Test Booklet Set No. **18**

SUBJECT : MATHEMATICS

GUJARAT COMMON ENTRANCE TEST (GUJCET) 2018

Date: 23 April, 2018 | Duration: 1 Hours | Max. Marks: 40

:: IMPORTANT INSTRUCTIONS ::

1. The Physics and Chemistry test consists of 40 questions. Each question carries 1 mark. For each correct response, the candidate will get 1 mark. For each incorrect response ¼ mark will be deducted. The maximum marks are 40.
2. This test is of 1 hr. duration.
3. Use Black Ball Point Pen only for writing particulars on OMR Answer Sheet and marking answer by darkening the circle '•'.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room/Hall. The candidates are allowed to take away this Test Booklet with them.
6. The Set No. for this Booklet is **18**. Make sure that the Set No. printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write you Seat No. anywhere else, except in the specified space in the Test Booklet/Answer Sheet.
9. Use of White fluid for correction is not permissible on the Answer Sheet.
10. Each candidate must show on demand his/her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her sent.
12. Use of Manual Calculator is permissible.
13. The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak - 01). Cases where a candidate has not signed the Attendance Sheet (Patrak - 01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candidates will write the Correct Test Booklet Set No. As given in the Test Booklet/Answer Sheet in the Attendance Sheet. (Patrak - 01)

Candidate's Name : ……………………………………………………………………………………………………

Exam. Seat No. (in figures)…………………………………….(in words)…………………………………………….

Name of Exam. Centre : ………………………………………Exam. Centre No. : …………………………………

Test Booklet Set No. : ……………………………………………Test Booklet No. : ………………………………..

Candidate's Sign……………………………………………Block Supervisor Sign……………………………………………

MATHEMATICS

1. If $x^4 + y^4 + z^4 = 0$, then $\begin{vmatrix} 1 & xy & yz \\ zx & 1 & xy \\ yz & zx & 1 \end{vmatrix}$ = (where $x,y,z \in R$)

(A) 0 (B) x + y + z + 3 (C) 1 (D) xyx + 2

Ans. **(C)**

Sol. x = y = z = 0

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

2. $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} =$

(A) 2(10!. 12!. 13!) (B) 2(10!. 11!. 12!) (C) –2(10!. 11!. 12!) (D) 2(10!. 13!)

Ans. **(B)**

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 11 & 12 & 13 \\ 11.12 & 12.13 & 13.14 \end{vmatrix}$

$$10!\ 11!\ 12! \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & 13 \\ -24 & -26 & 13.14 \end{vmatrix}$$

10! 11! 12! (26 – 24) = 2.10! 11! 12!

3. If s = p + q + r, then value of $\begin{vmatrix} s+r & p & q \\ r & s+p & q \\ r & p & s+q \end{vmatrix}$ is

(A) $3s^3$ (B) $2s^3$ (C) s^3 (D) $2s^2$

Ans. **(B)**

Sol. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} p+q+r+s & p & q \\ p+q+r+s & s+p & q \\ p+q+r+s & p & s+q \end{vmatrix} = 2s \begin{vmatrix} 1 & p & q \\ 1 & s+p & q \\ 1 & p & s+q \end{vmatrix} = 2s \begin{vmatrix} 0 & -s & 0 \\ 0 & s & -s \\ 1 & p & s+q \end{vmatrix} \quad R_1 \to R_1 - R_2 \ \& \ R_2 \to R_2 - R_3$$

$= 2s(s^2) = 2s^3$

4. If A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and B = ________ then AB = BA, where $B \neq I$.

(A) $\begin{bmatrix} x & 0 \\ y & y \end{bmatrix}$ (B) $\begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$ (C) $\begin{bmatrix} x & y \\ 0 & y \end{bmatrix}$ (D) $\begin{bmatrix} x & x \\ y & 0 \end{bmatrix}$

Ans. **(B)**

Sol. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ let $B = \begin{bmatrix} x & z \\ y & w \end{bmatrix}$

