Motion

TALENT & OLYMPIAD

Introduction

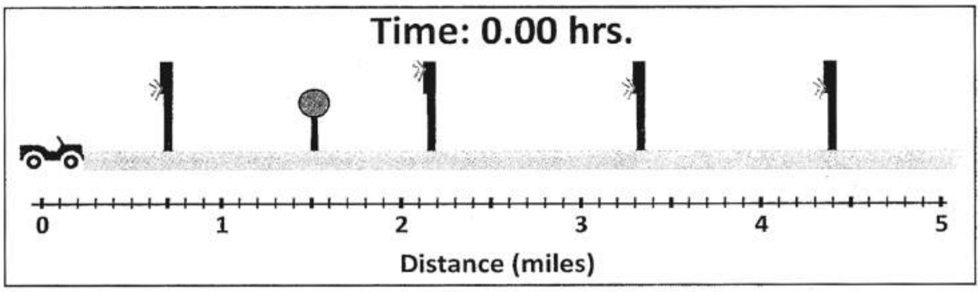
In everyday life we always come across some objects which are at rest and some objects in motion. The branch of Physics, which deals with the behavior of moving objects, is known as mechanics. Mechanics is further divided into two sections namely Kinematics and Dynamics. Kinematics deals with the study of motion without taking into account the cause of motion, while Dynamics is concerned with the cause of motion, namely force. This chapter covers only the different aspects of motion without considering the cause of motion.

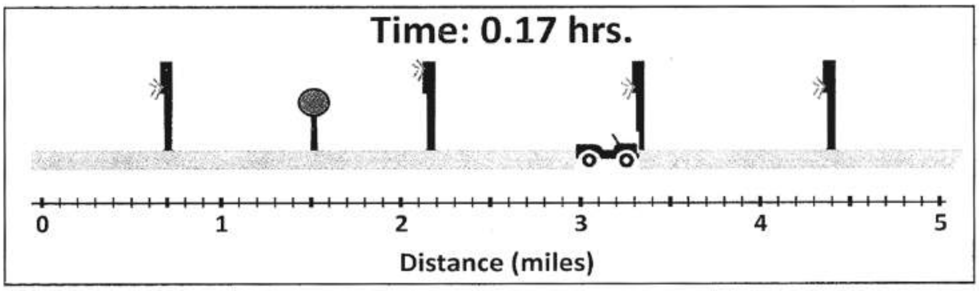
Motion

A body is said to be in motion if its position changes continuously, with respect to a stationary object, taken as reference point, with the passage of time.

Illustrative EXAMPLE

When we are sitting in a moving bus, we observe the continuous changes of position with respect to stationary object like houses, trees, lamp posts, etc. We say that the bus is moving or that the bus is in motion.





Motion and rest are Relative Terms

Suppose you are travelling in a train, which is in motion.

Observe 1: Is there any change in your position with respect to your co passengers? Answer: No

Observe 2: Is there any change of scene you view through the window?

Answer: Yes

Thus, we can say that an object is in, motion, if it changes its position continuously with respect to its surroundings in a given time. Since the position of desk and benches does not changes with time, hence we can say that they are not in motion, i.e. they are stationary. You must have observed sky at night: the position of stars changes with the passage of time, whereas the position of house and our surrounding remains the same. But in reality the earth is also moving, that means all the objects on the surface of the earth are also in motion. Thus, an object which appears to be at rest, may actually be in motion. Therefore, motion and rest are relative terms. Hence, to describe the motion of an object we have to specify how its position changes with respect to a fixed point called the **reference point or origin**.

Without frame of reference we cannot specify whether an object is in motion or at rest. A frame of reference is another object or scene with respect to which we compare an object's position.

Types of Motion

- Translatory motion
- Circular motion
- Rotatory motion and
- Vibratory motion

Translatory Motion

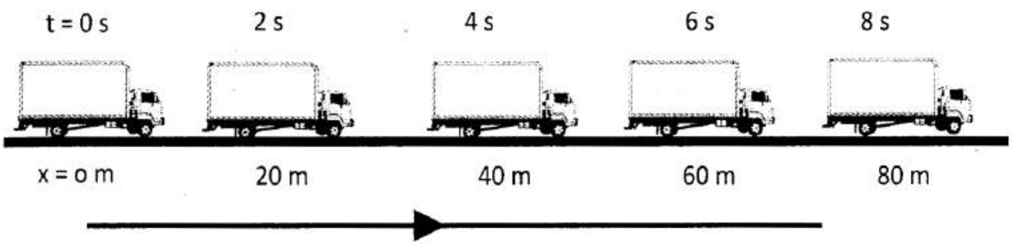
The continuous change of position of an object from one point to another in space is called the translatory motion. This motion may be linear or curvilinear.

Linear Motion

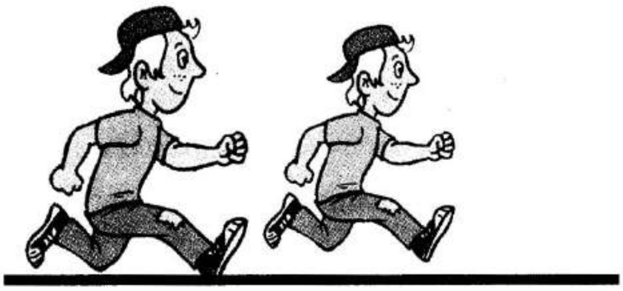
The motion of an object along a straight line is called **linear or rectilinear motion**.

Illustrative EXAMPLE

A car moving on a straight road



Rectilinear Motion

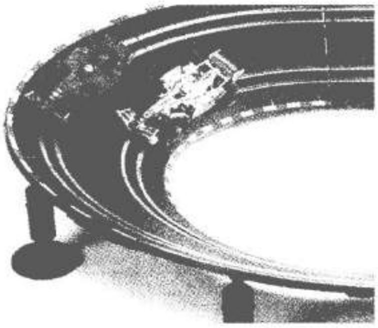


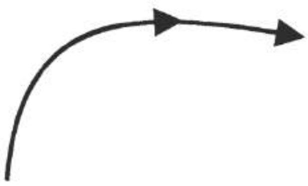
Curvilinear Motion

Motion along a curved path is called curvilinear motion.

Illustrative EXAMPLE

A car negotiating a curve or turn.





Curvilinear motion

Circular Motion

The motion of an object on a circular path is called circular motion.

Illustrative EXAMPLE

An athlete running on a circular track.

Rotatory Motion

The motion in which objects rotate on a given fixed given point is called the **rotatory motion**.

