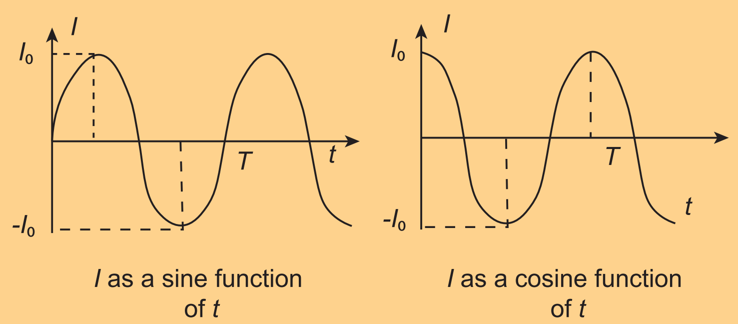
ALTERNATING CURRENT

ALTERNATING CURRENT AND VOLTAGE

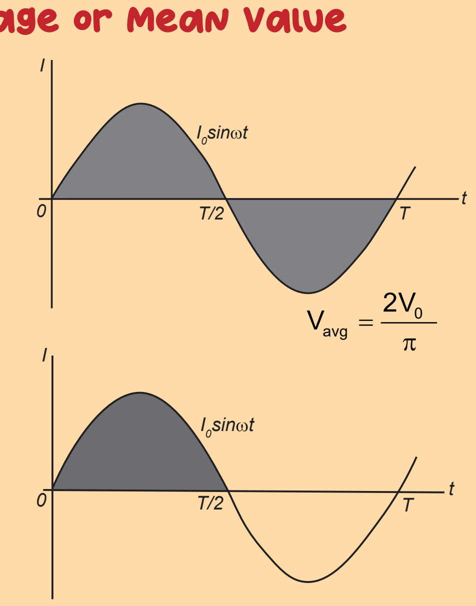
When the magnitude and direction of current and voltage change continuously with time, then current or voltage is said to be alternating.



$I = I_0 \sin(\omega t + \phi)$ or
$I = I_0 \cos(\omega t + \phi)$

I = instantaneous values of current
I_0 = peak value or amplitude
ω = angular frequency
ϕ = initial phase.

Average or Mean Value



$$I_{av} = \frac{2I_0}{\pi}$$

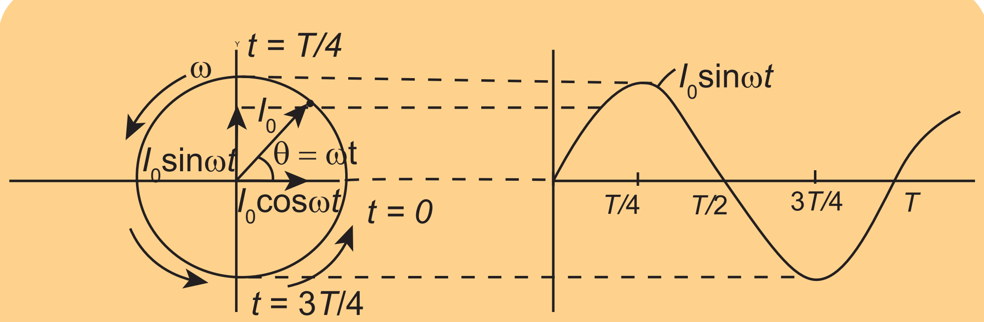
$$V_{avg} = \frac{2V_0}{\pi}$$

$$V_{avg} = \frac{2V_0}{\pi}$$

ROOT MEAN SQUARE VALUE

$$I_{av} = \frac{2I_0}{\pi}$$

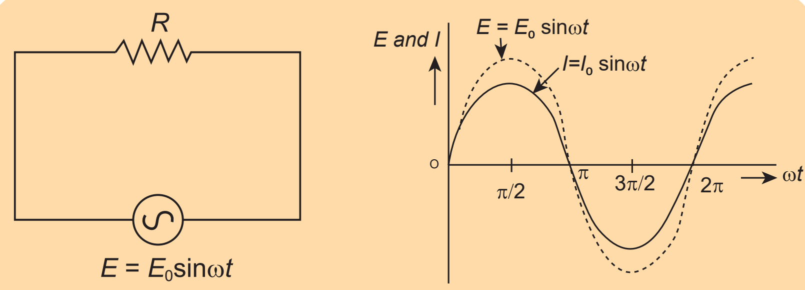
$$V_{av} = \frac{2V_0}{\pi}$$



- The projection phasor on x – axis or y – axis gives the instantaneous value of alternating current/voltage.
- A phase rotates with angular speed ω about the origin.
- Arrow length of this vector is equal to the peak value of alternating current/voltage.

