Chapter 6. Quadratic Equation

Definition

- In mathematics, a **quadratic equation** is a polynomial equation of the second degree. The general form is

$$ax^2 + bx + c = 0$$

- where *x* represents a variable or an unknown, and *a*, *b*, and *c* are constants with $a \neq 0$. (If $a = 0$, the equation is a linear equation.)
- The constants *a*, *b*, and *c* are called respectively, the quadratic coefficient, the linear coefficient and the constant term or free term.

Forms of a Quadratic Equation

- **Standard Form of a Quadratic Equation** $ax^2 + bx + c = 0, a \neq 0$
- **Factored Form of a Quadratic Equation** $a(x + p)(x + q) = 0, a \neq 0$

  Factoring means to write the terms in multiplication form (as a product).
- **Zero Product Property**

  If $ab = 0$ then either $a = 0$ or $b = 0$ (or both).

  The expression *must* be set equal to zero to use this property.

Zero Product Example: Quadratic in Factored Form $(x - 6)\ (x + 8) = 0$

$x - 6 = 0$ or $x + 8 = 0$

$x = 6$ or $x = 8$

Formulae

1. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are all real numbers and $a \neq 0$.
   e.g., equation $4x^2 + 5x - 6 = 0$ is a quadratic equation in standard form.
2. Every quadratic equation gives two values of the unknown variable and these values are called roots of the equation.
3. **Zero Product Rule:** Whenever the product of two expressions is zero; at least one of the

expressions is zero.

$$\text{If} \quad (x+3)(x-2) = 0$$
$$\Rightarrow \quad x+3 = 0, \text{ or } x-2=0$$
$$\Rightarrow \quad x = -3, \text{ or } x=2$$

4. **Solving quadratic equations using the formula:**
   The roots of the quadratic equation $ax^2 + bx + c = 0$; where $a \neq 0$ can be obtained by using the formula:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

5. **To examine the nature of the roots:**
   Examining the roots of a quadratic equation means to see the type of its roots i.e., whether they are real or imaginary, rational or irrational, equal or unequal.
   The nature of the roots of a quadratic equation depends entirely on the value of its discriminant $b^2 - 4ac$.
   Case I: If a, b and c are real numbers and $a \neq 0$, then discriminant:

   (i) $b^2 - 4ac = 0 \Rightarrow$ the roots are real and equal.
   (ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and unequal.
   (iii) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary (not real).
   Case II: If a, b and c are rational numbers and $a \neq 0$, then discriminant.
   (i) $b^2 - 4ac = 0 \Rightarrow$ the roots are rational and equal.
   (ii) $b^2 - 4ac > 0$ and $b^2 - 4ac$ is a perfect square $\Rightarrow$ the roots are rational and unequal.
   (iii) $b^2 - 4ac > 0$ and $b^2 - 4ac$ is not a perfect square $\Rightarrow$ the roots are irrational and unequal.
   (iv) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary.
6. Sum and product of the roots: If a and P are the roots of quadratic equation $ax^2 + bx + c = 0$ then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$\text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$\text{then } \alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow \alpha + \beta = \frac{-2b}{2a} \Rightarrow \boxed{\alpha + \beta = \frac{-b}{a}}$$

Product of the roots

$$\alpha\beta = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2}$$
$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$
$$\alpha\beta = \frac{4ac}{4a^2} \Rightarrow \boxed{\alpha\beta = \frac{c}{a}}$$

7. To form a quadratic equation with given roots: Let α, β be the roots of the required quadratic equation, then

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$x^2 -$ (sum of the roots)x + product of roots $= 0$ be the required quadratic equation.

Determine the Following

Question 1. Which of the following are quadratic equation:

(i) $(2x-3)(x+5) = 2-3x$

(ii) $\left(x - \frac{1}{x}\right)^2 = 0.$

Solution : (i) Given equation

$$(2x-3)(x+5) = 2-3x$$

$$\Rightarrow \quad 2x^2 + 10x - 3x - 15 - 2 + 3x = 0$$

$$\Rightarrow \quad 2x^2 + 10x - 17 = 0$$

It is quadratic equation.

(ii) Given equation is

$$\left(x - \frac{1}{x}\right)^2 = 0$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2x\frac{1}{x} = 0$$

$$\Rightarrow \quad x^4 + 1 - 2x^2 = 0$$

$$\Rightarrow \quad x^4 - 2x^2 + 1 = 0$$

It is not a quadratic equation.

Question 2. Determine, if 3 is a root of the given equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}.$$

Solution : Substituting $x = 3$ in the given equation

$$\text{L.H.S.} = \sqrt{(3)^2 - 4 \times 3 + 3} + \sqrt{(3)^2 - 9}$$

$$= \sqrt{9 - 12 + 3} + \sqrt{9 - 9}$$

$$= 0 + 0 = 0$$

$$\text{R.H.S.} = \sqrt{4(3)^2 - 14 \times 3 + 16}$$

$$= \sqrt{36 - 42 + 16}$$

$$= \sqrt{52 - 42} = \sqrt{10}$$

Since, L.H.S. ≠ R.H.S.

Therefore, $x = 3$ is not a root of the given equation. Ans.

Question 3. Examine whether the equation 5x² -6x + 7 = 2x² – 4x + 5 can be put in the form of a quadratic equation.

Solution : $5x^2 - 6x + 7 = 2x^2 - 4x + 5$

$\Rightarrow 5x^2 - 6x + 7 - 2x^2 + 4x - 5 = 0$

$\Rightarrow 3x^2 - 2x + 2 = 0.$ Ans.

Question 4. Find if x = – 1 is a root of the equation 2x² – 3x + 1 = 0.

Solution : $2x^2 - 3x + 1 = 0; x = -1.$

Putting $x = -1$ in L.H.S. of equation

L.H.S. $= 2(-1)^2 - 3 \times -1 + 1$

$= 2 + 3 + 1 = 6 \neq 0 \neq$ R.H.S.

Hence, $x = -1$ is not a root of the equation.

Ans.

Question 5. $(3x - 5)(2x + 7) = 0$

Solution : $(3x - 5)(2x + 7) = 0$

$2x + 7 = 0$

or $3x - 5 = 0$

$x = -\frac{7}{2}$

or $x = \frac{5}{3}$

Hence $x = \frac{5}{3}$ and $x = -\frac{7}{2}$ are two roots of the equation. Ans.

Question 6. 48x² – 13x -1 = 0

Solution : $48x^2 - 13x - 1 = 0$

$\Rightarrow 48x^2 - 16x + 3x - 1 = 0$

$\Rightarrow 16x(3x - 1) + 1(3x - 1) = 0$

$\Rightarrow (3x - 1)(16x + 1) = 0$

$\Rightarrow 3x - 1 = 0$

or $16x + 1 = 0$

$x = \frac{1}{3}$ or $x = \frac{-1}{16}$ are two roots of the equation.

Question 7. $10x - \frac{1}{x} = 3$

Solution :

$$10x - \frac{1}{x} = 3$$

$$\Rightarrow \quad \frac{10x^2 - 1}{x} = 3$$

$$\Rightarrow \quad 10x^2 - 3x - 1 = 0$$

$$\Rightarrow \quad 10x^2 - 5x + 2x - 1 = 0$$

$$\Rightarrow \quad 5x(2x - 1) + 1(2x - 1) = 0$$

$$\Rightarrow \quad (2x - 1)(5x + 1) = 0$$

$$2x - 1 = 0$$

or

$$5x + 1 = 0$$

$x = \frac{1}{2}$ or $x = -\frac{1}{5}$ are two roots of the equation.

Question 8. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

Solution :

$$\frac{2}{x^2} \frac{-5}{x} + 2 = 0$$

$$\Rightarrow \quad \frac{2 - 5x + 2x^2}{x^2} = 0$$

$$\Rightarrow \quad 2x^2 - 5x + 2 = 0$$

$$\Rightarrow \quad 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow \quad 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(2x - 1) = 0$$

$$\Rightarrow \quad x - 2 = 0$$

or

$$2x - 1 = 0$$

$$\Rightarrow \quad x = 2$$

or

$$x = \frac{1}{2}$$

are two roots of the equation.

Factoring

- Before today the only way we had for solving quadratics was to factor.

$$x^2 - 2x - 15 = 0$$

$$(x + 3)(x - 5) = 0$$

Zero-factor property

$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -3 \quad \text{or} \quad x = 5$$

$$x = \{-3, 5\}$$

Question 9. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

Solution : $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0$

$\Rightarrow \quad (x + 3\sqrt{3})(\sqrt{3}x + 2) = 0$

$x + 3\sqrt{3} = 0$

or $\quad \sqrt{3}x + 2 = 0$

$\Rightarrow \quad x = -3\sqrt{3}$

or $\quad x = -\dfrac{2}{\sqrt{3}}$

are two roots of the equation.

Question 10. $\sqrt{3}\, x^2 + 10x + 7\sqrt{3} = 0$

Solution : $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x\left(x + \sqrt{3}\right) + 7\left(x + \sqrt{3}\right) = 0$

$\Rightarrow \quad \sqrt{3}x + 7 = 0$ or $x + \sqrt{3} = 0$

$x = -\dfrac{7}{\sqrt{3}}$ or $-\sqrt{3}$. are two roots of equation.

Question 11. $ax^2 + (4a^2 - 3b)x - 12\,ab = 0$

Solution: Here $ax^2 + (4a^2 - 3b)x - 12ab = 0$

$\Rightarrow \quad ax^2 + 4a^2x - 3bx - 12ab = 0$

$\Rightarrow \quad ax\,(x + 4a) - 3b(x + 4a) = 0$

$\Rightarrow \quad (ax - 3b)\,(x + 4a) = 0$

$\Rightarrow x = \dfrac{3b}{a}$ or $-4a$ are two roots of equation.

Question 12. Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots: $x^2 + (p - 3)x + p = 0$

$x^2 + (p - 3)\,x + p = 0$

Solution : $x^2 + (p - 3)\,x + p = 0$

Here $a = 1, b = p - 3, c = p$

For real and equal roots

$D = b^2 - 4ac = 0$

$(p - 3)^2 - 4 \times 1 \times p = 0$

$p^2 - 6p + 9 - 4p = 0$

$p^2 - 10p + 9 = 0$

$\Rightarrow \quad p^2 - p - 9p + 9 = 0$

$p\,(p - 1) - 9\,(p - 1) = 0$

$\Rightarrow \quad (p - 1)\,(p - 9) = 0$

$\Rightarrow \quad p = 1$ or $p = +9$

Question 13. Find the value of k for which the following equation has equal roots:

$$(k-12)x^2 + 2(k-12)x + 2 = 0.$$

Solution : The given equation is

$$(k-12)x^2 + 2(k-12)x + 2 = 0$$

Here, $a = k-12$, $b = 2(k-12)$ and $c = 2$

Since, the given equation has two equal real roots

then we must have $b^2 - 4ac = 0$

$$\Rightarrow \{2(k-12)\}^2 - 4(k-12) \times 2 = 0$$

$$\Rightarrow 4(k-12)^2 - 8(k-12) = 0$$

$$\Rightarrow 4(k-12)\{k-12-2\} = 0$$

$$\Rightarrow (k-12)(k-14) = 0$$

$$\Rightarrow k-12 = 0 \text{ or } k-14 = 0$$

$$\Rightarrow k = 12 \text{ or } k = 14.$$

Note : But at $k = 12$, terms of x^2 and x in the equation vanish hence only $k = 14$ is acceptable.

Question 14. If one root of the equation $2x^2 - px + 4 = 0$ is 2, find the other root. Also find the value of p.

Solution : The given quadratic equation is

$$2x^2 - px + 4 = 0$$

$$\text{one root} = 2$$

Let the other root be α

Then sum of the roots $2 + \alpha = \frac{-(-p)}{2} = \frac{p}{2}$

$$\alpha = \frac{p}{2} - 2 \qquad \ldots(i)$$

The product of the roots

$$\alpha \times 2 = \frac{4}{2} = 2$$

$$\Rightarrow \alpha = 1$$

Now $2 + \alpha = \frac{p}{2}$

$$\Rightarrow 2 + 1 = \frac{p}{2}$$

$$\Rightarrow p = 6. \qquad \text{Ans.}$$

Question 15. Solve $x^{2/3} + x^{1/3} - 2 = 0$.

Solution : Given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Putting $x^{1/3} = y$, the given equation becomes

$$y^2 + y - 2 = 0$$
$$\Rightarrow \quad y^2 + 2y - y - 2 = 0$$
$$\Rightarrow \quad y(y+2) - 1(y+2) = 0$$
$$\Rightarrow \quad (y+2)(y-1) = 0$$
$$\Rightarrow \quad y+2 = 0 \text{ or } y - 1 = 0$$
$$\Rightarrow \quad y = -2 \text{ or } y = 1$$

But $\quad x^{1/3} = y$

$$\therefore \quad x^{1/3} = -2 \text{ or } x^{1/3} = 1$$
$$\Rightarrow \quad x = (-2)^3 \text{ or } x = (1)^3$$
$$\Rightarrow \quad x = -8 \text{ or } x = 1$$

Hence, roots are – 8, 1.

Question 16. The sum of two numbers is 15. If the sum of reciprocals is 3/10, find the numbers.

Solution : Let the numbers be x and $15 - x$.

Then according to problem

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$
$$\Rightarrow \quad \frac{15 - x + x}{x(15-x)} = \frac{3}{10}$$
$$\Rightarrow \quad 15 \times 10 = 3x(15-x)$$
$$\Rightarrow \quad 150 = 45x - 3x^2$$
$$\Rightarrow \quad 3x^2 - 45x + 150 = 0$$
$$\Rightarrow \quad x^2 - 15x + 50 = 0$$
$$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$$
$$\Rightarrow \quad x(x-10) - 5(x-10) = 0$$
$$\Rightarrow \quad x - 10 = 0 \text{ or } x - 5 = 0$$
$$\Rightarrow \quad x = 10 \text{ or } x = 5$$

Hence, the numbers are 10, 5.

Question 17. Find two consecutive natural numbers whose squares have the sum 221.