Illustrative EXAMPLE

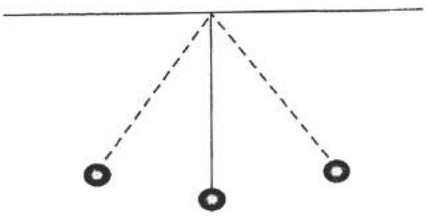
Motion of a top, motion of a fan in the celling etc.

Vibratory Motion

To and fro motion of the body about the mean position is called the **vibratory motion**.

Illustrative EXAMPLE

The motion of the pendulum about the mean position.



Suppose an auto, starting from an auto stand X, travels 1000 m to reach Anand Vihar Bus terminal. The distance covered by an auto is 1000 m. Now if the same auto returns to the auto stand X from Anand Vihar bus terminal with another passenger, then what is the distance covered by an auto during the return trip? The distance covered during the return trip is 1000 m. But the total distance covered by the an auto during the trip from X to Anand Vihar Bus terminal and then back to X is 1000 m + 1000 m = 2000 m.

Thus, the distance traveled by a body is the actual length of the path covered by a moving body, irrespective of the direction in which the body travels.

The shortest distance between the initial and final position of a moving object in a particular direction is called its displacement. It is path independent.

(i) **Distance is a scalar quantity:** (It has magnitude only, and has no specified directions).

(ii) **Displacement is a vector quantity** (It has magnitude as well as a specified direction).

Difference between Distance and Displacement

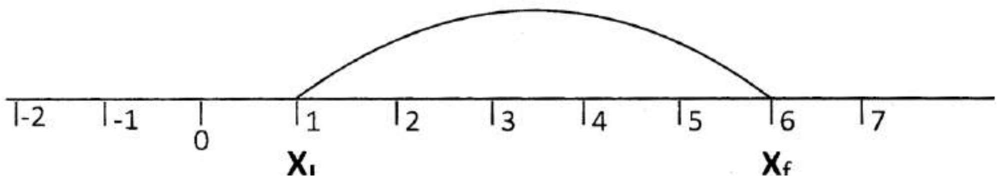
S. No.	Distance	S. No.	Displacement
1.	Distance is the length of the actual path traveled by an object in a given interval of time.	1.	Displacement is the shortest distance between the initial and final positions of a moving object in a particular direction.
2.	Distance traveled by an object depends upon the path followed by	2.	The displacement of an object between initial and final positions of the particle does not

	the particle in going from initial position to the final position.		depend upon the path followed by it.
3.	Distance traveled by an object in a given interval of time is always positive.	3.	Displacement of an object in a given interval of time may be positive, negative or zero.
4.	Distance is a scalar quantity.	4.	Displacement is a vector quantity.

Now let us consider an object changing its position, with respect to a fixed point called the origin 0. x_i and x_f are the initial position and final position of the object. Then the displacement of the object $= x_f - x_i$.

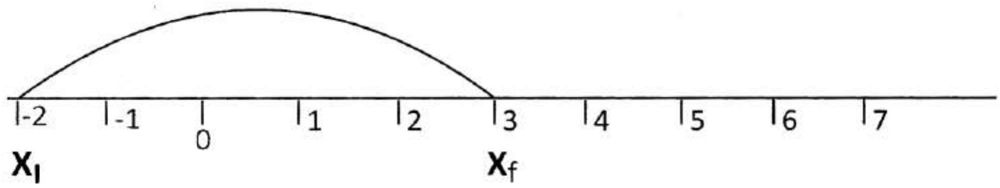
Case 1: Suppose the object is moving from + 1 to + 6 then displacement $-x_i$

$= +6 - (+1) = +5$



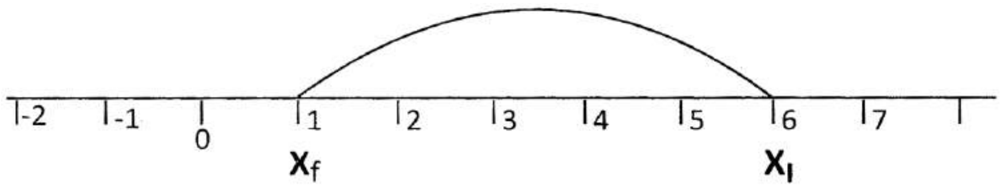
Case 2: If the object is moving from -2 to 3 then displacement $= x_f - x_i$

$= 3 - (-2) = +5$

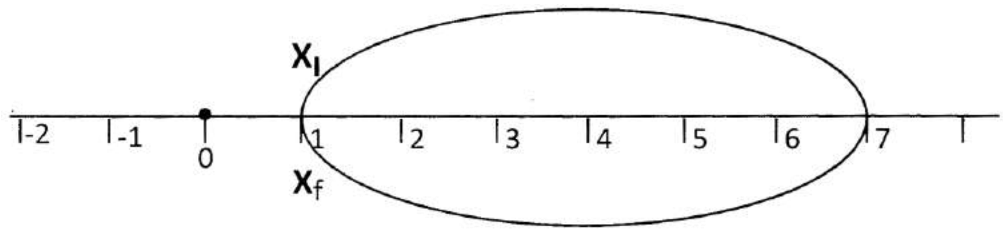


Case 3: if the object is moving from +6 to -1 then displacement $= x_f - xi$

$= +1 - (+6) = -5$



Case 4: if the follows the path as shown in the figure then the final position and the initial position is the same i.e., the displacement is zero



From the above examples, we can conclude that the displacement of a body is positive if its position lies on the right side of the initial position and negative if its final position is on the left side of its initial position. Whenever a moving object comes back to the original position then the displacement is zero.

Commonly Asked Questions

If a body moves 6 m towards south and then turns towards east and moves 8 m, then find the displacement of the body from the initial position?

(a) 12 m (b) 8 m

(c) 10 m (d) 6 m

(e) None of these

Answer: (c)

A man travels a distance of 2 m towards East, then 6 m towards South and finally 6 m towards East. Find the resultant displacement.

(a) 6 m (b) 9 m

(c) 11 m (d) 10 m

(e) None of these

Answer: (d)

Speed

It is defined as the distance traveled by the body per unit time. If the distance of the body is 'S' in time T, then its speed 'M' is given by:

$$V = \frac{s}{t}$$

Where V = Speed,

S = Distance,

and t = time taken (to travel that distance)

SI unit of Speed is m/s or ms^{-1}.

The small values of Speed are expressed in the units of centimeter per second which is written as cm/s or $cm\,s^{-1}$.