AB = BA

$\Rightarrow \begin{bmatrix} x+y & z+w \\ y & w \end{bmatrix} = \begin{bmatrix} x & x+z \\ y & w+y \end{bmatrix}$

$x + y = x \Rightarrow \quad y = 0$

$x = w$

$\Rightarrow \quad B = \begin{bmatrix} x & z \\ 0 & x \end{bmatrix}$

5. If $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, then $A^3 =$

(A) 729A (B) 81A (C) 243A (D) 27A

Ans. **(B)**

Sol. $A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 27 & 27 & 27 \\ 27 & 27 & 27 \\ 27 & 27 & 27 \end{bmatrix}$

$A^3 = \begin{bmatrix} 243 & 243 & 243 \\ 243 & 243 & 243 \\ 243 & 243 & 243 \end{bmatrix} = 81 \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 81A$

6. $\frac{d}{dx}\log_{|x|} e =$

(A) e^x (B) $\frac{1}{(\log x)^2}$ (C) $\frac{-1}{x(\log|x|)^2}$ (D) $\frac{1}{|x|}$

Ans. **(C)**

Sol. $\frac{d}{dx}\ln_{|x|} e = \frac{d}{dx}\left(\frac{1}{\ln|x|}\right] = -\frac{1}{x\ln^2|x|}$

7. $\frac{d}{dx}\tan^{-1}\left(\frac{1-x}{1+x}\right) =$

(A) $\frac{-2}{1+x^2}$ (B) $\frac{-1}{1+x^2}$ (C) $\frac{2}{1+x^2}$ (D) $\frac{1}{1+x^2}$

Ans. **(B)**

Sol. $\frac{1}{1+\left(\frac{1-x}{1+x}\right)^2} \times \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2} = \frac{(1+x)^2}{(1+x)^2+(1-x)^2} \times \frac{(-1-x)-1+x}{(1+x)^2} = \frac{-2}{2(1+x^2)} = \frac{-1}{1+x^2}$

8. If $x = at^2$, $y = 2at$, then $\frac{d^2x}{dy^2} =$

(A) $\frac{1}{2a}$ (B) $-2at^3$ (C) $\frac{-1}{2at^3}$ (D) $\frac{-1}{t^2}$

Ans. **(A)**

Sol. $\frac{dx}{dt} = 2\,at$, $\frac{dy}{dt} = 2a$

$\frac{dx}{dy} = t$

$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{dt}{dy} = \frac{1}{2a}$

9. $\int xe^{x^2\log 2}\cdot e^{x^2}\,dx =$ _____ + c

(A) $\frac{(2e)^{x^2}}{\log(2e)}$ (B) $\frac{2^{x^2}.e^{x^2}}{1+\log 2}$ (C) $\frac{2^{x^2}.e^{x^2}}{2(1+\log 2)}$ (D) $\frac{2^{x^2\log 2}.e^{x^2}}{\log 2}$

Ans. **(C)**

Sol. $\int x.2^{x^2}.e^{x^2}\,dx$

Let $x^2 = t$

$2xdx = dt$

$= \frac{1}{2}\int e^t.2^t dt = \frac{1}{2}\frac{(2e)^t}{\ln(2e)} + C = \frac{2^{x^2}.e^{x^2}}{2(\ln 2+1)} + C$

10. $\int\left(\frac{1}{x-3} - \frac{1}{x^2-3x}\right)dx =$ ________ + c, x > 3

(A) $\frac{1}{3}\log(x(x-3))$ (B) $\frac{1}{3}\log(\sqrt{x}\,(x-3))$ (C) $\frac{2}{3}\log(x(x-3))$ (D) $\frac{2}{3}\log(\sqrt{x}\,(x-3))$

