CHAPTER – 11

MID – POINT THEOREM

Exercise 11.1

1.

(a) In the figure (1) given below, D, E and F are mid – point of the sides BC, CA and AB respectively of ΔABC. If AB = 6 cm, BC = 4.8 cm and CA = 5.6 cm, find the perimeter of

(i) The trapezium FBCE

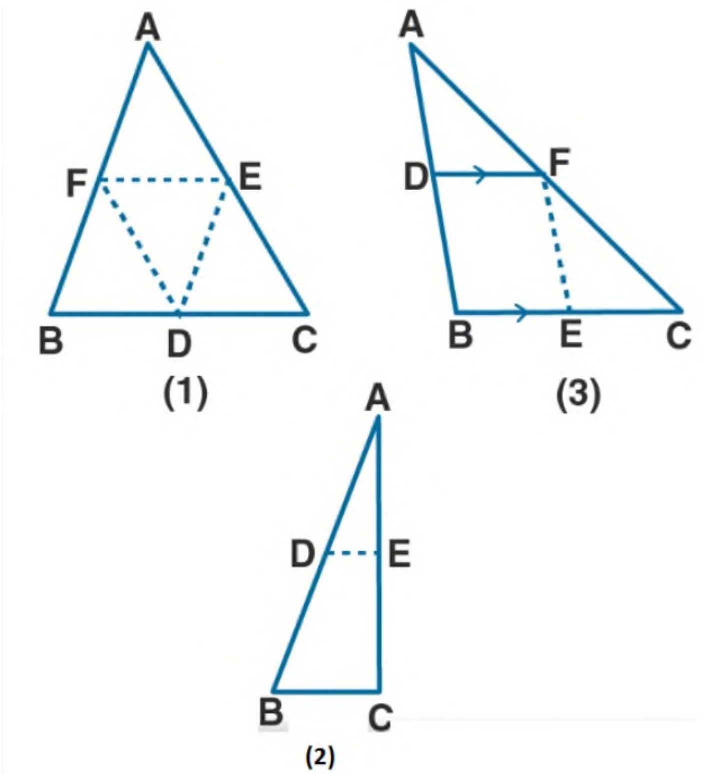
(ii) The triangle DEF.

(b) In the figure (2) given below, D and E are mid-points of the side AB and AC respectively. If BC = 5.6 cm and ∠B = 72°, compute

(i) DE

(ii) ∠ADE.

(c) In the figure (3) given below, D and E are mid-point of AB, BC respectively and DF || BC. Prove that DBEF is a parallelogram. Calculate AC if AF = 2.6 cm.

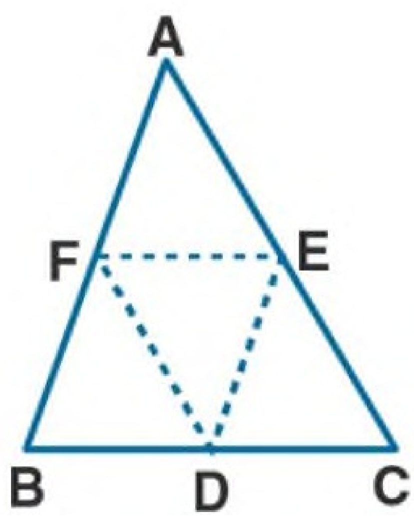


Solution:

(a)

(i) Given: AB = 6 cm, BC = 4.8 cm, and CA = 5.6 cm

To find: The perimeter of trapezium FBCA.



It is given that

F is the mid-point of AB

We know that

BF = $\frac{1}{2}$ AB = $\frac{1}{2} \times 6$ cm = 3 cm ... (1)

It is given that

E is the mid-point of AC

We know that

CE = $\frac{1}{2}$ AC = $\frac{1}{2} \times 5.6$ cm = 2.8 cm ... (2)

Here F and E are the mid-point of AB and CA

FE || BC

We know that

FE = $\frac{1}{2}$ BC = $\frac{1}{2} \times 4.8 = 2.4$ cm ... (3)

Here

Perimeter of trapezium FBCE = BF + BC + CE + EF

Now substituting the value from all the equations

= $3 + 4.8 + 2.8 + 2.4$

= 13 cm

Therefore, the perimeter of trapezium FBCE is 13 cm.

(ii) D, E and F are the midpoints of sides BC, CA and AB of ΔABC

Here EF || BC

EF = $\frac{1}{2}$ BC = $\frac{1}{2} \times 4.8 = 2.4$ cm

DE = $\frac{1}{2}$ AB = $\frac{1}{2} \times 6 = 3$ cm

$FD = \frac{1}{2} AC = \frac{1}{2} \times 5.6 = 2.8$ cm

We know that

Perimeter of ΔDEF = DE + EF + FD

Substituting the values

= 3 + 2.4 + 2.8

= 8.2 cm

(b) It is given that

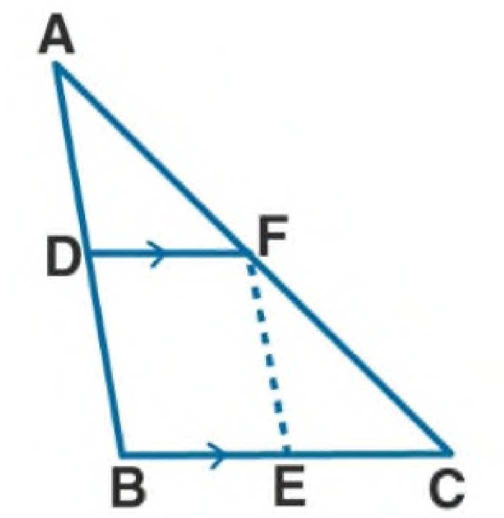
D and E are the mid-point of sides AB and AC

BC = 5.6 cm and ∠B = 72°

To find:

(i) DE

(ii) ∠ADE.



We know that

In ΔABC

D and E is the mid-point of the sides AB and AC

Using mid-point theorem.