To express high speed values, we use the unit of kilometer per hour, written as km. p.h. or km/h or $km\,h^{-1}$.

Speed is a scalar quantity, as it has only magnitude, no specific direction.

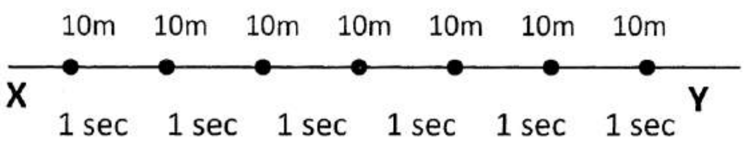
Illustrative Example

In a race different participants start running at the same time and cover the same distance in different time intervals. The participant who takes the minimum time to cover the distance is considered as the winner. Suppose Sidhant and Dhanajay take 15 minutes and 10 minutes respectively to cover a distance of 1000 m, who is faster? To find out who is faster we have to calculate the distance covered by Sidhant and Dhanajay in one minute i.e., Sidhant covers more distance in unit time or in other words we can conclude that Sidhant covered the distance with greater speed. Speed can be defined as the distance covered by a moving object in unit time. The speed of a body gives us an idea of how slow or fast that body is moving. Speed of a body is the distance traveled by it per unit time.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{time taken}}$$

Uniform Speed

Below figure represents the distance covered by the object per second between two fixed points X and Y.

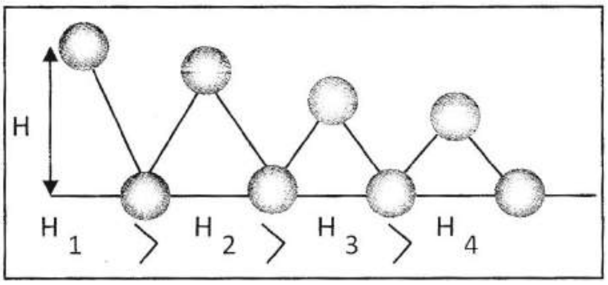


The object covers 10 meters in every one second, hence the speed of the object is 10 m/s between the points X and Y. Thus we see that the speed of object remains the same between any two points in between the given fixed points.

Hence we can say that the object is moving with the uniform speed. An object is said to be moving with uniform speed if it covers equal distances in equal interval of time.

Non-Uniform Speed

The above example holds good only if friction or resistance offered by the surface is ignored. The distance covered varies with time.



Illustrative EXAMPLE

A bouncing ball bounces off to a height less than the initial height and slowly the height decreases. The distance covered by the ball decreases with time. That means the speed of the ball varies from point to point. This type of speed is called a **variable speed**.

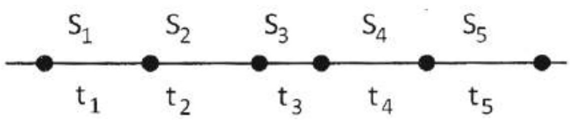
An object is said to be moving with variable speed or non-uniform speed if it covers equal distances in unequal intervals of time or vice-versa.

Average Speed and Instantaneous Speed

When we move on the road in our vehicle, we find that we have to change the speed of the vehicle several times before we reach our destination. Thus, we can say that our speed does not remain the same throughout the journey. In such case we calculate the speed of the journey by taking the ratio of the total distance traveled, by the total time taken to cover the distance.

Average Speed $= \dfrac{\text{Total Distance}}{\text{Total Time}}$

If a car covers a distance s_1 in time t_1, distance s_2 in time t_2 and distances s_n in time t_n then the average speed is given by,



The average speed of a body is the total distance covered by the body divided by the total time taken to cover that distance.

Average speed $= \dfrac{\text{Total Distance}}{\text{Total Time}} = \dfrac{(s_1 + s_2 + s_3 + s_4 + s_5)}{(t_1 + t_2 + t_3 + t_4 + t_5)}$

Instantaneous Speed

You must have observed two meters in the vehicles one is 'speedometer' and another one is 'odometer'. The speedometer gives the speed in kilometer per hour and the 'odometer' records the distance traveled in kilometers. If the 'speedometer' shows 80 km/h, it doesn't mean that the vehicle is moving with the speed of 80km/h throughout the journey. The actual speed of the vehicle may be less than or greater than that. The 'speedometer' shows the speed at a particular instant of time, and is known as instantaneous speed.

Distance - Time Graph for Uniform Motion

A public transport bus is traveling from terminus P to terminus Q. The following observation was recorded by a passenger in the bus.

Observations

Distance in km	0	10	20	30	40	50	60
Time	10.00 am	10.10 am	10.20 am	10.30 am	10.40 am	10.50 am	11.00 am

From the above table it is clear that the bus is covering equal distances in equal intervals of time i.e., the bus is moving with uniform speed. If the bus continues to move with uniform speed, we can calculate the distance covered by the bus at any time.

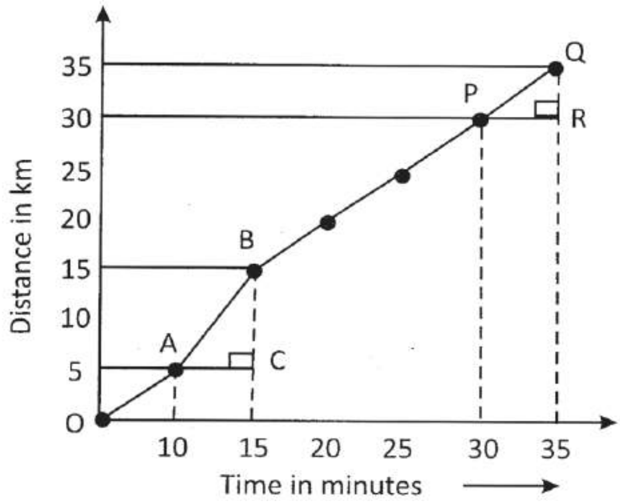
Let us consider an object moving with uniform speed v from its initial position S_1 to final positions S_2 in time t_1 and t_2.

then, uniform speed $= \dfrac{\text{Total Distance}}{\text{Total Time}}$

$$V = \frac{S_2 - S_1}{t_2 - t_1}$$

$S_2 - S_1 = V(t_2 - t_1)$ ....(1)

The equation (1) gives the relation between distance, time and average speed. Let us now plot a distance-time graph for the above illustration. A distance time graph is a line graph showing the variation of distance with time. In a distance-time graph, time is taken along x-axis and distance along y-axis.



- Take time along x-axis and distance along y-axis.
- Analyze the given data and make a proper choice of scale for time and distance.
- Plot the points.
- Consider any two points (A, B) on the straight line graph.
- Draw perpendiculars from A and B to x and y axes.
- Join A to C to get a right angled triangle ACB.