AC SERIES CIRCUIT ANALYSIS

RESISTIVE CIRCUIT

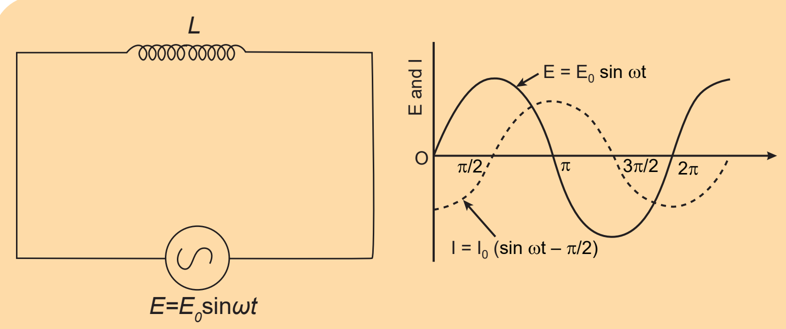


$\phi = 0°$

$P_{av} = \frac{V_0 I_0}{2}$

- Current and voltage are in same phase.

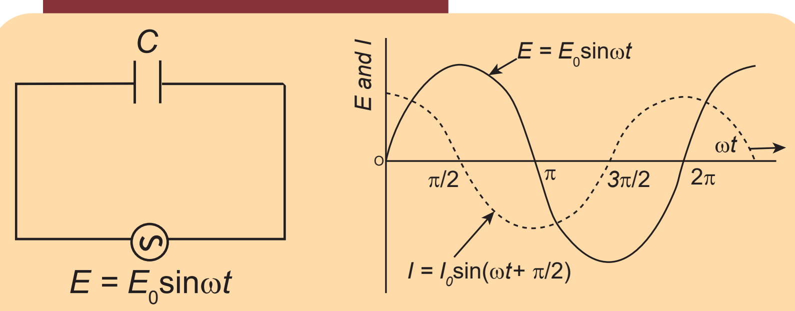
INDUCTIVE CIRCUIT



- $I = I_0 \sin(\omega t - \frac{\pi}{2})$

  $\phi = \frac{\pi}{2}$
- Voltage leads current by $\frac{\pi}{2}$.
- $P_{av} = 0$

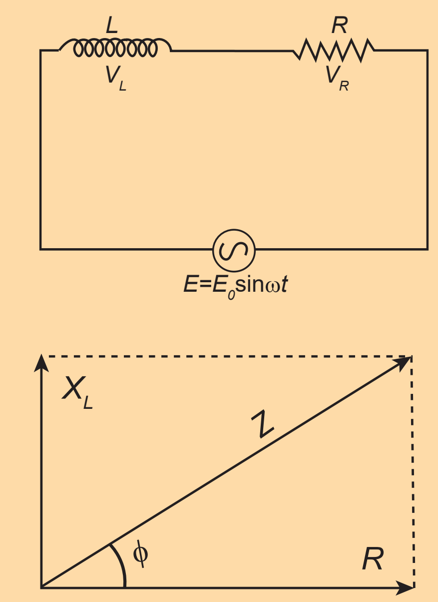
CAPACITIVE CIRCUIT



- $I = I_0 \sin(\omega t + \frac{\pi}{2})$

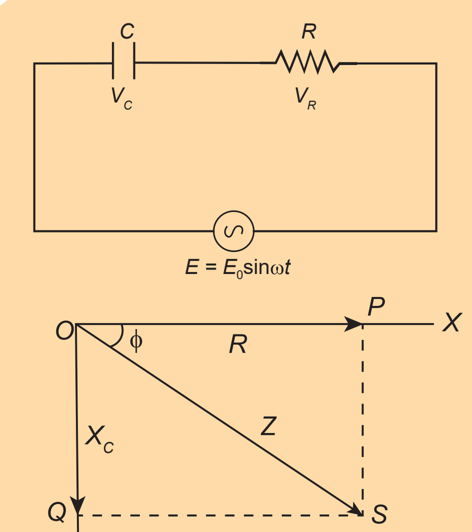
  $\phi = -\frac{\pi}{2}$ or $\frac{\pi}{2}$
- Current leads voltage by $\frac{\pi}{2}$.
- $P_{av} = 0$

