Logarithms

OLYMPIAD EXCELLENCE BOOK

QUESTIONS

1. **Evaluate:** $\log_4 3 \times \log_{27} 64$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{3}$

2. **Evaluate:** $log_{16}64 - log_{64}16$

(a) 6 (b) $\frac{1}{6}$ (c) $\frac{6}{5}$ (d) $\frac{5}{6}$

3. **Find the value of x which satisfies the relation** $log_{10}2 + log_{10}(4x+1) = \log_{10}(x+1) + 1$

(a) 4 (b) -4 (c) 1/4 (d) not defined

4. **Simplify:** $\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}\right]$

(a) 1 (b) 2 (c) 3 (d) 0

5. **The value of log, 81 is equal to:**

(a) –27 (b) –4 (c) 4 (d) 27

6. $\frac{\log \sqrt[3]{6}}{\log 6}$ **is equal to:**

(a) $\frac{1}{\sqrt{8}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

7. **Which of the following statements is correct?**

(a) $log_{10}10 = 0$ (b) $log(2-3) = log(2\times 3)$

(c) $log_{10}1 = 1$ (d) $log(1\times 2\times 3) = log1 + log2 + log3$

8. **If** $\log_2\left[\log_3\left(\log_2 x\right)\right] = 1$**, then x is equal to:**

(a) 0 (b) 12 (c) 128 (d) 512

9. $\log 160$ **is equal to:**

(a) $2log2 + 3log3$ (b) $3log2 + 2log3$

(c) $3log2 + 2log3 - log5$ (d) $5log2 + log5$

10. **If** $log_a(ab) = x$**, then** $log_b(ab)$ **is:**

(a) $\frac{1}{x}$ (b) $\frac{x}{1+x}$ (c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

11. **If** $log2 = x, log3 = y$ **and** $\log 7 = z$**, then the value of** $\log\left(8.\sqrt[3]{21}\right)$ **is:**

(a) $2x + \frac{2}{3}y - \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$ (c) $2x - \frac{2}{3}y + \frac{1}{3}z$ (d) $3x + \frac{1}{3}y + \frac{1}{3}z$

12. **If** $log_8 x + log_8 \frac{1}{6} = \frac{1}{3}$, **then the value of s is:**

(a) 12 (b) 16 (c) 18 (d) 24

13. **If** $log_5 x + 2\log_{25} x + 3log_{12} x = 9$, **then x = _____.**

(a) 6 (b) 36 (c) 125 (d) None of these

14. **If** $log_{10} 5 + log_{10}(5x+1) = log(x+5) + 1$, **then x is equal to:**

(a) 1 (b) 3 (c) 5 (d) 10

15. **if** $log_5(x^2 + x) - log_{10}(x+1) = 2$, **then the value of x is:**

(a) 5 (b) 10 (c) 25 (d) 32

16. **The value of** $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ **is:**

(a) 0 (b) 1 (c) 5 (d) 60

17. **The value of** $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$ **is :**

(a) 2 (b) 7 (c) 8 (d) 33

18. **The value of** $16^{\log_4 5}$ **is:**

(a) $\frac{5}{64}$ (b) 5 (c) 16 (d) 25

19. $\left[\frac{1}{(\log_x yz)+1} + \frac{1}{(\log_y zx)+1} + \frac{1}{(\log_z xz)+1}\right]$ **is equal to :**

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

20. **If** $log_{10} 8 = x$, **then** $log_{10}\left(\frac{1}{80}\right)$ **is equal to:**

(a) $-(1+x)$ (b) $(1+x)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

21. **If** $x = y^x, y = z^y$ **and** $z = x^y$, **then the value of xyz equal to;**

(a) -1 (b) 0 (c) 1 (d) xyz

22. $\frac{\log_5 6}{\log_5 2 + 1} =$

(a) $log_2 6$ (b) $log_2 5$ (c) $log_{10} 6$ (d) $log_{10} 30$

23. If $\mathbf{x = \log_3 27}$ and $\mathbf{y = \log_9 27}$ then $\mathbf{\frac{1}{x} + \frac{1}{y}} =$ _____.

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) 3 (d) 1

24. If $\mathbf{log(0.37) = \bar{1}.756}$, then the value of $\mathbf{log 37 + log(0.37)^3 + log\sqrt{0.37}}$ Is:

(a) 0.902 (b) $\bar{2}.146$ (c) 3.444 (d) $\bar{1}.146$

25. The value of $\frac{1}{1+\log_{ab} c} + \frac{1}{1+\log_{ac} b} + \frac{1}{1+\log_{bc} a}$ equals

(a) 2 (b) 0 (c) 1 (d) log abc

26. If $2^{log_3 9} + 25^{log_9 3} = 8^{log_x 9}$, then x = _____.

(a) 9 (b) 8 (c) 3 (d) 2

27. What is $log_{10}\left(\frac{3}{2}\right) + log_{10}\left(\frac{4}{3}\right) + log_{10}\left(\frac{5}{4}\right) + ...$ up to 10 terms equal to?

