we transpose 10 to RHS,

$$4x = 66 - 10$$

or $$4x = 56$$

or $$x = \frac{56}{4} = 14 \qquad \text{(solution)}$$

Thus, Sahil's present age is 14 years and his mother's age is 42 years. (You may easily check that 5 years from now the sum of their ages will be 66 years.)

Example 8: Bansi has 3 times as many two-rupee coins as he has five-rupee coins. If he has in all a sum of ₹ 77, how many coins of each denomination does he have?

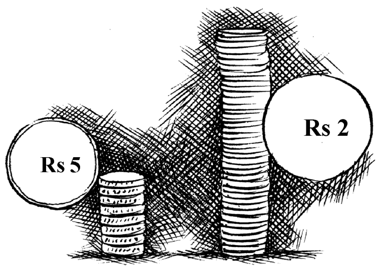
Solution: Let the number of five-rupee coins that Bansi has be x. Then the number of two-rupee coins he has is 3 times x or $3x$.

The amount Bansi has:

(i) from 5 rupee coins, ₹ $5 \times x$ = ₹ $5x$

(ii) from 2 rupee coins, ₹ $2 \times 3x$ = ₹ $6x$

Hence the total money he has = ₹ $11x$



But this is given to be ₹ 77; therefore,

$$11x = 77$$

or $$x = \frac{77}{11} = 7$$

Thus, number of five-rupee coins = x = 7

and number of two-rupee coins = $3x$ = 21 (solution)

(You can check that the total money with Bansi is ₹ 77.)

Example 9: The sum of three consecutive multiples of 11 is 363. Find these multiples.

Solution: If x is a multiple of 11, the next multiple is $x + 11$. The next to this is $x + 11 + 11$ or $x + 22$. So we can take three consecutive multiples of 11 as x, $x + 11$ and $x + 22$.

It is given that the sum of these consecutive multiples of 11 is 363. This will give the following equation:

$$x + (x + 11) + (x + 22) = 363$$

or $$x + x + 11 + x + 22 = 363$$

or $$3x + 33 = 363$$

or $$3x = 363 - 33$$

or $$3x = 330$$

> Alternatively, we may think of the multiple of 11 immediately before x. This is $(x - 11)$. Therefore, we may take three consecutive multiples of 11 as $x - 11$, x, $x + 11$.
>
> In this case we arrive at the equation
>
> $$(x - 11) + x + (x + 11) = 363$$
>
> or $$3x = 363$$

or $$x = \frac{330}{3}$$
$$= 110$$

Hence, the three consecutive multiples are 110, 121, 132 (answer).

or $x = \frac{363}{3} = 121$. Therefore,
$x = 121$, $x - 11 = 110$, $x + 11 = 132$
Hence, the three consecutive multiples are 110, 121, 132.

We can see that we can adopt different ways to find a solution for the problem.

Example 10: The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

Solution: Since the ratio of the two numbers is 2 : 5, we may take one number to be $2x$ and the other to be $5x$. (Note that $2x : 5x$ is same as 2 : 5.)

The difference between the two numbers is $(5x - 2x)$. It is given that the difference is 66. Therefore,

$$5x - 2x = 66$$
or $$3x = 66$$
or $$x = 22$$

Since the numbers are $2x$ and $5x$, they are 2×22 or 44 and 5×22 or 110, respectively.

The difference between the two numbers is $110 - 44 = 66$ as desired.

Example 11: Deveshi has a total of ₹ 590 as currency notes in the denominations of ₹ 50, ₹ 20 and ₹ 10. The ratio of the number of ₹ 50 notes and ₹ 20 notes is 3:5. If she has a total of 25 notes, how many notes of each denomination she has?

Solution: Let the number of ₹ 50 notes and ₹ 20 notes be $3x$ and $5x$, respectively.

But she has 25 notes in total.

Therefore, the number of ₹ 10 notes = $25 - (3x + 5x) = 25 - 8x$

The amount she has

from ₹ 50 notes : $3x \times 50$ = ₹ $150x$

from ₹ 20 notes : $5x \times 20$ = ₹ $100x$

from ₹ 10 notes : $(25 - 8x) \times 10$ = ₹ $(250 - 80x)$

Hence the total money she has $= 150x + 100x + (250 - 80x)$ = ₹ $(170x + 250)$

But she has ₹ 590. Therefore, $170x + 250 = 590$

or $$170x = 590 - 250 = 340$$

or $$x = \frac{340}{170} = 2$$

The number of ₹ 50 notes she has = $3x$
$= 3 \times 2 = 6$

The number of ₹ 20 notes she has = $5x = 5 \times 2 = 10$

The number of ₹ 10 notes she has = $25 - 8x$
$= 25 - (8 \times 2) = 25 - 16 = 9$

EXERCISE 3.2

1. If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?
2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?
3. The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?
4. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.
5. Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?
6. Three consecutive integers add up to 51. What are these integers?
7. The sum of three consecutive multiples of 8 is 888. Find the multiples.
8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.
9. The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?
10. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?
11. Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?
12. Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?
13. A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?
15. I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?
16. The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

3.4 Solving Equations having the Variable on both Sides

An equation is the equality of the values of two expressions. In the equation $2x - 3 = 7$, the two expressions are $2x - 3$ and 7. In most examples that we have come across so far, the RHS is just a number. But this need not always be so; both sides could have expressions with variables. For example, the equation $2x - 3 = x + 2$ has expressions with a variable on both sides; the expression on the LHS is $(2x - 3)$ and the expression on the RHS is $(x + 2)$.

- We now discuss how to solve such equations which have expressions with the variable on both sides.

Example 12: Solve $2x - 3 = x + 2$

Solution: We have

$$2x = x + 2 + 3$$

or $$2x = x + 5$$

or $$2x - x = x + 5 - x \quad \text{(subtracting } x \text{ from both sides)}$$

or $$x = 5 \quad \text{(solution)}$$

Here we subtracted from both sides of the equation, not a number (constant), but a term involving the variable. We can do this as variables are also numbers. Also, note that subtracting x from both sides amounts to transposing x to LHS.

Example 13: Solve $5x + \frac{7}{2} = \frac{3}{2}x - 14$

Solution: Multiply both sides of the equation by 2. We get

$$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$$

$$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$$

or $$10x + 7 = 3x - 28$$

or $$10x - 3x + 7 = -28 \quad \text{(transposing } 3x \text{ to LHS)}$$

or $$7x + 7 = -28$$

or $$7x = -28 - 7$$

or $$7x = -35$$

or $$x = \frac{-35}{7}$$ or $$x = -5 \quad \text{(solution)}$$

EXERCISE 3.3

Solve the following equations and check3your results.

1. $3x = 2x + 18$
2. $5t - 3 = 3t - 5$
3. $5x + 9 = 5 + 3x$
4. $4z + 3 = 6 + 2z$
5. $2x - 1 = 14 - x$
6. $8x + 4 = 3(x - 1) + 7$
7. $x = \frac{4}{5}(x + 10)$
8. $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$
9. $2y + \frac{5}{3} = \frac{26}{3} - y$
10. $3m = 5m - \frac{8}{5}$

3.5 Some More Applications

Example 14: The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

Solution: Take, for example, a two-digit number, say, 56. It can be written as $56 = (10 \times 5) + 6$.

If the digits in 56 are interchanged, we get 65, which can be written as $(10 \times 6) + 5$.

Let us take the two digit number such that the digit in the units place is b. The digit in the tens place differs from b by 3. Let us take it as $b + 3$. So the two-digit number is $10(b + 3) + b = 10b + 30 + b = 11b + 30$.

With interchange of digits, the resulting two-digit number will be

$$10b + (b + 3) = 11b + 3$$

> Could we take the tens place digit to be $(b - 3)$? Try it and see what solution you get.

If we add these two two-digit numbers, their sum is

$$(11b + 30) + (11b + 3) = 11b + 11b + 30 + 3 = 22b + 33$$

It is given that the sum is 143. Therefore, $22b + 33 = 143$

or $\quad 22b = 143 - 33$

or $\quad 22b = 110$

or $\quad b = \frac{110}{22}$

or $\quad b = 5$

> Remember, this is the solution when we choose the tens digits to be 3 more than the unit's digits. What happens if we take the tens digit to be $(b - 3)$?

The units digit is 5 and therefore the tens digit is $5 + 3$ which is 8. The number is 85.

Check: On interchange of digits the number we get is 58. The sum of 85 and 58 is 143 as given.

> The statement of the example is valid for both 58 and 85 and both are correct answers.

Example 15: Arjun is twice as old as Shriya. Five years ago his age was three times Shriya's age. Find their present ages.

Solution: Let us take Shriya's present age to be x years.

Then Arjun's present age would be $2x$ years.

Shriya's age five years ago was $(x-5)$ years.

Arjun's age five years ago was $(2x-5)$ years.

It is given that Arjun's age five years ago was three times Shriya's age.

Thus, $2x - 5 = 3(x - 5)$

or $2x - 5 = 3x - 15$

or $15 - 5 = 3x - 2x$

or $10 = x$

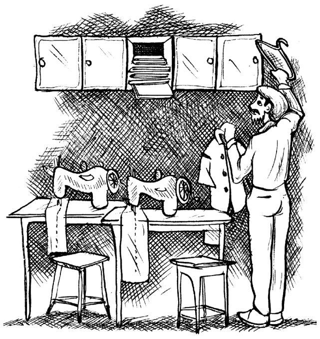
So, Shriya's present age $= x = 10$ years.

Therefore, Arjun's present age $= 2x = 2 \times 10 = 20$ years.

EXERCISE 3.4

1. Amina thinks of a number and subtracts $\frac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?
2. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?
3. Sum of the digits of a two-digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?
4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?
5. Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?
6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate ₹100 per metre it will cost the village panchayat ₹ 75000 to fence the plot. What are the dimensions of the plot?
7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trouser material that costs him ₹ 90 per metre.

For every 3 meters of the shirt material he buys 2 metres of the trouser material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹ 36,600. How much trouser material did he buy?



8. Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

9. A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.

10. Aman's age is three times his son's age. Ten years ago he was five times his son's age. Find their present ages.

3.6 Reducing Equations to Simpler Form

Example 16: Solve $\frac{6x+1}{3}+1=\frac{x-3}{6}$

Solution: Multiplying both sides of the equation by 6,

Why 6? Because it is the smallest multiple (or LCM) of the given denominators.

$$\frac{6\,(6x+1)}{3}+6\times 1 = \frac{6(x-3)}{6}$$

or $2\,(6x + 1) + 6 = x - 3$

or $12x + 2 + 6 = x - 3$ (opening the brackets)

or $12x + 8 = x - 3$

or $12x - x + 8 = -3$

or $11x + 8 = -3$

or $11x = -3 - 8$

or $11x = -11$

or $x = -1$ (required solution)

Check: LHS $=\frac{6(-1)+1}{3}+1=\frac{-6+1}{3}+1 = \frac{-5}{3}+\frac{3}{3}=\frac{-5+3}{3}=\frac{-2}{3}$

RHS $= \frac{(-1)-3}{6}=\frac{-4}{6}=\frac{-2}{3}$

LHS = RHS. (as required)

Example 17: Solve $5x - 2\,(2x - 7) = 2\,(3x - 1) + \frac{7}{2}$

Solution: Let us open the brackets,

LHS $= 5x - 4x + 14 = x + 14$

$$\text{RHS} = 6x - 2 + \frac{7}{2} = 6x - \frac{4}{2} + \frac{7}{2} = 6x + \frac{3}{2}$$

The equation is $x + 14 = 6x + \frac{3}{2}$

or $14 = 6x - x + \frac{3}{2}$

or $14 = 5x + \frac{3}{2}$

or $14 - \frac{3}{2} = 5x$ (transposing $\frac{3}{2}$)

or $\frac{28-3}{2} = 5x$

or $\frac{25}{2} = 5x$

or $x = \frac{25}{2} \times \frac{1}{5} = \frac{5 \times 5}{2 \times 5} = \frac{5}{2}$

Did you observe how we simplified the form of the given equation? Here, we had to multiply both sides of the equation by the LCM of the denominators of the terms in the expressions of the equation.

Therefore, required solution is $x = \frac{5}{2}$.

Check: $\text{LHS} = 5 \times \frac{5}{2} - 2\left(\frac{5}{2} \times 2 - 7\right)$

$$= \frac{25}{2} - 2(5-7) = \frac{25}{2} - 2(-2) = \frac{25}{2} + 4 = \frac{25+8}{2} = \frac{33}{2}$$

$$\text{RHS} = 2\left(\frac{5}{2} \times 3 - 1\right) + \frac{7}{2} = 2\left(\frac{15}{2} - \frac{2}{2}\right) + \frac{7}{2} = \frac{2 \times 13}{2} + \frac{7}{2}$$

$$= \frac{26+7}{2} = \frac{33}{2} = \text{LHS. (as required)}$$

Note, in this example we brought the equation to a simpler form by opening brackets and combining like terms on both sides of the equation.

EXERCISE 3.5

Solve the following linear equations.

1. $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ **2.** $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$ **3.** $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

4. $\frac{x-5}{3}=\frac{x-3}{5}$ **5.** $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$ **6.** $m-\frac{m-1}{2}=1-\frac{m-2}{3}$

Simplify and solve the following linear equations.

7. $3(t-3)=5(2t+1)$ **8.** $15(y-4)-2(y-9)+5(y+6)=0$

9. $3(5z-7)-2(9z-11)=4(8z-13)-17$

10. $0.25(4f-3)=0.05(10f-9)$

3.7 Equations Reducible to the Linear Form

Example 18: Solve $\frac{x+1}{2x+3}=\frac{3}{8}$

Solution: Observe that the equation is not a linear equation, since the expression on its LHS is not linear. But we can put it into the form of a linear equation. We multiply both sides of the equation by $(2x+3)$,

$$\left(\frac{x+1}{2x+3}\right)\times(2x+3)=\frac{3}{8}\times(2x+3)$$

Note that $2x+3\neq 0$ (Why?)

Notice that $(2x+3)$ gets cancelled on the LHS We have then,

$$x+1=\frac{3(2x+3)}{8}$$

We have now a linear equation which we know how to solve.
Multiplying both sides by 8

$$8(x+1)=3(2x+3)$$

This step can be directly obtained by 'cross-multiplication' $\frac{x+1}{2x+3}\nwarrow\!\!\!\nearrow\frac{3}{8}$

or $8x+8=6x+9$

or $8x=6x+9-8$

or $8x=6x+1$

or $8x-6x=1$

or $2x=1$

or $x=\frac{1}{2}$

The solution is $x=\frac{1}{2}$.

Check : Numerator of LHS = $\frac{1}{2}+1=\frac{1+2}{2}=\frac{3}{2}$

Denominator of LHS = $2x+3=2\times\frac{1}{2}+3=1+3=4$

LHS = numerator ÷ denominator = $\frac{3}{2} \div 4 = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$

LHS = RHS.

Example 19: Present ages of Anu and Raj are in the ratio 4:5. Eight years from now the ratio of their ages will be 5:6. Find their present ages.

Solution: Let the present ages of Anu and Raj be $4x$ years and $5x$ years respectively.

After eight years. Anu's age = $(4x + 8)$ years;

After eight years, Raj's age = $(5x + 8)$ years.

Therefore, the ratio of their ages after eight years = $\frac{4x+8}{5x+8}$

This is given to be 5 : 6

Therefore, $\frac{4x+8}{5x+8} = \frac{5}{6}$

Cross-multiplication gives $6(4x + 8) = 5(5x + 8)$

or $24x + 48 = 25x + 40$

or $24x + 48 - 40 = 25x$

or $24x + 8 = 25x$

or $8 = 25x - 24x$

or $8 = x$

Therefore, Anu's present age = $4x = 4 \times 8 = 32$ years

Raj's present age = $5x = 5 \times 8 = 40$ years

EXERCISE 3.6

Solve the following equations.

1. $\frac{8x-3}{3x} = 2$
2. $\frac{9x}{7-6x} = 15$
3. $\frac{z}{z+15} = \frac{4}{9}$
4. $\frac{3y+4}{2-6y} = \frac{-2}{5}$
5. $\frac{7y+4}{y+2} = \frac{-4}{3}$
6. The ages of Hari and Harry are in the ratio 5:7. Four years from now the ratio of their ages will be 3:4. Find their present ages.
7. The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

WHAT HAVE WE DISCUSSED?

1. An algebraic equation is an equality involving variables. It says that the value of the expression on one side of the equality sign is equal to the value of the expression on the other side.
2. The equations we study in Classes VI, VII and VIII are linear equations in one variable. In such equations, the expressions which form the equation contain only one variable. Further, the equations are linear, i.e., the highest power of the variable appearing in the equation is 1.
3. A linear equation may have for its solution any rational number.
4. An equation may have linear expressions on both sides. Equations that we studied in Classes VI and VII had just a number on one side of the equation.
5. Just as numbers, variables can, also, be transposed from one side of the equation to the other.
6. Occasionally, the expressions forming equations have to be simplified before we can solve them by usual methods. Some equations may not even be linear to begin with, but they can be brought to a linear form by multiplying both sides of the equation by a suitable expression.
7. The utility of linear equations is in their diverse applications; different problems on numbers, ages, perimeters, combination of currency notes, and so on can be solved using linear equations.

CHAPTER 4 Understanding Quadrilaterals

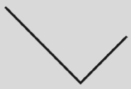
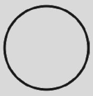
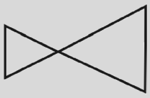
N1F1G9

4.1 Introduction

You know that the paper is a model for a **plane surface**. When you join a number of points without lifting a pencil from the paper (and without retracing any portion of the drawing other than single points), you get a **plane curve**.

Try to recall different varieties of curves you have seen in the earlier classes.

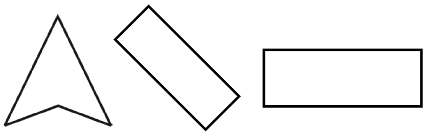
Match the following: (Caution! A figure may match to more than one type).

Figure		Type
(1)		(a) Simple closed curve
(2)		(b) A closed curve that is not simple
(3)		(c) Simple curve that is not closed
(4)		(d) Not a simple curve

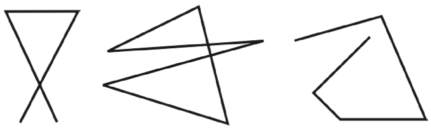
Compare your matchings with those of your friends. Do they agree?

4.2 Polygons

A simple closed curve made up of only line segments is called a **polygon**.



Curves that are polygons

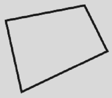
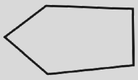
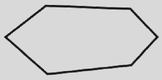
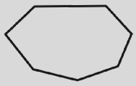
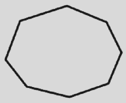
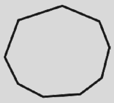
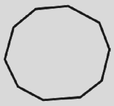


Curves that are not polygons

Try to give a few more examples and non-examples for a polygon.
Draw a rough figure of a polygon and identify its sides and vertices.

4.2.1 Classification of polygons

We classify polygons according to the number of sides (or vertices) they have.

Number of sides or vertices	Classification	Sample figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
⋮	⋮	⋮
n	*n*-gon	

4.2.2 Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon (Fig 4.1).

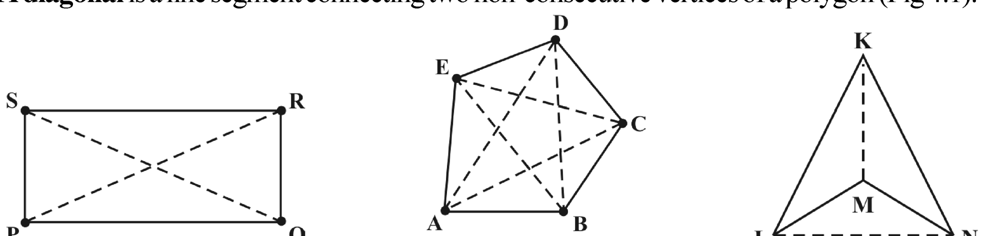
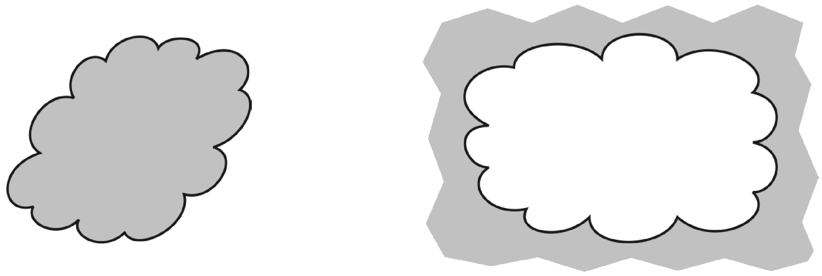


Fig 4.1

Can you name the diagonals in each of the above figures? (Fig 4.1)

Is $\overline{PQ}$ a diagonal? What about $\overline{LN}$?

You already know what we mean by **interior** and **exterior** of a closed curve (Fig 4.2).

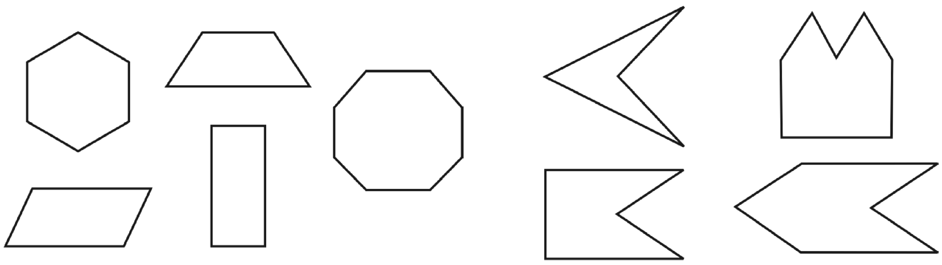


Interior **Fig 4.2** **Exterior**

The interior has a boundary. Does the exterior have a boundary? Discuss with your friends.