L – R CIRCUIT



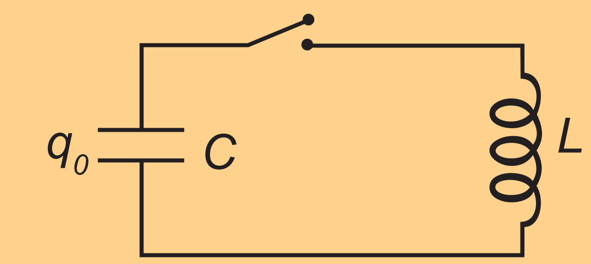
- $I = I_0 \sin(\omega t - \phi)$
- $Z = \sqrt{R^2 + X_L^2}$
- Inductive reactance, $X_L = \omega L$
- Phase angle $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

R – C CIRCUIT



- $I = I_0 \sin(\omega t + \phi)$
- $Z = \sqrt{R^2 + X_C^2}$
- Capacitive reactance, $X_C = \frac{1}{\omega C}$
- $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

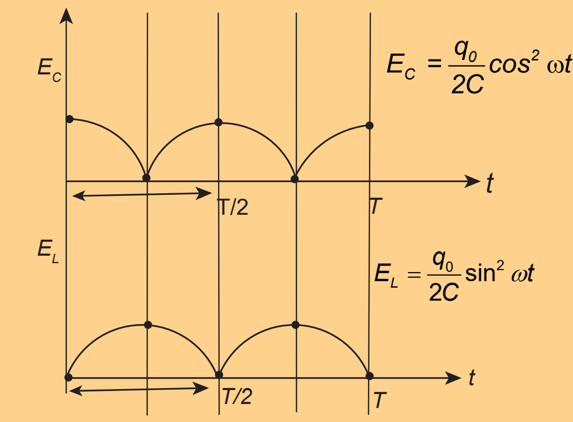
L C OSCILLATIONS



It is defined as the oscillation of energy between capacitor and inductor.

Frequency of Oscillation.

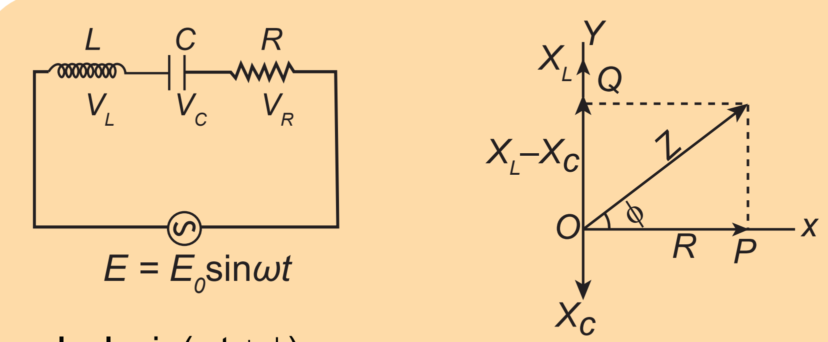
$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$



$$E_C = \frac{q_0}{2C}\cos^2\omega t$$

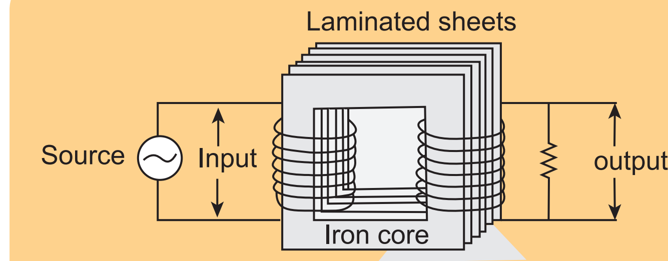
$$E_L = \frac{q_0}{2C}\sin^2\omega t$$

LCR CIRCUIT



- $I = I_0 \sin(\omega t \pm \phi)$
- $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

TRANSFORMER



- **Transformer ratio.**

$$K = \frac{N_s}{N_p} = \frac{\text{No. of turns in Secondary}}{\text{No. of turns in Primary}}$$

ASSUMPTIONS

- No magnetic flux leakage.