(a) 0 (b) $log_{10} 6$ (c) $log_{10} 5$ (d) None of these

28. What is the value of $[log_{10}(5log_{10}100)]^2{}_b$

(a) 4 (b) 3 (c) 2 (d) 1

29. What is the value of $\frac{1}{2} log_{10} 36 - 21 og_{10} 3 + log_{10} 15$?

(a) 2 (b) 3 (c) 1 (d) 0

30. What is the value of $\left(\frac{1}{2} 1 og_{10} 25 - 2 log_{10} 4 + log_{10} 32 + log_{10} 1\right)$?

(a) 0 (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

31. What is the value of $[log_{12}(10)] / [log_{144}(10)]$?

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $log_{10} 13$

32. If $\log_r 6 = m$ and $log_r 3 = n$ then is $log_r (r/2)$ is equal to

(a) $m - n + 1$ (b) $m + n - 1$ (c) $m - n - 1$ (d) $m - n + 1$

33. What is the value of $\mathbf{2log(5/8) + log(128/125) + log(5/2)}$?

(a) 0 (b) -1 (c) 2 (d) 5

34. What is the value of $\log_{100} 0.01$?

(a) 1/2 (B) -1 (c) $\frac{-1}{3}$ (d) -3

35. **What is the valve of** $\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right).....\left(\log_{\frac{1}{99}} 99\right)$

(a) 1 (b) -1 (c) 1 or -1 (d) 0

36. **Which is greatest among the following?**

(a) $\log_2 20$ (b) $\log_7 35$ (c) $\log_5 70$ (d) $\log_3 68$

37. **If** $\frac{\log x}{\log y} = \frac{\log 64}{\log 8}$**, then the relation between x and** γ **is.**

(a) $x = \sqrt{y}$ (b) $x = y^3$ (c) $y = x^2$ (d) $x = y^2$

38. **If** $\log_3 2 = x$ **then the value of** $\frac{\log_{10} 72}{\log_{10} 24}$ **Is**

(a) $\frac{1+x}{1-x}$ (b) $\frac{2+3x}{1+3x}$ (c) $\frac{2-3x}{2+3x}$ (d) $\frac{3x+1}{3x+2}$

39. $\log\left(\frac{1}{2}\right)+\log\left(\frac{2}{3}\right)+\log\left(\frac{3}{4}\right)+.....+\log\left(\frac{999}{10000}\right)=$_____.

(a) -3 (b) -1 (c) 0 (d) 2

40. $\log_2 1.\log_3 2.\log_4 3.\log_5 4.\log_6 5....\log_{100} 99 =$ _____.

(a) ∞ (b) 0 (c) 1 (d) Cannot be determined

ANSWER KEY & HINTS

1. (c): $log_4 3 \times \log_{27} 64 = \frac{\log 3}{\log 4} \times \frac{\log 64}{\log 27} = \frac{\log 3}{\log 4} \times \frac{\log\left(4^3\right)}{\log\left(3^3\right)} = \frac{\log 3}{\log 4} \times \frac{3\log 4}{3\log 3}$

2. (d): Let $log_{16}{}^{64} = n$

Then $16^n = 64$

$\Rightarrow 4^{2n} = 64 = 4^3$

$2n = 3$

$n = \frac{3}{2}$

Similarly $\therefore log_{64} 16 = \frac{2}{3}$

$\therefore log_{16} 64 - log_{64} 16 = (n - m) = \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{5}{6}$

3. (b): $log_{10} 2 + log_{10}(4x+1) = log_{10}(x+1) + 1$

$\Leftrightarrow log_{10} 2 + log_{10}(4x+1) = log_{10}(x+1) + log_{10} 10 \Leftrightarrow log_{10}\left[2(4x+1)\right] = log_{10}\left[10(x+1)\right]$

$\Leftrightarrow 2(4x+1) = 10(x+1)$

$\Leftrightarrow 10x + 2 = 8x + 10 \Leftrightarrow 2x = -8 \Leftrightarrow x = -4$