4.2.3 Convex and concave polygons

Here are some convex polygons and some concave polygons. (Fig 4.3)



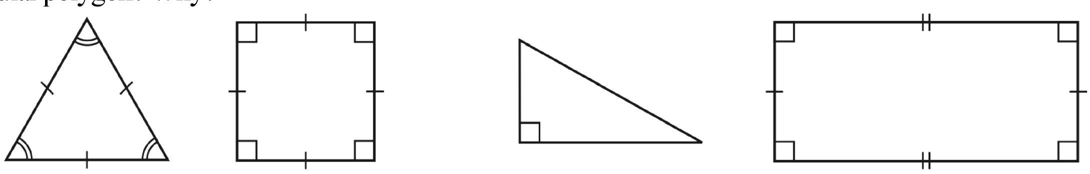
Convex polygons **Fig 4.3** **Concave polygons**

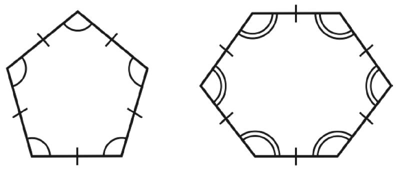
Can you find how these types of polygons differ from one another? Polygons that are convex have no portions of their diagonals in their exteriors or any line segment joining any two different points, in the interior of the polygon, lies wholly in the interior of it . Is this true with concave polygons? Study the figures given. Then try to describe in your own words what we mean by a convex polygon and what we mean by a concave polygon. Give two rough sketches of each kind.

In our work in this class, we will be dealing with convex polygons only.

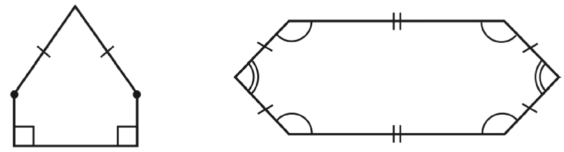
4.2.4 Regular and irregular polygons

A regular polygon is both 'equiangular' and 'equilateral'. For example, a square has sides of equal length and angles of equal measure. Hence it is a regular polygon. A rectangle is equiangular but not equilateral. Is a rectangle a regular polygon? Is an equilateral triangle a regular polygon? Why?





Regular polygons



Polygons that are not regular

[**Note:** Use of  or  indicates segments of equal length].

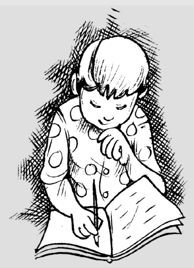
In the previous classes, have you come across any quadrilateral that is equilateral but not equiangular? Recall the quadrilateral shapes you saw in earlier classes – Rectangle, Square, Rhombus etc.

Is there a triangle that is equilateral but not equiangular?

4.2.5 Angle sum property

Do you remember the angle-sum property of a triangle? The sum of the measures of the three angles of a triangle is 180°. Recall the methods by which we tried to visualise this fact. We now extend these ideas to a quadrilateral.

DO THIS



1. Take any quadrilateral, say ABCD (Fig 4.4). Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

   Use the angle-sum property of a triangle and argue how the sum of the measures of ∠A, ∠B, ∠C and ∠D amounts to 180° + 180° = 360°.

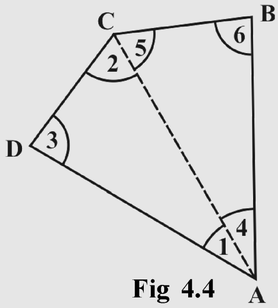


Fig 4.4

2. Take four congruent card-board copies of any quadrilateral ABCD, with angles as shown [Fig 4.5 (i)]. Arrange the copies as shown in the figure, where angles ∠1, ∠2, ∠3, ∠4 meet at a point [Fig 4.5 (ii)].

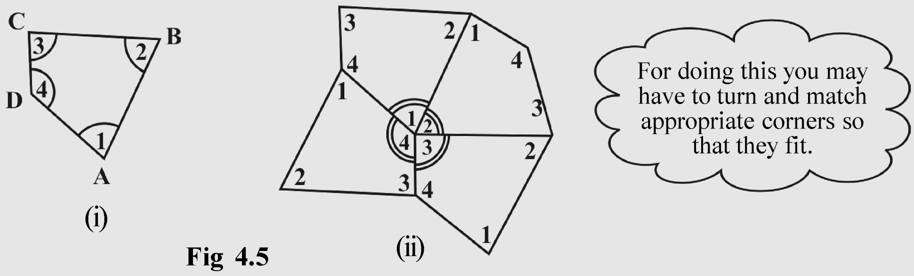


Fig 4.5

What can you say about the sum of the angles ∠1, ∠2, ∠3 and ∠4?
[**Note:** We denote the angles by ∠1, ∠2, ∠3, etc., and their respective measures by $m\angle 1$, $m\angle 2$, $m\angle 3$, etc.]
The sum of the measures of the four angles of a quadrilateral is___________.

You may arrive at this result in several other ways also.

3. As before consider quadrilateral ABCD (Fig 4.6). Let P be any point in its interior. Join P to vertices A, B, C and D. In the figure, consider ΔPAB. From this we see $x = 180° - m\angle 2 - m\angle 3$; similarly from ΔPBC, $y = 180° - m\angle 4 - m\angle 5$, from ΔPCD, $z = 180° - m\angle 6 - m\angle 7$ and from ΔPDA, $w = 180° - m\angle 8 - m\angle 1$. Use this to find the total measure $m\angle 1 + m\angle 2 + ... + m\angle 8$, does it help you to arrive at the result? Remember $\angle x + \angle y + \angle z + \angle w = 360°$.
4. These quadrilaterals were convex. What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles (Fig 4.7).

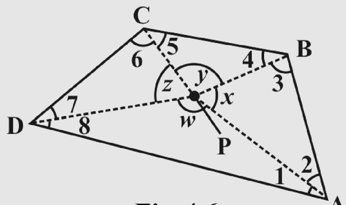


Fig 4.6

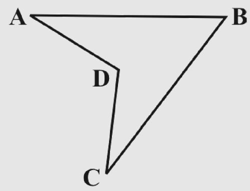
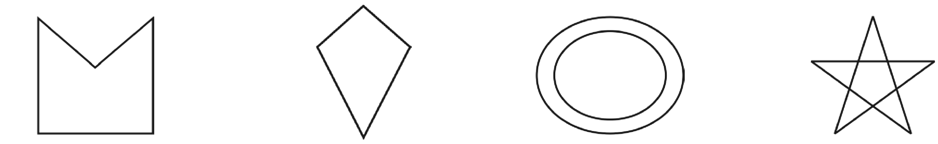


Fig 4.7

EXERCISE 4.1

1. Given here are some figures.



(1) (2) (3) (4)

(5) (6) (7) (8)

Classify each of them on the basis of the following.

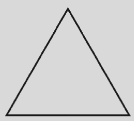
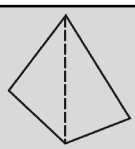
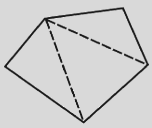
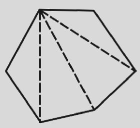
(a) Simple curve (b) Simple closed curve (c) Polygon
(d) Convex polygon (e) Concave polygon

2. How many diagonals does each of the following have?

(a) A convex quadrilateral (b) A regular hexagon (c) A triangle

3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180º	2 × 180° = (4 – 2) × 180°	3 × 180° = (5 – 2) × 180°	4 × 180° = (6 – 2) × 180°

What can you say about the angle sum of a convex polygon with number of sides?

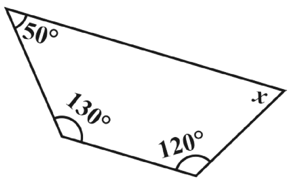
(a) 7 (b) 8 (c) 10 (d) n

5. What is a regular polygon?

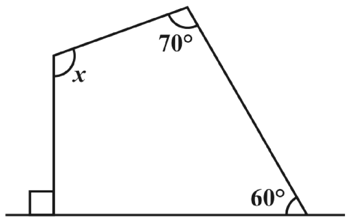
State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides

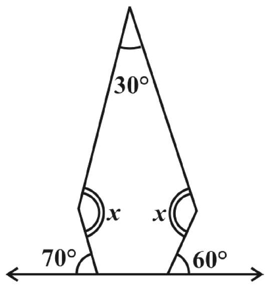
6. Find the angle measure x in the following figures.



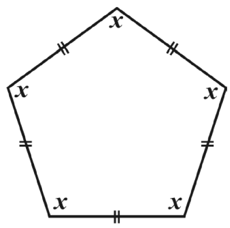
(a)



(b)

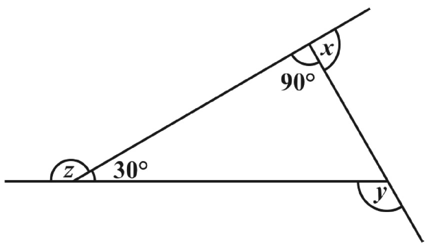


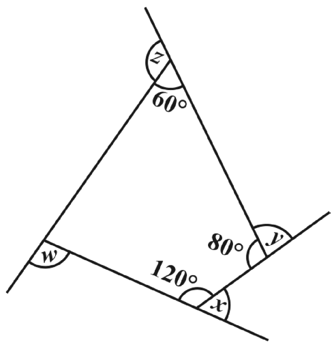
(c)



(d)

7.





(a) Find $x + y + z$ (b) Find $x + y + z + w$

4.3 Sum of the Measures of the Exterior Angles of a Polygon

On many occasions a knowledge of exterior angles may throw light on the nature of interior angles and sides.

DO THIS

Draw a polygon on the floor, using a piece of chalk. (In the figure, a pentagon ABCDE is shown) (Fig 4.8).

We want to know the total measure of angles, i.e, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$. Start at A. Walk along $\overline{AB}$. On reaching B, you need to turn through an angle of $m\angle 1$, to walk along $\overline{BC}$. When you reach at C, you need to turn through an angle of $m\angle 2$ to walk along $\overline{CD}$. You continue to move in this manner, until you return to side AB. You would have in fact made one complete turn.

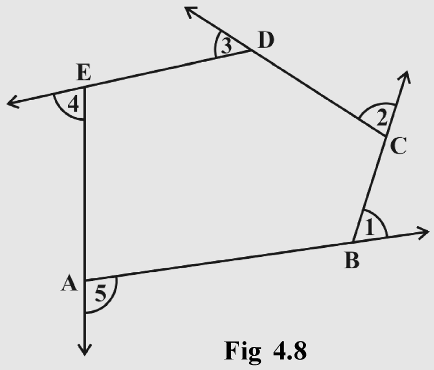


Fig 4.8

Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

This is true whatever be the number of sides of the polygon.