DE || BC

(i) DE $= \frac{1}{2}$ BC $= \frac{1}{2} \times 5.6 = 2.8$ cm

(ii) ∠ADE = ∠B are corresponding angles

It is given that

∠B = 72° and BC || DE

∠ADE = 72°

(c) It is given that

D and E are the midpoints of AB and BC respectively

DF || BC and AF = 2.6 cm

To find:

(i) BEF is a parallelogram
(ii) Calculate the value of AC

Proof:

(i) In ΔABC

D is the midpoint of AB and DF || BC

F is the midpoint of AC … (1)

F and E are the midpoints of AC and BC

EF || AB … (2)

Here DF || BC

DF || BE …. (3)

Using equation (2)

EF || AB

EF || DB …. (4)

Using equation (3) and (4)

DBEF is a parallelogram

(ii) F is the midpoint of AC

So we get

AC = $2 \times$ AF = $2 \times 2.6 = 5.2$ cm

2. Prove that the four triangle formed by joining in pairs the mid-point of the sides C of a triangle are congruent to each other.

Solution:

It is given that

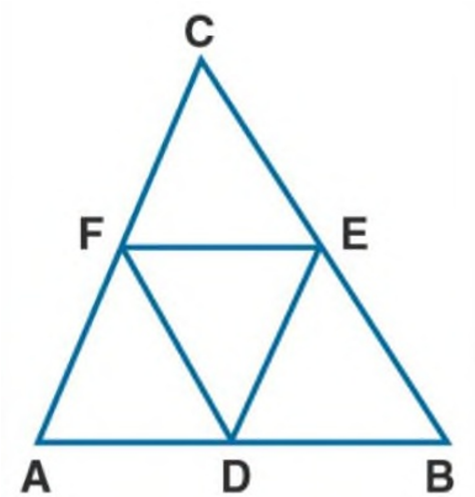
In ΔABC

D, E and F are the mid-point of AB, BC and CA

Now join DE, EF and FD

To find:

ΔADF ≅ΔDBE ≅ΔECF ≅ΔDEF



To prove:

In ΔABC

D and E are the mid-points of AB and BC

DE || AC or FC

Similarly DF || EC

DECF is a parallelogram

We know that

Diagonal FE divides the parallelogram DECF in two congruent triangle DEF and CEF

ΔDBE ≅ΔECF … (1)

Here we can prove that

ΔDBE ≅ΔDEF … (2)

$\Delta DEF \cong \Delta ADF$ … (3)

Using equation (1), (2) and (3)

$\Delta ADF \cong \Delta DBE \cong \Delta ECF \cong \Delta DEF$

3. If D, E and F are mid-point of sides AB, BC and CA respectively of an isosceles triangle ABC, probe that ΔDEF is also isosceles.

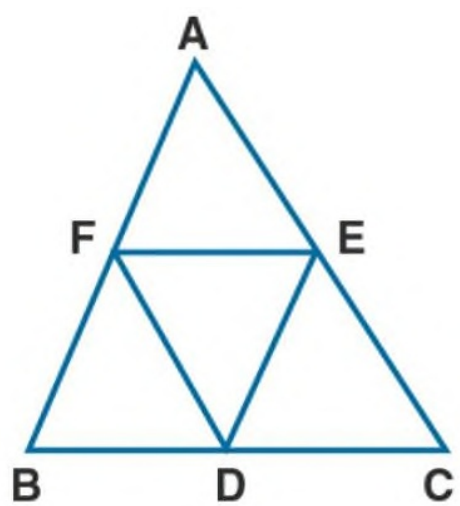
Solution:

It is given that

ABC is an isosceles triangle in which AB = AC

D, E and F are the midpoints of the sides BC, CA and AB

Now, D, E and F are joined.



To find:

ΔDEF is an isosceles triangle

Proof:

D and E are the midpoints of BC and AC.

Here DE || AB and DE = $\frac{1}{2}$ AB … (1)

D and F are the midpoints of BC and AB

Here DF || AC and DF = $\frac{1}{2}$ AC … (2)

It is given that

AB = BC and DE = DF

Hence, ΔDEF is an isosceles triangle.

4. The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the midpoint of AD, prove that

(i) PQ || AB

(ii) PO = $\frac{1}{2}$ CD.

Solution:

It is given that

ABCD is a parallelogram in which diagonals AC and BD intersect each other

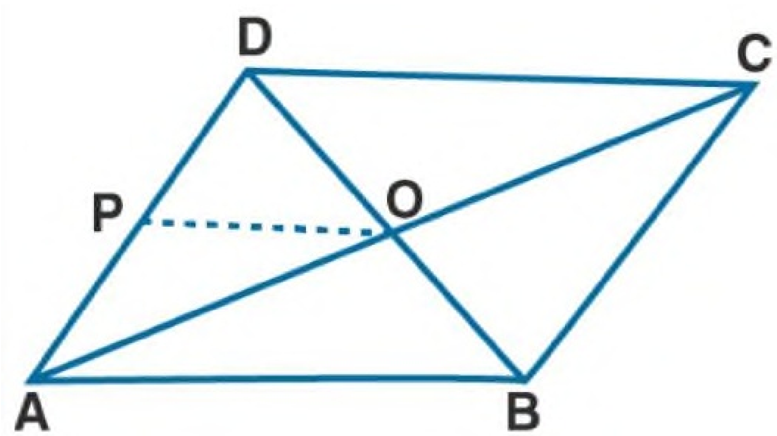
At the point O, P is the midpoint of AD

Join OP

To find:

(i) PQ || AB

(ii) PQ = $\frac{1}{2}$ CD



Proof:

(i) In parallelogram diagonals bisect each other

BO = OD

Here O is the midpoint of BD

In ΔABD

P and O is the midpoint of AD and BD

PO || AB and PO = $\frac{1}{2}$ AB …. (1)

Hence, it is proved that PO || AB.