The slope of the graph $=\frac{BC}{AC}=\frac{s}{t}=\text{speed}$

Calculation

Consider another two A and B on the graph and construct a right – angled triangle ABC.

Speed $=\frac{40-30}{10}=\frac{10}{10}=1\text{km/min}$

Consider another two points P and Q on the graph and construct a right angled triangle PRQ.

Slope = Speed $=\frac{QR}{PR}=\frac{35-30}{60-50}=\frac{15}{10}=1.5\text{ km/min}$

This shows that speed is not uniform.

Now, let us consider the nature of S-t graph for non-uniform motion.

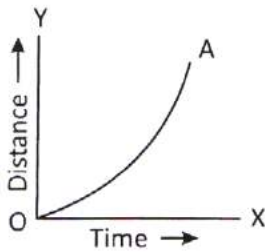


Figure (a)

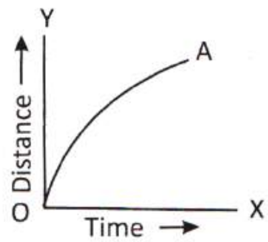
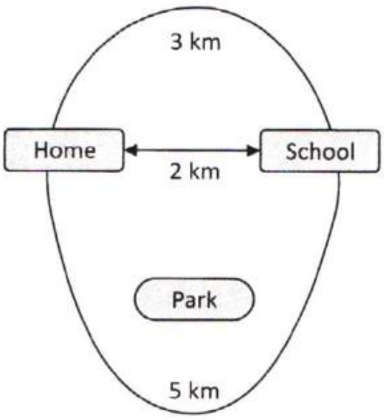


Figure (b)

From figure (a), the S-t graph represents the speed of a moving object increases with time.
From figure (b), the S-t graph represents the speed of a moving object decreases with time.
From the nature of S-t graph we can conclude whether the object is moving with uniform speed or non uniform speed.
The figure given below represents the different route a student can choose from his home to the school.



Martin goes to school on his bike with an average speed of 45 km/h. By using the relation. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ one, can find out the time required to reach the school. But if you are not sure of the route which one would have been taken/ then it is not possible.
Thus, by just giving the speed of a moving object it is not possible to locate the exact position of the object at a given time. So, there arises a need to define a quantity. which has both magnitude as well as direction. The physical quantity which has both magnitude and direction is called vector quantity Thus **Velocity is defined as the distance covered by a moving object in a particular direction in unit time or speed in a particular direction**.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

In a body travels a distance 's' in a given direction in time t, then its velocity 'v' is given by :

$$V = \frac{s}{t}$$

Where, V = velocity of the body, it is a vector quantity.
S = displacement
and t = time taken (to travel that distance;
SI unit of velocity is m/s (meter/second).
[$\because$ SI unit of distance is meter and that of time is second]

Thus we can say that the difference between speed and velocity is that speed has only magnitude but no direction, whereas velocity has magnitude as well as direction.

Average Velocity

It is defined as the ratio of the total displacement of the body to the total time taken by the body, to travel the distance.

Average Velocity = $\dfrac{\text{Total Placement}}{\text{Total Time}}$

Or, Average Velocity $\dfrac{\text{Initial velocity} + \text{final velocity}}{2}$

Or, $V_{av} = \dfrac{u+v}{2}$

Where, u = initial velocity and v = final velocity

Uniform Velocity

A body is said to have uniform velocity if it travels equal distance in equal interval of time, no matter how small is the time interval.

The velocity of a body can be changed in two ways:

- By changing the speed of the body, and
- By changing the direction.

Non-Uniform Velocity

A body is said to be moving with non uniform velocity if it covers unequal distance in equal time interval.

Illustrative EXAMPLE

1. A ball is falling from a certain height. When a ball is dropped from a certain height its direction remains the same, but its speed changes continuously.
2. The velocity of a car moving with a constant speed on a circular path changes because its direction is continuously changing.

Difference between Speed and Velocity

S. No.	Speed	S. No.	Velocity
1.	Distance traveled by an object per unit time is known as its speed.	**1.**	The distance traveled by an object in a particular direction (i.e. displacement) per unit time is known as its velocity.
2.	Average speed of a moving object cannot be zero.	**2.**	Average velocity of a moving object can be zero.
3.	Speed tells how fast an object moves.	**3.**	Velocity tells how fast an object moves and in which direction it moves;
4.	Speed is a scalar quantity.	**4.**	Velocity is a vector quantity.
5.	Speed of an object is always positive	**5.**	Velocity of an object can be positive and negative.

Thus, we can conclude that Speed and Velocity are not always equal in magnitude.

The magnitude of speed and velocity of a moving body is equal only if the body moves in a single straight line. If however, a body does not move in a single straight line, then the speed and velocity of the body are not equal.

Illustrative EXAMPLE

A bus travels a distance of 250000 meter from Mumbai to Nagpur in 300 minutes. Find the speed and velocity of the bus for the entire journey.

(a) 50km/h and50km/h due North

(b) 25 km/h and 50 km/h due South

(c) 50 km/h and-50 km/h due west

(d) 250 km/h and 250 km/h due North

(e) None of these

Answer: (a)

Explanation

(i) $Speed = \frac{Dis\tan ce}{Time} = \frac{250\,km}{5h} = 50\,km/h$

Thus, the speed (or average speed) of the car is 50 km/h.

(ii) Velocity $= \frac{\text{Displacement}}{\text{Time}}$

$= \frac{\text{250 km towards North}}{5\text{h}}$

= 50 km/h towards North

So, the velocity (or average velocity) of the car is 50 km/h towards North.

Illustrative EXAMPLE

A bus moving towards its destination travels first 40 km at the uniform speed of 55 km/h and nest 20 km at the uniform speed of 50 km/h. Find the total time taken by the bus to reach its destination.