Therefore, *the sum of the measures of the external angles of any polygon is 360°.*

Example 1: Find measure x in Fig 4.9.

Solution: $x + 90° + 50° + 110° = 360°$ (Why?)

$x + 250° = 360°$

$x = 110°$

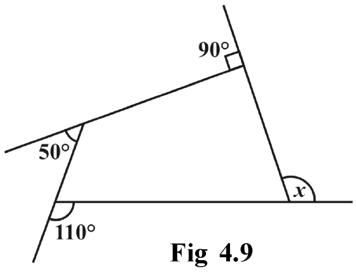


Fig 4.9

TRY THESE

Take a regular hexagon Fig 4.10.

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
   (i) exterior angle (ii) interior angle
4. Repeat this activity for the cases of
   (i) a regular octagon (ii) a regular 20-gon

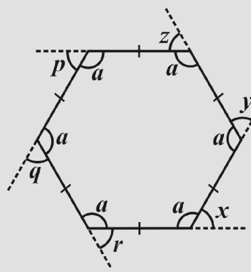


Fig 4.10

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Solution: Total measure of all exterior angles = 360°

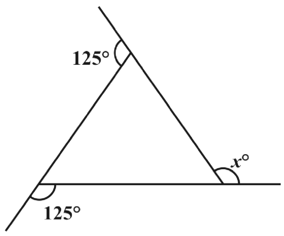
Measure of each exterior angle = 45°

Therefore, the number of exterior angles = $\frac{360°}{45°} = 8$

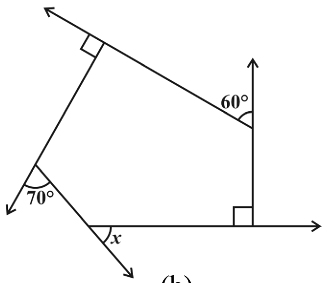
The polygon has 8 sides.

EXERCISE 4.2

1. Find x in the following figures.



(a)



(b)

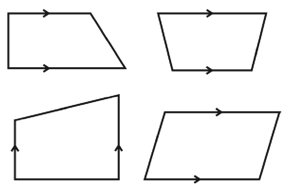
2. Find the measure of each exterior angle of a regular polygon of
   (i) 9 sides (ii) 15 sides
3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?
4. How many sides does a regular polygon have if each of its interior angles is 165°?
5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
   (b) Can it be an interior angle of a regular polygon? Why?
6. (a) What is the minimum interior angle possible for a regular polygon? Why?
   (b) What is the maximum exterior angle possible for a regular polygon?

4.4 Kinds of Quadrilaterals

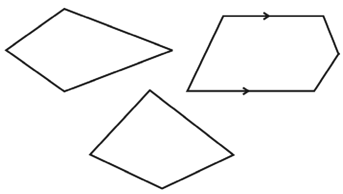
Based on the nature of the sides or angles of a quadrilateral, it gets special names.

4.4.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.



These are trapeziums



These are not trapeziums

Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

DO THIS

1. Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm, 5 cm. Arrange them as shown (Fig 4.11).

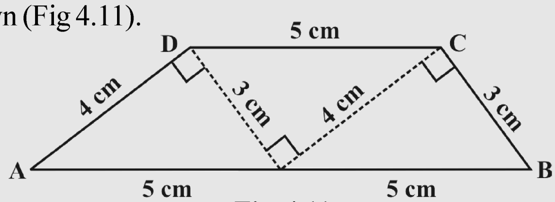


Fig 4.11

You get a trapezium. (Check it!) Which are the parallel sides here? Should the non-parallel sides be equal?

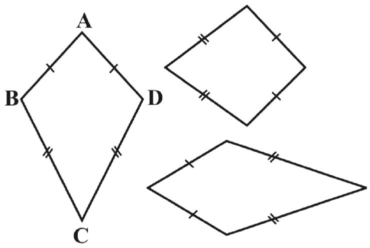
You can get two more trapeziums using the same set of triangles. Find them out and discuss their shapes.

2. Take four set-squares from your and your friend's instrument boxes. Use different numbers of them to place side-by-side and obtain different trapeziums.

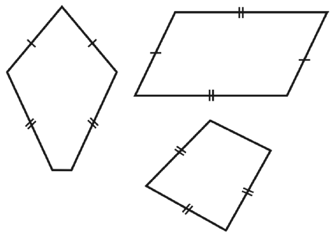
   If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*. Did you get an isoceles trapezium in any of your investigations given above?

4.4.2 Kite

Kite is a special type of a quadrilateral. The sides with the same markings in each figure are equal. For example AB = AD and BC = CD.



These are kites



These are not kites

Study these figures and try to describe what a kite is. Observe that

(i) A kite has 4 sides (It is a quadrilateral).

(ii) There are exactly two **distinct consecutive pairs** of sides of equal length.

Check whether a square is a kite.

DO THIS

Take a thick white sheet.

Fold the paper once.

Draw two line segments of different lengths as shown in Fig 4.12.

Cut along the line segments and open up.

You have the shape of a kite (Fig 4.13).

Has the kite any line symmetry?

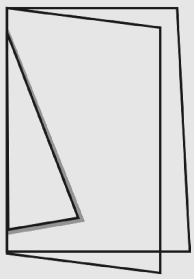


Fig 4.12

Show that ΔABC and ΔADC are congruent . What do we infer from this?

Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles. Are the diagonals equal in length?

Verify (by paper-folding or measurement) if the diagonals bisect each other.

By folding an angle of the kite on its opposite, check for angles of equal measure.

Observe the diagonal folds; do they indicate any diagonal being an angle bisector?

Share your findings with others and list them. A summary of these results are given elsewhere in the chapter for your reference.

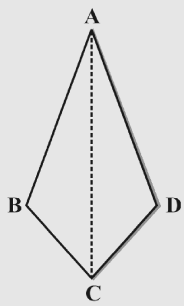
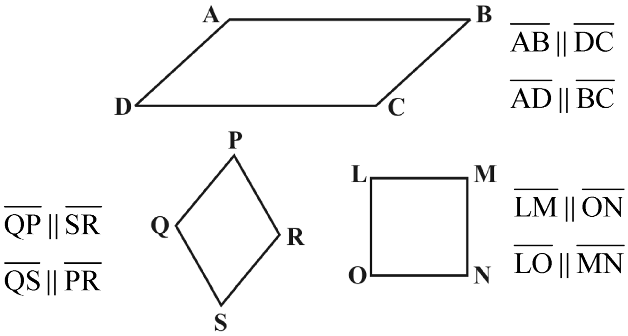


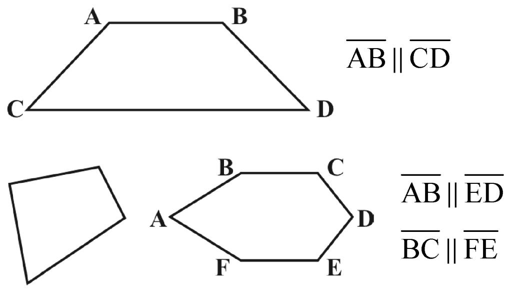
Fig 4.13

4.4.3 Parallelogram

A parallelogram is a quadrilateral. As the name suggests, it has something to do with parallel lines.



These are parallelograms



These are not parallelograms

Study these figures and try to describe in your own words what we mean by a parallelogram. Share your observations with your friends.

Check whether a rectangle is also a parallelogram.

DO THIS

Take two different rectangular cardboard strips of different widths (Fig 4.14).

Strip 1 **Fig 4.14** **Strip 2**

Place one strip horizontally and draw lines along its edge as drawn in the figure (Fig 4.15).

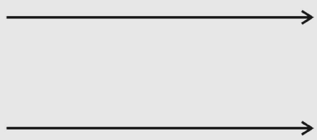


Fig 4.15

Now place the other strip in a slant position over the lines drawn and use this to draw two more lines as shown (Fig 4.16).

These four lines enclose a quadrilateral. This is made up of two pairs of parallel lines (Fig 4.17).

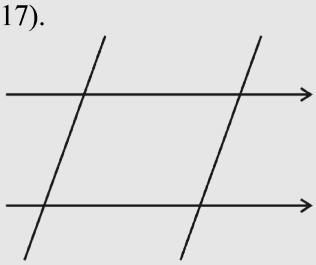


Fig 4.16

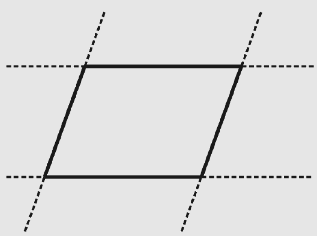


Fig 4.17

It is a parallelogram.

A parallelogram is a quadrilateral whose opposite sides are parallel.

4.4.4 Elements of a parallelogram

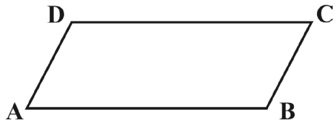


Fig 4.18

There are four sides and four angles in a parallelogram. Some of these are equal. There are some terms associated with these elements that you need to remember.

Given a parallelogram ABCD (Fig 4.18).

$\overline{AB}$ and $\overline{DC}$, are **opposite sides**. $\overline{AD}$ and $\overline{BC}$ form another pair of opposite sides.

$\angle$A and $\angle$C are a pair of **opposite angles**; another pair of opposite angles would be $\angle$B and $\angle$D.

$\overline{AB}$ and $\overline{BC}$ are **adjacent sides**. This means, one of the sides starts where the other ends. Are $\overline{BC}$ and $\overline{CD}$ adjacent sides too? Try to find two more pairs of adjacent sides.

$\angle$A and $\angle$B are **adjacent angles**. They are at the ends of the same side. $\angle$B and $\angle$C are also adjacent. Identify other pairs of adjacent angles of the parallelogram.

DO THIS

Take cut-outs of two identical parallelograms, say ABCD and A′B′C′D′ (Fig 4.19).

Fig 4.19

Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly the other corresponding sides are equal too.

Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? What can you now say about the lengths $\overline{AB}$ and $\overline{DC}$?

Similarly examine the lengths $\overline{AD}$ and $\overline{BC}$. What do you find?

You may also arrive at this result by measuring $\overline{AB}$ and $\overline{DC}$.

Property: *The opposite sides of a parallelogram are of equal length.*

TRY THESE

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in Fig 4.20. Does this help you to verify the above property?

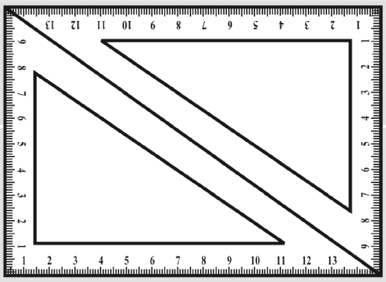


Fig 4.20

You can further strengthen this idea through a logical argument also.

Consider a parallelogram ABCD (Fig 4.21). Draw any one diagonal, say $\overline{AC}$.

