{2p} x^2 dx$

$= \sqrt{2p} \left[ \frac{2(x)^{3/2}}{3} \right]_0^{2p} - \frac{1}{2p} \left[ \frac{x^3}{3} \right]_0^{2p}$

$= \sqrt{2p} \left[ \frac{2}{3}.(2p)^{3/2} - 0 \right] - \frac{1}{2p} \left[ \frac{1}{3}(2p^3) - 0 \right]$

$= \sqrt{2p} \left( \frac{2}{3}.2\sqrt{2}p^{3/2} \right) - \frac{1}{2p} \left( \frac{1}{3} 8p^3 \right)$

$= \sqrt{2p} \left( \frac{4\sqrt{2}}{3} p^{3/2} \right) - \frac{1}{2p} \left( \frac{8}{3} p^3 \right)$

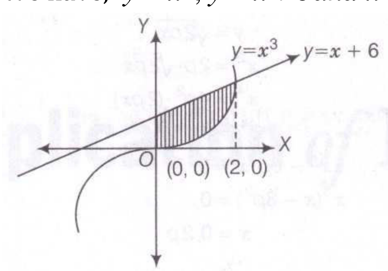
$= \frac{4\sqrt{2}}{3}.\sqrt{2}p^2 - \frac{8}{6} p^2$

$= \frac{(16-8)p^2}{6} = \frac{8p^2}{6}$

$= \frac{4p^2}{3} \, sq\, unit$

**3. Find the area of the region bounded by the curve $y = x^3$ and $y = x + 6$ and $x = 0$.**

Sol. We have, $y = x^3, y = x + 6 \, and \, x = 0$




$\therefore \quad x^3 = x + 6$

$\Rightarrow \quad x^3 - x = 6$

$\Rightarrow \quad x^3 - x - 6 = 0$

$\Rightarrow \quad x^2(x-2) + 2x(x-2) + 3(x-2) = 0$

$\Rightarrow \quad (x-2)(x^2 + 2x + 3) = 0$

$\Rightarrow \quad x = 2,$ with two imaginary points

$\therefore$ Required area of shaded region $= \int_0^2 (x + 6 - x^3) dx$

$= \left[ \frac{x^2}{2} + 6x - \frac{x^4}{4} \right]_0^2$

$= \left[ \frac{4}{2} + 12 - \frac{16}{4} - 0 \right]$

$= [2 + 12 - 4] = 10 \; sq\, units$

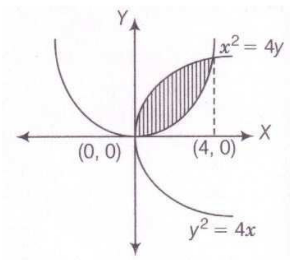
**4. Find the area of the region bounded by the curves $y^2 = 4x,\ x^2 = 4y$.**

Sol. Given equation of curves are

$y^2 = 4x$ ...(i)

and $x^2 = 4y$ ...(ii)

$\Rightarrow \left(\frac{x^2}{4}\right) = 4x$




$\Rightarrow \frac{x^4}{4.4} = 4x$

$\Rightarrow x^4 = 64x$

$\Rightarrow x^4 - 64x = 0$

$\Rightarrow x(x^3 - 4^3) = 0$

$\Rightarrow x = 4,\ 0$

$\therefore$ Area of shaded region, $A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[\frac{2x^{3/2}.2}{3} - \frac{1}{4}.\frac{x^3}{3}\right]_0^4$

$= \frac{2.2}{3}.8 - \frac{1}{4}.\frac{64}{3} - 0 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ *sq units*