(a) 1.2 hours

(b) 1.34 hours

(c) 1.127 hours

(d) 1.565 hours

(e) None of these

Answer: (c)

Explanation

For uniform speed, $S = V \times t$

Time taken to travel first 40 km $= \frac{40}{55} = \frac{8}{11}$ hours

Time taken to travel next 20 km $= \frac{20}{50} = \frac{2}{5}$ hours

The total time taken is $= \frac{2}{5} + \frac{8}{11} = \frac{62}{55}$ hours = 1.127 hours

Acceleration

When a bus starts from a stand its velocity increases for some time. When it was at the stand the velocity was zero and after some time it's velocity gradually starts increasing and reaches maximum after some time. Again when it approaches the second stand its velocity gradually decreases and become zero. This means that the velocity of the bus changes during the motion. The rate at which the velocity of the object changes with the time is called the **acceleration**. Or we can say that velocity per unit time is called the **acceleration**. It is denoted by 'a'. It is given by,

Acceleration $= \frac{\text{Change in Velocity}}{\text{Time}}$

The SI unit of acceleration is metre/sec^2 or m/s^2

Let us consider the motion of the object along the straight line in the same direction. If 'U' be the initial velocity and 'V be the final velocity, then change in velocity in the time 't' is given by,

$a = \frac{V - U}{t}$ or $V = U + at$

Observations

- As the bus starts from rest its speed increases from zero and it is said to be accelerating.
- After sometime the speed becomes constant, means speed of the bus stops increasing and it is said have uniform speed.
- As the bus approaches the next stand it slows down, then it is said have negative acceleration or retardation and finally at next stand it stops accelerating and comes to a halt.

So, it is clear from the above example that acceleration need not always remain the same. It means that the speed of a moving body may increase, it can also decrease or may remain the same or become zero. In general, when the velocity of a body is changing, the body is said to be accelerating. Suppose a car starts from rest (initial velocity is zero) and its velocity increases at a steady rate so that after 5 seconds its velocity is 10 meters per second. Now, in 5 seconds the velocity has increased by 10 - 0 = 10 meter per second and in 1 second the velocity increases by $\frac{10}{5} = 2$ meter per second.

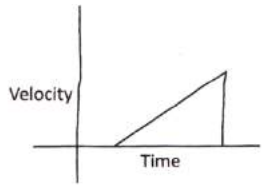
Uniform Acceleration

Consider a particle moving along a straight line in such a way that its velocity changes in equal amount, in equal time interval. If this happens, then the body is said to be moving with the **uniform acceleration**.

- The motion of a free falling body is an example of uniformly accelerated motion.
- The motion of a bicycle going down the slope of a road when the rider is not pedaling, and wind resistance is negligible, is also an example of uniformly accelerated motion.
- The motion of a ball rolling down an inclined plane is an example of uniformly accelerated motion.

Velocity Time Graph for Uniform Acceleration

Velocity time graph for uniform acceleration is as shown below. For example, when a ball is dropped from a certain height its velocity continuously increases and uniformly. The graph below represents the motion.



Thus area under the velocity time graph given the displacement of the object.

Area $=\frac{1}{2}\times base\times height$

The slope of the velocity time graph gives the acceleration of the object moving along the straight line.

Non- Uniform Acceleration

The object is said to have the non uniform acceleration, if the acceleration of the object moving along a straight line is not constant.

If the speed of an object decreases, its acceleration is negative. It is known as retardation. Since acceleration is a vector quantity, it has a direction associated with it.

The direction of the acceleration vector depends on two things:

- Whether the object is speeding up or slowing down
- Whether the object is moving in the +ve or -ve direction

For a free falling body, acceleration is in vertically downward direction.

If an object is slowing down, then its acceleration is in the opposite direction of its motion.

Example A	
Time (s)	**Velocity (m/s)**
0	0
1	4
2	8
3	16
4	32

Example B	
Time (s)	**Velocity (m/s)**
0	-8
1	-6
2	-4
3	-2
4	-0

Example C	
Time (s)	**Velocity (m/s)**
0	32
1	16
2	8
3	4
4	0

Example D	
Time (s)	**Velocity (m/s)**
0	0
1	-2
2	-4
3	-6
4	-8

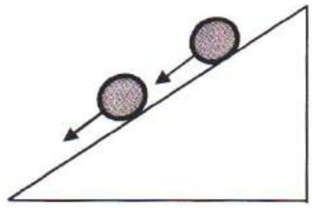
These are both examples of negative acceleration

If the velocity of a body increases along a straight line, the acceleration is positive, and if the velocity of a body decreases, the acceleration is negative.

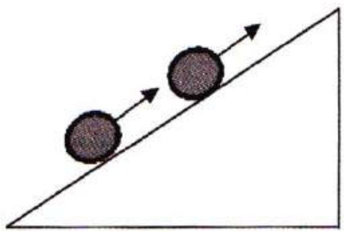
Retardation $=\frac{\text{change in velocity}}{\text{time taken}}$ and has the same unit as that of acceleration (m/s^2)

Types of Acceleration

- **Positive acceleration:** When a ball rolls down on an inclined plane the velocity of the ball increases and it is said to be moving with positive acceleration.



- **Negative acceleration :** When a ball rolls up on an inclined plane the velocity of the ball decreases and it said to be moving with negative acceleration or retardation or declaration.



A ball thrown vertically upwards is also an example of negative acceleration.

- **Zero acceleration:** A bus standing in the bus depot and a bus moving on a straight road with a constant speed of 50 km/h
- **Uniform acceleration:** A ball falling from a height towards the surface of the earth.

- Non-uniform acceleration: A car moving on a crowded road. It has to change its speed every now and then due to the traffic on the road.

Illustrative EXAMPLE

Find the acceleration of the bus whose speed changes from 35 m/s to 20 m/s in 5 seconds.

(a) $+3m/s^2$ (b) $-3m/s^2$

(c) $15m/s^2$ (d) $-25m/s^2$

(e) None of these

Answer: (b)

Explanation

(b) Initial velocity of car, $u = 35m/s$

Final velocity of car, $v = 20m/s$

And, time taken, $t = 5s$

Now, putting these values in the formula for acceleration :

$$a = \frac{V - U}{t}$$

We get, $a = \frac{20 - 35}{5} m/s^2 = \frac{-15}{5} = -3m/s^2$

The negative sign of acceleration means that it is retardation. So, we can also say that the car has a retardation of $-3m/s^2$.

Illustrative EXAMPLE

Which of the quantity distance, speed, velocity or acceleration decides the direction of motion of the body?

(a) Speed (b) Velocity

(c) Acceleration (d) Distance

(e) None of these

Answer: (b)

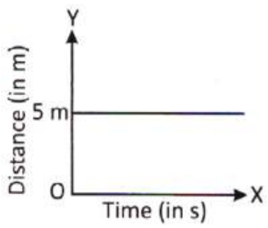
Explanation

It is the velocity, which decides the direction of motion of the body. The accelerartion simply tells the rate of change of velocity. When a body is thrown upwards, its direction of velocity is upwards, that is why the body goes upward, whereas its acceleration is downwards.