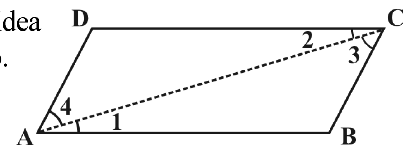


Fig 4.21

Looking at the angles,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (Why?)

Since in triangles ABC and ADC, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$

and $\overline{AC}$ is common, so, by ASA congruency condition,

Δ ABC $\cong$ Δ CDA (How is ASA used here?)

This gives AB = DC and BC = AD.

Example 3: Find the perimeter of the parallelogram PQRS (Fig 4.22).

Solution: In a parallelogram, the opposite sides have same length.

Therefore, PQ = SR = 12 cm and QR = PS = 7 cm

So, Perimeter = PQ + QR + RS + SP

= 12 cm + 7 cm + 12 cm + 7 cm = 38 cm

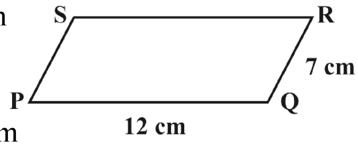
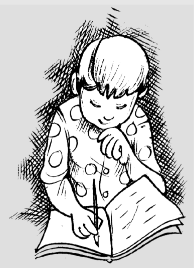


Fig 4.22

4.4.5 Angles of a parallelogram

We studied a property of parallelograms concerning the (opposite) sides. What can we say about the angles?

DO THIS



Let ABCD be a parallelogram (Fig 4.23). Copy it on a tracing sheet. Name this copy as A′B′C′D′. Place A′B′C′D′ on ABCD. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 180°. The parallelograms still concide; but you now find A′ lying exactly on C and vice-versa; similarly B′ lies on D and vice-versa.

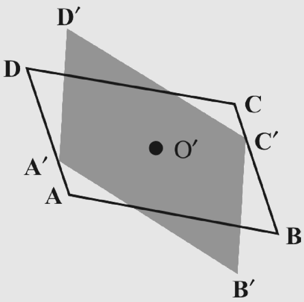


Fig 4.23

Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

Property: *The opposite angles of a parallelogram are of equal measure.*

TRY THESE

Take two identical 30° – 60° – 90° set-squares and form a parallelogram as before. Does the figure obtained help you to confirm the above property?

You can further justify this idea through logical arguments.

If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram, (Fig 4.24) you find that

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (Why?)

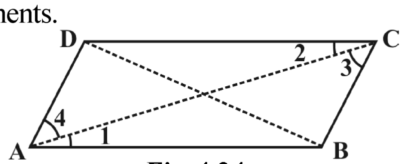


Fig 4.24

Studying ΔABC and ΔADC (Fig 4.25) separately, will help you to see that by ASA congruency condition,

$$\Delta ABC \cong \Delta CDA \qquad \text{(How?)}$$

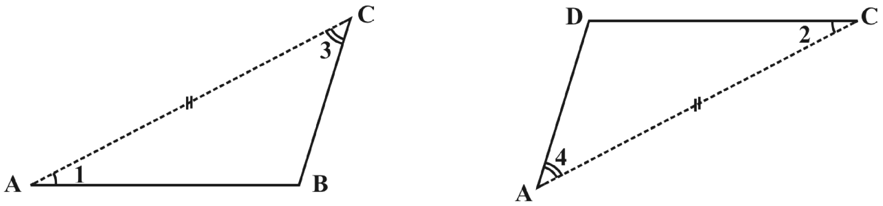


Fig 4.25

This shows that $\angle B$ and $\angle D$ have same measure. In the same way you can get $m\angle A = m\angle C$.

Alternatively, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, we have, $m\angle A = \angle 1 + \angle 4 = \angle 2 + \angle C \ \ m\angle C$

Example 4: In Fig 4.26, BEST is a parallelogram. Find the values x, y and z.

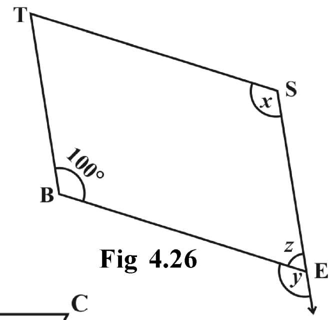


Fig 4.26

Solution: S is opposite to B.

So, $x = 100°$ (opposite angles property)

$y = 100°$ (measure of angle corresponding to $\angle x$)

$z = 80°$ (since $\angle y$, $\angle z$ is a linear pair)

We now turn our attention to adjacent angles of a parallelogram.
In parallelogram ABCD, (Fig 4.27).

$\angle A$ and $\angle D$ are supplementary since $\overline{DC} \parallel \overline{AB}$ and with transversal $\overline{DA}$, these two angles are interior opposite.

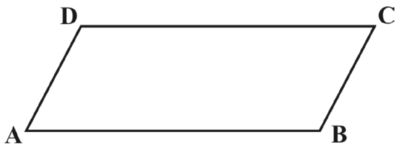


Fig 4.27

$\angle A$ and $\angle B$ are also supplementary. Can you say 'why'?

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making $\angle A$ and $\angle B$ interior opposite.

Identify two more pairs of supplementary angles from the figure.

Property: *The adjacent angles in a parallelogram are supplementary.*

Example 5: In a parallelogram RING, (Fig 4.28) if $m\angle R = 70°$, find all the other angles.

Solution: Given $m\angle R = 70°$

Then $m\angle N = 70°$

because $\angle R$ and $\angle N$ are opposite angles of a parallelogram.

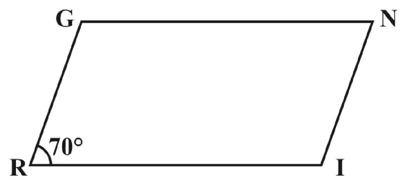


Fig 4.28

Since $\angle R$ and $\angle I$ are supplementary,

$$m\angle I = 180° - 70° = 110°$$

Also, $m\angle G = 110°$ since $\angle G$ is opposite to $\angle I$

Thus, $m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$

THINK, DISCUSS AND WRITE

After showing $m\angle R = m\angle N = 70°$, can you find $m\angle I$ and $m\angle G$ by any other method?

4.4.6 Diagonals of a parallelogram

The diagonals of a parallelogram, in general, are not of equal length. (Did you check this in your earlier activity?) However, the diagonals of a parallelogram have an interesting property.

DO THIS

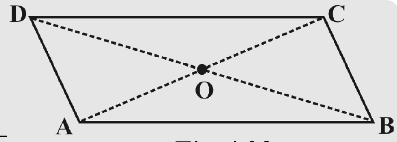


Fig 4.29

Take a cut-out of a parallelogram, say, ABCD (Fig 4.29). Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.

Find the mid point of $\overline{AC}$ by a fold, placing C on A. Is the mid-point same as O?

Does this show that diagonal $\overline{DB}$ bisects the diagonal $\overline{AC}$ at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of $\overline{DB}$ could lie.

Property: *The diagonals of a parallelogram bisect each other (at the point of their intersection, of course!)*

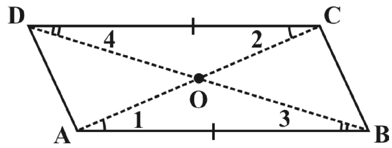


Fig 4.30

To argue and justify this property is not very difficult. From Fig 4.30, applying ASA criterion, it is easy to see that

Δ AOB $\cong$ Δ COD (How is ASA used here?)

This gives AO = CO and BO = DO

Example 6: In Fig 4.31 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

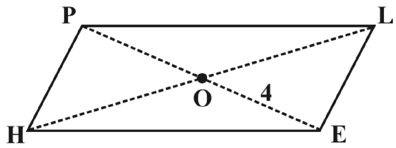


Fig 4.31

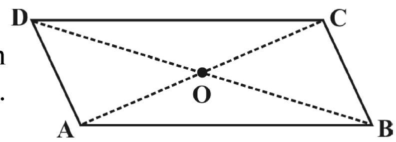
Solution : If OE = 4 then OP also is 4 (Why?)

So PE = 8, (Why?)

Therefore HL = 8 + 5 = 13

Hence $OH = \frac{1}{2} \times 13 = 6.5$ (cms)

EXERCISE 4.3

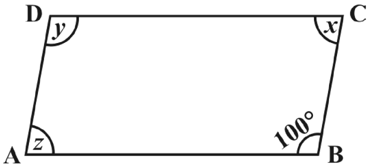


1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

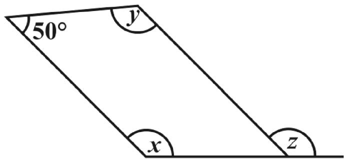
   (i) AD = ... (ii) $\angle$ DCB = ...

   (iii) OC = ... (iv) $m\angle DAB + m\angle CDA$ = ...

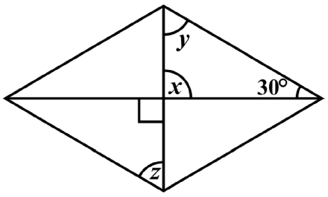
2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



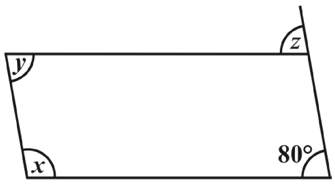
(i)



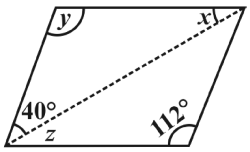
(ii)



(iii)



(iv)



(v)

3. Can a quadrilateral ABCD be a parallelogram if

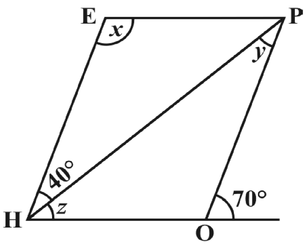
(i) $\angle D + \angle B = 180°$? (ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

(iii) $\angle A = 70°$ and $\angle C = 65°$?

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

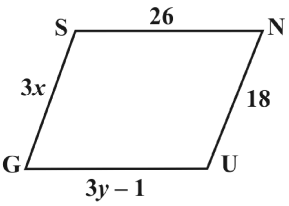
6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.



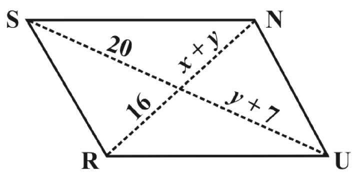
7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)

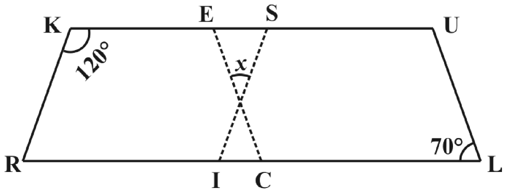
(i)



(ii)



9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 4.32)

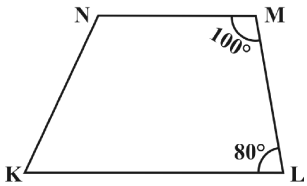


Fig 4.32

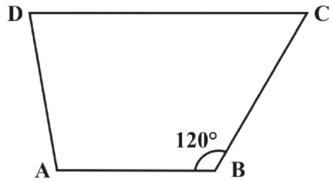


Fig 4.33

11. Find $m\angle C$ in Fig 4.33 if $\overline{AB} \parallel \overline{DC}$.

12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 4.34.
(If you find $m\angle R$, is there more than one method to find $m\angle P$?)