**5. Find the area of the region included between $y^2 = 9x$ and $y = x$.**

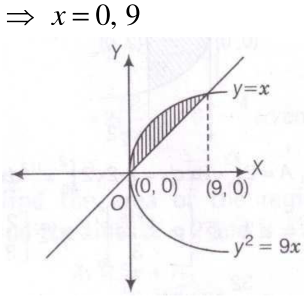
Sol. We have, $y^2 = 9x$ and $y = x$

$\Rightarrow x^2 = 9x$

$\Rightarrow x^2 - 9x = 0$

$\Rightarrow x(x - 9) = 0$

$\Rightarrow x = 0,\ 9$




$\therefore$ Area of shaded region, A = $\int_0^9 (\sqrt{9x} - x) dx = \int_0^9 3x^{1/2} dx - \int_0^9 x dx$

$$= \left[3.\frac{x^{3/2}}{3}\right]_0^9 - \left[\frac{x^2}{2}\right]_0^9$$

$$= \left[\frac{3.3^{\frac{3}{2}\times 2}}{3}.2 - 0\right] - \left[\frac{81}{2} - 0\right]$$

$$= 54 - \frac{81}{2} = \frac{108-81}{2} = \frac{27}{2}\ sq\,units$$

**6. Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$**

Sol. We have, $x^2 = y\ and\ y = x + 2$

$\Rightarrow x^2 = x + 2$

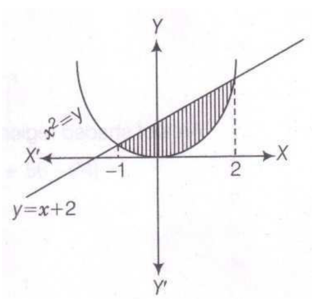
$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x-2) + 1(x-2) = 0$

$\Rightarrow (x+1)(x-2) = 0$

$\Rightarrow x = -1,\ 2$



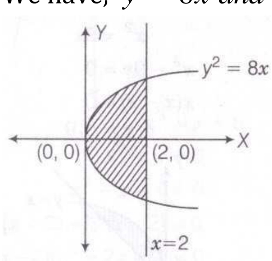

$\therefore$ Required area of shaded region, $= \int_{-1}^{2}(x + 2 - x^2)dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$$= \left[\frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}\right]$$

$$= 6 + \frac{3}{2} - \frac{9}{2} = \frac{36+9-18}{6} = \frac{27}{6} = \frac{9}{2}\,sq\,units$$

**7. Find the area of region bounded by the line $x = 2$ and the parabola $y^2 = 8x$**

Sol. We have, $y^2 = 8x\ and\ x = 2$




$\therefore$ Area of shaded region, $A = 2\int_0^2 \sqrt{8x}\,dx = 2.2\sqrt{2}\int_0^2 x^{1/2}dx$

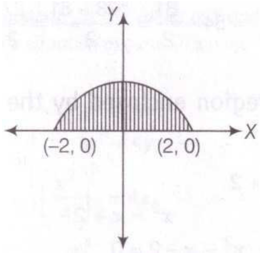
$$= 4.\sqrt{2}.\left[2.\frac{x^{3/2}}{3}\right]_0^2 = 4\sqrt{2}\left[\frac{2}{3}.2\sqrt{2} - 0\right]$$

$= \frac{32}{3}\ sq\,units$

**8.** **Sketch the region** $\{(x,0): y=\sqrt{4-x^2}\}$ **and x-axis. Find the area of the region using integration.**

Sol. Given region is $\{(x,0): y=\sqrt{4-x^2}\}$ and X-axis.

We have, $y=\sqrt{4-x^2} \Rightarrow y^2 = 4-x^2 \Rightarrow x^2+y^2=4$




$\therefore$ Area of shaded region, $A = \int_{-2}^{2}\sqrt{4-x^2}\,dx = \int_{-2}^{2}\sqrt{2^2-x^2}\,dx$

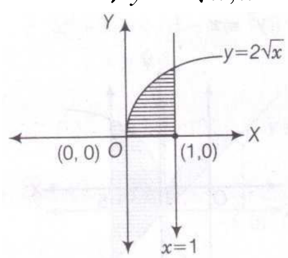
$= \left[\frac{x}{2}\sqrt{2^2-x^2} + \frac{2^2}{2}.\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$

$= \frac{2}{2}.0 + 2.\frac{\pi}{2} + \frac{2}{2}.0 - 2\sin^{-1}(-1) = 2.\frac{\pi}{2} + 2.\frac{\pi}{2}$