When it is putting $x = -4$ than $log(x+1)$) is not defined

4. (b): Given expression

$= log_{xyz}(xy) + log_{xyz}(yz) + log_{xyz}(zx)$

$= log_{xyz}(xy \times yz \times zx) = log_{xyz}(xyz)^2$

$= 21og_{xyz}(xyz) = 2 \times 1 = 2.$

5. (b): Let $log_{(-1/3)} 81 = x.$

Then, $\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$

$\therefore x = -4$ *i.e.*, $log_{(-1/3)} 81 = -4$

6. (c): $\dfrac{\log\sqrt[3]{6}}{\log 6} = \dfrac{\log(6)^{1/3}}{\log 6} = \dfrac{\frac{1}{3}\log 6}{\log 6} = \dfrac{1}{3}.$

7. (d): Not Available

8. (d): $log2\left[log_3\left(log_2 x\right)\right] = 1 = log_2 2$

$\Leftrightarrow log_3(log_2 x) = 2 \Leftrightarrow log_2 x = 3^2 = 9$

$\Leftrightarrow x = 2^9 = 512$

9. (d): $160 = 2\times 2\times 2\times 2\times 2\times 5$

So, $log160 = log(2^5\times 5) = log2^5 + log5$

$5log2 + log5$

10. (d): $log_a(ab) = x \Leftrightarrow \dfrac{\log ab}{\log a} = x$

$\Leftrightarrow \dfrac{\log a + \log b}{\log a} = x$

$\Leftrightarrow 1 + \dfrac{\log b}{\log a} = x \Leftrightarrow \dfrac{\log b}{\log a} = x - 1$

$\Leftrightarrow \dfrac{\log b}{\log a} = \dfrac{1}{x-1}x \Leftrightarrow 1 + \dfrac{\log a}{\log b} = 1 + \dfrac{1}{x-1}$

$\Leftrightarrow \dfrac{\log b}{\log b} + \dfrac{\log a}{\log b} = \dfrac{x}{x-1} \Leftrightarrow \dfrac{\log b + \log a}{\log b} = \dfrac{x}{x-1} \Leftrightarrow \dfrac{\log(ab)}{\log b} = \dfrac{x}{x-1} \Leftrightarrow \log_b(ab) = \dfrac{x}{x-1}$

11. (d): $log(8\sqrt[3]{21}) = log8 + log(\sqrt[3]{21})$

$log8 + log\left(21\right)^{1/3} = log\left(2^3\right) + log(7\times 3)^{1/3}$

$= 3\log 2 + \dfrac{1}{3}\log 7 + \dfrac{2}{3}\log 3 = 3x + \dfrac{1}{3}z + \dfrac{2}{3}y.$

12. (a): $\log_8 \dfrac{x}{6} = \dfrac{1}{3}$

$\dfrac{x}{6} = (8)^{\frac{1}{3}}$

$\dfrac{x}{6} = 2$

$x = 12$

13. (c): $log_5 x + 2log_{25} x + 3log_{125} x = 9$

$\Leftrightarrow log_5 x + log_5 x + log_5 x = 9$

$\Leftrightarrow 3log_5 x = 9 \Leftrightarrow log_5 x = 3$

$\Leftrightarrow x = 5^3$

14. (b): $\Leftrightarrow log_{10} 5 + log_{10}(5x+1) = log_{10}[(x+5)] + \log_{10} 10 \Leftrightarrow log_{10} 5 + log_{10}(5x+1) = log_{10}\left[10(x+5)\right]$

$\Leftrightarrow 5(5x+1) = 10(x+5)$

$\Leftrightarrow 5x+1 = 2x+10 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$

15. (c): $log_5(x^2+x) - log_5\left(x+1\right) = 2$

$\Rightarrow \log_5\left(\frac{x^2+x}{x+1}\right) = 2$

$\Rightarrow \log_5\left(\frac{x(x+1)}{x+1}\right) = 2$

$\Leftrightarrow log_5 x = 2 \quad \Leftrightarrow x = 5^2 = 25$

16. (b):Given expression

$= log_{60} 3 + log_{60} 4 + log_{60} 5$

$= log_{60}(3 \times 4 \times 5) = log_{60} 60 = 1.$

17. (a): Given expression $\left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}\right)$

$\frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{3\log 3}{\log 3} = 2.$

18. (d): $\therefore 16^{\log_4 5} = \left(4^2\right)^{log_4 5} = 4^{2log_1 5} = 4^{log_4(5^2)} = 4^{\log_4 25} = 25.$