Solution : Let the number be x, $x + 1$

Then $x^2 + (x+1)^2 = 221$

$$\Rightarrow \quad x^2 + x^2 + 1 + 2x - 221 = 0$$
$$\Rightarrow \quad 2x^2 + 2x - 220 = 0$$
$$\Rightarrow \quad x^2 + x - 110 = 0$$
$$\Rightarrow \quad x^2 + 11x - 10x - 110 = 0$$
$$\Rightarrow \quad x(x+11) - 10(x+11) = 0$$
$$\Rightarrow \quad (x+11)(x-10) = 0$$
$$\Rightarrow \quad x = -11 \text{ or } x = 10$$

But $x = -11$ is rejected

[∵ It cannot be – ve as it is a natural number]

$$\therefore \quad x = 10$$

Hence, required numbers are 10, 10 + 1.

i.e., 10 and 11. Ans.

Question 18. The sum of the squares of three consecutive natural numbers is 110. Determine the numbers.

Solution : Let three consecutive natural numbers be x, $x+1$ and $x+2$.

Then according to problem

$$(x)^2 + (x+1)^2 + (x+2)^2 = 110$$

$\Rightarrow \quad x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x - 110 = 0$

$\Rightarrow \quad 3x^2 + 6x - 105 = 0$

$\Rightarrow \quad x^2 + 2x - 35 = 0$

$\Rightarrow \quad x^2 + 7x - 5x - 35 = 0$

$\Rightarrow \quad x(x+7) - 5(x+7) = 0$

$\Rightarrow \quad (x+7)(x-5) = 0$

$\Rightarrow \quad x + 7 = 0 \text{ or } x - 5 = 0$

$\Rightarrow \quad x = -7 \text{ or } x = 5$

But $x = -7$ is rejected as it is not a natural number.

Then $x = 5$

Hence, required numbers are 5, (5 + 1), (5 + 2) *i.e.*, 5, 6 and 7. Ans.

Question 19. If an integer is added to its square the sum is 90. Find the integer with the help of a quadratic equation.

Solution : Let the required intetger be x.

Then according to the given condition

$\Rightarrow \quad x + x^2 = 90$

$\Rightarrow \quad x^2 + x - 90 = 0$

$\Rightarrow \quad x^2 + 10x - 9x - 90 = 0$

$\Rightarrow \quad x(x+10) - 9(x+10) = 0$

$\Rightarrow \quad (x+10)(x-9) = 0$

$\Rightarrow \quad x + 10 = 0 \text{ or } x - 9 = 0$

$\Rightarrow \quad x = -10 \text{ or } x = 9$

Hence, the required integer is 9 or – 10.

Question 20. Find two consecutive positive even integers whose squares have the sum 340.

Solution : Let two consecutive positive even integers be $2x$, $2x+2$

$\therefore \quad (2x)^2 + (2x+2)^2 = 340$

$\Rightarrow \quad 4x^2 + 4x^2 + 4 + 8x = 340$

$\Rightarrow \quad 8x^2 + 8x - 336 = 0$

$\Rightarrow \quad x^2 + x - 42 = 0$

$\Rightarrow \quad x^2 + 7x - 6x - 42 = 0$

$\Rightarrow \quad x(x+7) - 6(x+7) = 0$

$\Rightarrow \quad (x+7)(x-6) = 0$

$\Rightarrow \quad x + 7 = 0 \text{ or } x - 6 = 0$

$\Rightarrow \quad x = -7 \text{ or } x = 6$

Negative integer is not required, therefore, $x = 6$.

Hence, integers are 6×2, $(6 \times 2) + 2$.

i.e., 12 and 14. Ans.

Question 21. Divide 29 into two parts so that the sum of the square of the parts is 425.

Solution : Let the parts be x and $29 - x$

According to the problem

$$x^2 + (29 - x)^2 = 425$$
$$\Rightarrow x^2 + 841 + x^2 - 58x - 425 = 0$$
$$\Rightarrow 2x^2 - 58x + 416 = 0$$
$$\Rightarrow x^2 - 29x + 208 = 0$$
$$\Rightarrow x^2 - 16x - 13x + 208 = 0$$
$$\Rightarrow x(x - 16) - 13(x - 16) = 0$$
$$\Rightarrow (x - 16)(x - 13) = 0$$
$$\Rightarrow x - 16 = 0 \text{ or } x - 13 = 0$$
$$\Rightarrow x = 16 \text{ or } x = 13$$

When $x = 16$, Then $29 - x = 13$ | When $x = 13$, Then $29 - x = 16$

Hence, the parts are 16 and 13.

Question 22. In a two digit number, the unit's digit is twice the ten's digit. If 27 is added to the number, the digit interchange their places. Find the number.

Solution : Let ten's digit $= x$

Unit's digit $= 2x$

Required number $= 10x + 2x = 12x$

On interchanging the digit's

Number formed $= 10(2x) + x = 21x$

According given condition

$$12x + 27 = 21x$$
$$27 = 21x - 12x$$
$$27 = 9x$$
$$\therefore \quad x = \frac{27}{9}$$
$$x = 3$$
$$\therefore \quad \text{Required number} = 12 \times 3 = 36.$$

Question 23. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number.

Solution : Let the unit's digit be x then tens digit will be $\frac{6}{x}$, then two digit number is $\frac{60}{x}+x$.

From question,

$$\frac{60}{x}+x+9 = 10x+\frac{6}{x}$$
$$60+x^2+9x = 10x^2+6$$
$$9x^2-9x-54 = 0$$
$$x^2-x-6 = 0$$
$$x^2-3x+2x-6 = 0$$
$$x(x-3)+2(x-3) = 0$$
$$(x-3)(x+2) = 0$$
$$\Rightarrow \quad x = -2 \text{ or } 3$$

As x can't be – ve

So, required two digit number

$$= \frac{60}{3}+3$$
$$= 23.$$ Ans.

Question 24. Five years ago, a woman's age was the square of her son's age. Ten years later her age will be twice that of her son's age. Find:
(i) The age of the son five years ago.
(ii) The present age of the woman.

Solution : Let the age of son be x years five years ago.

$\therefore$ Mother's age be x^2 years five years ago.

After ten years son's age be $(x+15)$ years and woman's age be (x^2+15)

Given
$$x^2+15 = 2(x+15)$$
$$x^2+15 = 2x+30$$
$$x^2-2x-15 = 0$$
$$(x-5)(x+3) = 0$$
$$x = 5$$
or
$$x = -3 \text{ (not possible)}$$

$\therefore$ Son's age five years ago = 5 years.

Woman's present age = $25+5$ = 30 years. Ans.

Question 25. The length of verandah is 3m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.
(i) Taking x, breadth of the verandah write an equation in 'x' that represents the above statement.
(ii) Solve the equation obtained in above and hence find the dimension of verandah.

Solution : Let breadth = xm, length = $(x + 3)$m.

Area = $x(x + 3)$ sq. m.

Perimeter = $2(x + x + 3) = (4x + 6)$ m.

Accordinge to the question, $x(x + 3) = 4x + 6$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x + 2)(x - 3) = 0$

$\therefore x = 3$ and $x = -2$ (inadmissiable).

Hence breadth = 3m, length = 6m. Ans.

Question 26. A two digit number is such that the product of its digit is 14. When 45 is added to the number, then the digit interchange their places. Find the number.

Solution : Let the ten's digit be x, then unit's digit $= \left(\frac{14}{x}\right)$

Then, the number is $\left(10x + \frac{14}{x}\right)$

When 45 is added to the number, the digits get interchanged.

$\therefore \quad 10x + \frac{14}{x} + 45 = 10 \times \frac{14}{x} + x$

$\Rightarrow \quad x^2 + 5x - 14 = 0$

$\Rightarrow \quad (x + 7)(x - 2) = 0$

$\Rightarrow \quad x = 2$

and $\quad x = -7$ (inadmissible)

Hence, the number is $\left(10x + \frac{14}{x}\right)$

$= \left(10 \times 2 + \frac{14}{2}\right)$

$= 27.$

Question 27. In each of the following determine the; value of k for which the given value is a solution of the equation:

(i) $kx^2 + 2x - 3 = 0; x = 2$

(ii) $3x^2 + 2kx - 3 = 0; x = -\frac{1}{2}$

(iii) $x^2 + 2ax - k = 0; x = -a.$

Solution : (i) Since, $x = 2$ is a root of the given equation, therefore, it satisfies the equation *i.e.,*

$$k(2)^2 + 2 \times 2 - 3 = 0$$

$$\Rightarrow \quad 4k + 1 = 0 \Rightarrow k = -\frac{1}{4}. \quad \text{Ans.}$$

(ii) Since, $x = -\frac{1}{2}$ is a root of the given equation

$$3x^2 + 2kx - 3 = 0$$

Therefore,

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\Rightarrow \quad 3 \times \frac{1}{4} - k - 3 = 0$$

$$\Rightarrow \quad k = \frac{3}{4} - 3 = -\frac{9}{4}$$

$$\Rightarrow \quad k = -\frac{9}{4}. \quad \text{Ans.}$$

(iii) Since, $x = -a$ is a root of the equation

$$x^2 + 2ax - k = 0$$

$$\Rightarrow (-a)^2 + 2a \times (-a) - k = 0$$

$$\Rightarrow \quad a^2 - 2a^2 - k = 0$$

$$\Rightarrow \quad -k = a^2 \Rightarrow k = -a^2. \quad \text{Ans.}$$

Question 28. If x = 2 and x = 3 are roots of the equation $3x^2 - 2kx + 2m = 0$. Find the values of k and m.

Solution : $x = 2$ is a root of given equation substitute $x = 2$ in L.H.S.

$$\text{L.H.S.} = 3(2)^2 - 2k \times 2 + 2m = 0$$

$$12 - 4k + 2m = 0$$

$$4k - 2m = 12 \quad \ldots\text{(i)}$$

Similarly when $x = 2$ is root of given equation

Substitute $x = 3$ in L.H.S.

$$\text{L.H.S.} = 3(3)^2 - 2k \times 3 + 2m = 0$$

$$27 - 6k + 2m = 0$$

$$6k - 2m = 27 \quad \ldots\text{(ii)}$$

On solving equations (i) and (ii), we get

$$k = \frac{15}{2} \text{ and } m = 9.$$

Question 29. Solve the following equation and give your answer up to two decimal places:

$$x^2 - 5x - 10 = 0$$

Solution : Given equation is

$$x^2 - 5x - 10 = 0$$

On comparing with

$$ax^2 + bx + c = 0$$

$a = 1, b = -5, c = -10$

$$\because \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{5 \pm \sqrt{25 + 40}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8{\cdot}06}{2}$$

$$x = \frac{5 + 8{\cdot}06}{2} = \frac{13.06}{2} = 6{\cdot}53$$

and $$x = \frac{5 - 8{\cdot}06}{2} = \frac{-3{\cdot}06}{2} = -1{\cdot}53$$

$x = 6{\cdot}53, x = -1{\cdot}53$

Question 30. Determine whether the given values of x is the solution of the given quadratic equation below:

$$6x^2 - x - 2 = 0; x = \frac{2}{3}, -1.$$

Solution : $6x^2 - x - 2 = 0; x = \frac{2}{3}, -1.$

Now put $x = -1$ in L.H.S. of equation.

$$\text{L.H.S.} = 6 \times (-1)^2 - (-1) - 2$$

$$= 6 + 1 - 2$$

$$= 7 - 2 = 5 \neq 0 \neq \text{R.H.S.}$$

Hence, $x = -1$ is not a root of the equation.

Put $x = \frac{2}{3}$ in L.H.S. of equation.

$$\text{L.H.S.} = 6 \times \left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2$$

$$= \frac{24}{9} - \frac{2}{3} - 2$$

$$= \frac{8}{3} - \frac{2}{3} - 2 = 0$$

$$= 8 - 8 = 0$$

$$= \text{R.H.S.}$$

Hence, $x = \frac{2}{3}$ is a solution of given equation.

Question 31. Find whether the values $x = \frac{1}{a^2}$ and $x = \frac{1}{b^2}$ are the solutions of the equations :

$a^2b^2x^2 - (a^2 + b^2)x + 1 = 0, a \neq 0, b \neq 0.$

Solution :

$$a^2b^2x^2 - (a^2 + b^2)x + 1 = 0; x = \frac{1}{a^2}, x = \frac{1}{b^2}$$

By putting $x = \frac{1}{a^2}$ in L.H.S. of equation

$$\text{L.H.S.} = a^2b^2 \times \left(\frac{1}{a^2}\right)^2 - (a^2 + b^2) \times \frac{1}{a^2} + 1$$

$$= \frac{b^2}{a^2} - 1 - \frac{b^2}{a^2} + 1 = 0 = \text{R.H.S.}$$

By Putting $x = \frac{1}{b^2}$, in L.H.S. of equation

$$\text{L.H.S.} = a^2b^2 \times \left(\frac{1}{b^2}\right)^2 - (a^2 + b^2) \times \frac{1}{b^2} + 1$$

$$= \frac{a^2}{b^2} - \frac{a^2}{b^2} - 1 + 1 = 0 = \text{R.H.S.}$$

Hence, $x = \frac{1}{a^2}, \frac{1}{b^2}$ are the solution of the equation.

Ans.

Question 32. Solve using the quadratic formula x² – 4x + 1 = 0

Solution : $x^2 - 4x + 1 = 0$

$$a = 1, b = -4, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

Taking (+)

$$= \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$\therefore \quad x = 2 + 1{\cdot}732 = 3{\cdot}732$ Taking (–)

or $\quad x = \frac{4 - \sqrt{12}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$

$\therefore \quad x = 2 - 1{\cdot}732 = 0{\cdot}268$

Hence, $x = 2 + \sqrt{3}$ and $2 \div \sqrt{3}$

or $3{\cdot}732$ and $0{\cdot}268$

Ans.