(ii) ABCD is a parallelogram

AB = CD …. (2)

Using both (1) and (2)

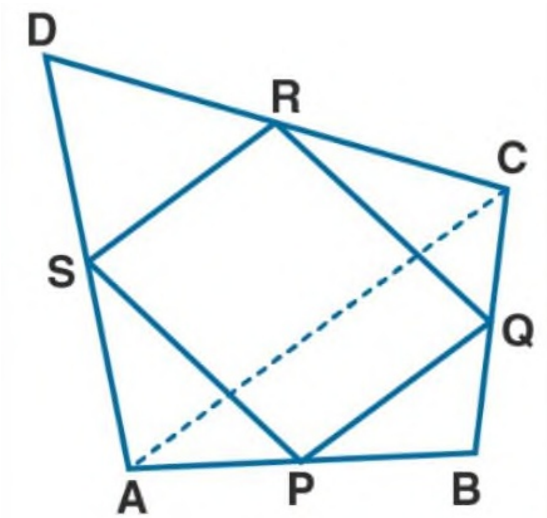
PO = $\frac{1}{2}$ CD

5. In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-point of AB, BC, CD and DA respectively. AC is its diagonal. Show that

(i) **SR || AC and SR = $\frac{1}{2}$ AC**

(ii) **PQ = SR**

(iii) **PQRS is a parallelogram**



Solution:

It is given that

In quadrilateral ABCD

P, Q, R and S are the midpoint of sides AB, BC, CD and DA

AC is the diagonal

To find:

(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Proof:

(i) In ΔADC

S and R are the midpoint of AD and DC

SR || AC and SR = $\frac{1}{2}$ AC …. (1)

Using the mid-point theorem

(ii) In ΔABC

P and Q are the midpoint of AB and BC

PQ || AC and PQ = $\frac{1}{2}$ AC …. (2)

Using equation (1) and (2)

PQ = SR and PQ || SR

(iii) PQ = SR and PQ || SR

Hence, PQRS is a parallelogram.

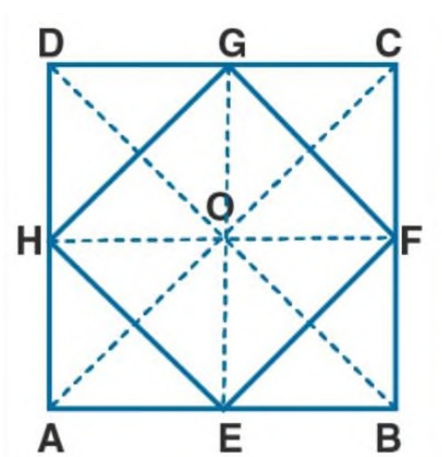
6. Show that the quadrilateral formed by joining the midpoint of the adjacent sides of a square, is also a square.

Solution:

It is given that

A square ABCD in which E, F, G and G are mid-points fo AB, BC, CD and DA

Join EF, FG, GH and HE.



To find:

EFGH is a square

Construct AC and BD

Proof:

In ΔACD

G and H are the midpoint of CD and AC

GH || AC and GH = $\frac{1}{2}$ AC …. (1)

In ΔABC, E and F are the midpoint of AB and BC

EF || AC and EF = $\frac{1}{2}$ AC …. (2)

Using both the equations

EF || AC and EF = GH = $\frac{1}{2}$ AC …. (3)

In the same way we can prove that

EF || GH and EH = GF = $\frac{1}{2}$ BD

We know that the diagonals of square are equal

AC = BD

By dividing both sides by 2

$\frac{1}{2}$ AC = $\frac{1}{2}$ BD ….. (4)

Using equation (3) and (4)

EF = GH = EH = GF …. (5)

Therefore, EFGH is a parallelogram

In ΔGOH and ΔGOF

OH and OF as the diagonals of parallelogram bisect each other

OG = OG is common

Using equation (5)

GF = GF

$\Delta GOH \cong \Delta GOF$ (SSS axiom of congruency)

$\angle GOH = \angle GOF$ (c.p.c.t.)

We know that

$\angle GOH + \angle GOF = 180°$ as it is a linear pair

$\angle GOH + \angle GOH = 180°$

So we get

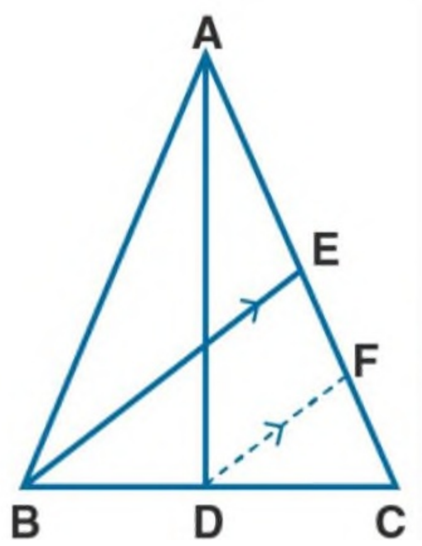
$2\angle GOF = 180°$

$\angle GOF = \frac{180°}{2} = 90°$

So the diagonals of a parallelogram ABCD bisect and perpendicular to each other

Hence, it is proved that EFGH is a square.

7. In the adjoining figure, AD and BE are medians of ΔABC. If DF || BE, Prove that CF = $\frac{1}{2}$ AC.



Solution:

It is given that

AD and BE are the medians of ΔABC

Construct DF || BE

To find:

$CF = \frac{1}{4}AC$

Proof:

In ΔBCE

D is the midpoint of BC and DF || BE

F is the midpoint of EC

$CF = \frac{1}{2}EC$ …. (1)

E is the midpoint of AC

$EC = \frac{1}{2}AC$ ….. (2)

Using both the equations

$CF = \frac{1}{2} EC = \frac{1}{2}\left(\frac{1}{2} AC\right)$

So we get

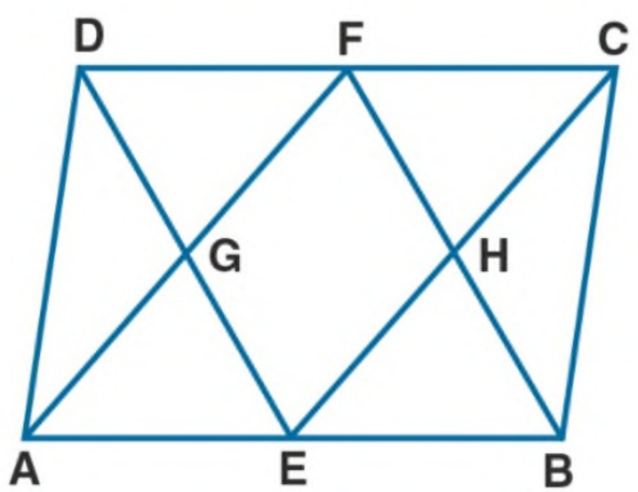
$CF = \frac{1}{4} AC$

Hence, it is proved.