$= 2\pi\ sq\,units$

**9.** **Calculate the area under the curve** $y = 2\sqrt{x}$ **included between the lines** $x=0\ and\ x=1$ **.**

**Sol.** **We have,** $y = 2\sqrt{x}, x = 0\, and\ x = 1$ **.**




$\therefore$ Area of shaded region, $A = \int_0^1 (2\sqrt{x})dx$

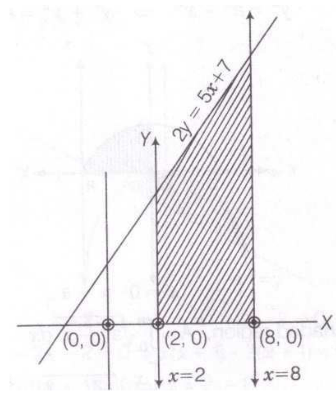
$= 2.\left[\frac{x^{3/2}}{3}.2\right]_0^1$

$= 2\left(\frac{2}{3}.1 - 0\right) = \frac{4}{3} sq\,units$

**10. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, $x - axis$ and the lines $x = 2$ *and* $x = 8$.**

Sol. We have $2y = 5x + 7$

$\Rightarrow y = \frac{5x}{2} + \frac{7}{2}$




$\therefore$ Area of shaded region=

$\frac{1}{2}\int_2^8 (5x+7)dx = \frac{1}{2}\left[5.\frac{x^2}{2} + 7x\right]_2^8$

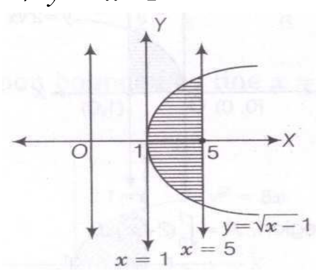
$= \frac{1}{2}[5 \cdot 32 + 7 \cdot 8 - 10 - 14] = \frac{1}{2}[160 + 56 - 24]$

$= \frac{192}{2} = 96\, sq\, units$

**11. Draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval $[1,\ 5]$. Find the area under the curve and between the lines $x = 1$ *and* $x = 5$.**

Sol. Given equation of the curve is $y = \sqrt{x-1}$

$\Rightarrow y^2 = x - 1$



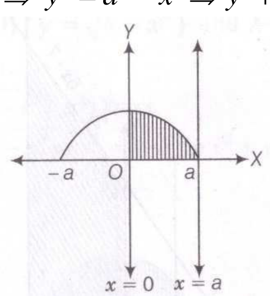

$\therefore$ Area of shaded region, $A = \int_1^5 (x-1)^{1/2} dx = \left[\frac{2.(x-1)^{3/2}}{3}\right]_1^5$

$= \left[\frac{2}{3} \cdot (5-1)^{3/2} - 0\right] = \frac{16}{3} sq\, unit$

**12. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ *and* $x = a$.**

Sol. Given equation of the curve is $y = \sqrt{a^2 - x^2}$

$\Rightarrow y^2 = a^2 - x^2 \Rightarrow y^2 + x^2 = a^2$




$\therefore$ Required area of shaded region, $A = \int_0^a \sqrt{a^2 - x^2}\, dx$

$$=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{2}\right]_0^a$$

$$=\left[0+\frac{a^2}{2}\sin^{-1}(1)-0-\frac{a^2}{2}\sin^{-1}0\right]$$

$$=\frac{a^2}{2}\cdot\frac{\pi}{2}=\frac{\pi a^2}{4}\,sq\,units$$

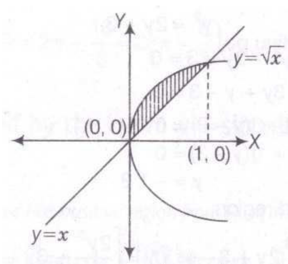
**13. Find the area of the region bounded by $y=\sqrt{x}$ $and$ $y=x$.**