Question 33. Solve the quadratic equation:

$$4\sqrt{5}x^2 + 7x - 3\sqrt{5} = 0.$$

Solution : The given equation is

$$4\sqrt{5}x^2 + 7x - 3\sqrt{5} = 0$$

$$\Rightarrow \quad 4\sqrt{5}x^2 + 12x - 5x - 3\sqrt{5} = 0$$

$$\Rightarrow 4x(\sqrt{5}x + 3) - \sqrt{5}(\sqrt{5}x + 3) = 0$$

$$\Rightarrow \quad (\sqrt{5}x + 3)(4x - \sqrt{5}) = 0$$

$$\Rightarrow \quad \sqrt{5}x + 3 = 0 \text{ or } 4x - \sqrt{5} = 0$$

$$\Rightarrow \quad \sqrt{5}x = -3 \text{ and } 4x = \sqrt{5}$$

$$\Rightarrow \quad x = -\frac{3}{\sqrt{5}} \text{ and } x = \frac{\sqrt{5}}{4}$$

so $\quad x = -\frac{3}{\sqrt{5}}, \frac{\sqrt{5}}{4}.$

Question 34. (i) $3a^2x^2 + 8abx + 4b^2 = 0$

(ii) $\left(x - \frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

Solution : (i) $3a^2x^2 + 8abx + 4b^2 = 0$

$$\Rightarrow \quad 3a^2x^2 + 6abx + 2abx + 4b^2 = 0$$

$$\Rightarrow \quad 3ax(ax + 2b) + 2b(ax + 2b) = 0$$

$$\Rightarrow \quad (3ax + 2b)(ax + 2b) = 0$$

$$\Rightarrow \quad x = -\frac{2b}{3a} \text{ or } -\frac{2b}{a}.$$

(ii) $\quad \left(x - \frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

$$\Rightarrow \quad x^2 + \frac{a^2}{b^2} - \frac{2ax}{b} = \frac{a^2}{b^2}$$

$$\Rightarrow \quad x^2 - \frac{2ax}{b} = 0$$

$$\Rightarrow \quad x\left(x - \frac{2a}{b}\right) = 0$$

$$\Rightarrow x = 0 \text{ or } x - \frac{2a}{b} = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = \frac{2a}{b}.$$

Question 35. Solve the equation $2x - \frac{1}{x} = 7$.

Write your answer correct to two decimal places.

Solution : Solve the equation $2x - \frac{1}{x} = 7$

$$2x^2 - 1 = 7x$$

$$2x^2 - 7x - 1 = 0$$

for quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 2, b = -7, c = -1$

Therefore,

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{7 \pm \sqrt{49 + 8}}{4} = \frac{7 \pm \sqrt{57}}{4}$$

$$x = \frac{7 + \sqrt{57}}{4} \quad \text{or} \quad x = \frac{7 - \sqrt{57}}{4}$$

$$x = \frac{7 + 7{\cdot}550}{4} \quad \text{or} \quad x = \frac{7 - 7{\cdot}550}{4}$$

$$x = 3{\cdot}64 \quad \text{or} \quad x = -{\cdot}14 \qquad \text{Ans.}$$

Question 36. Form the quadratic equation whose roots are:

(i) $\sqrt{3}$ and $3\sqrt{3}$ (ii) $2 + \sqrt{5}$ and $2 - \sqrt{5}$.

Solution : (i) Let α, β be the roots of the required quadratic equation :

Then, $\alpha = \sqrt{3}$ and $\beta = 3\sqrt{3}$

$\alpha + \beta = \sqrt{3} + 3\sqrt{3}$ and $\alpha\beta = \sqrt{3} \times 3\sqrt{3}$

$\therefore$ $\alpha + \beta = 4\sqrt{3}$ and $\alpha\beta = 9$

Required quadratic equation

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - 4\sqrt{3}x + 9 = 0. \qquad \text{Ans.}$$

(ii) Let α, β be the given roots.

Then $\alpha = 2 + \sqrt{5}$ and $\beta = 2 - \sqrt{5}$

$\alpha + \beta = 2 + \sqrt{5} + 2 - \sqrt{5} = 4$

and $\alpha\beta = (2 + \sqrt{5})(2 - \sqrt{5})$

$\Rightarrow$ $\alpha + \beta = 4$ and $\alpha\beta = (2)^2 - (\sqrt{5})^2$

$\Rightarrow$ $\alpha + \beta = 4$ and $\alpha\beta = 4 - 5$

$\Rightarrow$ $\alpha + \beta = 4$ and $\alpha\beta = -1$

Required quadratic equation

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - 4x - 1 = 0. \qquad \text{Ans.}$$

Question 37. Find the value of k for which the given equation has real roots:

(i) $kx^2 - 6x - 2 = 0$ (ii) $9x^2 + 3kx + 4 = 0$.

Solution : (i) The given equation is :

$$kx^2 - 6x - 2 = 0$$

Here, $a = k, b = -6$ & $c = -2$

This equation has real root if

$$b^2 - 4ac \geq 0$$

$\Rightarrow$ $(-6)^2 - 4 \times k \times (-2) \geq 0$

$\Rightarrow$ $36 + 8k \geq 0$

$\Rightarrow$ $8k \geq -36$

$\Rightarrow$ $k \geq -\frac{36}{8} \Rightarrow k \geq -\frac{9}{2}$. Ans.

(ii) The given quadratic equation is :

$$9x^2 + 3kx + 4 = 0$$

Here, $a = 9, b = 3k$ and $c = 4$.

This equation has real roots if

$$b^2 - 4ac \geq 0$$

$\Rightarrow$ $(3k)^2 - 4 \times 9 \times 4 \geq 0$

$\Rightarrow$ $9k^2 - 144 \geq 0$

$\Rightarrow$ $9k^2 \geq 144$

$\Rightarrow$ $k^2 \geq \frac{144}{9} \Rightarrow k \geq \frac{12}{3}$

$\Rightarrow$ $k \geq 4$. Ans.

Question 38. Without actually determining the roots comment upon the nature of the roots of each of the following equations:

(i) $3x^2 + 2x - 1 = 0$

(ii) $2\sqrt{3}x^2 - 2\sqrt{2}x - \sqrt{3} = 0$

(iii) $9a^2b^2x^2 - 48abc + 64c^2d^2 = 0,\ a \neq 0, b \neq 0$

(iv) $x^2 - 5x + 7 = 0$

(v) $x^2 - 4x + 1 = 0$

(vi) $x^2 + 5x + 15 = 0$.

Solution : (i) $3x^2 + 2x - 1 = 0$.

Here, $a = 3, b = 2$ and $c = -1$

$$D = b^2 - 4ac = 4 - 4 \times 3 \times (-1)$$

$\Rightarrow$ $D = 4 + 12 = 16 > 0$.

The given equation has real roots. Ans.

(ii) $2\sqrt{3}x^2 - 2\sqrt{2}x - \sqrt{3} = 0$.

Here, $a = 2\sqrt{3}, b = -2\sqrt{2}$ and $c = -\sqrt{3}$

$$D = b^2 - 4ac$$

$\Rightarrow$ $D = 8 - 4 \times 2\sqrt{3} \times -\sqrt{3}$

$\Rightarrow$ $D = 8 + 24 = 32 > 0$

The given equation has real roots. Ans.

(iii) $9a^2b^2x^2 - 48abcdx + 64c^2d^2 = 0$.

Here, $D = b^2 - 4ac$

$\Rightarrow$ $(-48abcd)^2 - 4 \times 9a^2b^2 \times 64c^2d^2$

$2304a^2b^2c^2d^2 - 2304a^2b^2c^2d^2 = 0$

$$D = 0$$

Roots are real and equal. Ans.

(iv) $x^2 - 5x + 7 = 0$.

Here, $a = 1, b = -5$ and $c = 7$

$D = b^2 - 4ac \Rightarrow 25 - 4 \times 1 \times 7$.

$\Rightarrow \quad 25 - 28 = -3$

Since, $D < 0$ roots are imaginary.

(v) $x^2 - 4x + 1 = 0$.

Here, $a = 1, b = -4$ and $c = 1$

$D = b^2 - 4ac \Rightarrow 16 - 4 \times 1 \times 1$

$\Rightarrow \quad 16 - 4 = 12 > 0$

The given equation has real roots.

(vi) $x^2 + 5x + 15 = 0$.

Here, $a = 1, b = 5$ and $c = 15$

$D = b^2 - 4ac = (5)^2 - 4 \times 1 \times 15$

$= 25 - 60 = -35 \Rightarrow D < 0$

roots are imaginary.

Question 39. Solve the equation 3x² – x – 7 = 0 and give your answer correct to two decimal places.

Solution : $3x^2 - x - 7 = 0$

$a = 3, b = -1, c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{(-1) \pm \sqrt{(-1)^2 - 4.3.(-7)}}{2 \times 3}$$

$$= \frac{1 \pm \sqrt{1 + 84}}{6}$$

$$= \frac{1 \pm \sqrt{85}}{6} = \frac{1 \pm 9{\cdot}216}{6}$$

$$x = \frac{1 + 9.216}{6} \text{ and } \frac{1 - 9{\cdot}216}{6}$$

$$= \frac{10{\cdot}216}{6} \text{ and } -\frac{8{\cdot}216}{6}$$

$= 1{\cdot}703$ and $-1{\cdot}37$.

Question 40. Solve for x using the quadratic formula. Write your answer correct to two significant figures (x -1)² – 3x + 4 = 0.

Solution : $(x-1)^2 - 3x + 4 = 0$

$$x^2 + 1 - 2x - 3x + 4 = 0$$

$$x^2 - 5x + 5 = 0$$

Comparing it with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = -5, c = 5$$

By using the formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm 2 \cdot 24}{2}$$

Taking +ve sign $x = \dfrac{5 + 2 \cdot 24}{2}$

$$x = 3 \cdot 62$$

Taking – ve sign $x = \dfrac{5 - 2 \cdot 25}{2}$

$$= \frac{2 \cdot 76}{2} = 1 \cdot 38$$

Thus required value are 3·62 and 1·38. Ans.

Question 41. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$$x^2 + 2(m-1)x + (m+5) = 0$$

Solution : $x^2 + 2(m-1)x + (m+5) = 0$

Equating with $ax^2 + bx + c = 0$

$$a = 1, b = 2(m-1), \ c = (m+5)$$

Since equation has real and equal roots.

so, $D = 0$

$\Rightarrow$ $b^2 - 4ac = 0$

$$[2(m-1)]^2 - 4 \times 1 \times (m+5) = 0$$

$\Rightarrow$ $4(m-1)^2 - 4(m+5) = 0$

$\Rightarrow$ $4[(m-1)^2 - (m+5)] = 0$

$\Rightarrow$ $4[m^2 - 2m + 1 - m - 5] = 0$

$\Rightarrow$ $m^2 - 3m - 4 = 0$

$\Rightarrow$ $(m+1)(m-4) = 0$

Either $m + 1 = 0$

$$m = -1$$

or $m - 4 = 0$

$$m = 4$$

$$m = -1, 4$$

Question 42. Solve the following by reducing them to quadratic equations:

(i) $x^4 - 26x^2 + 25 = 0$

(ii) $z^4 - 10z^2 + 9 = 0$.

Solution : (i) Given $x^4 - 26x^2 + 25 = 0$

Putting, $x^2 = y$, the given equation reduces to the form $y^2 - 26y + 25 = 0$

$\Rightarrow \quad y^2 - 25y - y + 25 = 0$

$\Rightarrow \quad y(y-25) - 1(y-25) = 0$

$\Rightarrow \quad (y-25)(y-1) = 0$

$\Rightarrow \quad y - 25 = 0 \text{ or } y - 1 = 0$

$\Rightarrow \quad y = 25 \text{ or } y = 1$

$\therefore \quad x^2 = 25$

$\Rightarrow \quad x = \pm 5$

or $\quad x^2 = 1$

$x = \pm 1$

Hence, the required roots are $\pm 5, \pm 1$. Ans.

(ii) Given equation $z^4 - 10z^2 + 9 = 0$

Putting $z^2 = x$, then given equation reduces to the form $x^2 - 10x + 9 = 0$

$\Rightarrow \quad x^2 - 9x - x + 9 = 0$

$\Rightarrow \quad x(x-9) - 1(x-9) = 0$

$\Rightarrow \quad (x-9)(x-1) = 0$

$\Rightarrow \quad x - 9 = 0 \text{ or } x - 1 = 1$

$\Rightarrow \quad x = 9 \text{ or } x = 1$

But $z^2 = x$

$\therefore \quad z^2 = 9$

$\Rightarrow \quad z = \pm 3$

or $\quad z^2 = 1$

$z = \pm 1$

Hence, the required roots are $\pm 3, \pm 1$.

Question 43. Solve for x : $9^{x+2} - 6.3^{x+1} + 1 = 0$.

Solution : Given equation $9^{x+2} - 6.3^{x+1} + 1 = 0$

$\Rightarrow \quad 9^x.9^2 - 6.3^x.3^1 + 1 = 0$

$\Rightarrow \quad 81.(3^2)^x - 18.3^x + 1 = 0$

$\Rightarrow \quad 81.3^{2x} - 18.3^x + 1 = 0$

Putting $3^x = y$, then it becomes $81y^2 - 18y + 1 = 0$

$\Rightarrow \quad 81y^2 - 9y - 9y + 1 = 0$

$\Rightarrow \quad 9y(9y-1) - 1(9y-1) = 0$

$\Rightarrow \quad (9y-1)(9y-1) = 0$

$$\Rightarrow \quad 9y = 1 \Rightarrow y = \frac{1}{9}$$

But $3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$$\therefore \quad x = -2$$

Hence, the required root is – 2.