8. In the figure (1) given below, ABCD is a parallelogram. E and F are midpoint of the sides AB and CO respectively. The straight lines ED and EC in points G and H respectively. Prove that

(i) **ΔHEB = ΔHCF**

(ii) **GEHF is a parallelogram**.



Solution:

It is given that

ABCD is a parallelogram

E and F are the midpoint of sides AB and CD

To prove:

(i) ΔHEB = ΔHCF

(ii) GEHF is a parallelogram.

Proof:

(i) We know that

ABCD is a parallelogram

FC || BE

∠CEB = ∠FCE are alternate angle

∠HEB = ∠FCH …. (1)

∠EBF = ∠CEB are alternate angle

∠EBH = ∠CFM …. (2)

Here E and F are midpoint of AB and CD

BE = $\frac{1}{2}$ AB …. (3)

CF = $\frac{1}{2}$ CD …. (4)

We know that

ABCD is a parallelogram

AB = CD

Now dividing both sides by $\frac{1}{2}$

$\frac{1}{2}$ AB = $\frac{1}{2}$ CD

Using equation (3) and (4)

BE = CF …. (5)

In ΔHEB and ΔHCF

∠HEB = ∠FCH using equation (1)

∠EBH = ∠CFH using equation (1)

BE = CF using equation (5)

So we get

$\Delta HEB \cong \Delta HCF$ (ASA axiom of congruency)

Hence, it is proved.

(ii) It is given that

E and F are the midpoint of AB and CD

AB = CD

So we get

AE = CF

Here AE || CF

AE = CF and AE || CF

So AECF is a parallelogram.

G and H are the midpoint of AF and CE

GF || EH …. (6)

In the same way we can prove that GFHE is a parallelogram

So G and H are the points on the line DE and BF

GE || HF …. (7)

Using equation (6) and (7) GEHF is a parallelogram.

Hence, it is proved.

9. ABC is an isosceles triangle AB = AC. D, E and F are midpoint of the sides BC, AB and AC respectively. Prove that the line segment AD is perpendicular to EF and is bisected by it.

Solution:

It is given that

ABC is an isosceles triangle with AB = AC

D, E and F are mid-point of the sides BC, AB and AC

To find:

AD is perpendicular to EF and is bisected by it.

Proof:

In ΔABD and ΔACD

ABC is an isosceles triangle

$\angle ABD = \angle ACD$

Here D is the midpoint of BC

BD = BD

It is given that AB = AC

$\Delta ABD \cong \Delta ACD$ (SAS axiom of congruency)

$\angle ADB + \angle AOC = 180°$ is a linear pair

$\angle ADB + \angle ADB = 180°$

By further calculation

$2\angle ADB = 180°$

So we get

$\angle ADB = \frac{180}{2} = 90°$

So AD is perpendicular to BC …. (1)

D and E are the midpoint of BC and AB

DE || AF …. (2)

D and F are the midpoint of BC and AC

EF || AD ..... (3)

Using equation (2) and (3)

AEDF is a parallelogram

Here the diagonals of a parallelogram bisect each other

AD and EF bisect each other

Using equation (1) and (3)

EF || BC

So AD is perpendicular to EF

Hence, it is proved.

10.

(a) In the quadrilateral (1) given below, AB || DC, E and F are the midpoints of AD and BD respectively. Prove that:

(i) G is midpoint of BC

(ii) $EG = \frac{1}{2}(AB + DC)$

(b) In the quadrilateral (2) given below, AB || DC || EG. If E is mid-point of AD prove that:

(i) G is the mid-point of BC

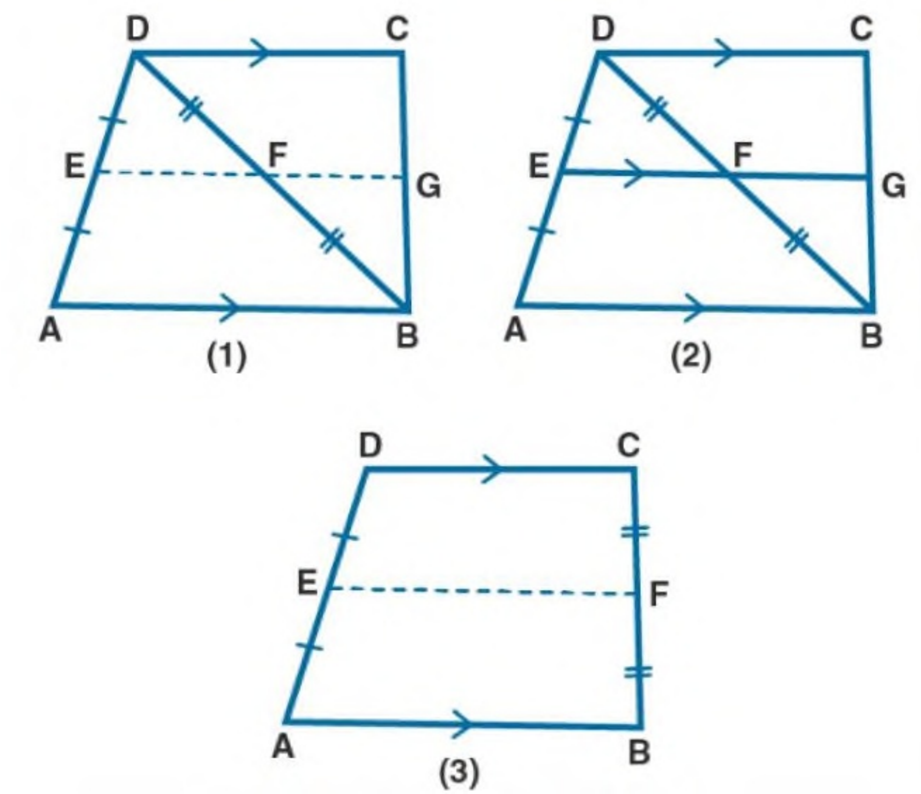
(ii) 2EG = AB + CD

(c) In the quadrilateral (3) given below, AB || DC

E and F are mid-point of non-parallel sides AD and BC respectively. Calculate:

(i) EF if AB = 6 cm and DC = 4 cm.