Ans. **(D)**

Sol. $\int\left(\frac{1}{x-3} - \frac{1}{x^2-3x}\right)dx$

$\int\left(\frac{1}{x-3} - \frac{1}{x(x-3)}\right)dx = \int\left(\frac{1}{x-3} + \frac{1}{3x} - \frac{1}{3(x-3)}\right)dx = \int\left(+\frac{2}{3(x-3)} + \frac{1}{3x}\right)dx$

$= \frac{2}{3}\ln(x-3) + \frac{2}{3}\ln\sqrt{x} + C = \frac{2}{3}\ln(\sqrt{x}\,(x-3)) + C$

11. What is the mean of f(x) = 3x + 2 where x is a random variable with probability distribution

X = x	1	2	3	4
P(X = x)	1/6	1/3	1/3	1/6

(A) $\frac{19}{2}$ (B) $\frac{15}{2}$ (C) $\frac{5}{2}$ (D) $\frac{5}{3}$

Ans. **(D)**

Sol. Mean $\frac{\sum p_i x_i}{\sum p_i} = \frac{1}{6} + \frac{2}{3} + 1 + \frac{4}{6} = \frac{1+4+6+4}{6} = \frac{15}{6} = \frac{5}{3}$

12. The probability that an event A occurs in a single trial of an experiment is 0.3. Six independent trials of the experiment are performed. What is the variance of probability distribution of occurrence of event A ?

(A) 1.8 (B) 0.18 (C) 12.6 (D) 1.26

Ans. (D)

Sol. $p = \frac{3}{10}, q = \frac{7}{10}, n = 6$

$\therefore$ Variable $= n\,p\,q = 6 \times \frac{3}{10} \times \frac{7}{10} = \frac{126}{100} = 1.26$

13. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{5}$. The probability of atleast one of atleast one of them is true when asked to speak on an event is

(A) $\frac{4}{25}$ (B) $\frac{2}{25}$ (C) $\frac{3}{25}$ (D) $\frac{23}{25}$

Ans. (D)

Sol. $P(A) = \frac{4}{5}, P(B) = \frac{3}{5}$

$P(A') = \frac{1}{5}\ P(B) = \frac{2}{5}$

$\therefore$ P(bath false) $= \frac{1}{5} \cdot \frac{2}{5} = \frac{2}{25}$

$\therefore$ P(at least one is true) = 1 – P(bath false) $= 1 - \frac{2}{25} = \frac{23}{25}$

14. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let z = px + qy where p, q > 0. Condition on p and q so that the maximum of z occurs at both the points (15, 15) and (0, 20) is

(A) q = 3p (B) p = 2q (C) q = 2p (D) p = q

Ans. (A)

Sol. z = px + qy

z(0, 10) = 10q

z(5,5) = 5p + 5q

z(15, 15) = 15p + 15q

z(0, 20) = 20q

maxi. at (15, 15) and (0, 20)

$\therefore$ 15p + 15q = 20q

$\Rightarrow$ 15p = 5q

$\Rightarrow$ q = 3p

15. What is approximate value of $\sqrt[5]{242.999}$?

(A) $\frac{1214999}{4050}$ (B) $\frac{1115}{405}$ (C) $\frac{1214999}{405000}$ (D) $\frac{121499}{40500}$

Ans. (C)

Sol. $\sqrt[5]{242.999}$

Let $f(x) = x^{1/5}$

$f(a + h) = f(a) + hf'(a)$ $f'(x) = \frac{1}{5} x^{\frac{-4}{5}}$

$a = 243, h = 0.001$

$f(243 - 0.001) = (243)^{1/5} - (0.001) \frac{1}{5} (243)^{\frac{-4}{5}}$

$f(242.999) = 3 - \frac{1}{1000} \times \frac{1}{5} \times \frac{1}{81} = 3 - \frac{1}{405000} = \frac{1214999}{405000}$