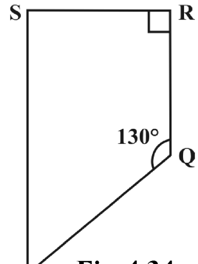


Fig 4.34

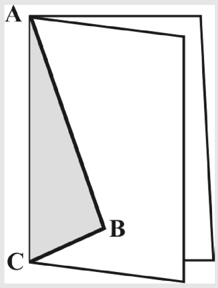
4.5 Some Special Parallelograms

4.5.1 Rhombus

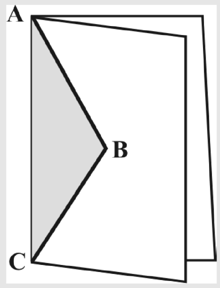
We obtain a Rhombus (which, you will see, is a parallelogram) as a special case of kite (which is not a a parallelogram).

DO THIS

Recall the paper-cut kite you made earlier.



Kite-cut



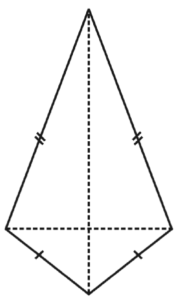
Rhombus-cut

When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a **rhombus**.

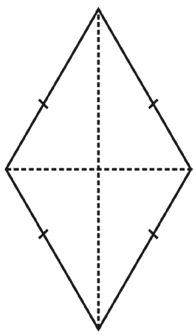
Note that the sides of rhombus are all of same length; this is not the case with the kite.

A rhombus is a quadrilateral with sides of equal length.

Since the opposite sides of a rhombus have the same length, it is also a parallelogram. So, *a rhombus has all the properties of a parallelogram and also that of a kite*. Try to list them out. You can then verify your list with the check list summarised in the book elsewhere.



Kite



Rhombus

The most useful property of a rhombus is that of its diagonals.

Property: *The diagonals of a rhombus are perpendicular bisectors of one another.*

DO THIS

Take a copy of rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Here is an outline justifying this property using logical steps.

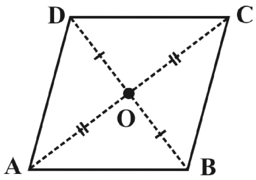


Fig 4.35

ABCD is a rhombus (Fig 4.35). Therefore it is a parallelogram too.

Since diagonals bisect each other, OA = OC and OB = OD.

We have to show that $m\angle AOD = m\angle COD = 90°$

It can be seen that by SSS congruency criterion

$$\Delta AOD \cong \Delta COD$$

Since AO = CO (Why?)
AD = CD (Why?)
OD = OD

Therefore, $m\angle AOD = m\angle COD$

Since $\angle AOD$ and $\angle COD$ are a linear pair,

$$m\angle AOD = m\angle COD = 90°$$

Example 7:

RICE is a rhombus (Fig 4.36). Find x, y, z. Justify your findings.

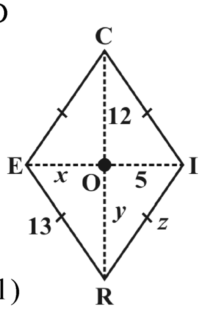


Fig 4.36

Solution:

x = OE	y = OR	z = side of the rhombus
= OI (diagonals bisect)	= OC (diagonals bisect)	= 13 (all sides are equal)
= 5	= 12	

4.5.2 A rectangle

A rectangle is a parallelogram with equal angles (Fig 4.37).

Fig 4.37

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

Let the measure of each angle be $x°$.

Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus each angle of a rectangle is a right angle.

So, a rectangle is a parallelogram in which every angle is a right angle.

Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

Property: *The diagonals of a rectangle are of equal length.*

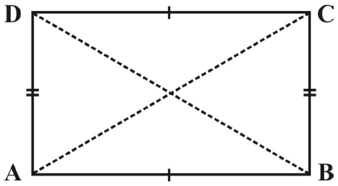


Fig 4.38

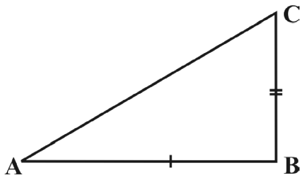


Fig 4.39

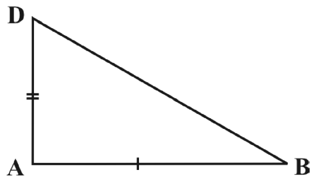


Fig 4.40

This is easy to justify. If ABCD is a rectangle (Fig 4.38), then looking at triangles ABC and ABD separately [(Fig 4.39) and (Fig 4.40) respectively], we have

$$\Delta ABC \cong \Delta ABD$$

This is because $AB = AB$ (Common)

$BC = AD$ (Why?)

$m\angle A = m\angle B = 90°$ (Why?)

The congruency follows by SAS criterion.

Thus $AC = BD$

and *in a rectangle the diagonals, besides being equal in length bisect each other* (Why?)

Example 8: RENT is a rectangle (Fig 4.41). Its diagonals meet at O. Find x, if $OR = 2x + 4$ and $OT = 3x + 1$.

Solution: $\overline{OT}$ is half of the diagonal $\overline{TE}$,

$\overline{OR}$ is half of the diagonal $\overline{RN}$.

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $3x + 1 = 2x + 4$

or $x = 3$

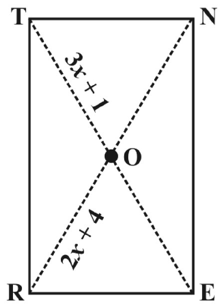


Fig 4.41

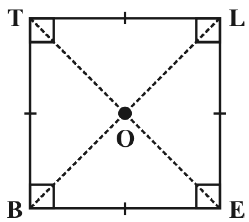
4.5.3 A square

A square is a rectangle with equal sides.

This means a square has all the properties of a rectangle with an additional requirement that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another, (Check this).



BELT is a square, $BE = EL = LT = TB$
$\angle B$, $\angle E$, $\angle L$, $\angle T$ are right angles.
$BL = ET$ and $\overline{BL} \perp \overline{ET}$.
$OB = OL$ and $OE = OT$.

In a square the diagonals.

(i) bisect one another (square being a parallelogram)

(ii) are of equal length (square being a rectangle) and

(iii) are perpendicular to one another.

Hence, we get the following property.

Property: *The diagonals of a square are perpendicular bisectors of each other.*

DO THIS

Take a square sheet, say PQRS (Fig 4.42).

Fold along both the diagonals. Are their mid-points the same?

Check if the angle at O is 90° by using a set-square.

This verifies the property stated above.

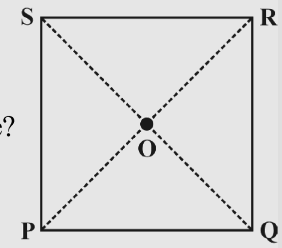


Fig 4.42

We can justify this also by arguing logically:

ABCD is a square whose diagonals meet at O (Fig 4.43).

OA = OC (Since the square is a parallelogram)

By SSS congruency condition, we now see that

Δ AOD $\cong$ Δ COD (How?)

Therefore, $m\angle AOD = m\angle COD$

These angles being a linear pair, each is right angle.

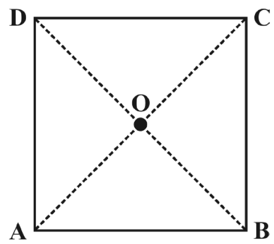


Fig 4.43

EXERCISE 4.4

1. State whether True or False.

(a) All rectangles are squares

(b) All rhombuses are parallelograms

(c) All squares are rhombuses and also rectangles

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

2. Identify all the quadrilaterals that have.

(a) four sides of equal length (b) four right angles

3. Explain how a square is.

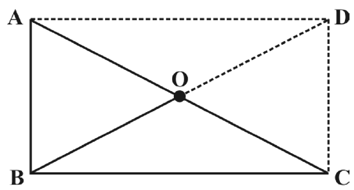
(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle

4. Name the quadrilaterals whose diagonals.

(i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal

5. Explain why a rectangle is a convex quadrilateral.

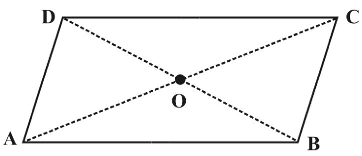
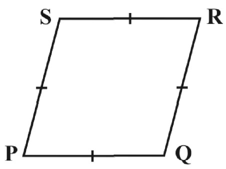
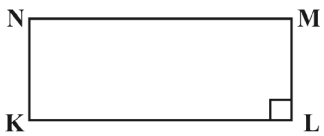
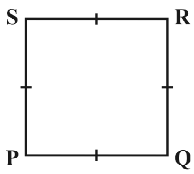
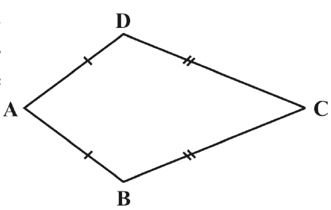
6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?
2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.
3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

WHAT HAVE WE DISCUSSED?

Quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length. 	(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle. 	(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square: A rectangle with sides of equal length. 	All the properties of a parallelogram, rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides 	(1) The diagonals are perpendicular to one another (2) One of the diagonals bisects the other. (3) In the figure $m\angle B = m\angle D$ but $m\angle A \neq m\angle C$.

CHAPTER

5 Squares and Square Roots

D7V5N4

5.1 Introduction

You know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers?

Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself.

Such numbers like 1, 4, 9, 16, 25, ... are known as **square numbers**.

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**. Is 32 a square number?

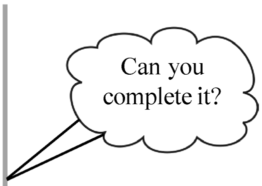
We know that $5^2 = 25$ and $6^2 = 36$. If 32 is a square number, it must be the square of a natural number between 5 and 6. But there is no natural number between 5 and 6.

Therefore 32 is not a square number.

Consider the following numbers and their squares.

Number	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$

3	3 × 3 = 9
4	4 × 4 = 16
5	5 × 5 = 25
6	-----------
7	-----------
8	-----------
9	-----------
10	-----------



From the above table, can we enlist the square numbers between 1 and 100? Are there any natural square numbers upto 100 left out?

You will find that the rest of the numbers are not square numbers.

The numbers 1, 4, 9, 16 ... are square numbers. These numbers are also called **perfect squares**.

TRY THESE

1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60

5.2 Properties of Square Numbers

Following table shows the squares of numbers from 1 to 20.

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Study the square numbers in the above table. What are the ending digits (that is, digits in the units place) of the square numbers? All these numbers end with 0, 1, 4, 5, 6 or 9 at units place. None of these end with 2, 3, 7 or 8 at unit's place.

Can we say that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a square number? Think about it.