Sol. Given equation of are $y=\sqrt{x}$ $and$ $y=x$

$\Rightarrow x=\sqrt{x} \Rightarrow x^2=x$

$\Rightarrow x^2-x=0 \Rightarrow x(x-1)=0$

$\Rightarrow x=0, 1$



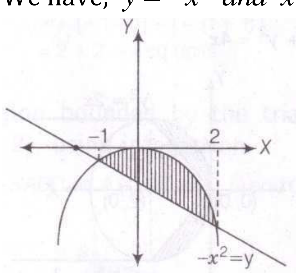

$\therefore$ Required area of shaded region, $A=\int_0^1(\sqrt{x})dx-\int_0^1 xdx$

$$=\left[2.\frac{x^{3/2}}{3}\right]_0^1-\left[\frac{x^2}{2}\right]_0^1$$

$$=\frac{2}{3}\cdot 1-\frac{1}{2}=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\,sq\,units$$

**14. Find the area enclosed by the curve $y=-x^2$ and the straight line $x+y+2=0$.**

Sol. We have, $y=-x^2$ $and$ $x+y+2=0$




$\Rightarrow -x-2=-x^2 \Rightarrow x^2-x-2=0$

$\Rightarrow x^2+x-2x-2=0 \Rightarrow x(x+1)-2(x+1)=0$

$\Rightarrow (x-2)(x+1)=0 \Rightarrow x=2,-1$

$\therefore$ Area of shaded region, $A=\left|\int_{-1}^{2}(-x-2+x^2)dx\right|=\left|\int_{-1}^{2}(x^2-x-2)dx\right|$

$$=\left|\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_{-1}^{2}\right|=\left|\left[\frac{8}{3}-\frac{4}{2}-4+\frac{1}{3}+\frac{1}{2}-2\right]\right|$$

$= \left|\frac{16-12-24+2+3-12}{6}\right| = \left|-\frac{27}{6}\right| = \frac{9}{2} sq\,units$

**15.** **Find the area bounded by the curve $y=\sqrt{x}, x=2y+3$ in the first quadrant and x-axis.**

Sol. Given equation of the curves are for $y=\sqrt{x}$ *and* $x=2y+3$ in the first quadrant.

On solving both the equation for y, we get

$y=\sqrt{2y+3}$

$\Rightarrow y^2 = 2y+3$

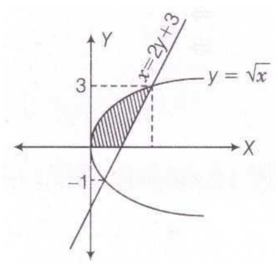
$\Rightarrow y^2-2y-3=0$

$\Rightarrow y^2-3y+y-3=0$

$\Rightarrow y(y-3)+1(y-3)=0$

$\Rightarrow (y+1)(y-3)=0$

$\Rightarrow y=-1,3$




$\therefore$ Required area of shaded region,

$$A=\int_0^3 (2y+3-y^2)dy = \left[\frac{2y^2}{2}+3y-\frac{y^3}{3}\right]_0^3$$

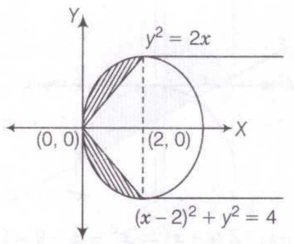
$$= \left[\frac{18}{2}+9-9-0\right] = 9\,sq\,units$$

# Application of Integrals

## Long Answer Type Questions

**16. Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$.**

Sol. We have, $y^2 = 2x \, and \, x^2 + y^2 = 4x$




$\Rightarrow x^2 + 2x = 4x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$\Rightarrow x = 0, 2$