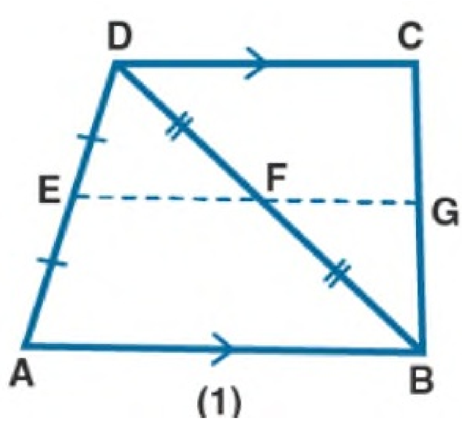
(ii) AB if DC = 8 cm and EF = 9 cm.



Solution:

(a) It is given that

AB || DC, E and F are midpoints of AD and BD



To Prove:

(i) G is midpoint of BC

(ii) $EG = \frac{1}{2}(AB + DC)$

Proof:

In ΔABD

F is the midpoint of BD

DF = BF

E is the midpoint of AD

EF || AB and $EF = \frac{1}{2} AB$ …. (1)

It is given that AB || CD

EG || CD

F is the midpoint of BD

FG || DC

G is the mid-point of BC

$FG = \frac{1}{2} DC$ …. (2)

By adding both the equations

$EF + FG = \frac{1}{2} AB + \frac{1}{2} DC$

Taking $\frac{1}{2}$ as common

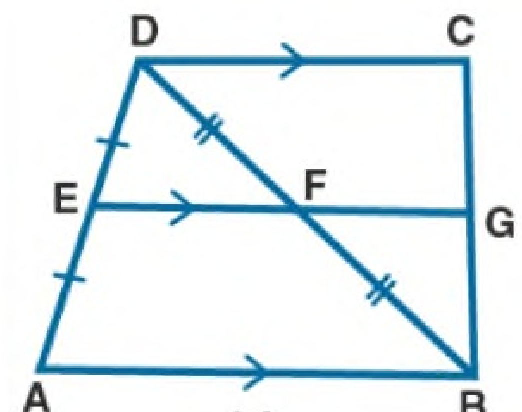
$EG = \frac{1}{2}(AB + DC)$

Therefore, it is proved.

(b) It is given that

Quadrilateral ABCD in which AB || DC || EG

E is the mid-point of AD



(2)

To Prove:

(i) G is the mid-point of BC

(ii) 2EG = AB + CD

Proof:

AB || DC

EG || AB

So we get

EG || DC

In ΔDAB,

E is the midpoint of BD and EF = $\frac{1}{2}$ AB …. (1)

In ΔBCD,

F is the midpoint of BD and FG || DC

FG = $\frac{1}{2}$ CD …. (2)

By adding both the equations

EF + FG = $\frac{1}{2}$ AB + $\frac{1}{2}$ CD

Talking out the common terms

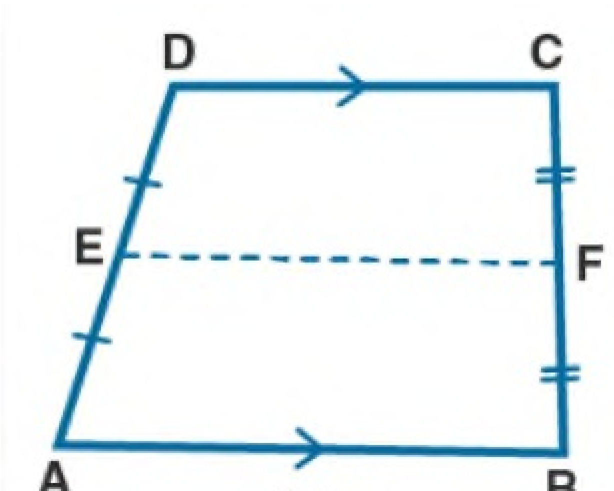
EG = $\frac{1}{2}$ (AB + CD)

Hence, it is proved.

(c) It is given that

A quadrilateral in which AB || DC

E and F are the midpoint of non-parallel sides AD and BC



(3)

To Prove

(i) EF if AB = 6 cm and DC = 4 cm.

(ii) AB if DC = 8 cm and EF = 9 cm.

Proof:

We know that

The length of line segment joining the midpoint of two non-parallel sides is half the sum of the lengths of the parallel sides.

E and F are the midpoint of AD and BC

$\text{EF} = \frac{1}{2}(\text{AB} + \text{CD})$ …. (1)

(i) AB = 6 cm and DC = 4 cm

Substituting in equation (1)

$\text{EF} = \frac{1}{2}(6 + 4)$

By further calculation

$\text{EF} = \frac{1}{2} \times 10 = 5 \text{ cm}$

(ii) DC = 8 cm and EF = 9 cm

Substituting in equation (1)

$\text{EF} = \frac{1}{2}(\text{AB} + \text{DC})$

By further calculation

$$9 = \frac{1}{2}(\text{AB} + 8)$$

$$18 = \text{AB} + 8$$

So we get

$$18 - 8 = \text{AB}$$

$\text{AB} = 10 \text{ cm}$

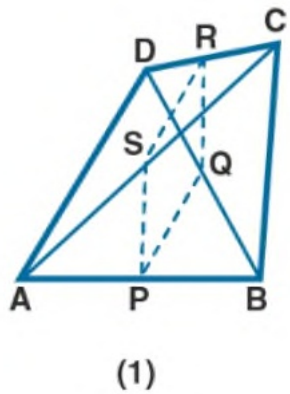
11.

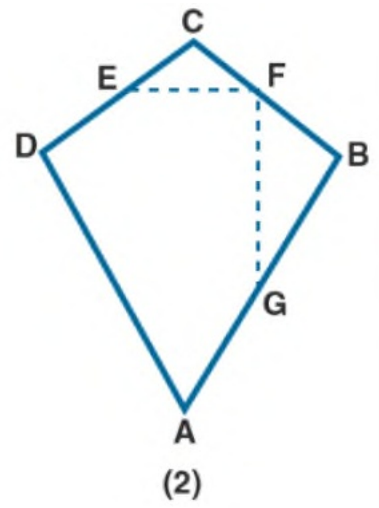
(a) In the quadrilateral (1) given below, AD = BC, P, Q, R and S are midpoint of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.