16. The length of subtangent at any point of the curve logy = 25x is

(A) constant (B) proportional to y (C) zero (D) proportional to x

Ans. **(A)**

Sol. $\ell ny = 25x$

$\therefore \quad \frac{1}{y}\frac{dy}{dx} = 25$

$\frac{dy}{dx} = 25y$

subtangent = $\frac{y}{dy/dx} = \frac{y}{25y} = \frac{1}{25}$

17. Where does $f(x) = x + \sqrt{1-x}$, $0 < x < 1$ decrease ?

(A) $\left(\frac{3}{4}, \infty\right)$ (B) (0, 1) (C) $\left(\frac{3}{4}, 1\right)$ (D) $\left(0, \frac{3}{4}\right)$

Ans. **(C)**

Sol. $f(x) = x + \sqrt{1-x}$

$f'(x) = 1 + \frac{1}{2\sqrt{1-x}}(-1) = 1 - \frac{1}{2\sqrt{1-x}} < 0$ for $x \in \left(\frac{3}{4}, 1\right)$

18. If $f'(x) = 2 - \frac{5}{x^4}$ and $f(1) = \frac{14}{3}$, then $f(-1) =$

(A) 0 (B) $\frac{11}{3}$ (C) $-\frac{14}{3}$ (D) $-\frac{8}{3}$

Ans. **(D)**

Sol. $f'(x) = 2 - \frac{5}{x^4}$, $f(1) = \frac{14}{3}$

$f(x) = 2x + \frac{5}{3x^3} + c$

$\frac{14}{3} = 2 + \frac{5}{3} + c$

$C = \frac{14}{3} - \frac{11}{3} = 1$

$\therefore \quad f(x) = 2x + \dfrac{5}{3x^3} + 1$

$\therefore \quad f(-1) = -2 - \dfrac{5}{3} + 1 = \dfrac{-8}{3}$

19. $\int \dfrac{\cos\alpha}{\sin x \cos(x-\alpha)} dx =$ ________ + c where $0 < x < \alpha < \dfrac{\pi}{2}$ and α-constant

(A) – log|cotx + tanα|
(B) log|cotx + tanα|
(C) – log|tanx + cotα|
(D) log|tanx + cotα|

Ans. **(A)**

Sol. $\int \dfrac{\cos\alpha}{\sin x \cos(\alpha - x)} dx = \int \dfrac{\cos((\alpha - x) + x)}{\sin x \cos(\alpha - x)} dx = \int \dfrac{\cos(\alpha - x)\cos x - \sin(\alpha - x)\sin x}{\sin x \cos(\alpha - x)} dx$

$= \int ((\cot x - \tan(\alpha - x)) \; dx = \ln|\sin x| + \ln|\sec(\alpha - x)| = \ln \sin x - \ln \cos(\alpha - x)$

$= -\ln\left(\dfrac{\cos(\alpha - x)}{\sin x}\right) = -\ln\left(\dfrac{\cos\alpha\cos x + \sin\alpha \sin x}{\sin x}\right) = -\ln|\cot x + \tan\alpha| + C$

20. $\int \dfrac{e^{\cot^{-1}x}}{1+x^2}(x^2 - x + 1)dx =$ ______ + c

(A) $-\; e^{\cot^{-1}x}$
(B) $x.e^{\cot^{-1}x}$
(C) $\dfrac{e^{\cot^{-1}x}}{1+x^2}$
(D) $e^{\cot^{-1}x}$

Ans. **(B)**

Sol. $\int \dfrac{e^{\cot^{-1}x}(x^2 - x + 1)}{1+x^2} dx = \int e^{\cot^{-1}x}\left(1 - \dfrac{x}{1+x^2}\right)dx = \int e^{\cot^{-1}x}.1\,dx - \int e^{\cot^{-1}x}\dfrac{x}{1+x^2}dx$

$= e^{\cot^{-1}x}.x - \int e^{\cot^{-1}x}\left(-\dfrac{1}{1+x^2}\right).xdx - \int e^{\cot^{-1}x}\dfrac{x}{1+x^2}dx = x\cot^{-1}x + C$

21. $\int_0^{\pi/2} (x - [\cos x])dx =$ ______(where [t] = greatest integer less or equal to t)