Also, $x^2 + y^2 = 4x$

$\Rightarrow x^2 - 4x = -y^2$

$\Rightarrow x^2 - 4x + 4 = -y^2 + 4$

$\Rightarrow (x-2)^2 - 2^2 = -y^2$

$\therefore$ Required area= $2.\int_0^2 \left[ \sqrt{2^2 - (x-2)^2} - \sqrt{2x} \right] dx$

$$= 2\left[ \left[ \frac{x-2}{2} \cdot \sqrt{2^2 - (x-2)^2} + \frac{2^2}{2} \sin^{-1}\left( \frac{x-2}{2} \right) \right]_0^2 - \left[ \sqrt{2} \cdot \frac{x^{3/2}}{3/2} \right]_0^2 \right]$$

$$= 2\left[ \left( 0 + 0 - 1.0 + 2.\frac{\pi}{2} \right) - \frac{2\sqrt{2}}{3}(2^{3/2} - 0 \right]$$

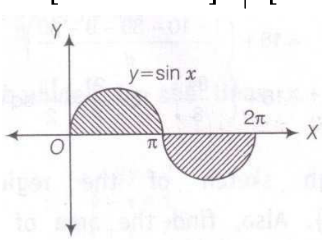
$$= \frac{4\pi}{2} - \frac{8 \cdot 2}{3} = 2\pi - \frac{16}{3} = 2\left( \pi - \frac{8}{3} \right) sq\,units$$

**17. Find the area bounded by the curve $y = sin\,x$ between $x = 0 \; and \; x = 2\pi$.**

Sol. Required area = $\int_0^{2\pi} \sin x \, dx = \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$

$= -[\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right|$

$= -[\cos \pi - \cos 0] + \left| -[\cos 2\pi - \cos \pi] \right|$

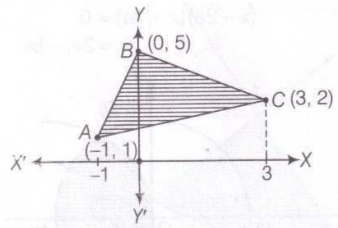



$= -[-1-1] + |-(1+1)|$

$= 2 + 2 = 4 \; sq \; units$

**18. Find the area of region bounded by the triangle whose vertices are ( -1 , 1), (0 , 5) and (3 , 2) using integration.**

Sol. Let we have the vertices of a $\Delta ABC$ as $A(-1, 1), B(0, 5)$ and $C(3, 2)$.




$\therefore$ Equation of AB is $y-1=\left(\frac{5-1}{0+1}\right)(x+1)$

$\Rightarrow y-1=4x+4$

$\Rightarrow y=4x+5 \quad ...(i)$

And equation of BC is $y-5=\left(\frac{2-5}{3-0}\right)(x-0)$

$\Rightarrow y-5=\frac{-3}{3}(x)$

$\Rightarrow y=5-x \quad ...(ii)$

Similarly, equation of AC is $y-1=\left(\frac{2-1}{3+1}\right)(x+1)$

$\Rightarrow y-1=\frac{1}{4}(x+1)$

$\Rightarrow 4y=x+5 \quad ...(iii)$

$\therefore$ Area of shaded region = $\int_{-1}^{0}(y_1-y_2)dx+\int_{0}^{3}(y_1-y_2)dx$

$=\int_{-1}^{0}\left[4x+5-\frac{x+5}{4}\right]dx+\int_{0}^{3}\left[5-x-\frac{x+5}{4}\right]dx$

$=\left[\frac{4x^2}{2}+5x-\frac{x^2}{8}-\frac{5x}{4}\right]_{-1}^{0}+\left[5x-\frac{x^2}{2}-\frac{x^2}{8}-\frac{5x}{4}\right]_{0}^{3}$

$=\left[0-\left(4.\frac{1}{2}+5(-1)-\frac{1}{8}+\frac{5}{4}\right)\right]+\left[\left(15-\frac{9}{2}-\frac{9}{8}-\frac{15}{4}\right)-0\right]$

$=\left[-2+5+\frac{1}{8}-\frac{5}{4}+15-\frac{9}{2}-\frac{9}{8}-\frac{15}{4}\right]$

$=18+\left(\frac{1-10-36-9-30}{8}\right)$

$=18+\left(-\frac{84}{8}\right)=18-\frac{21}{2}=\frac{15}{2}\, sq\, units$

**19. Draw a rough sketch of the region $\{(x, y): y^2 \le 6ax \text{ and } x^2+y^2 \le 16a^2\}$. Also, find the area of the region sketched using method of integration.**