TRY THESE

1. Can we say whether the following numbers are perfect squares? How do we know?
   (i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
   (v) 1069 (vi) 2061

Write five numbers which you can decide by looking at their units digit that they are not square numbers.
2. Write five numbers which you cannot decide just by looking at their units digit (or units place) whether they are square numbers or not.

- Study the following table of some numbers and their squares and observe the one's place in both.

Table 1

Number	Square	Number	Square	Number	Square
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	30	900
7	49	17	289	35	1225
8	64	18	324	40	1600
9	81	19	361	45	2025
10	100	20	400	50	2500

The following square numbers end with digit 1.

Square	Number
1	1
81	9
121	11
361	19
441	21

TRY THESE

Which of 123^2, 77^2, 82^2, 161^2, 109^2 would end with digit 1?

Write the next two square numbers which end in 1 and their corresponding numbers.
You will see that if a number has 1 or 9 in the units place, then it's square ends in 1.

- Let us consider square numbers ending in 6.

Square	Number
16	4
36	6
196	14
256	16

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2
(iv) 36^2 (v) 34^2

We can see that *when a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place*.

Can you find more such rules by observing the numbers and their squares (Table 1)?

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) 1234 (ii) 26387 (iii) 52698 (iv) 99880

(v) 21222 (vi) 9106

- Consider the following numbers and their squares.

We have one zero: $10^2 = 100$, $20^2 = 400$, $80^2 = 6400$ — But we have two zeros

We have two zeros: $100^2 = 10000$, $200^2 = 40000$, $700^2 = 490000$, $900^2 = 810000$ — But we have four zeros

If a number contains 3 zeros at the end, how many zeros will its square have ?
What do you notice about the number of zeros at the end of the number and the number of zeros at the end of its square?
Can we say that square numbers can only have even number of zeros at the end?

- See Table 1 with numbers and their squares.
What can you say about the squares of even numbers and squares of odd numbers?

TRY THESE

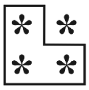
1. The square of which of the following numbers would be an odd number/an even number? Why?
   (i) 727 (ii) 158 (iii) 269 (iv) 1980
2. What will be the number of zeros in the square of the following numbers?
   (i) 60 (ii) 400

5.3 Some More Interesting Patterns

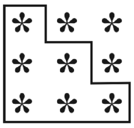
1. **Adding triangular numbers.**
   Do you remember triangular numbers (numbers whose dot patterns can be arranged as triangles)?

```
                              *
                     *        * *
              *      * *      * **
       *      * *    * * *    * ***
*      * *    * * *  * * * *  * ****
1      3      6      10       15
```

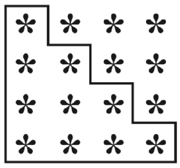
If we combine two consecutive triangular numbers, we get a square number, like



$1 + 3 = 4$
$= 2^2$



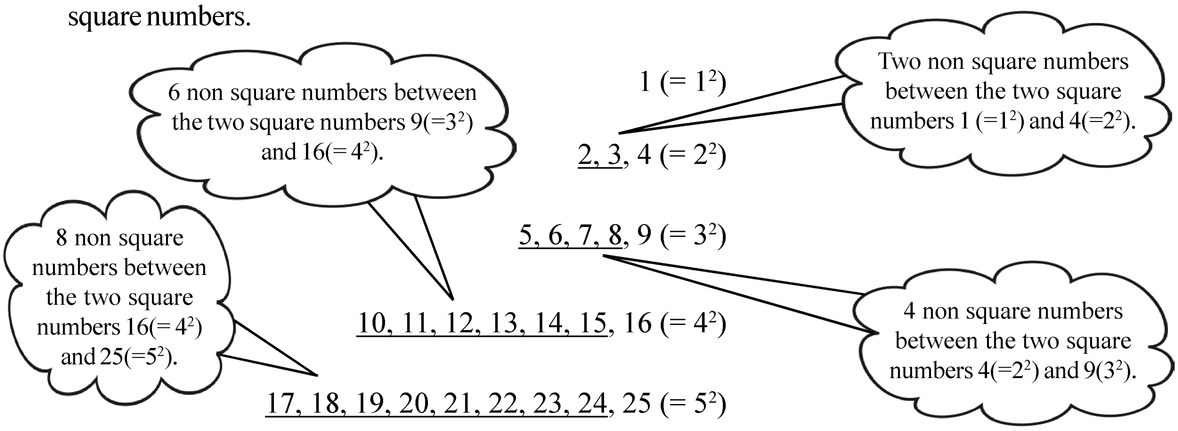
$3 + 6 = 9$
$= 3^2$



$6 + 10 = 16$
$= 4^2$

2. Numbers between square numbers

Let us now see if we can find some interesting pattern between two consecutive square numbers.



Between $1^2(=1)$ and $2^2(= 4)$ there are two (i.e., 2×1) non square numbers 2, 3.

Between $2^2(= 4)$ and $3^2(= 9)$ there are four (i.e., 2×2) non square numbers 5, 6, 7, 8.

Now, $3^2 = 9$, $4^2 = 16$

Therefore, $4^2 - 3^2 = 16 - 9 = 7$

Between $9(=3^2)$ and $16(= 4^2)$ the numbers are 10, 11, 12, 13, 14, 15 that is, six non-square numbers which is 1 less than the difference of two squares.

We have $4^2 = 16$ and $5^2 = 25$

Therefore, $5^2 - 4^2 = 9$

Between $16(= 4^2)$ and $25(= 5^2)$ the numbers are 17, 18, ... , 24 that is, eight non square numbers which is 1 less than the difference of two squares.

Consider 7^2 and 6^2. Can you say how many numbers are there between 6^2 and 7^2? If we think of any natural number n and $(n + 1)$, then,

$$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1.$$

We find that between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

Thus, in general we can say that *there are 2n non perfect square numbers between the squares of the numbers n and* $(n + 1)$. Check for $n = 5$, $n = 6$ etc., and verify.

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2?
2. How many non square numbers lie between the following pairs of numbers
   (i) 100^2 and 101^2 (ii) 90^2 and 91^2 (iii) 1000^2 and 1001^2

3. Adding odd numbers

Consider the following

1 [one odd number]	$= 1 = 1^2$
1 + 3 [sum of first two odd numbers]	$= 4 = 2^2$
1 + 3 + 5 [sum of first three odd numbers]	$= 9 = 3^2$
1 + 3 + 5 + 7 [...]	$= 16 = 4^2$
1 + 3 + 5 + 7 + 9 [...]	$= 25 = 5^2$
1 + 3 + 5 + 7 + 9 + 11 [...]	$= 36 = 6^2$

So we can say that the *sum of first n odd natural numbers is* n^2.

Looking at it in a different way, we can say: 'If the number is a square number, it has to be the sum of successive **odd** numbers starting from 1.

Consider those numbers which are not perfect squares, say 2, 3, 5, 6, ... Can you express these numbers as a sum of successive odd natural numbers beginning from 1?

You will find that these numbers cannot be expressed in this form.

Consider the number 25. Successively subtract 1, 3, 5, 7, 9, ... from it

(i) 25 – 1 = 24 (ii) 24 – 3 = 21 (iii) 21 – 5 = 16 (iv) 16 – 7 = 9
(v) 9 – 9 = 0

This means, 25 = 1 + 3 + 5 + 7 + 9. Also, 25 is a perfect square.

Now consider another number 38, and again do as above.

(i) 38 – 1 = 37 (ii) 37 – 3 = 34 (iii) 34 – 5 = 29 (iv) 29 – 7 = 22
(v) 22 – 9 = 13 (vi) 13 – 11 = 2 (vii) 2 – 13 = – 11

This shows that we are not able to express 38 as the sum of consecutive odd numbers starting with 1. Also, 38 is not a perfect square.

So we can also say that *if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square*.

We can use this result to find whether a number is a perfect square or not.

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 (ii) 55 (iii) 81
(iv) 49 (v) 69

4. A sum of consecutive natural numbers

Consider the following

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

First Number $= \frac{3^2 - 1}{2}$

Second Number $= \frac{3^2 + 1}{2}$

$9^2 = 81 = 40 + 41$
$11^2 = 121 = 60 + 61$
$15^2 = 225 = 112 + 113$

TRY THESE

1. Express the following as the sum of two consecutive integers.
   (i) 21^2 (ii) 13^2 (iii) 11^2 (iv) 19^2
2. Do you think the reverse is also true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

5. Product of two consecutive even or odd natural numbers

$11 \times 13 = 143 = 12^2 - 1$

Also $11 \times 13 = (12 - 1) \times (12 + 1)$

Therefore, $11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

Similarly, $13 \times 15 = (14 - 1) \times (14 + 1) = 14^2 - 1$

$29 \times 31 = (30 - 1) \times (30 + 1) = 30^2 - 1$

$44 \times 46 = (45 - 1) \times (45 + 1) = 45^2 - 1$

So in general we can say that $(a + 1) \times (a - 1) = a^2 - 1$.

6. Some more patterns in square numbers

Observe the squares of numbers; 1, 11, 111 ... etc. They give a beautiful pattern:

1^2 = 1
11^2 = 1 2 1
111^2 = 1 2 3 2 1
1111^2 = 1 2 3 4 3 2 1
11111^2 = 1 2 3 4 5 4 3 2 1
11111111^2 = 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1

Another interesting pattern.

$7^2 = 49$
$67^2 = 4489$
$667^2 = 444889$
$6667^2 = 44448889$
$66667^2 = 4444488889$
$666667^2 = 444444888889$

The fun is in being able to find out why this happens. May be it would be interesting for you to explore and think about such questions even if the answers come some years later.

TRY THESE

Write the square, making use of the above pattern.
(i) 111111^2 (ii) 1111111^2

TRY THESE

Can you find the square of the following numbers using the above pattern?
(i) 6666667^2 (ii) 66666667^2

EXERCISE 5.1

1. What will be the unit digit of the squares of the following numbers?

(i) 81 (ii) 272 (iii) 799 (iv) 3853
(v) 1234 (vi) 26387 (vii) 52698 (viii) 99880
(ix) 12796 (x) 55555

2. The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
(v) 64000 (vi) 89722 (vii) 222000 (viii) 505050

3. The squares of which of the following would be odd numbers?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004

4. Observe the following pattern and find the missing digits.

$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$100001^2 = 1 \text{ .... } 2 \text{ .... } 1$$
$$10000001^2 = \text{...........................}$$

5. Observe the following pattern and supply the missing numbers.

$$11^2 = 1\ 2\ 1$$
$$101^2 = 1\ 0\ 2\ 0\ 1$$
$$10101^2 = 102030201$$
$$1010101^2 = \text{...........................}$$
$$\text{...........}^2 = 10203040504030201$$

6. Using the given pattern, find the missing numbers.

$$1^2 + 2^2 + 2^2 = 3^2$$
$$2^2 + 3^2 + 6^2 = 7^2$$
$$3^2 + 4^2 + 12^2 = 13^2$$
$$4^2 + 5^2 + \_^2 = 21^2$$
$$5^2 + \_^2 + 30^2 = 31^2$$
$$6^2 + 7^2 + \_^2 = \_\_^2$$

To find pattern
Third number is related to first and second number. How?
Fourth number is related to third number. How?

7. Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9
(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 +19
(iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

8. (i) Express 49 as the sum of 7 odd numbers.
(ii) Express 121 as the sum of 11 odd numbers.

9. How many numbers lie between squares of the following numbers?

(i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

5.4 Finding the Square of a Number

Squares of small numbers like 3, 4, 5, 6, 7, ... etc. are easy to find. But can we find the square of 23 so quickly?

The answer is not so easy and we may need to multiply 23 by 23.

There is a way to find this without having to multiply 23×23.

We know $23 = 20 + 3$

Therefore $23^2 = (20 + 3)^2 = 20(20 + 3) + 3(20 + 3)$

$= 20^2 + 20 \times 3 + 3 \times 20 + 3^2$

$= 400 + 60 + 60 + 9 = 529$

Example 1: Find the square of the following numbers without actual multiplication.

(i) 39 (ii) 42

Solution: (i) $39^2 = (30 + 9)^2 = 30(30 + 9) + 9(30 + 9)$

$= 30^2 + 30 \times 9 + 9 \times 30 + 9^2$

$= 900 + 270 + 270 + 81 = 1521$

(ii) $42^2 = (40 + 2)^2 = 40(40 + 2) + 2(40 + 2)$

$= 40^2 + 40 \times 2 + 2 \times 40 + 2^2$

$= 1600 + 80 + 80 + 4 = 1764$

5.4.1 Other patterns in squares

Consider the following pattern:

$25^2 = 625 = (2 \times 3)$ hundreds + 25

$35^2 = 1225 = (3 \times 4)$ hundreds + 25

$75^2 = 5625 = (7 \times 8)$ hundreds + 25

$125^2 = 15625 = (12 \times 13)$ hundreds + 25

Now can you find the square of 95?

Consider a number with unit digit 5, i.e., $a5$

$(a5)^2 = (10a + 5)^2$

$= 10a(10a + 5) + 5(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred + 25

TRY THESE

Find the squares of the following numbers containing 5 in unit's place.

(i) 15 (ii) 95 (iii) 105 (iv) 205

5.4.2 Pythagorean triplets

Consider the following

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

The collection of numbers 3, 4 and 5 is known as **Pythagorean triplet**. 6, 8, 10 is also a Pythagorean triplet, since

$$6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

Again, observe that

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$. The numbers 5, 12, 13 form another such triplet.

Can you find more such triplets?

For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$. So, $2m$, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Try to find some more Pythagorean triplets using this form.

Example 2: Write a Pythagorean triplet whose smallest member is 8.

Solution: We can get Pythagorean triplets by using general form $2m$, $m^2 - 1$, $m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives $m = 3$

Therefore, $2m = 6$ and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try $2m = 8$

then $m = 4$

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example 3: Find a Pythagorean triplet in which one member is 12.

Solution: If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$. Again $m^2 = 11$ will not give an integer value for m.

So, let us take $2m = 12$

then $m = 6$

Thus, $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note: All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

EXERCISE 5.2

1. Find the square of the following numbers.

(i) 32 (ii) 35 (iii) 86 (iv) 93

(v) 71 (vi) 46

2. Write a Pythagorean triplet whose one member is.

(i) 6 (ii) 14 (iii) 16 (iv) 18

5.5 Square Roots

Study the following situations.

(a) Area of a square is 144 cm^2. What could be the side of the square?

We know that the area of a square = side^2

If we assume the length of the side to be 'a', then $144 = a^2$

To find the length of side it is necessary to find a number whose square is 144.

(b) What is the length of a diagonal of a square of side 8 cm (Fig 5.1)?

Can we use Pythagoras theorem to solve this ?

We have, $AB^2 + BC^2 = AC^2$

i.e., $8^2 + 8^2 = AC^2$

or $64 + 64 = AC^2$

or $128 = AC^2$

Again to get AC we need to think of a number whose square is 128.

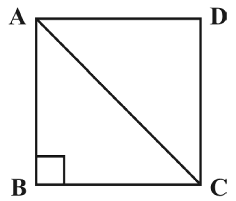


Fig 5.1

(c) In a right triangle the length of the hypotenuse and a side are respectively 5 cm and 3 cm (Fig 5.2).

Can you find the third side?

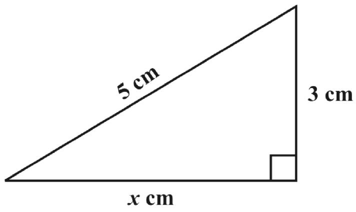


Fig 5.2

Let x cm be the length of the third side.

Using Pythagoras theorem $5^2 = x^2 + 3^2$

$$25 = x^2 + 9$$

$$25 - 9 = x^2$$

$$16 = x^2$$

Again, to find x we need a number whose square is 16.

In all the above cases, we need to find a number whose square is known. Finding the number with the known square is known as finding the square root.

5.5.1 Finding square roots

The inverse (opposite) operation of addition is subtraction and the inverse operation of multiplication is division. Similarly, finding the square root is the inverse operation of squaring.

We have, $1^2 = 1$, therefore square root of 1 is 1

$2^2 = 4$, therefore square root of 4 is 2

$3^2 = 9$, therefore square root of 9 is 3

> Since $9^2 = 81$,
> and $(-9)^2 = 81$
> We say that square roots of 81 are 9 and –9.

TRY THESE

(i) $11^2 = 121$. What is the square root of 121?

(ii) $14^2 = 196$. What is the square root of 196?

THINK, DISCUSS AND WRITE

$(-1)^2 = 1$. Is –1, a square root of 1? $(-2)^2 = 4$. Is –2, a square root of 4?

$(-9)^2 = 81$. Is –9 a square root of 81?

From the above, you may say that there are two integral square roots of a perfect square number. In this chapter, we shall take up only positive square root of a natural number. Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

For example: $\sqrt{4} = 2$ (not –2); $\sqrt{9} = 3$ (not –3) etc.

Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$

Statement	Inference
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

5.5.2 Finding square root through repeated subtraction

Do you remember that the sum of the first n odd natural numbers is n^2? That is, every square number can be expressed as a sum of successive odd natural numbers starting from 1.

Consider $\sqrt{81}$. Then,

(i) $81 - 1 = 80$ (ii) $80 - 3 = 77$ (iii) $77 - 5 = 72$ (iv) $72 - 7 = 65$
(v) $65 - 9 = 56$ (vi) $56 - 11 = 45$ (vii) $45 - 13 = 32$ (viii) $32 - 15 = 17$
(ix) $17 - 17 = 0$

From 81 we have subtracted successive odd numbers starting from 1 and obtained 0 at 9th step.

Therefore $\sqrt{81} = 9$.

Can you find the square root of 729 using this method? Yes, but it will be time consuming. Let us try to find it in a simpler way.

TRY THESE

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121
(ii) 55
(iii) 36
(iv) 49
(v) 90

5.5.3 Finding square root through prime factorisation

Consider the prime factorisation of the following numbers and their squares.

Prime factorisation of a Number	Prime factorisation of its Square
$6 = 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$
$8 = 2 \times 2 \times 2$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$	$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$15 = 3 \times 5$	$225 = 3 \times 3 \times 5 \times 5$

How many times does 2 occur in the prime factorisation of 6? Once. How many times does 2 occur in the prime factorisation of 36? Twice. Similarly, observe the occurrence of 3 in 6 and 36 of 2 in 8 and 64 etc.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given square number, say 324.

We know that the prime factorisation of 324 is

2	324
2	162
3	81
3	27
3	9
	3

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

By pairing the prime factors, we get

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} = 2^2 \times 3^2 \times 3^2 = (2 \times 3 \times 3)^2$$

So, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Similarly can you find the square root of 256? Prime factorisation of 256 is

$$256 = \underline{2 \times 2} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

By pairing the prime factors we get,

$$256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = (2 \times 2 \times 2 \times 2)^2$$

Therefore, $\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

Is 48 a perfect square?

We know $48 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

Since all the factors are not in pairs so 48 is not a perfect square.

Suppose we want to find the smallest multiple of 48 that is a perfect square, how should we proceed? Making pairs of the prime factors of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Can you tell by which number should we divide 48 to get a perfect square?

The factor 3 is not in pair, so if we divide 48 by 3 we get $48 \div 3 = 16 = \underline{2 \times 2} \times \underline{2 \times 2}$ and this number 16 is a perfect square too.

Example 4: Find the square root of 6400.

Solution: Write $6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

Therefore $\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5 = 80$

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Example 5: Is 90 a perfect square?

Solution: We have $90 = 2 \times 3 \times 3 \times 5$

2	90
3	45
3	15
	5

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square.
That 90 is not a perfect square can also be seen from the fact that it has only one zero.

Example 6: Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.

Solution: We have $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

2	2352
2	1176
2	588
2	294
3	147
7	49
	7

As the prime factor 3 has no pair, 2352 is not a perfect square.
If 3 gets a pair then the number will become perfect square. So, we multiply 2352 by 3 to get,

$$2352 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Now each prime factor is in a pair. Therefore, $2352 \times 3 = 7056$ is a perfect square. Thus the required smallest multiple of 2352 is 7056 which is a perfect square.

And, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

Example 7: Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

Solution: We have, $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

If we divide 9408 by the factor 3, then

$9408 \div 3 = 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$ which is a perfect square. (Why?)

Therefore, the required smallest number is 3.

And, $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56.$

2	6, 9, 15
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

Example 8: Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.

Solution: This has to be done in two steps. First find the smallest common multiple and then find the square number needed. The least number divisible by each one of 6, 9 and 15 is their LCM. The LCM of 6, 9 and 15 is $2 \times 3 \times 3 \times 5 = 90$.

Prime factorisation of 90 is $90 = 2 \times \underline{3 \times 3} \times 5$.

We see that prime factors 2 and 5 are not in pairs. Therefore 90 is not a perfect square.

In order to get a perfect square, each factor of 90 must be paired. So we need to make pairs of 2 and 5. Therefore, 90 should be multiplied by 2×5, i.e., 10.

Hence, the required square number is $90 \times 10 = 900$.

EXERCISE 5.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?
   (i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025
2. Without doing any calculation, find the numbers which are surely not perfect squares.
   (i) 153 (ii) 257 (iii) 408 (iv) 441
3. Find the square roots of 100 and 169 by the method of repeated subtraction.
4. Find the square roots of the following numbers by the Prime Factorisation Method.
   (i) 729 (ii) 400 (iii) 1764 (iv) 4096
   (v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
   (ix) 529 (x) 8100
5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.
   (i) 252 (ii) 180 (iii) 1008 (iv) 2028
   (v) 1458 (vi) 768
6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.
   (i) 252 (ii) 2925 (iii) 396 (iv) 2645
   (v) 2800 (vi) 1620
7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.