(A) $\dfrac{\pi}{4}$
(B) $\dfrac{\pi^2}{8} - \dfrac{\pi}{8}$
(C) $\dfrac{\pi^2}{8} - 1$
(D) $\dfrac{\pi^2}{8}$

Ans. **(D)**

Sol. $\int_0^{\pi/2} (x - [\cos x])dx = \int_0^{\pi/2} xdx = \left[\dfrac{x^2}{2}\right]_0^{\pi/2} = \dfrac{\pi^2}{8}$

22. If $\int_{\log 2}^{a} \dfrac{e^x}{\sqrt{e^x - 1}} dx = 2$, then a =

(A) 0
(B) 2log2
(C) log5
(D) log2

Ans. **(C)**

Sol. $\int_{\log 2}^{a} \frac{e^x}{e^x - 1} dx = 2$

$\therefore \int_{1}^{e^a - 1} \frac{dt}{\sqrt{t}} = 2$

$\Rightarrow \left[2\sqrt{t}\right]_1^{e^a - 1} = 2$

$\sqrt{e^a - 1} - 1 = 1$

$e^a - 1 = 4, e^a = 5$

$a = \ln 5$

23. $\int_0^{\sqrt{2}} \sqrt{2 - x^2}\ dx =$ __________.

(A) $\frac{\pi}{2}$ (B) π (C) 0 (D) $-\frac{\pi}{2}$

Ans. **(A)**

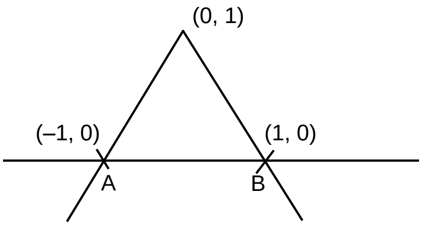
Sol. $\int_0^{\sqrt{2}} \sqrt{2 - x^2}\ dx = \left(\frac{x}{2}\sqrt{2 - x^2} + \frac{2}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)_0^{\sqrt{2}} = \sin^{-1}(1) = \pi/2$

24. Area of the region bounded by rays | x | + y = 1 and X-axis is ___________

(A) $\frac{1}{4}$ (B) 2 (C) $\frac{1}{2}$ (D) 1

Ans. **(D)**

Sol.



$y = 1 - |x|$

Area of region = Area of $\Delta ABC = \frac{1}{2} \times 1 \times 2 = 1$

25. If area bounded by the curves $x = ay^2$ and $y = ax^2$ is 1, then a = _________

(A) 3 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{3}}$

Ans. **(D)**

Sol. $x = ay^2$ & $y = ax^2$

Area of region = $\frac{1}{3a^2} = 1$ {$\therefore$ Area bounded by $y^2 = 4ax$ & $x^2 = 4by = \frac{16}{3}ab$}

$\Rightarrow a = \frac{1}{\sqrt{3}}$

26. The solution of the differential equation $2x\dfrac{dy}{dx} - y = 0$; y (1) = 2 represents ________

(A) ellipse (B) parabola (C) circle (D) straight line

Ans. **(B)**

Sol. 2x dy = y dx

$2\dfrac{dy}{y} = \dfrac{dx}{x}$

2 lny = lnx + ln c

$\ln y^2 = \ln x c$

$y^2 = xc$

at x = 1, y = 2

$\Rightarrow$ 4 = c

curve $y^2 = 4x$

27. Particular solution of differential equation $e^{\frac{dy}{dx}} = x$: y(1) = 3 ; x > 0 is __________

(A) $2y = x^2 + 5$ (B) $y = \log x - x + 4$ (C) $\log y = x^2 + 4$ (D) $y^2 = \log x + 4$

Ans. **(BONUS)**

Sol. $e^{\frac{dy}{dx}} = x$

$\dfrac{dy}{dx} = \ln x$

dy = lnxdx

y = lnx.x – x + C

3 = 0 – 1 + C

C = 4

y = xlnx – x + 4

28. The population of a city increases at the rate 3% per year. If at time t the population of city is p, then find equation of p in time t.