Sol. We have, $y^2=6ax$ and $x^2+y^2=16a^2$

$\Rightarrow x^2+6ax=16a^2$

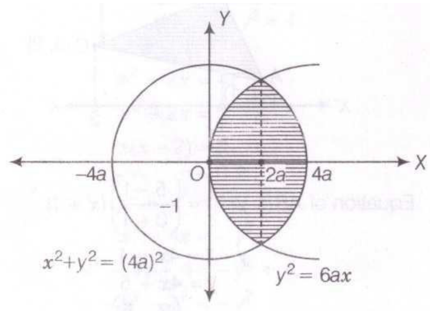
$\Rightarrow x^2+6ax-16a^2=0$

$\Rightarrow x^2+8ax-2ax-16a^2=0$

$\Rightarrow x(x+8a)-2a(x+8a)=0$

$\Rightarrow (x-2a)(x+8a)=0$

$\Rightarrow x=2a,-8a$




$\therefore$ Area of required region= $2\left[\int_0^{2a}\sqrt{6ax}\,dx+\int_{2a}^{4a}\sqrt{(4a)^2-x^2}\,dx\right]$

$=2\left[\int_0^{2a}\sqrt{6a}\,x^{1/2}dx+\int_{2a}^{4a}\sqrt{(4a^2)-x^2}\,dx\right]$

$=2\left[\sqrt{6a}\left[\frac{x^{3/2}}{3/2}\right]_0^{2a}+\left(\frac{x}{2}\sqrt{(4a)^2-x^2}+\frac{(4a)^2}{2}\sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$

$=2\left[\sqrt{6a}.\frac{2}{3}((2a)^{\frac{3}{2}}-0)+\frac{4a}{2}.0+\frac{16a^2}{2}\cdot\frac{\pi}{2}-\frac{2a}{2}\sqrt{16a^2-4a^2}-\frac{16a^2}{2}\cdot\sin^{-1}\frac{2a}{4a}\right]$

$=2\left[\sqrt{6a}\frac{2}{3}\cdot 2\sqrt{2}a^{3/2}+0+4\pi a^2-\frac{2a}{2}\cdot 2\sqrt{3}a-8a^2\cdot\frac{\pi}{6}\right]$

$=2\left[\sqrt{12}\cdot\frac{4}{3}a^2+4\pi a^2-2\sqrt{3}a^2-\frac{4a^2\pi}{3}\right]$