Question 44. Solve for x:

$$(x^2 - 5x)^2 - 7(x^2 - 5x) + 6 = 0; x \in R.$$

Solution : Given equation

$$(x^2 - 5x)^2 - 7(x^2 - 5x) + 6 = 0$$

Put $x^2 - 5x = y$

$\therefore$ The given equation becomes

$$y^2 - 7y + 6 = 0$$

$$\Rightarrow \quad y^2 - 6y - y + 6 = 0$$

$$\Rightarrow \quad y(y-6) - 1(y-6) = 0$$

$$\Rightarrow \quad y = 1, 6$$

But $x^2 - 5x = y$

$\therefore \; x^2 - 5x = 1$	$x^2 - 5x = 6$
$x^2 - 5x - 1 = 0$	$\Rightarrow x^2 - 5x - 6 = 0$
Here $a = 1, b = -5, c = -1$	$\Rightarrow x^2 - 6x + x - 6 = 0$
$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\Rightarrow x(x-6) + 1(x-6) = 0$
$x = \frac{-(-5) \pm \sqrt{25 + 4}}{2}$	$\Rightarrow (x-6)(x+1) = 0$
$x = \frac{5 \pm \sqrt{29}}{2}$	$\Rightarrow x = 6$ or $x = -1$

Hence, the roots are $-1, 6, \frac{5 \pm \sqrt{29}}{2}$. Ans.

Question 45. Solve the following equation by reducing it to quadratic equation:

$$\sqrt{3x^2 - 2} + 1 = 2x.$$

Solution : $\sqrt{3x^2 - 2} + 1 = 2x$

$$\Rightarrow \quad \sqrt{3x^2 - 2} = 2x - 1$$

On squaring both sides, we get

$$3x^2 - 2 = 4x^2 + 1 - 4x$$

$$\Rightarrow \quad -x^2 + 4x - 3 = 0$$

$$\Rightarrow \quad x^2 - 4x + 3 = 0$$

$$\Rightarrow \quad x^2 - 3x - x + 3 = 0$$

$$\Rightarrow \quad x(x-3) - 1(x-3) = 0$$

$$\Rightarrow \quad (x-3)(x-1) = 0$$

$$\Rightarrow \quad x = 3 \text{ or } x = 1$$

Hence, the solutions are {3, 1}.

Question 46. Solve:

$$(x + 2)(x - 5)(x - 6)(x + 1) = 144.$$

Solution : Given equation

$$(x + 2)(x - 5)(x - 6)(x + 1) = 144$$

$$\Rightarrow (x + 2)(x - 6)(x - 5)(x + 1) = 144$$

$$\Rightarrow (x^2 - 4x - 12)(x^2 - 4x - 5) = 144$$

Put $x^2 - 4x = y$

Then $(y - 12)(y - 5) = 144$

$$\Rightarrow y^2 - 17y + 60 - 144 = 0$$

$$\Rightarrow y^2 - 17y - 84 = 0$$

$$\Rightarrow y^2 - 21y + 4y - 84 = 0$$

$$\Rightarrow y(y - 21) + 4(y - 21) = 0$$

$$\Rightarrow (y - 21)(y + 4) = 0$$

$$\Rightarrow y - 21 = 0 \text{ or } y + 4 = 0$$

$$\Rightarrow y = 21 \text{ or } y = -4$$

But $x^2 - 4x = y$

$\therefore\ x^2 - 4x = 21$	or $x^2 - 4x = -4$
$\Rightarrow x^2 - 4x - 21 = 0$	$\Rightarrow x^2 - 4x + 4 = 0$
$\Rightarrow x^2 - 7x + 3x - 21 = 0$	$\Rightarrow (x - 2)^2 = 0$
$\Rightarrow x(x - 7) + 3(x - 7) = 0$	$\Rightarrow x - 2 = 0$
$\Rightarrow (x - 7)(x + 3) = 0$	$\Rightarrow x = 2$
$\Rightarrow x - 7 = 0$ or $x + 3 = 0$	
$\Rightarrow x = 7$ or $x = -3$	

Hence, $x = 7, -3$ and 2.

Question 47. A two digit number is such that the product of the digits is 12. When 36 is added to this number the digits interchange their places. Determine the number.

Solution : Let a digit at unit's place be x and at ten's place by y.

Then according to problem

$$\text{Required no.} = 10y + x$$

On interchanging the digits

$$\text{Number formed} = 10x + y$$

$$xy = 12$$

$$\therefore \quad x = \frac{12}{y}$$

$$10y + x + 36 = 10x + y$$

$$10y + x - 10x - y = -36$$

$$9y - 9x = -36$$

$$9(y - x) = -36$$

$$y - x = \frac{-36}{9}$$

$$y - x = -4$$

On substituting value of $x = \frac{12}{y}$

$$y - \frac{12}{y} = -4$$

$$\frac{y^2 - 12}{y} = -4$$

$$y^2 + 4y - 12 = 0$$

$$y^2 + 6y - 2y - 12 = 0$$

$$y(y + 6) - 2(y + 6) = 0$$

$$(y + 6)(y - 2) = 0$$

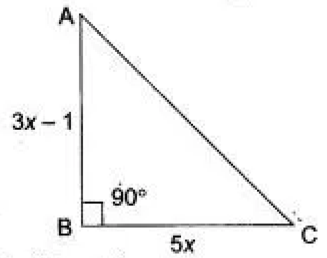
$$y = -6, 2$$

$$\text{When} \quad y = 2$$
$$x = \frac{12}{2} = 6$$
$$\text{Required no.} = 10y + x$$
$$= 10 \times 2 + 6$$
$$= 20 + 6$$
$$= 26.$$

Question 48. The side (in cm) of a triangle containing the right angle are 5x and 3x – 1. If the area of the triangle is 60 cm². Find the sides of the triangle.

Solution : Area of the right triangle ABC

$$= \frac{5x(3x-1)}{2}$$



$$\therefore \quad \frac{5x(3x-1)}{2} = 60$$
$$\Rightarrow \quad 15x^2 - 5x = 120$$
$$\Rightarrow \quad 3x^2 - x = 24$$
$$\Rightarrow \quad 3x^2 - x - 24 = 0$$
$$\Rightarrow \quad 3x^2 - 9x + 8x - 24 = 0$$
$$\Rightarrow \quad 3x(x-3) + 8(x-3) = 0$$
$$\Rightarrow \quad (x-3)(3x+8) = 0$$
$$\Rightarrow \quad x - 3 = 0 \text{ or } 3x + 8 = 0$$
$$\Rightarrow \quad x = 3 \quad \text{or} \quad x = \frac{-8}{3}$$

But $x = \frac{-8}{3}$ is not possible as side cannot be – ve.

Then $x = 3$.

Hence, sides are $AB = 3x - 1 = 8$ cm

$$BC = 5x = 15 \text{ cm}$$
$$\text{Also from fig.} AC = \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{64 + 225}$$
$$= \sqrt{289} = 17 \text{ cm.} \qquad \text{Ans.}$$

Question 49. Rs. 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got Rs. 12 less. Find 'x'.

Solution :

$$\text{Share of each child} = ₹ \frac{480}{x}$$

Now, number of children

$$= x + 20$$
$$\because \quad \text{Share of each child} = ₹ \frac{480}{x+20}$$

Now, According to the question

$$\frac{480}{x} - \frac{480}{x+20} = 12$$

$$\Rightarrow \frac{480x + 9,600 - 480x}{x(x+20)} = 12$$

$$\Rightarrow 9,600 = 12x(x+20)$$

$$\Rightarrow 800 = x^2 + 20x$$

$$\Rightarrow x^2 + 20x - 800 = 0$$

$$\Rightarrow x^2 + 40x - 20x - 800 = 0$$

$$\Rightarrow x(x+40) - 20(x+40) = 0$$

$$\Rightarrow (x-20)(x+40) = 0$$

$$\Rightarrow x = 20$$

or $x = -40$ (not possible)

$$\therefore x = 20$$

Question 50. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km. is reduced by 36 minutes. Find the original speed of the car.

Solution : Let original speed be x km/hr.

$$\therefore \text{Time} = \frac{72}{x}\text{hr.}$$

$$\text{New speed} = x + 10 \text{ km/hr.}$$

$$\therefore \text{New time} = \frac{72}{x+10}\text{hr.}$$

$$\text{Difference in time} = 36 \text{ mins.}$$

$$\therefore \frac{72}{x} - \frac{72}{x+10} = \frac{36}{60}$$

$$\frac{72x + 720 - 72x}{x(x+10)} = \frac{3}{5}$$

$$5 \times 720 = 3(x^2 + 10x)$$

$$1,200 = x^2 + 10x$$

$$x^2 + 10x - 1,200 = 0$$

$$x^2 + 40x - 30x - 1,200 = 0$$

$$x(x+40) - 30(x+40) = 0$$

$$(x-30)(x+40) = 0$$

$$\therefore x = 30$$

as $x = -40$ is not acceptable

$\therefore$ Original speed = 30km/hr.

Question 51. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/hr more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car.

Solution : Let the original speed of the car be x km/hr,

so, Time taken by car $= \frac{400}{x}$ hrs.

Again, Speed $= (x + 12)$ km/hr

Time taken by car $= \frac{400}{x+12}$

so, $\frac{400}{x} - \frac{400}{x+12} = 1 \text{ hr} + \frac{40}{60}$

$$400 \frac{(x+12-x)}{x(x+12)} = \frac{5}{3}$$

$$\frac{4800}{x^2+12x} = \frac{5}{3}$$

$\Rightarrow$ $5(x^2 + 12x) = 14,400$

$\Rightarrow$ $x^2 + 12x - 2,880 = 0$

$\Rightarrow$ $x^2 + 60x - 48x - 2,880 = 0$

$\Rightarrow$ $x(x+60) - 48(x+60) = 0$

$\Rightarrow$ $(x+60)(x-48) = 0$

Either, $x + 60 = 0$

$x = -60$

(Neglect, speed can't be negative)

or $x - 48 = 0$

$x = 48$

$\Rightarrow$ Original speed of the car is 48 km/hr.

Question 52. The speed of an express train is x km/hr arid the speed of an ordinary train is 12 km/hr less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train.

Solution : Let the speed of express train is x km/hr. Speed of ordinary train is $(x - 12)$ km/hr.

Time require to cover for each train is $\frac{240}{x}$ and $\frac{240}{x-12}$ respectively.

According to question

$$\frac{240}{x-12} - \frac{240}{x} = 1$$

$$\frac{240x - 240(x-12)}{(x-12)(x)} = 1$$

$$240x - 240(x-12) = x(x-12)$$

$$x^2 - 12x - 2880 = 0$$

$$(x-60)(x+48) = 0$$

$\therefore$ $x = 60$ km/hr.

Speed of the express train is 60 km/hr. Ans.

Question 53. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic.

Solution : Let the total no. of students be x.

Cost of food for each = ₹ $\frac{480}{x}$

When 8 students failed to join, then cost of food for each = $\frac{480}{x-8}$

According to question

$$\frac{480}{x-8} - \frac{480}{x} = 10$$

$$\frac{480x - 480(x-8)}{x(x-8)} = 10.$$

$$\frac{480(x - x + 8)}{x(x-8)} = 10$$

$$x^2 - 8x - 384 = 0$$

$$x^2 - 24x + 16x - 384 = 0$$

$$(x - 24)(x + 16) = 0$$

$$x = 24$$

The no. of students went for picnic

$$= 24 - 8 = 16.$$

Question 54. Two pipes flowing together can fill a cistern in 6 minutes. If one pipe takes 5 minutes more than the other to fill the cistern, find the time in which each pipe would fill the cistern.

Solution : Let the time taken by the two pipes to fill the cistern be x and $x + 5$ min. respectively.

In 1 min., the first pipe can fill $\frac{1}{x}$ of the cistern. In 1 min., the second pipe can fill $\frac{1}{x+5}$ of the cistern then

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$\Rightarrow \frac{x + 5 + x}{x(x+5)} = \frac{1}{6}$$

$$\Rightarrow \frac{2x + 5}{x^2 + 5x} = \frac{1}{6}$$

$$\Rightarrow x^2 + 5x = 12x + 30$$

$$\Rightarrow x^2 - 7x - 30 = 0$$

$$\Rightarrow x^2 - 10x + 3x - 30 = 0$$

$$\Rightarrow x(x - 10) + 3(x - 10) = 0$$

$$\Rightarrow (x - 10)(x + 3) = 0$$

$$\Rightarrow x - 10 = 0 \text{ or } x = -3$$

$$\Rightarrow x = 10 \text{ or } x = -3$$

Since, time can not be negative.

So, $x = 10$ and $x + 5 = 10 + 5 = 15$.

Question 55. One fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Solution : Let x be the total number of camels.

Number of camels seen in the forest $= \frac{x}{4}$

Number of camels gone to mountains $= 2\sqrt{x}$

Number of camels on the bank of river = 15

Total number of camels $= \frac{x}{4} + 2\sqrt{x} + 15 = x$

$\Rightarrow \quad x + 8\sqrt{x} + 60 = 4x$

$\Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$

Put $\quad \sqrt{x} = y$

$\Rightarrow \quad 3y^2 - 8y - 60 = 0$

$\Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y-6) + 10(y-6) = 0$

$\Rightarrow \quad (y-6)(3y+10) = 0$

$\Rightarrow \quad y = 6 \text{ or } 3y + 10 = 0$

$\Rightarrow \quad y = 6 \quad \text{or} \quad y = -\frac{10}{3}$

Now $\quad y = 6$

$\Rightarrow \quad \sqrt{x} = 6$

On squaring $\quad x = 36.$

Hence, total number of camels = 36.

Question 56. An aeroplane travelled a distance of 400 km at an average speed of x km/hr. On the return journey the speed was increased by 40 km/hr. Write down the expression for the time taken for
(i) The outward journey (ii) the return Journey. If the return journey took 30 minutes less than the onward journey write down an equation in x and find its value.

Solution : (i) Time taken for the onward journey

$= \frac{400}{x}$ hours.

(ii) Time taken for the return journey $= \frac{400}{x+40}$ hours.

According to the question,

$$\frac{400}{x+40} = \frac{400}{x} - \frac{1}{2}$$

$\Rightarrow \quad 800x = 800(x+40) - x(x+40)$

$\Rightarrow \quad x^2 + 40x - 32,000 = 0$

$\Rightarrow \quad (x+200)(x-160) = 0$

$\Rightarrow \quad x = -200 \text{ (inadmissible) or } 160$

Hence, the required value of x is 160. Ans.