(A) $p = \dfrac{3}{100}e^{3t}$ (B) $p = 3e^{\frac{3t}{100}}$ (C) $p = c\,e^{\frac{3t}{100}}$ (D) $p = e^{\frac{3t}{100}}$

Ans. **(C)**

Sol. Let population of city is p

$\dfrac{dp}{dt} = 3\% = \dfrac{3p}{100} \Rightarrow \dfrac{dp}{p} = \dfrac{3}{100}dt \Rightarrow \ln p = \dfrac{3}{100}t + C$

$\Rightarrow p = e^{\frac{3}{100}t + C} = \lambda e^{\frac{3}{100}t}$

29. If $\bar{a}$ is unit vector, then $|\bar{a} \times \hat{i}|^2 + |\bar{a} \times \hat{j}|^2 + |\bar{a} \times \hat{k}|^2 =$ ___________

(A) 3 (B) 1 (C) 2 (D) 0

Ans. **(C)**

Sol. $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$|a \times \hat{i}|^2 = |-a_2\hat{k} \times a_3\hat{j}|^2 + |a_1\hat{k} - a_3\hat{j}|^2 + |-a_1\hat{j} + a_2\hat{i}|^2$

$= (a_2^2 + a_3^2) + (a_1^2 + a_3^2) + (a_1^2 + a_2^2) = 2|a|^2 = 2$

30. If for unit vectors $\bar{a}$ and $\bar{b}$, $\bar{a} + 2\bar{b}$ and $5\bar{a} - 4\bar{b}$ are perpendicular to each-other, then $(\bar{a} \wedge \bar{b})$ = ____________

(A) $\cos^{-1}\frac{2}{7}$ (B) $\frac{\pi}{3}$ (C) $\cos^{-1}\frac{1}{3}$ (D) $\frac{\pi}{4}$

Ans. **(B)**

Sol. $|\vec{a}| = 1$ $|\vec{b}| = 1$

$(\vec{a} + 2\vec{b}).(5\vec{a} - 4\vec{b}) = 0$

$5 - 4\vec{a}.\vec{b} + 10\vec{a}.\vec{b} - 8 = 0$

$-3 = -6\ \vec{a}.\vec{b} \Rightarrow \vec{a}.\vec{b} = \frac{1}{2}$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

31. If a vector $\bar{x}$ makes angles with measure $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ with positive directions of X-axis and Y-axis respectively. then $\bar{x}$ made angle of measure __________ with positive direction of Z-axis.

(A) $\frac{5\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans. **(C)**

Sol. $\alpha = \frac{\pi}{4}, \beta = \frac{5\pi}{4}$

Let angle with z-axis is γ

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1 \Rightarrow y = \frac{\pi}{2}$

32. If a plane has X-intercept *l*, Y-intercept m and Z-intercept n, and perpendicular distance of plane from origin is k, then

(A) $\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = k^2$ (B) $\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{k^2}$ (C) $l^2 + m^2 + n^2 = \frac{1}{k^2}$ (D) $l^2 + m^2 + n^2 = k^2$

Ans. **(B)**

Sol. Equation of plane $\frac{x}{\ell} + \frac{y}{m} + \frac{z}{n} = 1$

$\perp$ dist. from (0,0,0) is $k = \frac{1}{\sqrt{\frac{1}{\ell^2} + \frac{1}{m^2} + \frac{1}{n^2}}}$

$\Rightarrow \frac{1}{\ell^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{k^2}$

33. Lines $\bar{r} = (3 + t)\hat{i} + (1 - t)\hat{j} + (-2 - 2t)\hat{k}$, $t \in R$ and $x = 4 + k$, $y = -k$, $z = -4 - 2k$, $k \in R$, then relation between lines is__________.

