

Solution Of Triangles

Exercise 18A

Q. 1. In any ΔABC , prove that

$$a(b \cos C - c \cos B) = (b^2 - c^2)$$

Answer : Left hand side,

$$a(b \cos C - c \cos B)$$

$$= ab \cos C - ac \cos B$$

$$= ab \frac{a^2 + b^2 - c^2}{2ab} - ac \frac{a^2 + c^2 - b^2}{2ac} \text{ [As, } \cos C = \frac{a^2 + b^2 - c^2}{2ab} \text{ \& } \cos B = \frac{a^2 + c^2 - b^2}{2ac}]$$

$$= \frac{a^2 + b^2 - c^2}{2} - \frac{a^2 + c^2 - b^2}{2}$$

$$= \frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2}$$

$$= \frac{2(b^2 - c^2)}{2}$$

$$= b^2 - c^2$$

= Right hand side. [Proved]

Q. 2. In any ΔABC , prove that

$$ac \cos B - bc \cos A = (a^2 - b^2)$$

Answer : Left hand side,

$$ac \cos B - bc \cos A$$

$$= ac \frac{a^2 + c^2 - b^2}{2ac} - bc \frac{b^2 + c^2 - a^2}{2bc} \left[\text{As, } \cos B = \frac{a^2 + c^2 - b^2}{2ac} \text{ \& } \cos A = \frac{b^2 + c^2 - a^2}{2bc} \right]$$

$$= \frac{a^2 + c^2 - b^2}{2} - \frac{b^2 + c^2 - a^2}{2}$$

$$= \frac{a^2 + c^2 - b^2 - b^2 - c^2 + a^2}{2}$$

$$= \frac{2(a^2 - b^2)}{2}$$

$$= a^2 - b^2$$

= Right hand side. [Proved]

Q. 3. In any ΔABC , prove that

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{(a^2 + b^2 + c^2)}{2abc}$$

Answer :

Need to prove: $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{(a^2+b^2+c^2)}{2abc}$

Left hand side

$$= \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$

$$= \frac{b^2 + c^2 - a^2}{2abc} + \frac{c^2 + a^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc}$$

$$= \frac{a^2 + b^2 + c^2}{2abc}$$

= Right hand side. [Proved]

Q. 4. In any ΔABC , prove that

$$\frac{c - b \cos A}{b - c \cos A} = \frac{\cos B}{\cos C}$$

Answer :

Need to prove: $\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$

Left hand side

$$= \frac{c - b \cos A}{b - c \cos A}$$

$$= \frac{c - b \frac{b^2 + c^2 - a^2}{2bc}}{b - c \frac{b^2 + c^2 - a^2}{2bc}}$$

$$= \frac{\frac{2c^2 - b^2 - c^2 + a^2}{2c}}{\frac{2b^2 - b^2 - c^2 + a^2}{2b}}$$

$$= \frac{\frac{c^2 + a^2 - b^2}{2c}}{\frac{b^2 + a^2 - c^2}{2b}}$$

$$= \frac{\frac{c^2 + a^2 - b^2}{2ac}}{\frac{b^2 + a^2 - c^2}{2ab}} \text{ [Multiplying the numerator and denominator by } \frac{1}{a}]$$

$$= \frac{\cos B}{\cos C}$$

= Right hand side. [Proved]

Q. 5. In any ΔABC , prove that

$$2(bc \cos A + ca \cos B + ab \cos C) = (a^2 + b^2 + c^2)$$

Answer : Need to prove: $2(bc \cos A + ca \cos B + ab \cos C) = (a^2 + b^2 + c^2)$

Left hand side

$$2(bc \cos A + ca \cos B + ab \cos C)$$

$$2\left(bc \frac{b^2 + c^2 - a^2}{2bc} + ca \frac{c^2 + a^2 - b^2}{2ca} + ab \frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2$$

$$a^2 + b^2 + c^2$$

Right hand side. [Proved]

Q. 6. In any ΔABC , prove that

$$4\left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2}\right) = (a + b + c)^2$$

Answer :

Need to prove: $4\left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2}\right) = (a + b + c)^2$