Graphical Representation of Motion

Distance Time Graphs

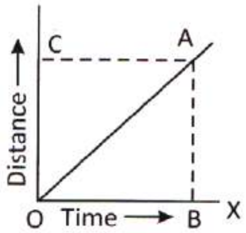
- When body is at rest.



Distance - time graph for a stationary body is straight line parallel to time - axis. From graph, at t = 0, body is at 5 m from reference point. It will remain at 5 m for all times.

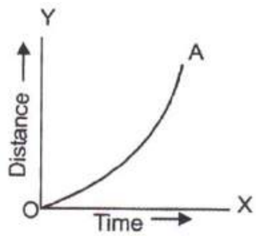
- When body is moving with uniform velocity.

(a) If position - time graph is a straight line it indicates uniform or constant velocity. velocity = slope of straight line e.g. slope $= \frac{AB}{OB}$

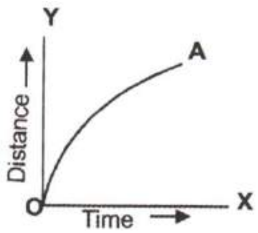


- Distance time graph for non - uniform motion

For non - uniform motion, position time graph is not a straight line, it is a curve line. Slope of the graph is different at different point

- 

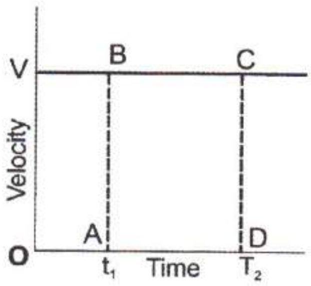
  Velocity of the increases as slope is increasing

- 

  Velocity of body decreases as slope is decreasing

Velocity time graph

- **Velocity-time** graph when the velocity remains constant.



If the Velocity-time graph of a body is a straight line parallel to the time axis, then the speed of the body remains constant (or uniform).

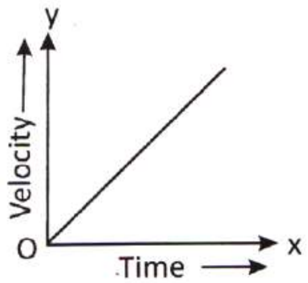
Distance covered = Area under graph

From graph, distance covered in time interval t_1 to t_2

S = area ABCD = v $(t_2 - t_1)$

- Velocity time Graph when Velocity changes at uniform Rate (Uniform Acceleration).

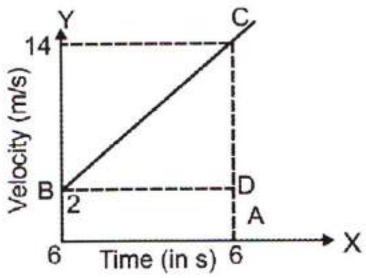
(a) If velocity time graph is a straight line, it indicates uniform acceleration



(b) Slope = Acceleration

(c) Area under graph = distance or displacement

- Velocity-time Graph when the initial velocity of the body is not zero from graph, initially at t = 0, velocity = 2ms^{-1}.

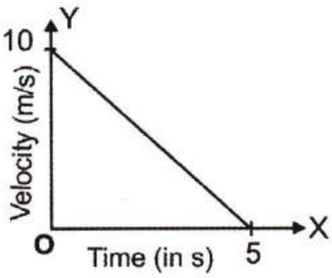


Acceleration = slope $= \frac{CD}{BD} = \frac{14-2}{6}$

$= 2m/s$

- **Velocity -time** graph when the velocity decreases uniformly.

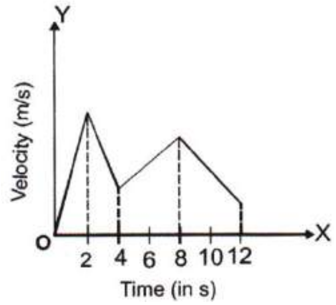
Acceleration = slope $= \frac{0-10}{5-0}$



$= -2\ m/s^2$

Negative acceleration indicates retardation

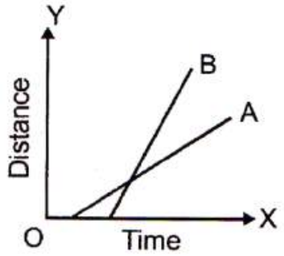
- **Velocity- time** graph of non uniform velocity



From the graph, velocity of the body is increasing at constant rate upto 2 s, and then the velocity is decreasing at a constant rate upto 4 s. Again velocity is increasing upto 8 s and then decreasing upto to 12 s.

Illustrative EXAMPLE

Figure shows distance - time graphs of two objects A and B. Which object is moving with a greater speed when both are moving?



(a) A is taster than B
(b) B is faster than A
(c) Both A and B have same speed
(d) All of these
(e) None of these

Answer: (b)

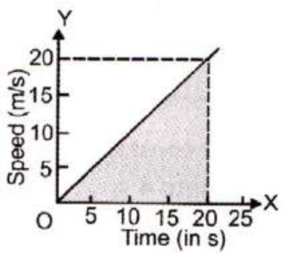
Expiations

The line for object B makes a larger angle with the time-axis. Its slope is, therefore, larger than the slope of the line for obejct A.

Thus, the speed of B is greater than that of A.

Illustrative EXAMPLE

What is the distance covered by a particle during the time interval t = 0 to t = 20 for which the speed - time graph is shown in Figure.



(a) 400m (b) 100 m
(c) 200m (d) All of these
(e) None of these
Answer: (c)

Explanation

The distance covered in the time interval 0 to 20 s = area of the triangle. Distance $=\frac{1}{2}\times$ base x height

$=\frac{1}{2}\times(20s)\times(20m/s)=200m$

Equations of Motion

The motion of the body moving along a straight line with uniform acceleration can be described by three equations of motion. These equations can be derived as follows.

First Equation of Motion

Let us consider an object moving with a initial velocity 'u'. If it is subjected to a uniform accleration 'a' such that it attains a velocity of V after time 't', then

Acceleration $=\frac{Final\,velocity-initial\,velocity}{Time\,taken}$

So, $a=\frac{v-u}{t}$

$\Rightarrow at=v-u,$ or $v=u+at$

where, v = final velocity of the body
u = Initial velocity of the body
a = acceleration
and, t= time taken

Second Equation of Motion

The second equation of motion is: $s=ut+\frac{1}{2}at^2$. It gives the distance traveled by a body in time t.