(b) In the figure (2) given below, ABCD is a kite in which BC = CD, AB = AD, E, F, G are midpoint of CD, BC and AB respectively. Prove that:

(i) **∠EFG = 90°**

(ii) **The line drawn through G and parallel to FE bisectes DA.**





Solution:

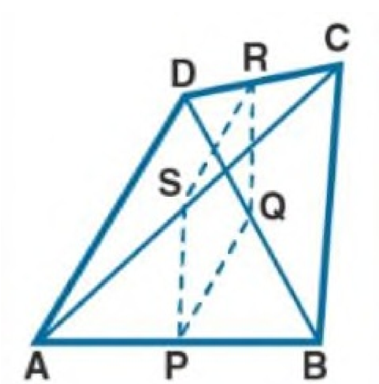
(a) It is given that

A quadrilateral ABCD in which AD = C

P, Q, R and S are midpoint of AB, BD, CD and AC

To prove:

PQRS is a rhombus



Proof:

In ΔABD

P and Q are midpoint of AB and BD

PQ || AD and PQ = $\frac{1}{2}$ AB ….. (1)

In ΔBCD,

R and Q are midpoint of DC and BD

RQ || BC and RQ = $\frac{1}{2}$BC ….. (2)

P and S are mid-point of AB and AC

PS || BC and PS = $\frac{1}{2}$ BC ….. (3)

AD = BC

Using all the equations

PS || RQ and PQ = PS = RQ

Here PS || RQ and PS = RQ

PQRS is a parallelogram

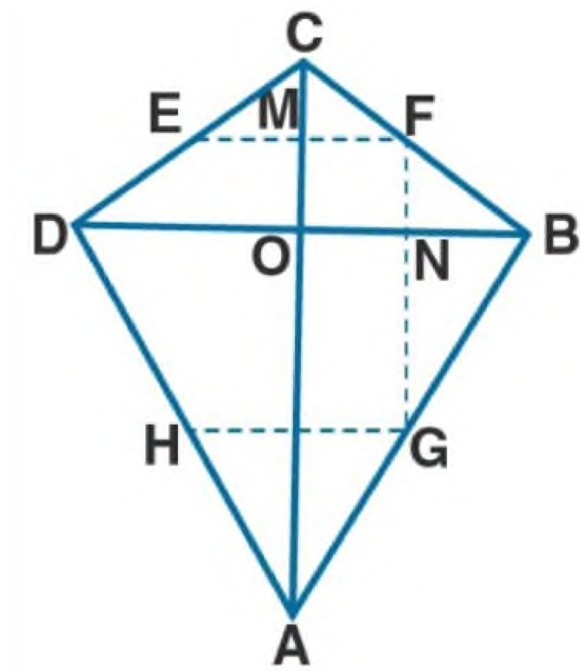
PQ = RS = PS = RQ

PQRS is a parallelogram

Therefore, it is proved.

(ii) It is given that

ABCD is a kite in which BC = CD, AB = AD, E, F, G are midpoint fo CD, BC and AB



To prove:

(i) $\angle EFG = 90°$

(ii) The line drawn through G and parallel to FE bisects DA

Construction:

Join AC and BD

Construct GH through G parallel to FE

Proof:

(i) We know that

Diagonals of a kite interest at right angles

$\angle MON = 90°$ …. (1)

In ΔBCD,

E and F are midpoint of CD and BC

EF || DB and $EF = \frac{1}{2}DB$ ….. (2)

EF || DB

MF || ON

Here

$\angle MON + \angle MFN = 180°$

$90° + \angle MFN = 180°$

By further calculation

$\angle MFN = 180 - 90 = 90°$

So $\angle EFG = 90°$

Hence, it is proved.

(ii) In ΔABD,

G is the midpoint of AB and HG || DB

Using equation (2)

EF || DB and EF || HG

HG || DB

Here H is the midpoint of DA

Therefore, the line drawn through G and parallel to FE bisects DA.

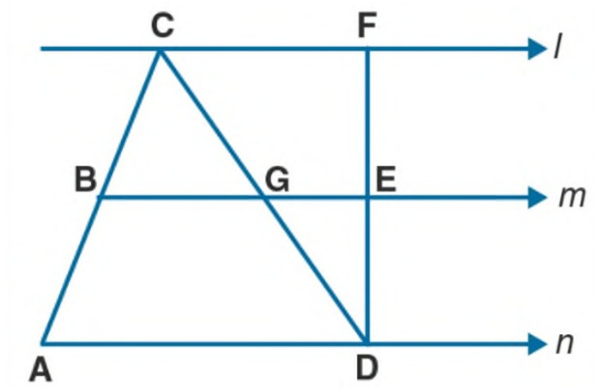
12. In the adjoining figure, the lines l, m and n are parallel to each other, and G is midpoint of CD. Calculate:

(i) BG if AD = 6 cm

(ii) CF if GE = 2.3 cm

(iii) AB if BC = 2.4 cm

(iv) ED if FD = 4.4 cm



Solution:

It is given that

The straight line l, m and n are parallel to each other

G is the midpoint of CD

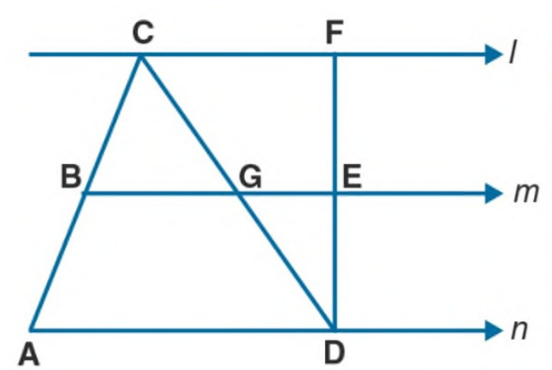
To find:

(i) BG if AD = 6 cm

(ii) CF if GE = 2.3 cm

(iii) AB if BC = 2.4 cm

(iv) ED if FD = 4.4 cm



Proof:

(i) In ΔACD,

G is the midpoint of CD

BG || AD as m || n

Here B is the midpoint of AC and BG = $\frac{1}{2}$ AD

So we get

BG = $\frac{1}{2} \times 6 = 3$ cm

(ii) In ΔCDF,

G is the midpoint of CD

GE || CF as m || l

Here E is the midpoint of DF and GE = $\frac{1}{2}$CF

So we get

CF = 2GE

CF = $2 \times 2.3 = 4.6$ cm

(iii) From (i)

B is the midpoint of AC

AB = BC

We know that

BC = 2.4 cm

So AB = 2.4 cm

(iv) From (ii)

E is the mid-point of FD

ED = $\frac{1}{2}$ FD

We know that

FD = 4.4 cm

ED = $\frac{1}{2} \times 4.4 = 2.2$ cm.