(A) perpendicular (B) coincident (C) skew (D) parallel

Ans. **(B)**

Sol. $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - \hat{j} - 2\hat{k})$. $t \in R$ be a line

Passes through the point (3, 1, –2) and || to vector $\hat{i} - \hat{j} - 2\hat{k}$

other line $\frac{x-4}{1} = \frac{y}{-1} = \frac{z+4}{-2} = k \in R$

Clealy both lines are parallel

also point (3, 1, –2) lies on $\frac{x-4}{1} = \frac{y}{-1} = \frac{z+4}{-2}$

Hence both lines are coincident (B)

34. The equation of plane containing intersecting lines $\frac{x+3}{3} = \frac{y}{1} = \frac{z-2}{2}$ and $\frac{x-3}{4} = \frac{y-2}{2} = \frac{z-6}{3}$ is________.

(A) x + 2y – 2z + 9 = 0 (B) 2x –y + z + 9 = 0 (C) x + y + z + 5 = 0 (D) x + y – 2z + 7 = 0

Ans. **(D)**

Sol. Equation of plane containing both lines is

$$\begin{vmatrix} x+3 & y & z-2 \\ 3 & 1 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 0 \quad \Rightarrow -(x+3) - y + 2(z-2) = 0 \quad \Rightarrow x + y - 2z + 7 = 0$$

35. The number of binary operations on the set {1, 2, 3} is_________.

(A) 3! (B) 9^3 (C) 3^9 (D) 27

Ans. **(C)**

Sol. Number of binary operations = $n^{n^2} = 3^9$

36. Function f : N → f(n) = $\begin{cases} \frac{n}{2}, & n-\text{even} \\ -\left(\frac{n-1}{2}\right), & n-\text{odd} \end{cases}$ is________.

(A) not one-one and not onto (B) one-one but not onto

(C) not one-one but onto (D) one-one and onto

Ans. **(D)**

Sol. f(n) = $\begin{cases} \frac{n}{2} & n-\text{even} \\ -\left(\frac{n-1}{2}\right) & n-\text{odd} \end{cases}$

Clealy f is one-one function as for each even natural number we have different image as positive integer and for odd natural number image set is set of non-positive integer.

also their are no any integer for which pre image does not exists. Hence one-one onto (D)

37. The relation S = {(3, 3), 4, 4} on the set A = {3, 4, 5} is_________.

(A) an equivalence relation (B) reflexive only

(C) not reflexive but symmetric and transitive (D) symmetric only

Ans. **(C)**

Sol. S = {(3, 3) , (4, 4)}

(5, 5) ∉ S Hence not reflexive

But symmetric and transitive

38. $\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = +$________.

(A) $\frac{\pi}{2}-\frac{1}{2}\cot^{-1}x$ (B) $\cot^{-1}x$ (C) $-\frac{1}{2}\tan^{-1}x$ (D) $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}x$

Ans. **(D)**

Sol. Put $x = \tan\theta$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \cot^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \cot^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \cot^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\pi}{2} - \frac{\theta}{2} = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}x$$

39. If $\cos(2\tan^{-1}x) = \frac{1}{2}$, then value of x is_________.

(A) $1-\frac{1}{\sqrt{3}}$ (B) $\pm\sqrt{3}$ (C) $\sqrt{3}-1$ (D) $\pm\frac{1}{\sqrt{3}}$

Ans. **(D)**

Sol. $\cos(2\tan^{-1}x) = \frac{1}{2}$

$2\tan^{-1}x = \frac{\pi}{3}, -\frac{\pi}{3}$

$\tan^{-1}x = \frac{\pi}{6}, -\frac{\pi}{6}$

$\Rightarrow$ $x = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

40. $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x)) =$__________.

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) 0

Ans. **(C)**

Sol. $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$

$= \sin^{-1}\left(\sqrt{1-x^2}\right) + \cos^{-1}\left(\sqrt{1-x^2}\right) = \frac{\pi}{2}$