19. (a): Given expression =

$= \frac{1}{\log_x yz + \log_x x} + \frac{1}{\log_y zx + \log_y y} + \frac{1}{\log_z xy + \log_z z} = \frac{1}{\log_x(xyz)} + \frac{1}{\log_y(xyz)} + \frac{1}{\log_z(xyz)}$

$= \log_{xyz} x + \log_{xyz} y + \log_{xyz} z$

$= \log_{xyz}\left(xyz\right) = 1$

20. (a): $\log_{10}\left(\frac{1}{80}\right)=\log_{10}1-\log_{10}80$

$=-log_{10}(8\times10)=(log_{10}8+log_{10}10)=-(x+1)$

21. (c): $x=y^x, y=z^y, z=x^y$

$\Rightarrow x=log_y x, y=log_z y, z=log_x z$

$\Rightarrow xyz=\left(\log_y x\right)\times\left(\log_z y\right)\times\left(\log_x z\right)$

$\Leftrightarrow xyz=\left(\frac{\log x}{\log y}\times\frac{\log y}{\log z}\times\frac{\log z}{\log x}\right)=1.$

22. (c): $\frac{\log_5 6}{\log_5 2+1}=\frac{\log_5 6}{\log_5 2+\log_5 5}=\frac{\log_5 6}{\log_5\left(2\times5\right)}=\frac{\log_5 6}{\log_5 10}=\log_{10}6$

23. (d): $x=log_3 27 \qquad y=log_9 27$

$\Rightarrow x=log_3 3^3 \;\; y=log_{3^2}3^3$

$\Rightarrow x=3 \;\; y=\frac{3}{2}$

$\therefore \frac{1}{x}+\frac{1}{y}=\frac{1}{3}+\frac{1}{\left(\frac{3}{2}\right)}=\frac{1}{3}+\frac{2}{3}=1.$

24. (c): $log(0.37)=\bar{1}.756\Rightarrow log37=1.756$

$\therefore log37+log(0.37)^3+log\sqrt{0.37}$

$=log37+3log\left(\frac{37}{100}\right)+log\left(\frac{37}{100}\right)^{1/2}$

$=log37+3log37-3log100+\frac{1}{2}log37-\frac{1}{2}log100=\frac{9}{2}log37-\frac{7}{2}log100=\frac{9}{2}\times1.756-\frac{7}{2}\times2$

$=7.902-7=0.902$

25. (a): $\frac{1}{1+\log_{ab}c}=\frac{1}{\log_{ab}ab+\log_{ab}c}=\frac{1}{\log_{ab}abc}=\log_{abc}ab$

$\frac{1}{1+\log_{ac}b}=\frac{1}{\log_{ac}ac+\log_{ac}b}=\frac{1}{\log_{ac}abc}=\log_{abc}ac$

$\frac{1}{1+\log_{bc}a}=\frac{1}{\log_{bc}bc+\log_{bc}a}=\frac{1}{\log_{bc}abc}=\log_{abc}bc$

Hence the value of the required expression

$= \log_{abc} ab + \log_{abc} ac + \log_{abc} bc$

$\log_{abc}\left[(ab)(ac)(bc)\right] = \log_{abc}(abc)^2 = 2.$

26. (b): $2^{\log_3 9} + 25^{\log_9 3} = 8^{\log_x 9}$

$\Rightarrow 2^{\log_3 3^2} + 25^{\log_{3^2} 3} = 8^{\log_x 9}$

$\Rightarrow 2^2 + 25^{1/2} = 8^{\log_x 9}$

$\Rightarrow 9 = 8^{\log_x 9}$

$\Rightarrow \log_x 9 = \log_8 9$

$\Rightarrow x = 8.$

27. (b): $\log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots\ldots + 10th \text{ term}$

$= \log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots\ldots \log_{10}\left(\frac{12}{11}\right) = \log_{10}\left(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots\ldots \times \frac{12}{11}\right)$

$= \log_{10}\left(\frac{12}{2}\right) = \log_{10} 6$

28. (d): $\left[log_{10}(51 log_{10} 100)\right]^2 = \left[log_{10}(5 log_{10} 10^2)\right]^2$