  $\frac{E_s}{E_p} = \frac{N_s}{N_p}$
- No power loss, efficiency (n) = 100%.

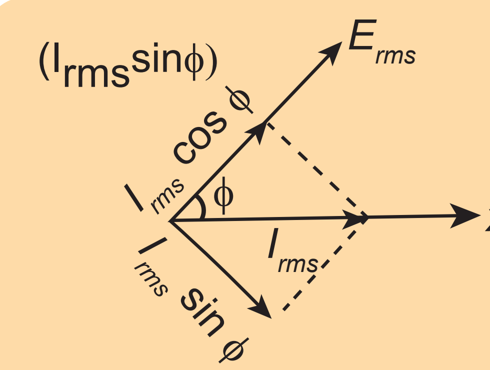
  $n = \frac{P_{Out}}{P_{in}} \times 100\%$, $P_{in} = P_{Out}$

  $\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{N_s}{N_p}$

POWER CONSUMED IN AC CIRCUIT

- Average Power dissipation, $\langle P \rangle = E_{rms} I_{rms} \cos\phi$
- Power factor, $\cos\phi = \frac{\text{Average Power}}{\text{rms Power}} = \frac{R}{Z}$

Wattless Current



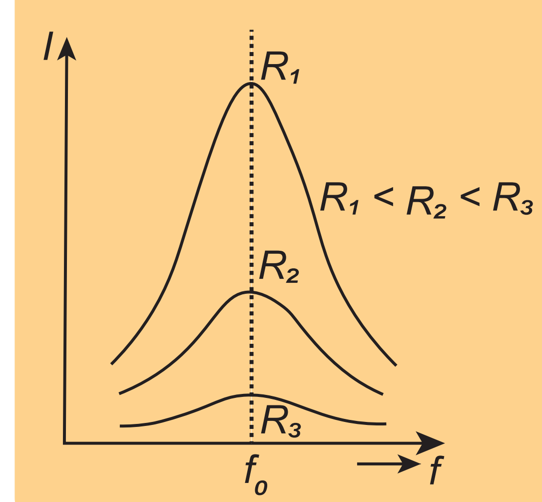
- When the power consumption in AC circuit is zero, then current is said to be wattless current.
- Wattless current is a sine component of current

Half power frequency

- Frequency at which power becomes half of its maximum value.
- At half power frequency,

  $\cos\phi = \frac{1}{2}$ or $\phi = 60°$

Quality Factor



- It represents sharpness curve (I vs f).
- It is unitless and dimensionless.
- $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
- $Q = \frac{f_0}{\text{band width } (\Delta f)}$
- Sharpness $\propto$ Q

RESONANCE IN SERIES LCR CIRCUIT

Conditions for resonance

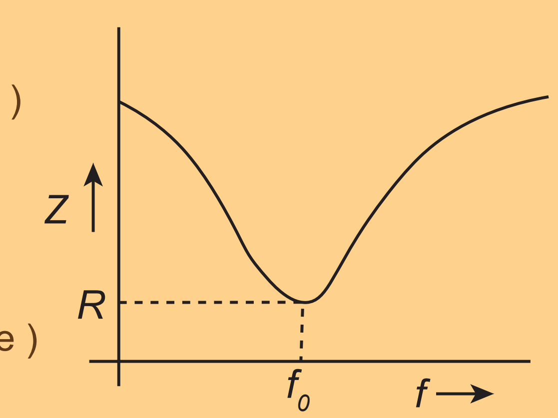
- $Z_{min} = R$
- $I_{max} = \frac{E}{Z_{min}} = \frac{E}{R}$
- $X_L = X_C$
- $V_L = V_C$
- $\cos\phi = 1$
- $\phi = 0°$

Resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Variation of Z with f

1) $f < f_0$, $X_L < X_C$ ➤ ϕ (negative)
➤ Capacitive in nature.
2) $f = f_0$, $X_L = X_C$ ➤ $\phi = 0$
➤ Resistive in nature
3) $f > f_0$, $X_L > X_C$ ➤ ϕ (Positive)
➤ Inductive in nature.



Variation of I with f

- As frequency (f) increases current (I) decreases
- Band width, $\Delta f = f_2 - f_1$