CHAPTER TEST

1. ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD respectively. Prove that PQ ⊥ QR.

Solution:

It is given that

ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD

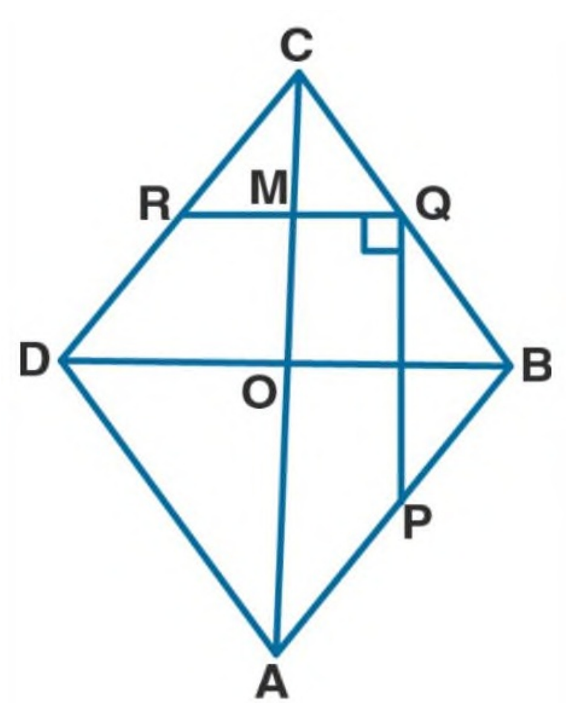
To prove:

PQ ⊥ QR

Construction: Join AC and BD

Proof:

Diagonals of rhombus intersect at right angle



$\angle MON = 90°$ .... (1)

In ΔBCD

Q and R are midpoint of BC and CD.

RQ || DB and RQ = $\frac{1}{2}$ DB ….. (2)

Here

RQ || DB

MQ || ON

We know that

∠MON + ∠MON = 180°

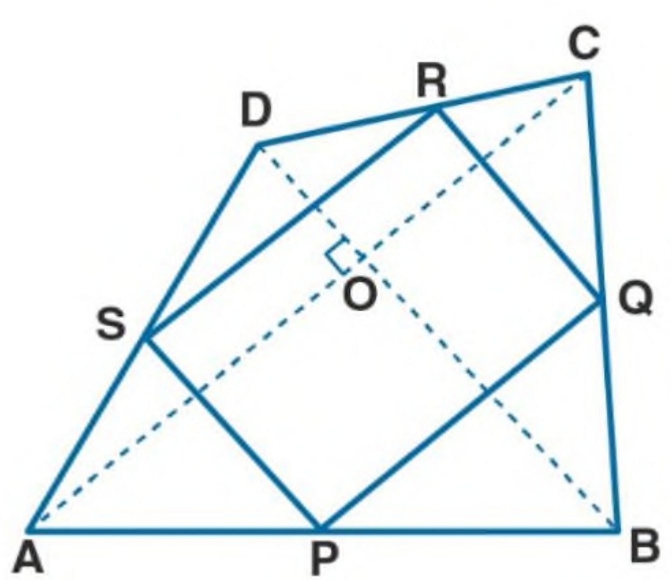
∠MON = 180 – 90 = 90°

So NQ ⊥ MQ or PQ ⊥ QR

Hence, it is proved.

2. The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the midpoint of its adjacent sides is a rectangle.

Solution:



It is given that

ABCD is a quadrilateral in which diagonals AC and BD are perpendicular to each other

P, Q, R and S are mid-point of AB, BC, CD and DA

To prove:

PQRS is a rectangle

Proof:

We know that

P and Q are the midpoint of AB and BC

PQ || AC and PQ = $\frac{1}{2}$AC …. (1)

S and R are midpoint of AD and DC

SR || AC and SR = $\frac{1}{2}$AC ….. (2)

Using both the equations

PQ || SR and PQ = SR

So PQRS is a parallelogram

AC and BD intersect at right angles

SP || BD and BD ⊥ AC

So SP ⊥ AC i.e. SP ⊥ SR

∠RSP = 90°

∠RSP = ∠SRQ = ∠RQS = ∠SPQ = 90°

Hence, PQRS is a rectangle.

3. If D, E, F are midpoint of the sides BC, CA and AB respectively of a ΔABC, prove that AD and FE bisect each other.

Solution:

It is given that

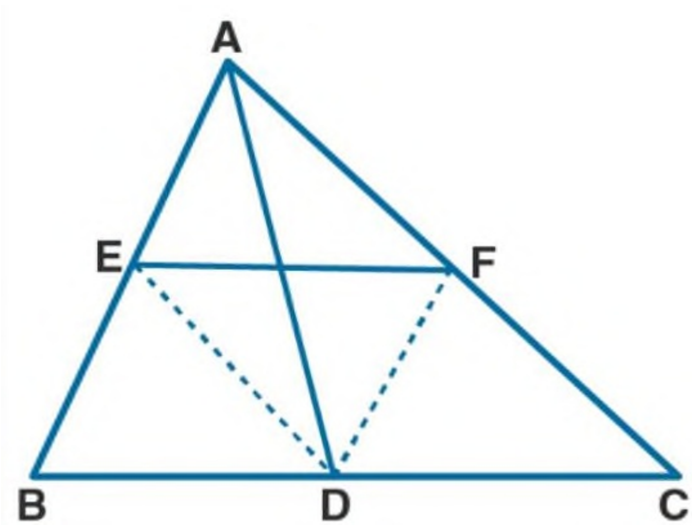
D, E, F are midpoint of sides BC, CA and AB of a ΔABC

To prove:

AD and FE bisect each other

Construction:

Join ED and FD



Proof:

We know that

D and E are the midpoint of BC and AB

DE || AC and DE || AF … (1)

D and F are the midpoint of BC and AC

DF || AB and DF || AE … (2)

Using both the equations

ADEF is a parallelogram

Here the diagonals of a parallelogram bisect each other

AD and EF bisect each other.