$=2\left[\frac{8\sqrt{3}a^2+12\pi a^2-6\sqrt{3}a^2-4a^2\pi}{3}\right]$

$=\frac{2}{3}a^2[8\sqrt{3}+12\pi-6\sqrt{3}-4\pi]$

$=\frac{2}{3}a^2[2\sqrt{3}+8\pi]=\frac{4}{3}a^2[\sqrt{3}+4\pi]$

**20. Compute the area bounded by the lines $x+2y=2,\ y-x=1$ and $2x+y=7$.**

Sol. We have,

$x+2y=2\ \ \ldots(i)$

$y-x=1\ \ldots(ii)$

and $2x+y=7\ \ \ldots(iii)$

On solving Eqs. (i) and (ii), we get

$y-(2-2y)=1\Rightarrow 3y-2=1\Rightarrow y=1$

$2(y-1)+y=7$

On solving Eqs. (ii) and (iii), We get

$\Rightarrow\ 2y-2+y=7$

$\Rightarrow\ y=3$

On solving Eqs. (i) and (iii), we get

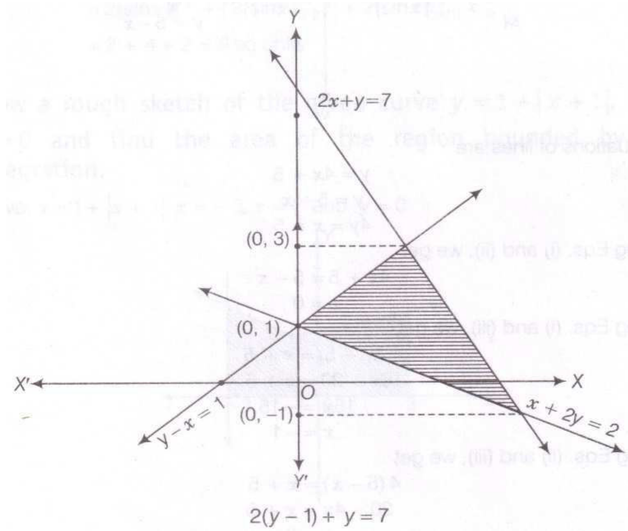
$2(2-2y)+y=7$

$\Rightarrow\ 4-4y+y=7$

$\Rightarrow\ -3y=3$

$\Rightarrow\ y=-1$

$\therefore$ Required area =




$$\int_{-1}^{1}(2-2y)dy+\int_{-1}^{3}\frac{(7-y)}{2}dy-\int_{1}^{3}(y-1)dy$$

$$=\left[-2y+\frac{2y^2}{2}\right]_{-1}^{1}+\left[\frac{7y}{2}-\frac{y^2}{2.2}\right]_{-1}^{3}-\left[\frac{y^2}{2}-y\right]_{1}^{3}$$

$$=\left[-2+\frac{2}{2}-2-\frac{2}{2}\right]+\left[\frac{21}{2}-\frac{9}{4}+\frac{7}{2}+\frac{1}{4}\right]-\left[\frac{9}{2}-3-\frac{1}{2}+1\right]$$

$$=[-4]+\left[\frac{42-9-14+1}{4}\right]-\left[\frac{9-6-1+2}{2}\right]$$

$$=-4+12-2=6\,sq\,units$$

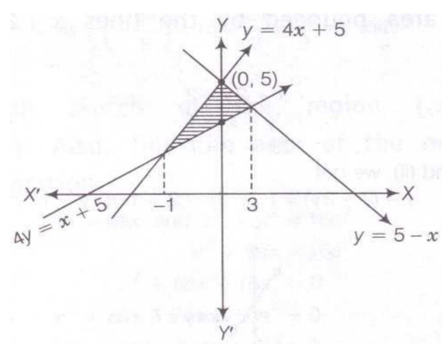
**21. Find the area bounded by the lines $y=4x+5,\ y=5-x$ and $4y=x+5$.**

Sol. Given equations of lines are

$y=4x+5\ \ldots(i)$

$y=5-x\ \ldots.(ii)$ *and*

$4y=x+5\ \ldots(iii)$




On solving Eqs. (i) and (ii), we get

$4x+5=5-x$

$\Rightarrow\ x=0$

On solving Eqs. (i) and (iii)

$4(4x+5)=x+5$

$\Rightarrow 16x+20=x+5$

$\Rightarrow 15x=-15$

$\Rightarrow\ x=-1$

On solving Eqs. (ii) and (iii), we get

$4(5-x)=x+5$

$\Rightarrow 20-4x=x+5$

$\Rightarrow\ x=3$

$\therefore$ Required area = $\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\frac{1}{4}\int_{-1}^{3}(x+5)dx$

$$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$$

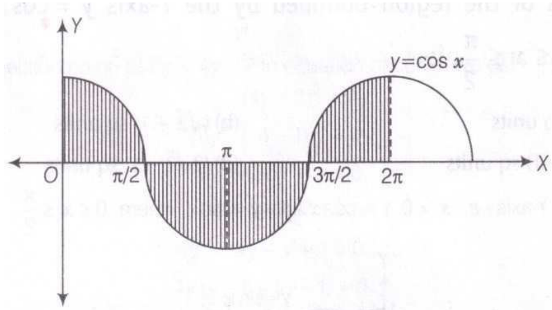
$$= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$$