Question 57. Car A travels x km for every litre of petrol, while car B travels (x + 5) km for every litre of petrol.
(i) Write down the number of litres of petrol used by car A and car B in covering a distance of 400 km.

(ii) If car A use 4 litre of petrol more than car B in covering the 400 km, write down and equation in x and solve it to determine the number of litre of petrol used by car B for the journey.

Solution: Given Distance = 400 km

Car A travels x km/litre.

Car B travels $(x + 5)$ km/litre.

(i) No, of litre used by car

$$A = \frac{\text{Distance}}{\text{Speed of car A}} = \frac{400}{x} \text{ litre}$$

No. of litre used by car $B = \frac{\text{Distance}}{\text{Speed of car B}} = \frac{400}{x+5}$ liter.

(ii) Car A uses 4 litre more than car B

$$\therefore \quad \frac{400}{x} - \frac{400}{x+5} = 4$$

$$400(x+5) - 400x = 4x(x+5)$$

$$400x + 2000 - 400x = 4x^2 + 20x$$

$$4x^2 + 20x - 2000 = 0$$

$$4(x^2 + 5x - 500) = 0$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x+25) - 20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$\therefore \quad x = 20 - 25($$

inadmissible)

No. of litre of petrol used by car B

$$= \frac{400}{20+5} = \frac{800}{25} = 16$$

Question 58. A shopkeeper purchases a certain number of books for Rs. 960. If the cost per book was Rs. 8 less, the number of books that could be purchased for Rs. 960 would be 4 more. Write an equation, taking the original cost of each book to be Rs. x, and Solve it to find the original cost of the books.

Solution : Original cost of each book

$$= ₹\, x$$

$\therefore$ Number of books for ₹ 960 $= \frac{960}{x}$

Now, If cost of each book $=$ ₹ $(x-8)$

Number of books for ₹ 960 $= \frac{960}{x-8}$

According to the question

$$\frac{960}{x} + 4 = \frac{960}{x-8}$$

or

$$\frac{960}{(x-8)} - \frac{960}{x} = 4$$

$$\frac{960x - 960x + 7{,}680}{x(x-8)} = 4$$

or

$$7{,}680 = 4x^2 - 32x$$

or

$$x^2 - 8x - 1{,}920 = 0$$

$$x^2 + 40x - 48x - 1{,}920 = 0$$

$$x(x+40) - 48(x+40) = 0$$

$$(x+40)(x-48) = 0$$

$\Rightarrow$ $x = -40, 48$

as cost can't be – ve $x = 48$

Question 59. Two pipes running together can 1 fill a cistern in 11 1/9 minutes. If one pipe takes 5 minutes more than the other to fill the cistern find the time when each pipe would fill the cistern.

Solution : Let x minutes be time taken by the larger pipe to fill the cistern then the smaller pipe taken $(x + 5)$ minutes. These two pipes would fill $\frac{1}{x}$ and $\frac{1}{x+5}$ of the cistern in a minute, respectively.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$$

$\Rightarrow$ $9x^2 - 155x - 500 = 0$

$\Rightarrow$ $9x^2 + 25x - 180x - 500 = 0$

$\Rightarrow$ $x(9x+25) - 20(9x+25) = 0$

$\Rightarrow$ $(9x+25)(x-20) = 0$

$\Rightarrow$ $x - 20 = 0$

and $9x + 25 = 0$

$x = 20$

and $x = -\frac{25}{9}$ (negligible)

Hence the time taken by the pipes to fill the cistern in 20 minutes and 25 minutes. Ans.

Question 60. In each of the following find the values of k of which the given value is a solution of the given equation:

(i) $7x^2 + kx - 3 = 0; x = \frac{2}{3}$

(ii) $x^2 - x(a + b) + k = 0, x = a$

(iii) $kx^2 + \sqrt{2}x - 4 = 0; x = \sqrt{2}$

(iv) $x^2 + 3ax + k = 0; x = a.$

Solution : (i) $7x^2 + kx - 3 = 0; x = \frac{2}{3}$.

Putting $x = \frac{2}{3}$ in L.H.S. of equation

$$\text{L.H.S.} = 7 \times \left(\frac{2}{3}\right)^2 + \frac{2}{3}k - 3 = 0$$

$$\Rightarrow \quad \frac{28}{9} + \frac{2}{3}k - 3 = 0$$

$$\Rightarrow \quad \frac{28 + 6k - 27}{9} = 0$$

$$\Rightarrow \quad 6k + 1 = 0$$

Hence, $k = -\frac{1}{6}$

(ii) $x^2 - x(a + b) + k = 0; x = a.$

Putting $x = a$ in L.H.S. of equation

$$\Rightarrow \quad (a)^2 - a(a + b) + k = 0$$

$$\Rightarrow \quad a^2 - a^2 - ab + k = 0$$

Hence, $k = ab.$

(iii) $kx^2 + \sqrt{2}x - 4 = 0; x = \sqrt{2}.$

Putting $x = \sqrt{2}$ in L.H.S. of equation.

$$\text{L.H.S.} = k.(\sqrt{2})^2 + \sqrt{2} \times \sqrt{2} - 4 = 0$$

$$\Rightarrow \quad 2k + 2 - 4 = 0$$

$$\Rightarrow \quad 2k - 2 = 0$$

Hence, $k = \frac{2}{2} =$

(iv) $x^2 + 3ax + k = 0; x = -a.$

Putting $x = -a$ in L.H.S. of equation

$$\text{L.H.S.} = (-a)^2 + 3a \times (-a) + k = 0$$

$$\Rightarrow \quad a^2 - 3a^2 + k = 0$$

$$\Rightarrow \quad -2a^2 + k = 0$$

Hence, $k = 2a^2.$

Question 61. Solve the following quadratic equation by factorisation:

(i) $(x-4)(x+2)=0$

(ii) $(2x+3)(3x-7)=0$

(iii) $x^2+3x-18=0$

(iv) $x^2-3x-10=0$

(v) $9x^2-3x-2=0$

(vi) $2x^2+ax-a^2=0$ where $a \in R$.

Solution : (i) The given quadratic equation is

$$(x-4)(x+2) = 0$$

$$\Rightarrow \quad x-4 = 0 \text{ or } x+2=0$$

$$\Rightarrow \quad x = \{4, -2\}. \qquad \text{Ans.}$$

(ii) The given quadratic equation is

$$(2x+3)(3x-7) = 0$$

$$\Rightarrow \quad 2x+3 = 0 \text{ or } 3x-7=0$$

$$\Rightarrow \quad x = \left\{\frac{-3}{2}, \frac{7}{3}\right\}. \qquad \text{Ans.}$$

(iii) The given quadratic equation is

$$x^2+3x-18 = 0$$

$$\Rightarrow \quad x^2+6x-3x-18 = 0$$

$$\Rightarrow \quad x(x+6)-3(x+6) = 0$$

$$\Rightarrow \quad (x+6)(x-3) = 0$$

when $x+6 = 0$

$$x = -6$$

when $x-3 = 0$

$$x = 3$$

$$\therefore \quad x = \{-6, 3\}. \qquad \text{Ans.}$$

(iv) $x^2-3x-10 = 0$

$$\Rightarrow \quad x^2-5x+2x-10 = 0$$

$$\Rightarrow \quad x(x-5)+2(x-5) = 0$$

$$\Rightarrow \quad (x-5)(x+2) = 0$$

$$\Rightarrow \quad x-5 = 0 \text{ or } x+2=0$$

$$\Rightarrow \quad x = 5 \text{ and } x=-2.$$

(v) The given quadratic equation is

$$9x^2-3x-2 = 0$$

$$\Rightarrow \quad 9x^2-6x+3x-2 = 0$$

$$\Rightarrow 3x(3x-2)+1(3x-2) = 0$$

$$\Rightarrow \quad (3x-2)(3x+1) = 0$$

$$\Rightarrow \quad 3x-2 = 0 \text{ or } 3x+1=0$$

$$\Rightarrow \quad 3x = 2 \text{ and } 3x=-1$$

$$\Rightarrow \quad x = \frac{2}{3} \text{ and } x=-\frac{1}{3}.$$

(vi) $2x^2+ax-a^2 = 0$

$$\Rightarrow \quad 2x^2+2ax-ax-a^2 = 0$$

$$\Rightarrow \quad 2x(x+a)-a(x+a) = 0$$

$$\Rightarrow \quad (x+a)(2x-a) = 0$$

$$\Rightarrow \quad x+a = 0 \text{ or } 2x-a=0$$

$$\Rightarrow \quad x = -a \text{ and } x=\frac{a}{2}.$$

Question 62. Solve the following quadratic equation by factorisation method:

(i) $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}, \quad x \neq 0, x \neq -1$

(ii) $\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$.

Solution : (i) We have

$$\frac{x}{x+1}+\frac{x+1}{x} = \frac{34}{15}$$

$$\Rightarrow \frac{x^2+(x+1)^2}{x(x+1)} = \frac{34}{15}$$

$$\Rightarrow \frac{x^2+x^2+1+2x}{x^2+x} = \frac{34}{15}$$

$$\Rightarrow \frac{2x^2+2x+1}{x^2+x} = \frac{34}{15}$$

$$\Rightarrow 34x^2+34x = 30x^2+30x+15$$

$$\Rightarrow 4x^2+4x-15 = 0$$

$$\Rightarrow 4x^2+10x-6x-15 = 0$$

$$\Rightarrow 2x(2x+5)-3(2x+5) = 0$$

$$\Rightarrow (2x+5)(2x-3) = 0$$

$$\Rightarrow 2x+5 = 0 \text{ or } 2x-3=0$$

$$\Rightarrow 2x = -5 \text{ and } 2x=3$$

$$\Rightarrow x = -\frac{5}{2}, x=\frac{3}{2}. \quad \text{Ans.}$$

(ii)
$$\frac{x+3}{x-2}-\frac{1-x}{x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x-(x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x-(x-x^2-2+2x)}{x^2-2x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x-(-x^2+3x-2)}{x^2-2x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x+x^2-3x+2}{x^2-2x} = \frac{17}{4}$$

$$\Rightarrow \frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$

$$\Rightarrow 17x^2-34x = 8x^2+8$$

$$\Rightarrow 9x^2-34x-8 = 0$$

$$\Rightarrow 9x^2-36x+2x-8 = 0$$

$$\Rightarrow 9x(x-4)+2(x-4) = 0$$

$$\Rightarrow (x-4)(9x+2) = 0$$

$$\Rightarrow x-4 = 0 \text{ or } 9x+2=0$$

$$\Rightarrow x = 4 \text{ or } x=-\frac{2}{9}.$$

Question 63. Solve the following quadratic equation:

(i) $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$, $a+b\neq 0$

(ii) $4x^2-4ax+(a^2-b^2)=0$ where $a, b \in R$.

Solution : (i) $\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\Rightarrow \quad \frac{1}{a+b+x}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$

$\Rightarrow \quad \frac{x-a-b-x}{x(a+b+x)} = \frac{1}{a}+\frac{1}{b}$

$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow \quad x(a+b+x)(a+b) = -(a+b)ab$

$\Rightarrow \quad x(a+b+x)(a+b)+ab(a+b) = 0$

$\Rightarrow \quad (a+b)\{x(a+b+x)+ab\} = 0$

$\Rightarrow \quad a+b \text{ or } x(a+b+x)+ab = 0$

But $a+b \neq 0$

So $x(a+b+x)+ab = 0$

$\Rightarrow \quad x(a+b)+x^2+ab = 0$

$\Rightarrow \quad x^2+ax+bx+ab = 0$

$\Rightarrow \quad x(x+a)+b(x+a) = 0$

$\Rightarrow \quad (x+a)(x+b) = 0$

$\Rightarrow \quad x=-a \text{ or } x = -b.$ Ans.

(ii) $4x^2-4ax+(a^2-b^2) = 0$

where $a, b \in R$

$\Rightarrow \quad 4x^2-\{2(a+b)x+2(a-b)x\}+a^2-b^2=0$

$\Rightarrow \quad \{4x^2-2(a+b)x\}-\{2(a-b)x-(a^2-b^2)\}=0$

$\Rightarrow \quad 2x\{2x-(a+b)\}-(a-b)\{2x-(a+b)\}=0$

$\Rightarrow \quad \{2x-(a+b)\}\{2x-(a-b)\}=0$

$\Rightarrow \quad 2x-(a+b)=0$

or $\quad 2x-(a-b)=0$

$\Rightarrow \quad x=\frac{a+b}{2} \text{ or } x=\frac{a-b}{2}.$ Ans.

Question 64. Determine whether the given quadratic equations have equal roots and if so, find the roots:

(i) $x^2 + 5x + 5 = 0$
(ii) $x^2 + 2x + 4 = 0$
(iii) $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$
(iv) $3x^2 - 6x + 5 = 0.$

Solution : (i) The given quadratic equation is

$$x^2 + 5x + 5 = 0$$

Here, $a = 1$, $b = 5$ and $c = 5$

$$\text{Discriminant} = b^2 - 4ac$$
$$= (5)^2 - 4 \times 1 \times 5$$
$$= 25 - 20 = 5 > 0$$

so the given equation has real roots

given by $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-5 + \sqrt{25 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-5 + \sqrt{5}}{2}$$

and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-5 - \sqrt{25 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-5 - \sqrt{5}}{2}.$$

(ii) The given quadratic equation is

$$x^2 + 2x + 4 = 0$$

Here, $a = 1$, $b = 2$ and $c = 4$

$$\text{Descriminant} = b^2 - 4ac$$
$$= (2)^2 - 4 \times 1 \times 4$$
$$= 4 - 16 = -12 < 0$$

Hence, the given equation has no real roots.

Ans.

(iii) We have

$$\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$$

Here, $a = \frac{4}{3}$, $b = -2$ and $c = \frac{3}{4}$

$$\text{Discriminant} = b^2 - 4ac$$
$$= (-2)^2 - 4 \times \frac{4}{3} \times \frac{3}{4}$$
$$= 4 - 4 = 0$$

So, the given equation has two real and equal roots given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{+2 + 0}{2 \times \frac{4}{3}} = \frac{3}{4}.$$

and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{+2 - 0}{2 \times \frac{4}{3}} = \frac{3}{4}$. Ans.