Right hand side

$$= 4\left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2}\right)$$

$$= 4\left(bc \frac{s(s-a)}{bc} + ca \frac{s(s-b)}{ca} + ab \frac{s(s-c)}{ab}\right), \text{ where } s \text{ is half of perimeter of triangle.}$$

$$= 4(s(s-a) + s(s-b) + s(s-c))$$

$$= 4(3s^2 - s(a+b+c))$$

We know, $2s = a + b + c$

$$\text{So, } 4\left(3\left(\frac{a+b+c}{2}\right)^2 - \frac{(a+b+c)^2}{2}\right)$$

$$= 4\left(3\frac{(a+b+c)^2}{4} - \frac{(a+b+c)^2}{2}\right)$$

$$= 4\left(\frac{3(a+b+c)^2 - 2(a+b+c)^2}{4}\right)$$

$$= 3(a+b+c)^2 - 2(a+b+c)^2$$

$$= (a+b+c)^2$$

= Right hand side. [Proved]

Q. 7. In any ΔABC , prove that

$$a \sin A - b \sin B = c \sin (A - B)$$

Answer : Need to prove: $a \sin A - b \sin B = c \sin (A - B)$

Left hand side,

$$= a \sin A - b \sin B$$

$$= (b \cos C + c \cos B) \sin A - (c \cos A + a \cos C) \sin B$$

$$= b \cos C \sin A + c \cos B \sin A - c \cos A \sin B - a \cos C \sin B$$

$$= c(\sin A \cos B - \cos A \sin B) + \cos C(b \sin A - a \sin B)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$= c(\sin A \cos B - \cos A \sin B) + \cos C(2R \sin B \sin A - 2R \sin A \sin B)$$

$$= c(\sin A \cos B - \cos A \sin B)$$

$$= c \sin (A - B)$$

$$= \text{Right hand side. [Proved]}$$

Q. 8. In any ΔABC , prove that

$$a^2 \sin (B - C) = (b^2 - c^2) \sin A$$

Answer : Need to prove: $a^2 \sin (B - C) = (b^2 - c^2) \sin A$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

$$\text{Similarly, } b = 2R \sin B \text{ and } c = 2R \sin C$$

From Right hand side,

$$= (b^2 - c^2) \sin A$$

$$= \{(2R \sin B)^2 - (2R \sin C)^2\} \sin A$$

$$= 4R^2 (\sin^2 B - \sin^2 C) \sin A$$

$$\text{We know, } \sin^2 B - \sin^2 C = \sin(B + C) \sin(B - C)$$

So,

$$= 4R^2 (\sin(B + C) \sin(B - C)) \sin A$$

$$= 4R^2 (\sin(\pi - A) \sin(B - C)) \sin A \quad [\text{As, } A + B + C = \pi]$$

$$= 4R^2 (\sin A \sin(B - C)) \sin A \quad [\text{As, } \sin(\pi - \theta) = \sin \theta]$$

$$= 4R^2 \sin^2 A \sin(B - C)$$

$$= a^2 \sin(B - C) \quad [\text{From (a)}]$$

$$= \text{Left hand side. [Proved]}$$

Q. 9. In any ΔABC , prove that

$$\frac{\sin(A - B)}{\sin(A + B)} = \frac{(a^2 - b^2)}{c^2}$$

Answer :

$$\text{Need to prove: } \frac{\sin(A - B)}{\sin(A + B)} = \frac{(a^2 - b^2)}{c^2}$$

$$\text{We know that, } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{ where } R \text{ is the circumradius.}$$

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

From Right hand side,

$$= \frac{a^2 - b^2}{c^2}$$

$$= \frac{4R^2 \sin^2 A - 4R^2 \sin^2 B}{4R^2 \sin^2 C}$$

$$= \frac{4R^2 (\sin^2 A - \sin^2 B)}{4R^2 \sin^2 C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2 C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2(\pi - (A + B))}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2(A + B)}$$

$$= \frac{\sin(A - B)}{\sin(A + B)}$$

= Left hand side. [Proved]

Q. 10. In any ΔABC , prove that

$$\frac{(b - c)}{a} \cos \frac{A}{2} = \sin \frac{(B - C)}{2}$$

Answer :

$$\text{Need to prove: } \frac{(b-c)}{a} \cos \frac{A}{2} = \sin \frac{(B-C)}{2}$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