Let us derive this second equation of motion:
Let us consider an object moving with a initial velocity **'u'** and a uniform acceleration **"a"**. Let it attains a velocity V after some time **'t'**. Let the distance traveled by the object in this time be **'s'**. The distance traveled by a moving body in time **'t'** can be found out by considering its average velocity. Since the initial velocity of the body is **'u'** and its final velocity is V, the average velocity is given by:

Average velocity $=\frac{\text{Initial velocity } + \text{ final velocity}}{2}$

i.e., Average velocity $=\frac{u+v}{2}$

Also, **Distance traveled = Average velocity** $\times$ **Time**

so, $S=\left(\frac{u+v}{2}\right)\times t$ ...(1)

From the first equation of motion we have, **v = u + at**. Putting this value of **'v'** in equation (1), we get:

Or $S = \left(\frac{u+v}{2}\right) \times t$

Or $S = \left(\frac{2u+at}{2}\right) \times t$

Or $S = \frac{2ut+at^2}{2}$

Where s = distance traveled by the body in time t

u = Initial velocity of the body and, a = Acceleration

Third Equation of Motion

The third equation of motion is: v- u^ 2as. It gives the velocity acquired by a body in traveling a distance 's'. The third equation of motion can be obtained by eliminating -f from the first two equations of motion and using the second equation of motion

From the second equation of motion we have:

$s = ut + \frac{1}{2}at^2$ ...(1)

And from the first equation of motion we have :

$v = u + at$

Or, $at = v - u$

Or, $t = \frac{v-u}{a}$

Putting this value of t in equation (1), we get:

$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$

Or $S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}\frac{(v-u)^2}{a}$

Or $S = \frac{uv - u^2 + v^2 + u^2 - 2uv}{2a}$

Or $2as = v^2 - u^2$

Or $v^2 = u^2 + 2as$

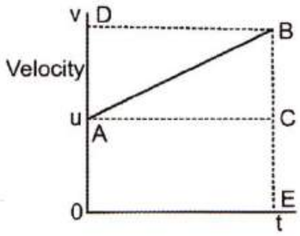
Where, v = final velocity,

u = initial velocity,

a = acceleration

and s = distance traveled

Graphical Method of Finding Equations of Motion



We can derive the equation of motions using the velocity time graph. Consider the motion of the object moving along a straight line with a uniform acceleration. The above graph shows the motion of an object along a straight line with a uniform accleration **'a'**. The point A and B on the graph corresponds to time 0 and t respectively.

Let **u** be the velocity of the object at time t = 0 represented by line OA on the graph. Also let the velocity of the particle be vat time t represented by line OD on the graph.

First Equation of Motion

The slope of the velocity-time graph gives the acceleration of an object moving along a straight line.

For line AB, the slope is given by

Slope $= \dfrac{BC}{AC}$

$\Rightarrow a = \dfrac{BC}{AC} = \dfrac{BE - CF}{OE}$

$\Rightarrow a = \dfrac{v-u}{t-0} = \dfrac{v-u}{t}$

$\Rightarrow v = u + at$

$= \mathbf{v = u + at}$

Second equation of Motion

The area under the velocity-time graph is equal to the displacement. In the time interval 0 - t, displacement = area OABE

S = area OABE = area of the rectangle OACE + area of the triangle ABC

$\Rightarrow\ S = OA \times OE + \dfrac{1}{2} \times AC \times BC$

$\Rightarrow\ S = OA \times OE + \dfrac{1}{2} \times AC \times (BC - CE)$

$\Rightarrow\ S = u \times t + \dfrac{1}{2} \times t \times (v-u)$ (using first equation of motion)

$\Rightarrow\ S = u \times t + \dfrac{1}{2} \times a \times t^2$

or $\Rightarrow S = ut + \dfrac{1}{2}at^2$

Third equation of motion

The area under the $v-t$ graph is =area of trapezium AOEB Area of trapezium $= \dfrac{1}{2}$ (sum of parallel sides) $\times$ (altitude)

Or, Area = $= \frac{1}{2} \times (OA + BE) \times AC$

Or, $S = \frac{1}{2} \times (u+v) \times t$

Or, $S = \frac{1}{2} \times (u+v) \times \left(\frac{v-u}{a}\right)$

Or, $v^2 = u^2 + 2as$

Average, velocity for uniformly Accelerated Motion

The average velocity of an object can be derived using the first and second equation of the motion. Let us consider an object moving along a straight line with a uniform accleration 'a'. The average velocity can be defined as,

Average velocity $= \frac{\text{Total Displacement}}{\text{Total Time}}$

Or, $\frac{S}{t} = \frac{v^2 - u^2}{2at}$

$V_{av} = \frac{v+u}{2}$, (using first equation of motion)

Illustrative EXAMPLE

An object moving with a velocity of 15 m/s decelerate at the rate of 1.5 m/ s^2 Find the time taken by the objects to come to rest.

(a) 10 sec
(b) 9.5 sec
(c) 11 sec
(d) 12.2 sec
(e) None of these

Answer: (c)

Explanation

We have, from first equation of motion

$t = \frac{v-u}{a}$

Or, $t = \frac{0-15}{-1.5} = \frac{15}{1.5} = 10 \text{ sec}$

Illustrative EXAMPLE

A car accelerates from 15 km/h to 60 km/h in 300 seconds. Find the distance traveled by the car during this time.

(a) 3.35km
(b) 3.33km
(c) 4.33km
(d) 5km
(e) None of these

Answer: (d)

Explanation

We have, u = 15 km/h, v = 60 km/h and $t = 300 \text{ sec} = \frac{1}{12}$

From first equation of motion,

$v = u + at,$

At $t = 0,$ the velocity is u = 20 km/h

At t = 4 min $= \frac{1}{15}h,$ the velocity is v = 80 km/h

Using $v = u + at,$

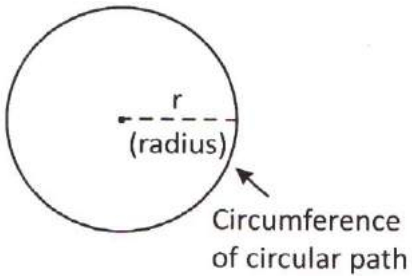
$$a = \frac{v-u}{t} = a = \frac{60-15}{\frac{1}{12}}$$

Now the distance covered by the car is given by,

$$S = ut + \frac{1}{2}at^2$$

$$S = 15 \times \frac{1}{12} + \frac{1}{2} \times 540 \times \frac{1}{12} \times \frac{1}{12} = 5\text{km}$$

Circular Motion



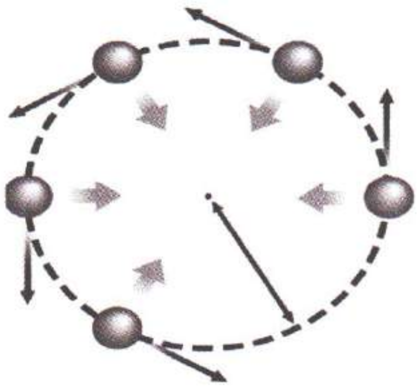
Where, T is the time taken by the object to move around the circular path.