(iv) The given equation is

$$3x^2 - 6x + 5 = 0$$

Here, $a = 3, b = -6$ and $c = 5$

$$\text{Discriminant} = b^2 - 4ac$$
$$= (-6)^2 - 4 \times 3 \times 5$$
$$= 36 - 60 = -24 < 0$$

imaginary roots.

Question 65. Find the value of k so that sum of the roots of the quadratic equation is equal to the product of the roots:

(i) $kx^2 + 6x - 3k = 0, k \neq 0$

(ii) $(k+1)x^2 + (2k+1)x - 9 = 0, k + 1 \neq 0.$

Solution : (i) The given quadratic equation is

$$kx^2 + 6x - 3k = 0$$

Here, $a = k, b = 6$ and $c = -3k$

Sum of the roots $\alpha + \beta = \frac{-b}{a} = \frac{-6}{k}$

andproduct of the roots $\alpha\beta = \frac{c}{a} = \frac{-3k}{k}$

Since, Sum of the roots = product of the roots

$$\Rightarrow \quad \frac{-6}{k} = -3$$

$$\Rightarrow \quad k = \frac{+6}{+3} \Rightarrow k = 2. \quad \text{Ans.}$$

(ii) The given equation is

$$(k+1)x^2 + (2k+1)x - 9 = 0$$

Here, $a = k + 1, b = (2k + 1)$ and $c = -9$.

Sum of the roots $\alpha + \beta = \frac{-(2k+1)}{k+1}$

and $\alpha\beta = \frac{c}{a} = \frac{-9}{k+1}$

Since,Sum of the roots = Product of the roots

$$\text{Then,} \quad \left(\frac{2k+1}{k+1}\right) = \frac{9}{k+1}$$

$$\Rightarrow \quad 2k + 1 = 9$$
$$\Rightarrow \quad 2k = 9 - 1$$
$$\Rightarrow \quad 2k = 8$$
$$\Rightarrow \quad k = \frac{8}{2} = 4$$
$$\Rightarrow \quad k = 4. \quad \text{Ans.}$$

Question 66. Find the values of k so that the sum of tire roots of the quadratic equation is equal to the product of the roots in each of the following:

(i) $kx^2 + 2x + 3k = 0$

(ii) $2x^2 - (3k + 1)x - k + 7 = 0.$

Solution : (i) $kx^2 + 2x + 3k = 0.$

Here, $a = k$, $b = 2$, and $c = 3k$.

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{2}{k}$$

$$\text{Product of root} = \frac{c}{a}$$

$$= \frac{3k}{k} = 3$$

$$\text{Sum of roots} = \text{Product of roots}$$

$$-\frac{2}{k} = 3$$

$$\Rightarrow \quad 3k = -2$$

$$\Rightarrow \quad k = -\frac{2}{3}.$$

(ii) $2x^2 - (3k + 1)\,x - k + 7 = 0.$

Here, $a = 2,$

$b = -(3k + 1)$

$c = -k + 7$

$$\text{Sum of roots} = \frac{-b}{a}$$

$$= \frac{3k+1}{2}$$

$$\text{Product of roots} = \frac{c}{a}$$

$$= \frac{-k+7}{2}$$

$$\text{Sum of roots} = \text{Product of roots}$$

$$\frac{3k+1}{2} = \frac{-k+7}{2}$$

$$6k + 2 = -2k + 14$$

$$8k = 12, \Rightarrow k = \frac{12}{8}$$

$$\therefore \quad k = \frac{3}{2}.$$

Question 67. Solve the following by reducing them to quadratic equations:

(i) $\left(\frac{7y-1}{y}\right)^2 - 3\left(\frac{7y-1}{y}\right) - 18 = 0, y \neq 0$

(ii) $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$.

Solution : (i) The given equation

$$\left(\frac{7y-1}{y}\right)^2 - 3\left(\frac{7y-1}{y}\right) - 18 = 0, y \neq 0$$

Putting $\frac{7y-1}{y} = z$, then given equation becomes

$$z^2 - 3z - 18 = 0$$
$$\Rightarrow \quad z^2 - 6z + 3z - 18 = 0$$
$$\Rightarrow \quad z(z-6) + 3(z-6) = 0$$
$$\Rightarrow \quad (z-6)(z+3) = 0$$
$$\Rightarrow \quad z - 6 = 0 \text{ or } z + 3 = 0$$
$$\Rightarrow \quad z = 6 \text{ or } z = -3$$

But $\frac{7y-1}{y} = z$

$$\therefore \quad \frac{7y-1}{y} = 6 \Rightarrow 7y - 1 = 6y$$
$$\Rightarrow \quad 7y - 6y = 1$$
$$\Rightarrow \quad y = 1$$

Also $\frac{7y-1}{y} = -3 \Rightarrow 7y - 1 = -3y$

$$\Rightarrow \quad 7y + 3y - 1 = 0$$
$$\Rightarrow \quad 10y = 1$$
$$\Rightarrow \quad y = \frac{1}{10}$$

Hence, the required roots are $\frac{1}{10}$, 1. Ans.

(ii) Given equation $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$

Putting $\sqrt{\frac{x}{1-x}} = y$, then given equation reducible to the form $y + \frac{1}{y} = \frac{13}{6}$

$$\Rightarrow \quad \frac{y^2+1}{y} = \frac{13}{6}$$
$$\Rightarrow \quad 6y^2 + 6 = 13y$$
$$\Rightarrow \quad 6y^2 - 13y + 6 = 0$$
$$\Rightarrow \quad 6y^2 - 9y - 4y + 6 = 0$$
$$\Rightarrow \quad 3y(2y-3) - 2(2y-3) = 0$$
$$\Rightarrow \quad (2y-3)(3y-2) = 0$$
$$\Rightarrow \quad 2y - 3 = 0 \text{ or } 3y - 2 = 0$$

$\Rightarrow \quad y = 3/2$ or $y = 2/3$

But $\sqrt{\dfrac{x}{1-x}} = y$

$\therefore \quad \sqrt{\dfrac{x}{1-x}} = \dfrac{3}{2}$ or $\sqrt{\dfrac{x}{1-x}} = \dfrac{2}{3}$

Squaring $\dfrac{x}{1-x} = \dfrac{9}{4}$	Squaring $\dfrac{x}{1-x} = \dfrac{4}{9}$
$\Rightarrow \quad 4x = 9 - 9x$	$9x = 4 - 4x$
$\Rightarrow \quad 13x = 9$	$\Rightarrow \quad 9x + 4x = 4$
$\Rightarrow \quad x = \dfrac{9}{13}$	$\Rightarrow \quad 13x = 4$
	$\Rightarrow \quad x = \dfrac{4}{13}$

Hence, the required roots are $\left\{\dfrac{9}{13}, \dfrac{4}{13}\right\}$.

Question 68. Solve $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$.

Solution : Given equation is

$$(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$$

Putting $x^2 + 3x = y$, the given equation becomes

$$y^2 - y - 6 = 0$$

$$\Rightarrow \quad y^2 - 3y + 2y - 6 = 0$$

$$\Rightarrow \quad y(y-3) + 2(y-3) = 0$$

$$\Rightarrow \quad (y-3)(y+2) = 0$$

$$\Rightarrow \quad y - 3 = 0 \text{ or } y + 2 = 0$$

$$\Rightarrow \quad y = 3 \text{ or } y = -2$$

But $x^2 + 3x = y$

$x^2 + 3x = 3$	or $x^2 + 3x = -2$
$\Rightarrow x^2 + 3x - 3 = 0$	$x^2 + 3x + 2 = 0$
Here $a = 1, b = 3, c = -3$	$x^2 + 2x + x + 2 = 0$
Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x(x+2) + 1(x+2) = 0$
$x = \dfrac{-3 \pm \sqrt{9 + 12}}{2}$	$(x+2)(x+1) = 0$
$x = \dfrac{-3 \pm \sqrt{21}}{2}$	$x + 2 = 0$ or $x + 1 = 0$
	$x = -2$ or $x = -1$

Hence, roots are $\dfrac{-3 \pm \sqrt{21}}{2}, -2, -1$. Ans.

Question 69. Solve the following by reducing them to quadratic form:

(i) $\sqrt{y+1}+\sqrt{2y-5}=3, y \in R$

(ii) $\sqrt{x^2-16}-(x-4)=\sqrt{x^2-5x+4}$.

Solution : (i) Given equation

$$\sqrt{y+1}+\sqrt{2y-5} = 3$$

$$\Rightarrow \quad \sqrt{y+1} = 3-\sqrt{2y-5}$$

Squaring both sides, we get

$$y+1 = 9+2y-5-6\sqrt{2y-5}$$

$$\Rightarrow \quad y-2y+1-4 = -6\sqrt{2y-5}$$

$$-y-3 = -6\sqrt{2y-5}$$

$$\Rightarrow \quad y+3 = 6\sqrt{2y-5}$$

On Squaring again, we get

$$y^2+9+6y = 36(2y-5)$$

$$\Rightarrow \quad y^2+9+6y = 72y-180$$

$$\Rightarrow \quad y^2+6y-72y+9+180 = 0$$

$$\Rightarrow \quad y^2-66y+189 = 0$$

$$\therefore \quad y^2-66y+189 = 0$$

Hence, $a=1, b=-66, c=189$

Then $D = b^2-4ac = (66)^2-4(1)(189)$

$= 4356-756$

$= 3600 > 0$

Roots are real.

$$\therefore \quad y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$y = \frac{-(-66) \pm \sqrt{3600}}{2 \times 1}$$

$$y = \frac{66 \pm 60}{2}$$

Squaring again, $4(x^2-16) = x^2+2x+1$

$$\Rightarrow \quad 4x^2-64-x^2-2x-1 = 0$$

$$\Rightarrow \quad 3x^2-2x-65 = 0$$

$$\Rightarrow \quad 3x^2-15x+13x-65 = 0$$

$$\Rightarrow \quad 3x(x-5)+13(x-5) = 0$$

$$\Rightarrow \quad (x-5)+(3x+13) = 0$$

$$\Rightarrow \quad x-5=0 \text{ or } 3x+13 = 0$$

$$\Rightarrow \quad x=5 \text{ or } x = \frac{-13}{3}$$

$$x = 5.$$

Hence, the solutions are 4, 5.

Question 70. Solve: x(x + 1) (x + 3) (x + 4) = 180.

Solution : Given equation

$$x(x+1)(x+3)(x+4) = 180$$

$$\Rightarrow [(x+0)(x+4)][(x+1)(x+3)] = 180$$

$$\Rightarrow (x^2+4x)(x^2+4x+3) - 180 = 0$$

Put $x^2 + 4x = y$,

then it becomes $y(y + 3) - 180 = 0$

$$\Rightarrow y^2 + 3y - 180 = 0$$

$$\Rightarrow y^2 + 15y - 12y - 180 = 0$$

$$\Rightarrow y(y+15) - 12(y+15) = 0$$

$$\Rightarrow (y+15)(y-12) = 0$$

$$\Rightarrow y + 15 = 0 \text{ or } y - 12 = 0$$

$$\Rightarrow y = -15 \text{ or } y = 12$$

But $x^2 + 4x = y$

Then $x^2 + 4x = -15$	or $x^2 + 4x = 12$
$x^2 + 4x + 15 = 0$	$\Rightarrow x^2 + 4x - 12 = 0$

$$\therefore x^2 + 4x + 15 = 0 \text{ gives } x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 15}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 60}}{2}$$

$$= \frac{-4 \pm \sqrt{-34}}{2}$$

$\therefore$ Roots of the equation are imaginary hence not acceptable.

or $x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow x(x+6) - 2(x+6) = 0$

$\Rightarrow (x+6)(x-2) = 0 \Rightarrow x + 6 = 0$ or $x - 2 = 0 \Rightarrow$

$\boxed{x = -6}$ or $\boxed{x = 2}$ Ans.

Question 71. Solve the equation:

$$6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0.$$

Solution : Given equation

$$6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0$$

Put $x-\frac{1}{x}=y$, squaring $\left(x-\frac{1}{x}\right)^2=y^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}-2 = y^2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = y^2+2$

Now, given equation becomes

$6(y^2+2)-25y+12 = 0$

$\Rightarrow \quad 6y^2+12-25y+12 = 0$

$\Rightarrow \quad 6y^2-25y+24 = 0$

$\Rightarrow \quad 6y^2-16y-9y+24 = 0$

$\Rightarrow \quad 2y(3y-8)-3(3y-8) = 0$

$\Rightarrow \quad (3y-8)(2y-3) = 0$

$\Rightarrow \quad 3y-8=0$ or $2y-3 = 0$

$\Rightarrow \quad 3y=8$ or $2y = 3$

$\Rightarrow \quad y=\frac{8}{3}$ or $y = \frac{3}{2}$

But $x-\frac{1}{x}=y$

$\therefore x-\frac{1}{x}=\frac{8}{3}$	or	$x-\frac{1}{x}=\frac{3}{2}$
$\Rightarrow \frac{x^2-1}{x}=\frac{8}{3}$	$\Rightarrow$	$\frac{x^2-1}{x}=\frac{3}{2}$
$\Rightarrow 3x^2-3=8x$	$\Rightarrow$	$2x^2-2=3x$
$\Rightarrow 3x^2-8x-3=0$	$\Rightarrow$	$2x^2-3x-2=0$
$\Rightarrow 3x^2-9x+x-3=0$	$\Rightarrow$	$2x^2-4x+x-2=0$
$\Rightarrow 3x(x-3)+1(x-3)=0$	$\Rightarrow$	$2x(x-2)+1(x-2)=0$
$\Rightarrow (x-3)(3x+1)=0$	$\Rightarrow$	$(x-2)(2x+1)=0$
$\Rightarrow x-3=0$ or $3x+1=0$	$\Rightarrow$	$x-2=0$
	or	$2x+1=0$
$\Rightarrow x=3$ or $x=\frac{-1}{3}$	$\Rightarrow$	$x=2$ or $x=\frac{-1}{2}$

Hence, $x=3, \frac{-1}{3}, 2$ and $\frac{-1}{2}$.