From Left hand side,

$$= \frac{2R \sin B - 2R \sin C}{2R \sin A} \cos \frac{A}{2}$$

$$= \frac{2 \cos\left(\frac{B+C}{2}\right) \sin\left(\frac{B-C}{2}\right)}{\sin A} \cos \frac{A}{2}$$

$$= \frac{2 \sin\left(\frac{B-C}{2}\right) \cos\left(\frac{\pi}{2} - \frac{A}{2}\right)}{\sin A} \cos \frac{A}{2}$$

$$= \frac{2 \cos^2 \frac{A}{2} \sin\left(\frac{B-C}{2}\right)}{\sin A}$$

$$= \frac{\sin A \sin\left(\frac{B-C}{2}\right)}{\sin A}$$

$$= \sin \frac{B-C}{2}$$

= Right hand side. [Proved]

Q. 11. In any ΔABC , prove that

$$\frac{(a+b)}{c} \sin \frac{C}{2} = \cos \frac{(A-B)}{2}$$

Answer :

$$\text{Need to prove: } \frac{(a+b)}{c} \sin \frac{C}{2} = \cos \frac{(A-B)}{2}$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

$$\text{Now, } \frac{a+b}{c} = \frac{2R(\sin A + \sin B)}{2R \sin C} = \frac{\sin A + \sin B}{\sin C}$$

$$\text{Therefore, } \frac{c}{a+b} = \frac{\sin C}{\sin A + \sin B} = \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}$$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\sin(\frac{\pi-C}{2}) \cos \frac{A-B}{2}}$$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\cos \frac{C}{2} \cos \frac{A-B}{2}}$$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin \frac{C}{2}}{\cos \frac{A-B}{2}}$$

$$\Rightarrow \frac{a+b}{c} \sin \frac{C}{2} = \cos \frac{A-B}{2} \text{ [Proved]}$$

Q. 12. In any ΔABC , prove that

$$\frac{(b+c)}{a} \cdot \cos \frac{(B+C)}{2} = \cos \frac{(B-C)}{2}$$

Answer :

Need to prove: $\frac{(b+c)}{a} \cdot \cos \frac{(B+C)}{2} = \cos \frac{(B-C)}{2}$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

$$\text{Now, } \frac{a}{b+c} = \frac{2R \sin A}{2R \sin B + 2R \sin C} = \frac{\sin A}{\sin B + \sin C}$$

$$\Rightarrow \frac{a}{b+c} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{2 \sin \frac{B+C}{2} \cos \frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{\sin \frac{A}{2} \cos \frac{A}{2}}{\sin(\frac{\pi}{2} - \frac{A}{2}) \cos \frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{\sin \frac{A}{2} \cos \frac{A}{2}}{\cos \frac{A}{2} \cos \frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{\pi-A}{2})}{\cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{\pi-A}{2})}{\cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{B+C}{2})}{\cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{b+c}{a} \cos(\frac{B+C}{2}) = \cos \frac{B-C}{2} \text{ [Proved]}$$

Q. 13. In any ΔABC , prove that

$$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$$

Answer : Need to prove: $a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$

From left hand side,

$$= a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B)$$

$$= a^2((1 - \sin^2 B) - (1 - \sin^2 C)) + b^2((1 - \sin^2 C) - (1 - \sin^2 A)) + c^2((1 - \sin^2 A) - (1 - \sin^2 B))$$

$$= a^2(-\sin^2 B + \sin^2 C) + b^2(-\sin^2 C + \sin^2 A) + c^2(-\sin^2 A + \sin^2 B)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

So,

$$= 4R^2[\sin^2 A(-\sin^2 B + \sin^2 C) + \sin^2 B(-\sin^2 C + \sin^2 A) + \sin^2 C(-\sin^2 A + \sin^2 B)]$$

$$= 4R^2[- \sin^2 A \sin^2 B + \sin^2 A \sin^2 C - \sin^2 B \sin^2 C + \sin^2 A \sin^2 B - \sin^2 A \sin^2 C + \sin^2 B \sin^2 C]$$

$$= 4R^2 [0]$$

$$= 0 \text{ [Proved]}$$

Q. 14. In any ΔABC , prove that

$$\frac{(\cos^2 B - \cos^2 C)}{b + c} + \frac{(\cos^2 C - \cos^2 A)}{c + a} + \frac{(\cos^2 A - \cos^2 B)}{a + b} = 0$$