$$= 3 + \frac{21}{2} - \frac{1}{4} \cdot 24$$

$$= -3 + \frac{21}{2} = \frac{15}{2}\, sq\, units$$

**22.** **Find the area bounded by the curve $y = 2cos\, x$ and the $x - axis$ from x = 0 to $x = 2\pi$.**

Sol. Required area of shaded region= $\int_0^{2\pi} 2\cos x dx$




=

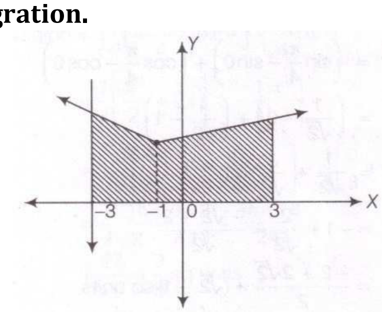
$$\int_0^{\pi/2} 2\cos x dx + \left|\int_{\pi/2}^{3\pi/2} 2\cos x dx\right| + \int_{3\pi/2}^{2\pi} 2\cos x\, dx$$

$$= 2[\sin x]_0^{\pi/2} + \left|2(\sin x)_{\pi/2}^{3\pi/2}\right| + 2[\sin x]_{3\pi/2}^{2\pi}$$

$$= 2 + 4 + 2 = 8\, sq\, units$$

**23.** **Draw a rough sketch of the given curve $y = 1 + |x+1|,\ x = -3,\ x = 3,\ y = 0$ and find the area of the region bounded by them, using integration.**

Sol. We have, $y = 1 + |x+1|,\ x = -3,\ x = 3,\ y = 0$




$$\therefore\ y = \begin{cases} -x, & if\ x < -1 \\ x + 2, & if\ x \geq -1 \end{cases}$$

$\therefore$ Area of shaded region,

$$A = \int_{-3}^{-1} -x dx + \int_{-1}^{3} (x+2) dx$$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\left[\frac{1}{2} - \frac{9}{2}\right] + \left[\frac{9}{2} + 6 - \frac{1}{2} + 2\right]$$

$$= -[-4] + [8 + 4]$$

$$= 12 + 4 = 16\, sq\, units$$

# Application of Integrals

## Objective Type Questions

**Choose the correct answer from the given four options in each of the Exercises 24 to 34.**

**24. The area of the region bounded by the** $y-axis,\ y = cos\ x\ and\ y = sin\,x,\ 0 \le x \le \frac{\pi}{2}$ **is**

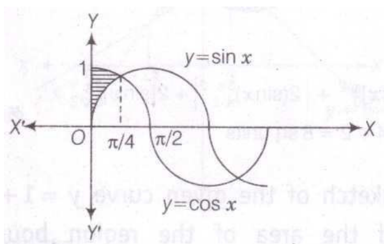
**(A)** $\sqrt{2}\ sq\,units$

**(B)** $(\sqrt{2}+1)\ sq\,units$

**(C)** $(\sqrt{2}-1)\ sq\,units$

**(D)** $(2\sqrt{2}-1)\ sq\,units$

Sol. (C) We have, $y-axis\ i.e.,\ x=0,\ y = cos\ x\ and\ y = sin\,x,$ where $0 \le x \le \frac{\pi}{2}$




$\therefore$ Required area $= \int_0^{\pi/4} (\cos x - \sin x)dx$

$= [\sin x]_0^{\pi/4} + [\cos x]_0^{\pi/4}$

$= \left(\sin\frac{\pi}{4} - \sin 0\right) + \left(\cos\frac{\pi}{4} - \cos 0\right)$

$= \left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - 1\right)$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$

$= -1 + \frac{2}{\sqrt{2}} = \frac{-\sqrt{2}+2}{\sqrt{2}}$

$= \frac{-2+2\sqrt{2}}{2} = \left(\sqrt{2}-1\right) sq\,units$

**25. The area of the region bounded by the curve** $x^2 = 4y$ **and the straight line x= 4y - 2 is**

**(A)** $\frac{3}{8} sq\,units$

**(B)** $\frac{5}{8} sq\,units$

**(C)** $\frac{7}{8} sq\,units$

**(D)** $\frac{9}{8} sq\,units$