Question 72. Solve for x:

$$\left(x+\frac{1}{x}\right)^2-\frac{3}{2}\left(x-\frac{1}{x}\right)-4=0.$$

Solution : Given equation

$$\left(x+\frac{1}{x}\right)^2-\frac{3}{2}\left(x-\frac{1}{x}\right)-4=0$$

Put $x-\frac{1}{x}=y$, squaring $x^2+\frac{1}{x^2}-2=y^2$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = y^2+2$$

Then given equation becomes :

$$x^2+\frac{1}{x^2}+2-\frac{3}{2}\left(x-\frac{1}{x}\right)-4 = 0$$

$$\Rightarrow \quad y^2+2+2-\frac{3}{2}y-4 = 0$$

$$\Rightarrow \quad y^2+4-\frac{3}{2}y-4 = 0$$

$$\Rightarrow \quad 2y^2-3y = 0$$

$$\Rightarrow \quad y(2y-3) = 0$$

$$\Rightarrow \quad y=0 \text{ or } 2y-3=0 \text{ i.e., } y = \frac{3}{2}$$

But $\quad x-\frac{1}{x}= y$

Then $x-\frac{1}{x}= 0$

$$\Rightarrow \quad x^2-1=0$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

or $\quad x-\frac{1}{x}=\frac{3}{2}$

$$\Rightarrow \quad \frac{x^2-1}{x}=\frac{3}{2}$$

$$\Rightarrow \quad 2x^2-2=3x$$

$$\Rightarrow \quad 2x^2-3x-2=0$$

$$\Rightarrow \quad 2x^2-4x+x-2=0$$

$$\Rightarrow \quad 2x(x-2)+1(x-2)=0$$

$$\Rightarrow \quad (x-2)(2x+1)=0$$

$$\Rightarrow \quad x-2=0 \text{ or } 2x+1=0$$

$$\Rightarrow \quad x=2 \text{ or } x=-\frac{1}{2}$$

Hence, $x=\pm 1, 2, -\frac{1}{2}$. Ans.

Question 73. Solve the equation

$$x^4+2x^3-13x^2+2x+1=0.$$

Solution : Given equation

$$x^4+2x^3-13x^2+2x+1 = 0$$

Dividing both sides by x^2, we get

$$x^2+2x-13+\frac{2}{x}+\frac{1}{x^2} = 0$$

$$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) - 13 = 0$$

Put $x + \frac{1}{x} = y$, squaring $x^2 + \frac{1}{x^2} + 2 = y^2$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 - 2$$

Then $y^2 - 2 + 2y - 13 = 0$

$\Rightarrow \quad y^2 + 2y - 15 = 0$

$\Rightarrow \quad y^2 + 5y - 3y - 15 = 0$

$\Rightarrow \quad y(y + 5) - 3(y + 5) = 0$

$\Rightarrow \quad (y + 5)(y - 3) = 0$

$\Rightarrow \quad y + 5 = 0$ or $y = -5$

or $y - 3 = 0$ or $y = 3$

But $x + \frac{1}{x} = y$

Then $x + \frac{1}{x} = -5$

$\Rightarrow \quad x^2 + 1 = -5x$

$\Rightarrow \quad x^2 + 5x + 1 = 0$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - 4}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - 4}}{2}$$

$$x = \frac{-5 \pm \sqrt{21}}{2}$$

or $x + \frac{1}{x} = 3$

$\Rightarrow \quad x^2 + 1 = 3x$

$\Rightarrow \quad x^2 - 3x + 1 = 0$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

Hence $x = \frac{-5 \pm \sqrt{21}}{2}, \frac{3 \pm \sqrt{5}}{2}$.

Prove the Following

Question 1. Given that one root of the quadratic equation $ax^2 + bx + c = 0$ is three times the other, show that $3b^2 – 16ac$.

Solution : The given qradratic equation is

$$ax^2 + bx + c = 0$$

Let α be the one root

Then other root = 3α

Now, Sum of the root $= \frac{-b}{a}$

$$\Rightarrow \alpha + 3\alpha = \frac{-b}{a} \Rightarrow 4\alpha = \frac{-b}{a}$$

$$\Rightarrow \alpha = \frac{-b}{4a} \quad \ldots(i)$$

Also product of the root $\alpha \times 3\alpha$

$$= \frac{c}{a}$$

$$3\alpha^2 = \frac{c}{a}$$

From equation (i) $3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$

$$\Rightarrow 3 \times \frac{b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow 3b^2 = 16ac \quad \text{Proved.}$$

Question 2. If one root of the quadratic equation $ax^2 + bx + c = 0$ is double the other, prove that $2b^2 = 9$ ac.

Solution : $ax^2 + bx + c = 0.$

Let the roots be α and 2α

$$\text{Sum of roots} = \frac{-b}{a}$$

$$\Rightarrow \alpha + 2\alpha = \frac{-b}{a}$$

$$\Rightarrow 3\alpha = \frac{-b}{a}$$

$$\Rightarrow \alpha = -\frac{b}{3a} \quad \ldots(i)$$

$$\text{Product of root} = \frac{c}{a}$$

$$\Rightarrow 2\alpha^2 = \frac{c}{a}$$

$$\alpha^2 = \frac{c}{2a}, \ \alpha = \sqrt{\frac{c}{2a}} \quad \ldots(ii)$$

Equation (i) = (ii)

$$\frac{-b}{3a} = \sqrt{\frac{c}{2a}}$$

(Squaring both side)

$$\frac{b^2}{9a^2} = \frac{c}{2a}$$

$$\Rightarrow 2b^2 = 9ac. \quad \text{Hence Proved.}$$

Question 3. If the ratio of the roots of the equation

$lx^2 + nx + n = 0$ is $p : q$, prove that

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0.$$

Solution : Let α, β be the roots of

$$lx^2 + nx + n = 0, \alpha + \beta = -\frac{n}{l} \text{ and } \alpha\beta = \frac{n}{l}.$$

$$\frac{\alpha}{\beta} = \frac{p}{q} \text{ (given)}$$

Now

$$\text{L. H. S.} = \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}$$

$$= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{n}{l}}$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{n}{l}}$$

$$= \frac{-n/l}{\sqrt{n/l}} + \sqrt{\frac{n}{l}}, \quad \left[\because \begin{array}{l} \alpha + \beta = \frac{-n}{l} \\ \alpha\beta = \frac{n}{l} \end{array}\right]$$

$$= -\sqrt{\frac{n}{l}} + \sqrt{\frac{n}{l}} = 0 = \text{R. H. S.}$$

Hence proved

Question 4. In each of the following determine whether the given values are solutions of the equation or not.

(i) $3x^2 - 2x - 1 = 0; x = 1$

(ii) $x^2 + 6x + 5 = 0; x = -1, x = -5$

(iii) $2x^2 - 6x + 3 = 0; x = \frac{1}{2}$

(iv) $6x^2 - x - 2 = 0; x = -\frac{1}{2}, x = \frac{2}{3}$

(v) $x^2 + \sqrt{2}x - 4 = 0; x = \sqrt{2}, x = -2\sqrt{2}$

(vi) $9x^2 - 3x - 2 = 0; x = -\frac{1}{3}, x = \frac{2}{3}$

(vii) $x^2 + x + 1 = 0; x = 1, x = -1.$

Solution : (i) Given equation is

$$3x^2 - 2x - 1 = 0; x = 1$$

Put $x = 1$ in the L.H.S.

$$\text{L.H.S.} = 3(1)^2 - 2 \times 1 - 1$$
$$= 3 - 3 = 0 = \text{R.H.S.}$$

Hence, $x = 1$ is a solution of the given equation.

Ans.

(ii) Given equation is

$$x^2 + 6x + 5 = 0; x = -1, x = -5$$

Substitute $x = -1$ in L.H.S.

$$\text{L.H.S.} = (-1)^2 + 6 \times (-1) + 5$$
$$= 1 - 6 + 5 = 6 - 6 = 0$$

Hence, $x = -1$ is a solution of the given equation.

Again put $x = -5$ in L.H.S.

$$\text{L.H.S.} = (-5)^2 + 6 \times (-5) + 5$$
$$= 25 - 30 + 5$$
$$= 30 - 30 = 0$$

Hence, $x = -5$ is also a solution of the given equation. Ans.

(iii) Given equation is

$$2x^2 - 6x + 3 = 0; x = \frac{1}{2}$$

Substitute $x = \frac{1}{2}$ in L.H.S.

$$\text{L.H.S.} = 2 \times \left(\frac{1}{2}\right)^2 - 6 \times \frac{1}{2} + 3$$
$$= 2 \times \frac{1}{4} - 3 + 3 = \frac{1}{2} \neq 0$$

Hence, $x = \frac{1}{2}$ is not a solution of the given equation. Ans.

(iv) Given equation

$$6x^2 - x - 2 = 0; x = -\frac{1}{2}, x = \frac{2}{3}$$

Substitute $x = -\frac{1}{2}$ in L.H.S.

$$\text{L.H.S.} = 6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 2$$
$$= 6 \times \frac{1}{4} + \frac{1}{2} - 2$$
$$= 2 - 2 = 0$$

Hence, $x = -\frac{1}{2}$ is a solution of the given equation.

Also put $x = \frac{2}{3}$ in L.H.S.

$$\text{L.H.S.} = 6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2$$

$$= 6 \times \frac{4}{9} - \frac{2}{3} - 2 = \frac{8}{3} - \frac{2}{3} - 2$$

$$= \frac{6}{3} - 2 = 2 - 2 = 0$$

Hence, $x = \frac{2}{3}$ is a solution of the given equation.

(v) Given equation

$$x^2 + \sqrt{2}x - 4 = 0; x = \sqrt{2}; x = -2\sqrt{2}$$

Substitute $x = \sqrt{2}$ in the L.H.S.

$$\text{L.H.S.} = (\sqrt{2})^2 + \sqrt{2} \times \sqrt{2} - 4$$

$$= 2 + 2 - 4 = 4 - 4 = 0$$

Hence $x = \sqrt{2}$ is a solution of the given equation.

Again substitute $x = -2\sqrt{2}$ in the L.H.S.

$$\text{L.H.S.} = (-2\sqrt{2})^2 + \sqrt{2}(-2\sqrt{2}) - 4$$

$$= 8 - 4 - 4 = 0 = \text{R.H.S.}$$

Hence, $-2\sqrt{2}$ is also a solution of the given equation. Ans.

(vi) Given equation is

$$9x^2 - 3x - 2 = 0; x = -\frac{1}{3}, x = \frac{2}{3}$$

Substitute $x = -\frac{1}{3}$ in the L.H.S.

$$\text{L.H.S.} = 9\left(-\frac{1}{3}\right)^2 - 3 \times \left(-\frac{1}{3}\right) - 2$$

$$= 9 \times \frac{1}{9} + 1 - 2$$

$$= 2 - 2 = 0 = \text{R.H.S.}$$

Hence, $x = -\frac{1}{3}$ is a solution of the equation.

Again put $x = \frac{2}{3}$

$$\text{L.H.S.} = 9\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right) - 2$$

$$= 9 \times \frac{4}{9} - 2 - 2$$

$$= 4 - 4 = 0 = \text{R.H.S.}$$

Hence, $x = \frac{2}{3}$ is a solution of the equation. Ans.

(vii) Given equation is

$$x^2 + x + 1 = 0; x = 1, x = -1$$

Substitute $x = 1$ in L.H.S.

$$\text{L.H.S.} = (1)^2 + (1) + 1$$

$$= 3 \neq \text{R.H.S} \neq 0$$

Hence, $x = 1$ is not a solution of the given equation. Ans.

Now substitute $x = -1$ in L.H.S.

$$\text{L.H.S.} = (-1)^2 + (-1) + 1 = 1 - 1 + 1$$
$$= 1 \neq \text{R.H.S.} \neq 0$$

Hence, $x = -1$ is not a solution of the given equation.

Question 5. In each of the following, determine whether the given values are solution of the given equation or not:

(i) $x^2 - 3x + 2 = 0; x = 2, x = -1$

(ii) $x^2 + x + 1 = 0; x = 0; x = 1$

(iii) $x^2 - 3\sqrt{3}x + 6 = 0; x = \sqrt{3}, x = -2\sqrt{3}$

(iv) $x + \frac{1}{x} = \frac{13}{6}, x = \frac{5}{6}, x = \frac{4}{3}$

(v) $2x^2 - x + 9 = x^2 + 4x + 3; x = 2, x = 3$

(vi) $x^2 - \sqrt{2}x - 4 = 0; x = -\sqrt{2}, x = -2\sqrt{2}$

(vii) $a^2x^2 - 3abx + 2b^2 = 0; x = \frac{a}{b}, x = \frac{b}{a}$.

Solution : (i) Substitute $x = 2$ in L.H.S. of given equation

$$\text{L.H.S.} = (2)^2 - 3 \times 2 + 2$$
$$= 6 - 6 = 0$$
$$\Rightarrow \quad \text{L.H.S.} = 0 = \text{R.H.S.}$$

Substitute $x = -1$ in L.H.S. of given equation.

$$\text{L.H.S.} = (-1)^2 - 3 \times -1 + 2 = 0$$
$$= 1 + 3 + 2 \neq 0 \neq \text{R.H.S.}$$

$x = 2$ is a solution and $x = -1$ is not a solution of the given equation. Ans.

(ii) Now Substitute $x = 0$ in given equation

$$\text{L.H.S.} = (0)^2 + 0 + 1 \neq 0 \neq \text{R.H.S.}$$

on substituting $x = 1$ in L.H.S. of given equation

$$\Rightarrow (1)^2 + 1 + 1 \neq 0 \neq \text{R.H.S.}$$

Hence $x = 0$ and $x = 1$ are not solutions of the given equation. Ans.

(iii) $x^2 - 3\sqrt{3}x + 6; x = \sqrt{3}, x = -2\sqrt{3}$.