Answer :

$$\text{Need to prove: } \frac{(\cos^2 B - \cos^2 C)}{b + c} + \frac{(\cos^2 C - \cos^2 A)}{c + a} + \frac{(\cos^2 A - \cos^2 B)}{a + b} = 0$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$a = 2R \sin A \text{ ---- (a)}$$

Similarly, $b = 2R \sin B$ and $c = 2R \sin C$

From left hand side,

$$= \frac{(\cos^2 B - \cos^2 C)}{b + c} + \frac{(\cos^2 C - \cos^2 A)}{c + a} + \frac{(\cos^2 A - \cos^2 B)}{a + b}$$

$$= \frac{(1 - \sin^2 B - 1 + \sin^2 C)}{b + c} + \frac{(1 - \sin^2 C - 1 + \sin^2 A)}{c + a} + \frac{(1 - \sin^2 A - 1 + \sin^2 B)}{a + b}$$

$$= \frac{\sin^2 C - \sin^2 B}{b + c} + \frac{\sin^2 A - \sin^2 C}{c + a} + \frac{\sin^2 B - \sin^2 A}{a + b}$$

Now,

$$= \frac{1}{2R} \left[\frac{(\sin B + \sin C)(\sin C - \sin B)}{\sin B + \sin C} + \frac{(\sin A + \sin C)(\sin A - \sin C)}{\sin A + \sin C} + \frac{(\sin A + \sin B)(\sin B - \sin A)}{\sin A + \sin B} \right]$$

$$= \frac{1}{2R} [\sin C - \sin B + \sin A - \sin C + \sin B - \sin A]$$

$$= 0 \text{ [Proved]}$$

Q. 15. In any ΔABC , prove that

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$$

Answer :

Need to prove: $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$

Left hand side,

$$= \frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$$

$$= \frac{1 - 2 \sin^2 A}{a^2} - \frac{1 - 2 \sin^2 B}{b^2}$$

$$= \frac{1}{a^2} - \frac{1}{b^2} + 2 \left(\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2} \right)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2} = \frac{1}{4R^2} - \frac{1}{4R^2} = 0$$

Hence,

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2} \text{ [Proved]}$$

Q. 16. In any ΔABC , prove that

$$(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$$

Answer : Need to prove: $(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$

We know,

$$\tan A = \frac{abc}{R} \frac{1}{b^2 + c^2 - a^2} \text{ ----- (a)}$$

$$\text{Similarly, } \tan B = \frac{abc}{R} \frac{1}{c^2 + a^2 - b^2} \text{ and } \tan C = \frac{abc}{R} \frac{1}{a^2 + b^2 - c^2}$$

Therefore,

$$(b^2 + c^2 - a^2) \tan A = \frac{abc}{R} \text{ [from (a)]}$$

Similarly,

$$(c^2 + a^2 - b^2) \tan B = \frac{abc}{R} \text{ and } (a^2 + b^2 - c^2) \tan C = \frac{abc}{R}$$

Hence we can conclude comparing above equations,

$$(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$$

[Proved]

Q. 17.

If in a $\triangle ABC$, $\angle C = 90^\circ$, then prove that $\sin(A - B) = \frac{(a^2 - b^2)}{(a^2 + b^2)}$.

Answer : Given: $\angle C = 90^\circ$

Need to prove: $\sin(A - B) = \frac{(a^2 - b^2)}{(a^2 + b^2)}$

Here, $\angle C = 90^\circ$; $\sin C = 1$

So, it is a Right-angled triangle.

And also, $a^2 + b^2 = c^2$

Now,

$$\frac{a^2 + b^2}{a^2 - b^2} \sin(A - B) = \frac{c^2}{a^2 - b^2} \sin(A - B)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$= \frac{4R^2 \sin^2 C}{4R^2 \sin^2 A - 4R^2 \sin^2 B} \sin(A - B) = \frac{\sin(A - B)}{\sin^2 A - \sin^2 B} [\text{As, } \sin C = 1]$$

$$= \frac{\sin(A - B)}{(\sin A + \sin B)(\sin A - \sin B)} = \frac{\sin(A - B)}{[2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}][2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}]}$$