Illustrative EXAMPLE

1. The motion of the blades of an electric fan around the axle.
2. The motion of an electron around the nuclear of an atom.
3. The motion of a satellite around the earth.
4. A stone tied at one end of the string and whirled above the head of a person in circular path.

Uniform Circular Motion

When a body moves along a circular path, with constant speed then its direction of motion (or direction of speed) keeps changing continuously.



Circle can be considered to be polygon with infinite sides.

Since the velocity changes due to continuous change in direction, therefore the motion along a circular path is said to be accelerated. When a body moves in a circular path with uniform speed (constant speed), its motion is called **uniform circular motion**.

Illustrative EXAMPLE

1. **1.** The tip of a seconds' hand of a watch exhibits uniform circular motion on the circular dial of the watch. Please note that though the speed of the tip of seconds' hand is constant, but its velocity is not constant, because the direction of motion of tip of seconds' hand changes continuously. Thus, the motion of the tip of seconds' hand of a watch is accelerated.
2. **2.** An anthele (or cyclist) moving on a circular track with a constant speed exhibits uniform circular motion. This motion is accelerated because of a continuous changes.

Illustrative EXAMPLE

An artificial satellite is moving in a circular orbit of radius 32,000 km. If it takes 30 hours to complete one revolution around the earth, then find the velocity of the satellite.

(a) 9428.57 km/h (b) 6704.76 km/h

(c) 9500 km/h (d) 7256.68 km/h

(e) None of these

Answer: (b)

Explanation

We have, R =32,000 km and t = 30 hours

Circumference of the orbit $= 2\pi\ r b = 2 \times \frac{22}{7} \times 32,000 = 201{,}142.86\,\text{km}$

Now, velocity $= \frac{2\pi R}{T} = \frac{201142.86}{30} = 6704.76\,km/h$

Illustrative EXAMPLE

A cyclist takes 180 second to complete one round of the circular track. A the radius of the circular track is 90 metres, then calculate his speed. (Given $\pi = \frac{22}{7}$)

(a) 3.14 m/sec (b) 3.56 m/sec

(c) 4.25 m/sec (d) 5.14 m/sec

(e) None of these

Answer: (a)

Explanation

We have $t = 180 \text{ sec}$ and $r = 90$ metres

$$V = \frac{2\pi r}{t}$$

Or, $V = \dfrac{2 \times \frac{22}{7} \times 90}{180} = 3.14\ m/\text{sec}$

SUMMARY

- Continous change of position of an object is called **motion**.
- The actual length of path covered by an object is called **distance**.
- The rate of change of velocity is called **acceleration**.
- If an object covers equal distance in equal time interval, it is called **uniform motion**.
- The distance time graph for uniform motion is a straight line.
- The motion of an object on a circular path is called **circular motion**.

Self Evaluation TEST

Duration
10 Minutes

---

1. **Ramu rides on a cycle in a park of radius 1km. If he takes 3.5 rounds of the park, what is the magnitude of displacement and the total distance covered by Ramu?**

(a) 22 km and 2 km (a) 44 km and 4 km
(c) 3.5 km and 2 km (d) 4.2 km and 5 km
(e) None of these

---

2. **A man walks on a straight road from his home to a market 3km away with a speed of 6 km/h. The market was closed; he instantly turns and walks back with a speed of 9km/h. What is the magnitude of average velocity of the man between time interval 0 to 40 min?**

(a) 2.25 km/h (b) 6.75 km/h
(c) 4.5 km/h (d) 7.5 km/h
(e) None of these

---

3. **What is the average speed of the man between time interval 0 to 40 min?**

(a) 2.25 km/h (b) 6.75 km/h
(c) 4.5 km/h (d) 7.5 km/h
(e) None of these

---

4. **The train 'A' traveled a distance of 120 km in 1.5 hours whereas another train 'B' traveled a distance of 180 km in 4 hours. Which train traveled faster?**

(a) A is faster than B (b) B is faster than A
(c) Both A and B have same speed (d) Insufficient data
(e) None of these

---

5. **The distance, s, a car has travelled from its starting point on a trip is shown in the table below as a function of time t, since the trip started.**

Hour	0	1	2	3	4	5
Km	0	45	135	220	300	400

What is the average velocity between $t = 1\text{h}$ and t = 3 h?

(a) 110km/h (b) 95 km/h
(c) 87.5 km/h (d) 85 km/h (e) None of these

6. **A bus starts from rest and attains a speed of 36 km h-1 in 10 minutes while moving with uniform acceleration. What is the acceleration of the bus?**

(a) - 0.0167 m/s^2 (b) 0.75 m/s^2

(c)-0.27m/s^2 (d) +0.0167 m/s^2

(e) None of these

---

7. **A car starts from rest and attains a velocity of 10 m/s in 40 s. The driver applies brakes and slow down the car to 5 m/s in 10 s. Find the acceleration of the car in both the cases.**

(a) -0.25 m/s2 and-0.5 m/s2 (b) 0.25 m/s^2 and-0.5 m/s^2

(c) 0.35 m/s2 and-0.15 m/s2 (d) 0.45 m/s^2 and-0.25 m/s^2

(e) None of these

---

8. **Identify wrong statement related to the velocity - time graph of a particle, shown in figure.**

(a) The motion is non uniform

(b) The acceleration is non uniform

(c) The particle change its direction of motion

(d) The distance covered = area under the velocity - time graph.

(e) None of these

---

9. **A car moving along a straight line at speed of 54 km/h stops in 5 seconds after the brakes are applied, what is the distance covered by the car after the brakes are applied?**

(a) 37.5m (b) 30m

(c) 31.5m (d) 34.2m

(e) None of these

---

10. **The velocity of a particle increases from u to v in a time t during which the particle has a uniform acceleration, which of the following equation apply to the motion?**

(a) $S = v \times t$ (b) $2s = (u + v) \times t$

(c) $a = \frac{v - u}{t}$ (d) $v^2 = u^2 + 2as$

(e) None of these

Answers – Self Evaluation Test																			
1.	A	**2.**	A	**3.**	B	**4.**	A	**5.**	C	**6.**	D	**7.**	B	**8.**	c	**9.**	A	**10.**	C