Now substitute $x = \sqrt{3}$ in L.H.S. of given equation

$$\text{L.H.S.} = (\sqrt{3})^2 - 3\sqrt{3} \times \sqrt{3} + 6 = 0$$
$$= 3 - 9 + 6 = 0 = \text{R.H.S.}$$

$x = \sqrt{3}$ is a solution of the given equation.

Substitute $x = -2\sqrt{3}$ in L.H.S. of given equation

$$\Rightarrow \quad (-2\sqrt{3})^2 - 3\sqrt{3} \times -2\sqrt{3} + 6 = 0$$
$$\Rightarrow \quad \text{L.H.S.} = 12 + 18 + 6 \neq 0 \neq \text{R.H.S.}$$

$x = -2\sqrt{3}$ is not a solution of the given equation.

(iv) $x+\frac{1}{x}=\frac{13}{6}$; $x=\frac{5}{6}$, $x=\frac{4}{3}$.

$$\frac{x^2+1}{x} = \frac{13}{6}$$

$\Rightarrow$ $6x^2-13x+6 = 0$

Now on substitute $x=\frac{5}{6}$ in equation

$$\text{L. H. S.} = 6\times\left(\frac{5}{6}\right)^2-13\times\frac{5}{6}+6$$

$$\Rightarrow \quad = \frac{25}{6}-\frac{65}{6}+6$$

$$\Rightarrow \quad = \frac{61}{6}-\frac{65}{6}\neq 0 \neq \text{R.H.S.}$$

$\therefore$ $x=\frac{5}{6}$ is not a solution of the given equation.

on substituting $x = \frac{4}{3}$ in L.H.S. of given equation

$$\Rightarrow \quad \text{L.H.S.} = 6\times\left(\frac{4}{3}\right)^2-13\times\frac{4}{3}+6$$

$$= \frac{32}{3}-\frac{52}{3}+6$$

$$= \frac{50}{3}-\frac{52}{3}\neq 0\neq \text{R.H.S.}$$

Hence, $\frac{4}{3}$ is not a solution of the given equation.

Ans.

(v) $2x^2-x+9=x^2+4x+3$; $x=2$, $x=3$,

Solution : $2x^2-x+9 = x^2+4x+3$

$2x^2-x^2-x-4x+(9-3) = 0$

$x^2-5x+6 = 0$ ...(1)

Now $x = 2$

L.H.S. $=(2)^2-5\times2+6 = 0$

$10-10 = 0=$ R.H.S.

$\therefore$ $x=2$ is the solution of the given equation.

On substituting $x=3$ in L.H.S. of equation (1)

$\Rightarrow$ L.H.S. $=(3)^2-5\times3+6$

$\Rightarrow$ $=15-15 = 0 =$ R.H.S.

$\therefore$ $x=3$ is a solution of the given equation.

Ans.

(vi) $x^2-\sqrt{2}x-4=0$; $x=-\sqrt{2}$, $x=-2\sqrt{2}$.

Now $x =-\sqrt{2}$

$\Rightarrow$ L.H.S. $=$ $(-\sqrt{2})^2-\sqrt{2}\times(-\sqrt{2})-4=0$

$= 2+2-4 = 0=$ R.H.S.

$\therefore$ $x=-\sqrt{2}$ is a solution of the given equation.

Now $x = -2\sqrt{2}$

$\Rightarrow$ L.H.S. $= (-2\sqrt{2})^2 - \sqrt{2}(-2\sqrt{2}) - 4 = 0$

$= 8 + 4 - 4 \neq 0 \neq$ R.H.S.

$\therefore$ $x = -2\sqrt{2}$ is not a solution of the equation. Ans.

(vii) $a^2x^2 - 3abx + 2b^2 = 0; x = \frac{a}{b}, x = \frac{b}{a}$.

Now on substituting $x = \frac{a}{b}$ in L.H.S.

$$\text{L.H.S.} = a^2x^2 - 3abx + 2b^2$$

$$= a^2 \times \left(\frac{a}{b}\right)^2 - 3ab \times \frac{a}{b} + 2b^2$$

$$= \frac{a^4}{b^2} - 3a^2 + 2b^2$$

$$= \frac{a^4 - 3a^2b^2 + 2b^4}{b^2}$$

$$= a^4 - 3a^2b^2 + 2b^4 \neq 0 \neq \text{R.H.S.}$$

$\therefore x = \frac{a}{b}$ is not a solution of the equation

Put $x = b/a$ in L.H.S. of given equation

$$\text{L.H.S.} = a^2 \times \left(\frac{b}{a}\right)^2 - 3ab \times \frac{b}{a} + 2b^2$$

$$= b^2 - 3b^2 + 2b^2$$

$$3b^2 - 3b^2 = 0 = \text{R.H.S.}$$

$\therefore x = \frac{b}{a}$ is a solution of the given equation.

Question 6. If $\frac{8a - 5b}{8c - 5d} = \frac{8a + 5b}{8c + 5d}$, prove that

$$\frac{a}{b} = \frac{c}{d}.$$

Solution : $\frac{8a - 5b}{8c - 5d} = \frac{8a + 5b}{8c + 5d}$

Applying alternendo

$$\frac{8a - 5b}{8a + 5d} = \frac{8c - 5d}{8c + 5d}$$

Applying componendo and Dividendo

$$\frac{8a - 5b + 8a + 5d}{8a - 5b - 8a - 5d} = \frac{8c - 5d + 8c + 5d}{8c - 5d - 8c - 5d}$$

$$\frac{16a}{-10b} = \frac{16c}{-10d}$$

$$\frac{a}{b} = \frac{c}{d}$$ Hence Proved.

Question 7. Show, that a, b, c, d are in proportion if:

(i) $(6a + 7b) : (6c + 7d) : : (6a - 7b) : (6c - 7d)$

(ii) $(a + b + c + d)(a - b - c + d)$

$$= (a + b - c - d)(a - b + c - d).$$

Solution : (i) We have,

$$\frac{a}{b} = \frac{c}{d}$$

$$\left(\text{Both sides are multiplied by } \frac{6}{7}\right)$$

$$\Rightarrow \quad \frac{6a}{7b} = \frac{6c}{7d}$$

Applying componendo and dividendo

$$\frac{6a + 7b}{6a - 7b} = \frac{6c + 7d}{6c - 7d}$$

Applying alternendo

$$\frac{6a + 7b}{6c + 7d} = \frac{6a - 7b}{6c - 7d}$$

$(6a + 7b) : (6c + 7d) :: (6a - 7b) (6c - 7d).$

(ii) We have $\dfrac{a}{b} = \dfrac{c}{d}$

Applying componendo and dividendo

$$\Rightarrow \quad \frac{a + b}{a - b} = \frac{c + d}{c - d}$$

Applying alternendo

$$\Rightarrow \quad \frac{a + b}{c + d} = \frac{a - b}{c - d}$$

Again applying componendo and dividendo

$$\frac{a + b + c + d}{a + b - c - d} = \frac{a - b + c - d}{a - b - c + d}$$

$\Rightarrow (a + b + c + d)(a - b - c + d)$

$$= (a + b - c - d)(a - b + c - d).$$

Hence proved.

Question 8. If $\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2}$ then show that each ratio is equal to $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$.

Solution : Each of the given ratio

$$= \frac{(by+cz)+(cz+ax)+(ax+by)}{(b^2+c^2)+(c^2+a^2)+(a^2+b^2)}$$

$$= \frac{ax+by+cz}{a^2+b^2+c^2}$$

Now $\frac{by+cz}{b^2+c^2} = \frac{ax+by+cz}{a^2+b^2+c^2}$

$$\Rightarrow \frac{a^2+b^2+cz}{b^2+c^2} = \frac{ax+by+cz}{by+cz}$$

$$\Rightarrow \frac{a^2}{b^2+c^2} = \frac{zx}{by+cz} \quad \text{(App. dividendo)}$$

$$\Rightarrow \frac{b^2+c^2}{a^2} = \frac{by+c^2}{ax} \quad \text{(App. invertends)}$$

$$\Rightarrow \frac{a^2+b^2+c^2}{a^2} = \frac{ax+by+cz}{ax}$$

(by componendo)

$$\Rightarrow \frac{x}{a} = \frac{ax+by+cz}{a^2+b^2+c^2}. \quad \text{(similarly)}$$

$$\therefore \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{ax+by+cz}{a^2+b^2+c^2}.$$

Hence proved.

Question 9. If $y = \frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$, show that $3by^2 - 2ay + 3b = 0$.

Solution : We have

$$\frac{y}{1} = \frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$$

Applying componendo and dividendo

$$\frac{y+1}{y-1} = \frac{\sqrt{a+3b}+\sqrt{a-3b}+\sqrt{a+3b}-\sqrt{a-3b}}{\sqrt{a+3b}+\sqrt{a-3b}-\sqrt{a+3b}+\sqrt{a-3b}}$$

$$\frac{y+1}{y-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

Squaring both side

$$\frac{(y+1)^2}{(y-1)^2} = \frac{a+3b}{a-3b}$$

$$\Rightarrow \frac{y^2+1+2y}{y^2+1-2y} = \frac{a+3b}{a-3b}$$

Again applying componendo and dividendo

$$\Rightarrow \frac{y^2+1+2y+y^2+1-2y}{y^2+1+2y-y^2-1+2y} = \frac{a+3b+a-3b}{a+3b-a+3b}$$

$$\Rightarrow \frac{2(y^2+1)}{4y} = \frac{2a}{6b}$$

$$\Rightarrow 3by^2 + 3b = 2ay$$

$$\Rightarrow 3by^2 - 2ay + 3b = 0.$$

Hence proved.

Question 10. If $y = \frac{(p+1)^{1/3} + (p-1)^{1/3}}{(p+1)^{1/3} - (p-1)^{1/3}}$ find that $y^3 - 3py^2 + 3y - p = 0$.

Solution : We have

$$\frac{y}{1} = \frac{(p+1)^{1/3} + (p-1)^{1/3}}{(p+1)^{1/3} - (p-1)^{1/3}}$$

Applying componendo and dividendo

$$\frac{y+1}{y-1} = \frac{(p+1)^{1/3} + (p-1)^{1/3} + (p+1)^{1/3} - (p-1)^{1/3}}{(p+1)^{1/3} + (p-1)^{1/3} - (p+1)^{1/3} + (p-1)^{1/3}}$$

$$\Rightarrow \quad \frac{y+1}{y-1} = \frac{2(p+1)^{1/3}}{2(p-1)^{1/3}}$$

Cubing both side :

$$\frac{(y+1)^3}{(y-1)^3} = \frac{p+1}{p-1}$$

$$\Rightarrow \quad \frac{y^3 + 1 + 3y^2 + 3y}{y^3 - 1 - 3y^2 + 3y} = \frac{p+1}{p-1}$$

Again applying componendo and dividendo

$$\Rightarrow \frac{y^3 + 1 + 3y^2 + 3y + y^3 - 1 - 3y^2 + 3y}{y^3 + 1 + 3y^2 + 3y - y^3 + 1 + 3y^2 - 3y} = \frac{p+1+p-1}{p+1-p+1}$$

$$\Rightarrow \quad \frac{2y^3 + 6y}{6y^2 + 2} = \frac{2p}{2}$$

$$\Rightarrow \quad \frac{2(y^3 + 3y)}{2(3y^2 + 1)} = p$$

$$\Rightarrow \quad y^3 + 3y = 3py^2 + p$$

$$\Rightarrow \quad y^3 - 3py^2 + 3y - p = 0.$$

Hence proved.

Question 11. If $x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, using properties of proportion show that

$$x^2 - 2ax + 1 = 0$$

Solution :

$$x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

$$\Rightarrow \frac{x}{1} = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

By componendo and dividendo

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+1}+\sqrt{a-1}+\sqrt{a+1}-\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}-\sqrt{a+1}+\sqrt{a-1}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$

(by duplicate ratio)

$$\Rightarrow \frac{x^2+2x+1}{x^2-2x+1} = \frac{a+1}{a-1}$$

Again by componendo and dividendo

$$\Rightarrow \frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{a+1+a-1}{a+1-a+1}$$

$$\Rightarrow \frac{x^2+1}{2x} = \frac{a}{1}$$

$$\Rightarrow x^2 + 1 = 2ax$$

$$\Rightarrow x^2 - 2ax + 1 = 0$$

Hence proved.

Concept Based Questions

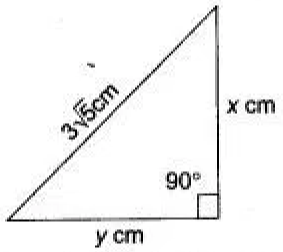
Question 1. The hypotenuse of a right angled triangle is 3√5. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side.

Solution : Let the smaller side of the right triangle be x cm and the longer side by y cm.

Using Pythagoras theorem, we have

$$x^2 + y^2 = (3\sqrt{5})^2$$

$$\Rightarrow \quad x^2 + y^2 = 45 \quad \ldots(i)$$



If the smaller side is tripled and larger side is doubled, then

The smaller side = $3x$ cm

Larger side = $2y$ cm

New hypotenuse = 15 cm

Then by Pythagoras theorem, we have

$$(3x)^2 + (2y)^2 = (15)^2$$

$$\Rightarrow \quad 9x^2 + 4y^2 = 225 \quad \ldots(ii)$$

From (i), $y^2 = 45 - x^2$ and putting in (ii) we get

$$9x^2 + 4(45 - x^2) = 225$$

$$\Rightarrow \quad 9x^2 + 180 - 4x^2 = 225$$

$$\Rightarrow \quad 5x^2 = 225 - 180 = 45$$

$$\Rightarrow \quad x^2 = 9$$

$$\Rightarrow \quad x = \pm 3.$$

But $x = -3$ is not possible as length can't be – ve. Then $x = 3$ cm

From (i), we have

$$x^2 + y^2 = 45$$

$$\Rightarrow \quad 9 + y^2 = 45$$

$$\Rightarrow \quad y^2 = 36$$

$$\Rightarrow \quad y = \pm 6$$

Rejecting – ve sign then $y = 6$

Hence, the length of the smaller side = 3 cm

The length of the longer side = 6 cm. Ans.