$$= \frac{\sin(A - B)}{2 \sin \frac{A + B}{2} \cos \frac{A + B}{2} \cdot 2 \sin \frac{A - B}{2} \cos \frac{A - B}{2}} = \frac{\sin(A - B)}{\sin(A + B) \sin(A - B)}$$
$$= \frac{1}{\sin(A + B)}$$

$$= \frac{1}{\sin(\pi - C)} = \frac{1}{\sin C} = 1$$

Therefore,

$$\Rightarrow \frac{a^2 + b^2}{a^2 - b^2} \sin(A - B) = 1$$

$$\Rightarrow \sin(A - B) = \frac{a^2 - b^2}{a^2 + b^2} \text{ [Proved]}$$

Q. 18. In a $\triangle ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, show that the triangle is isosceles.

Answer :

$$\text{Given: } \frac{\cos A}{a} = \frac{\cos B}{b}$$

Need to prove: $\triangle ABC$ is isosceles.

$$\frac{\cos A}{a} = \frac{\cos B}{b}$$

$$\Rightarrow \frac{\sqrt{1 - \sin^2 A}}{a} = \frac{\sqrt{1 - \sin^2 B}}{b}$$

$$\Rightarrow \frac{1 - \sin^2 A}{a^2} = \frac{1 - \sin^2 B}{b^2} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{1}{a^2} - \frac{\sin^2 A}{a^2} = \frac{1}{b^2} - \frac{\sin^2 B}{b^2}$$

$$\text{We know, } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Therefore, $\frac{\sin^2 A}{a^2} = \frac{\sin^2 B}{b^2}$

So,

$$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}$$

$$\Rightarrow a = b$$

That means a and b sides are of same length. Therefore, the triangle is isosceles.
[Proved]

Q. 19. In a $\triangle ABC$, if $\sin^2 A + \sin^2 B = \sin^2 C$, show that the triangle is right-angled.

Answer : Given: $\sin^2 A + \sin^2 B = \sin^2 C$

Need to prove: The triangle is right-angled

$$\sin^2 A + \sin^2 B = \sin^2 C$$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

So,

$$\sin^2 A + \sin^2 B = \sin^2 C$$

$$\frac{a^2}{4R^2} + \frac{b^2}{4R^2} = \frac{c^2}{4R^2}$$

$$a^2 + b^2 = c^2$$

This is one of the properties of right angled triangle. And it is satisfied here. Hence, the triangle is right angled. [Proved]

Q. 20. Solve the triangle in which $a = 2$ cm, $b = 1$ cm and $c = \sqrt{3}$ cm.

Answer : Given: $a = 2$ cm, $b = 1$ cm and $c = \sqrt{3}$ cm

$$\text{Perimeter} = a + b + c = 3 + \sqrt{3} \text{ cm}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3+\sqrt{3}}{2} \left(\frac{3+\sqrt{3}}{2} - 2\right) \left(\frac{3+\sqrt{3}}{2} - 1\right) \left(\frac{3+\sqrt{3}}{2} - \sqrt{3}\right)}$$

$$= \sqrt{\frac{3+\sqrt{3}}{2} \cdot \frac{\sqrt{3}-1}{2} \cdot \frac{\sqrt{3}+1}{2} \cdot \frac{3-\sqrt{3}}{2}}$$

$$= \sqrt{\frac{(9-3)(3-1)}{16}}$$

$$= \sqrt{\frac{12}{16}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \text{ cm}^2 \text{ [Proved]}$$

Q. 21. In a ΔABC , if $a = 3$ cm, $b = 5$ cm and $c = 7$ cm, find $\cos A$, $\cos B$, $\cos C$.

Answer : Given: $a = 3$ cm, $b = 5$ cm and $c = 7$ cm

Need to find: $\cos A$, $\cos B$, $\cos C$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 7^2 - 3^2}{2 \cdot 5 \cdot 7} = \frac{65}{70} = \frac{13}{14}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{7^2 + 3^2 - 5^2}{2 \cdot 7 \cdot 3} = \frac{33}{42}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3^2 + 5^2 - 7^2}{2 \cdot 3 \cdot 5} = \frac{-15}{30} = -\frac{1}{2}$$

Q. 22. If the angles of a triangle are in the ratio 1 : 2 : 3, prove that its corresponding sides are in the ratio $1:\sqrt{3}:2$.