Therefore, it is proved.

4. In ΔABC, D and E are midpoint of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC ant f. Prove that BDEF is a parallelogram. If AB = 8 cm and BC = 9 cm, find the perimeter of the parallelogram BDEF.

Solution:

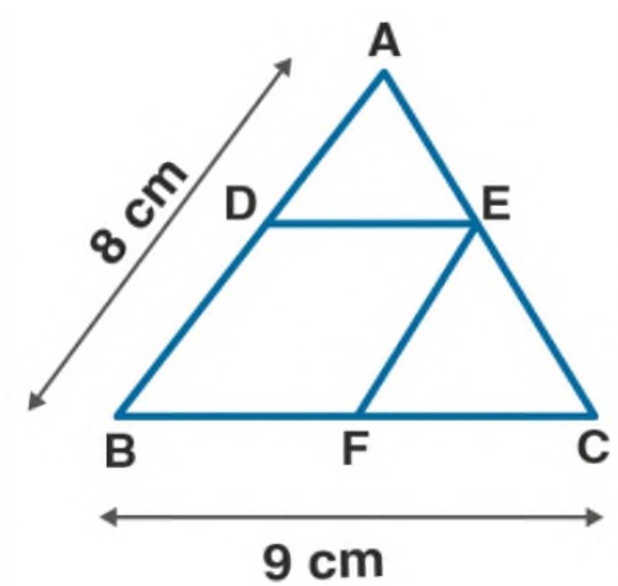
It is given that

In ΔABC

D and E are the midpoint of sides AB and AC

DE is joined from E

EF || AB is drawn AB = 8 cm and BC = 9 cm



To Prove:

(i) BDEF is parallelogram

(ii) Find the perimeter of BDEF

Proof:

In ΔABC

B and E are the mid-point of AB and AC

Here DE || BC and DE = $\frac{1}{2}$ BC

So EF || AB

DEFB is a parallelogram

DE = BF

So we get

DE = $\frac{1}{2}$ BD = $\frac{1}{2} \times 9 = 4.5$ cm

EF = $\frac{1}{2}$AB = $\frac{1}{2} \times 8 = 4$ cm

We know that

Perimeter of BDEF = 2 (DE + EF)

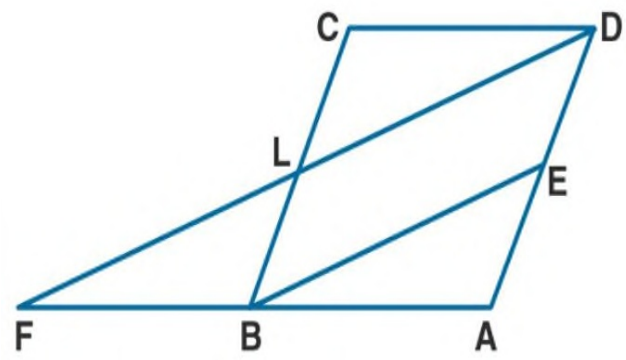
Substituting the value

= $2(4.5 + 4)$

= 2×8.5

= 17 cm

5. In the given figure, ABCD is a parallelogram and E is mid-point of AD, DL || EB meets AB produced at F. Prove that B is midpoint of AB and EB – LF.



Solution:

It is given that

ABCD is a parallelogram

E is the midpoint of AD

DL || EB meets AB produced at F

To prove:

EB = LF

B is the midpoint of AF

Proof:

We know that

BC || AD and BE || LD

BEDL is a parallelogram

BE = LD and BL = AE

Here E is the midpoint of AD

L is the midpoint of BC

In ΔFAD

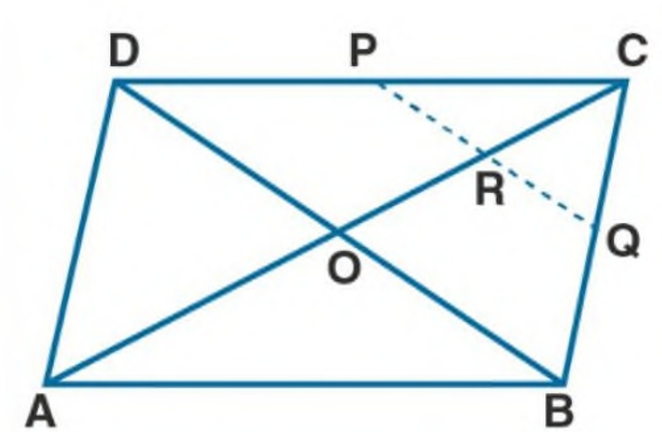
E is the mid-point of AD and BE || LD at FLD

So B is the midpoint of AF

Here

EB = $\frac{1}{2}$ FD = LF.

6. In the given figure, ABCD is a parallelogram. If P and Q are mid-point of sides CD and BC respectively, show that CR = $\frac{1}{2}$ AC.



Solution:

It is given that

ABCD is a parallelogram

P and Q are midpoint of CD and BC

To prove: CR = $\frac{1}{4}$ AC

Construction: Join AC and BD

Proof:

In parallelogram ABCD

Diagonals AC and BD bisect each other at O

AO = OC or OC = $\frac{1}{2}$ AC .... (1)

In ΔBCD

P and Q are midpoint of CD and BC

To prove: CR = $\frac{1}{4}$ AC

Construction: Join AC and BD

Proof:

In parallelogram ABCD

Diagonals AC and BD bisect each other at O

AO = OC or OC = $\frac{1}{2}$ AC   …. (1)

In ΔBCO

Q is the midpoint of BC and PQ || OB

Here is the midpoint of CO

So we get

CR = $\frac{1}{2}$OC = $\frac{1}{2}\left(\frac{1}{2}\ \text{BC}\right)$

CR = $\frac{1}{4}$ BC

Hence, it is proved.