$= \left[log_{10}(10 log_{10} 10)\right]^2$

$= \left[\log_{10} 10\right]^2 \qquad (\because log_{10} 10 = 1)$

$= 1^2 = 1$

29. (c): $\frac{1}{2} log_{10} 36 - 2 log_{10} 3 + log_{10} 15$

$= log_{10} 36^{1/2} - log_{10} 3^2 + log_{10} 15$

$= \log_{10} 6 - log_{10} 9 + log_{10} 15$

$= \log_{10} \frac{6 \times 15}{9} = \log_{10} \frac{90}{9} = \log_{10} 10 = 1$

30. (c): $\frac{1}{2} log_{10} 25 - 2 log_{10} 4 + log_{10} 32 + log_{10} 1$

$$= \frac{1}{2}\log_{10}(5)^2 - 21og_{10}(2)^2 + log_{10}(2)^5 + 0$$

$$= log_{10}5 - 41og_{10}2 + 51og_{10}2$$

$$= log_{10}5 + log_{10}2 = log_{10}10 = 1$$

31. (b) $\frac{\log_{12}(10)}{\log_{144}(10)} = \frac{\log_{12}(10)}{\log_{12^2}(10)}$

$\left(\therefore \log_{a^b} c = \frac{1}{b}\log_a c\right)$

$$= \frac{\log_{12}10}{\frac{1}{2}\log_{12}10} = \frac{1}{1/2} = 2$$

32. (d):Given, $log_r 6 = m \text{ and } log_r 3 = n$

$\therefore log_r 6 - log_r(2\times 3)$

$= log_r 2 + log_r 3$

$\therefore log_r 3 + log_r 2 = m$

$\Rightarrow n + log_r 2 = m$

$\Rightarrow log_r 2 = m - n$

$= 1 - m + n$

33. (a): $2log\left(\frac{5}{8}\right) + log\left(\frac{128}{125}\right) + log\left(\frac{5}{2}\right)$

$$= log\left(\frac{5}{8}\right)^2 + log\left(\frac{128}{125}\right) + log\left(\frac{5}{2}\right)$$

$$= \log\frac{5^2\times 128\times 5}{8^2\times 125\times 2}$$

$$= log\,1 = 0$$

34. (b): $log_{100}0.01$

$$\frac{1}{2}log_{10}(10)^{-2}$$

$$= \frac{-2}{2}log_{10}10$$

$$= -1$$

35. (a): $\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right).....\left(\log_{\frac{1}{99}} 99\right)\left(\frac{\log 2}{\log_{\frac{1}{2}} 2}\right)\left(\frac{\log 3}{\log_{\frac{1}{3}} 3}\right)\left(\frac{\log 4}{\log_{\frac{1}{4}} 4}\right).....\left(\frac{\log 99}{\log_{\frac{1}{99}} 99}\right)$

$\left(\therefore \log_b a = \frac{\log a}{\log b}\right) = \left(\frac{\log 2}{-\log 2}\right)\left(\frac{\log 3}{-\log 3}\right)\left(\frac{\log 4}{-\log 4}\right).....\left(\frac{\log 99}{-\log 99}\right) = (-1)\times(-1)\times(-1)\times,....\times(-1)$

($\therefore$ number of terms is even)

$= 1$

36. (a): Not Available

37. (d) (i) use the identity $\frac{\log a}{\log b} = \log_b a$

(ii) $\frac{\log x}{\log y} = 2 \Rightarrow \log_y x = 2$

(iii) $\log_b a = n \Rightarrow a = b^n$

38. (b): Given, $\log_3 2 = x$

$\frac{\log_{10} 72}{\log_{10} 24} = \log_{24} 72 = \log_{24}(24\times 3)$

$= \log 24 + \log_{24} 3 = 1 + \frac{\log 3}{\log 24}$

$= 1 + \frac{\log_3 3}{\log_3 24} \quad = 1 + \frac{\log_3 3}{\log_3 (3\times 8)}$

$= 1 + \frac{1}{\log_3 3 + \log_3 8}$

$= 1 + \frac{1}{1 + \log_3 2^3}$

$= 1 + \frac{1}{1 + 3\log_3 2}$

$= \frac{1 + 3\log_3 2 + 1}{1 + 3\log_3 2} = \frac{2 + 3x}{1 + 3x}$

39. (a): $\log\left(\frac{1}{2}\right) + \log\left(\frac{2}{3}\right) + ... + \log\left(\frac{999}{1000}\right)$

$= \log 1 - \log 2 + \log 2 - \log 3 + ... + \log 999 - \log 1000 = \log 1 - \log 1000 = 0 - \log 10^3$

$= -3\log 10 = -3.$

40. (b): As $\log_2 1 = 0, \log_2 1.\log_3 2.1\log_4 3....1\log_{99} 100 = 0$