Answer : Given: Angles of a triangle are in the ratio 1 : 2 : 3

Need to prove: Its corresponding sides are in the ratio $1:\sqrt{3}:2$

Let the angles are x , $2x$, $3x$

Therefore, $x + 2x + 3x = 180^\circ$

$$6x = 180^\circ$$

$$x = 30^\circ$$

So, the angles are 30° , 60° , 90°

So, the ratio of the corresponding sides are:

$$= \sin 30^\circ : \sin 60^\circ : \sin 90^\circ$$

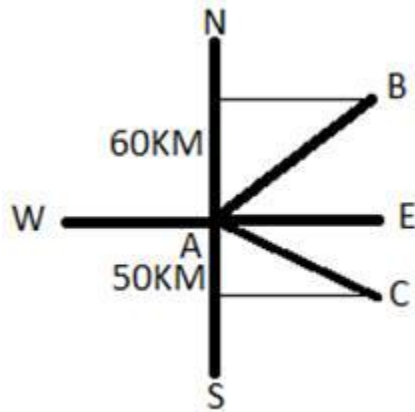
$$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$$

$$= 1:\sqrt{3}:2 \text{ [Proved]}$$

Exercise 18B

Q. 1. Two boats leave a port at the same time. One travels 60 km in the direction N 50° E while the other travels 50 km in the direction S 70° E. What is the distance between the boats?

Answer :



Both the boats start from A and boat 1 reaches at B and boat 2 reaches at C.

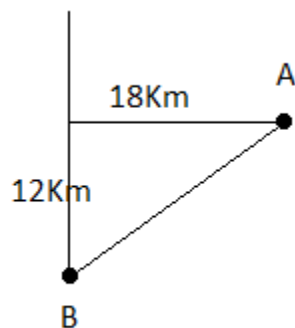
Here, AB = 60Km and AC = 50Km

So, the net distance between the two boats is:

$$\begin{aligned}
 |\overrightarrow{BC}| &= |\overrightarrow{AC} - \overrightarrow{AB}| \\
 &= \sqrt{60^2 + 50^2 - 2 \cdot 60 \cdot 50 \cdot \cos 60^\circ} \\
 &= \sqrt{3600 + 2500 - 3000} \\
 &= 55.67 \text{ Km}
 \end{aligned}$$

Q. 2. A town B is 12 km south and 18 km west of a town A. Show that the bearing of B from A is S $56^\circ 20'$ W. Also, find the distance of B from A.

Answer :



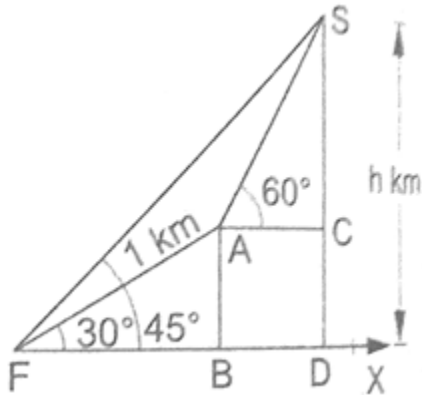
Distance from A to B is $= \sqrt{12^2 + 18^2} = \sqrt{468} = 21.63 \text{ Km}$

Let, bearing from A to B is θ .

$$\text{So, } \tan \theta = \frac{18}{12} = \frac{3}{2}$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.31^\circ = 56^\circ 20'$$

Q. 3. At the foot of a mountain, the angle of elevation of its summit is 45° . After ascending 1 km towards the mountain up an incline of 30° , the elevation changes to 60° (as shown in the given figure). Find the height of the mountain. [Given : $\sqrt{3} = 1.73$]



Answer : After ascending 1 km towards the mountain up an incline of 30° , the elevation changes to 60°

So, according to the figure given, $AB = AF \times \sin 30^\circ = (1 \times 0.5) = 0.5 \text{ Km}$.

At point A the elevation changes to 60° .

In this figure, $\triangle ABF \cong \triangle ACS$

Comparing these triangles, we get $AB = AC = 0.5 \text{ Km}$

Now, $CS = AC \times \tan 60^\circ = (0.5 \times 1.73) = 0.865 \text{ Km}$

Therefore, the total height of the mountain is $= CS + DC$

$= CS + BA$

$= (0.865 + 0.5) \text{ Km}$

= 1.365 Km